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Long-range rapidity correlations in hadron-nucleus interactions

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Long-range rapidity correlations between particles produced in proton-nucleus interactions at 200 GeV/c are studied in the multichain dual parton model. A large long-range correlation between particles produced in two rapidity intervals is predicted, provided these two rapidity intervals are properly chosen. The predicted effect is easily measurable. Predictions at 1 TeV are also given.

I. INTRODUCTION

Experiments at CERN ISR and especially at the CERN SPS colliders have revealed the existence of dynamical long-range correlations (LRC's) in pp and $\bar{p}p$ interactions. This LRC, measured by a parameter b $(0<b<1)$, is consistent with a value of zero at low ISR energies, is about 0.15 at top ISR energies,[1] and grows to as much as 0.40 at $\sqrt{s}=540$ GeV.[2]

Such a strong LRC is clearly in conflict with the property of short-range order (SRO) which is the main feature emerging from all the basic time-honored schemes of low-p_T hadronic interactions (Regge-Mueller, Feynman's parton approach, etc.).

The Regge-Mueller approach is formulated in the framework of an S-matrix theory, in which unitarity is known to play a very important role. Unitarity implies the existence of multiple inelastic collisions which produce dynamical LRC.[3]

It has been shown[3,4] that standard unitarity corrections, computed in the eikonal or perturbative Reggeon calculus unitarization schemes, together with SRO within each inelastic collision, lead to LRC's which are in quantitative agreement with the ISR and SPS data. Such an approach has the peculiar feature of introducing LRC, without modifying[3] the local compensation of charge, which is another characteristic feature of SRO. An experimental verification, at collider energies, of this local compensation of charge (which is known to work quite well at lower energies[5]), would provide a strong indication that multiple inelastic scattering is indeed the mechanism responsible for the observed LRC.

Another clear-cut consequence of the multiple-scattering approach is the existence of strong LRC in hadron-nucleus interactions—much stronger than the ones in pp scattering at the same energy per nucleon. So far, attempts to observe this LRC in hadron-hadron interactions at presently available energies have failed.[6]

In this paper we compute the strength of the LRC in hadron-nucleus collisions at presently available energies. The calculations are performed in the multichain dual parton model.[7] As far as we know this is the only version of the multiple-scattering model (and as a matter of fact the only model we are aware of) which can reproduce[4] the observed LRC at SPS energies.[2]

It turns out that the LRC strength is quite large—certainly large enough to be measured. We also show that the predicted LRC between the particles produced in the two rapidity intervals chosen in Ref. 6 is small, and compatible with the result obtained there. However, by choosing the two rapidity intervals in an appropriate way, we predict a larger LRC.

II. THE ORIGIN OF THE LONG-RANGE CORRELATION

We are going to use the complete version of the multichain dual parton model as formulated in the first paper of Ref. 7. The formulas needed to compute the LRC are given in Appendix A. However, in order to make the discussion more transparent, we present in this section a somewhat simpler form of the model. It has the advantage of simplifying the formalism considerably, and making very transparent the physical mechanism responsible for the appearance of LRC.

In this model the charged-particle rapidity density in a proton-nucleus (p-A) interaction is given by

$$\frac{1}{\sigma}\frac{d\sigma}{dy}^{p-A}(y)\equiv N^{p-A}(y)$$
$$=N^{qq_A-q_p^v}(y)+N^{q_A^v-qq_p}(y)$$
$$+(\langle\nu\rangle-1)[N^{qq_A-q_p^s}(y)+N^{q_A^v-\bar{q}_p^s}(y)]\ .\quad(1)$$

Here $\langle\nu\rangle=A\sigma_{\rm in}^{pp}/\sigma_{\rm in}^{pA}$ is the average number of inelastic collisions in the nucleus. For $\langle\nu\rangle=1$ one has only two terms, which correspond to an inelastic collision between the proton and a nucleon n_A^i of the nucleus. According to the model, in such a collision two chains of hadrons are produced: one of the chains (qq_A-q_r^v) links the diquark of the nucleon n_A^i with the valence quark of the proton, and the other one (q_A^v-qq_p) links the valence quark of the nucleon n_A^i with the diquark of the proton. Each new inelastic collision produces two extra chains qq_A-q_p^s and q_A^v-$\bar{q}_p^s$ linking, respectively, the diquark and the valence quark of the newly struck (wounded) nucleon n_A^j with a quark and antiquark of the proton sea.

The rapidity distributions $N^i(y)$ of the individual chains are obtained[7] from a convolution of momentum-distribution functions of the proton constituents (which can be determined within the model) with standard quark and diquark fragmentation functions. The main feature of the dual parton model which results from the momentum-distribution functions is the fact that the valence quarks are slow in average (with a $1/\sqrt{x}$ distribution near $x=0$), the sea quarks are even slower (in $1/x$), while the diquarks are fast and carry the proton quantum numbers (leading-particle effect). The resulting rapidity structure for the chains, which depends little on the detailed form of the fragmentation functions, is shown in Fig. 1.

Let us consider the correlation between particles produced in two rapidity intervals, $(y_1, y_1+\Delta_1)$ and $(y_2-\Delta_2, y_2)$ with $y_1 > y_2$ (the first interval will be denoted by F and the second one by B). In order to measure this correlation one plots the average multiplicity in, say, the second interval $\langle N_B^{p\text{-}A}\rangle$ as a function of the multiplicity $N_F^{p\text{-}A}$ in the first one. This type of plot is quite standard in probability theory where it is known as a linear regression. Under very general conditions one always obtains a linear dependence:

$$\langle N_B^{p\text{-}A}\rangle = a + b N_F^{p\text{-}A} ,$$

where

$$a = \langle N_B^{p\text{-}A}\rangle = b\langle N_F^{p\text{-}A}\rangle$$

and the slope b, which measures the strength of the correlation, is given by

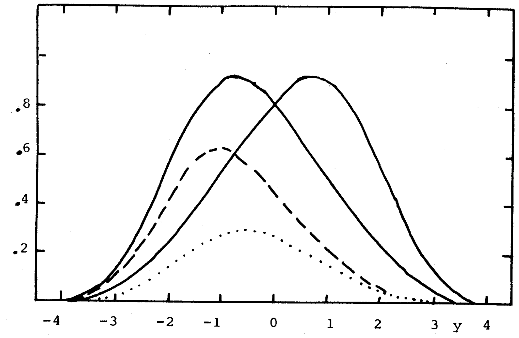

FIG. 1. The rapidity distributions of the four chains in Eq. (1). The solid curves correspond to the valence chains qq_A-q_p^v and q_A^v-qq_p. The dashed and dotted curves are the rescattering chains qq_A-q_p^s and q_A^v-$\bar{q}_p^s$, respectively. The valence chains are computed for $\nu=1$ and the rescattering chains for $\nu=2$ (the latter are present only for $\nu \geq 2$). The ν dependence of the chains, responsible for the attenuation effect (Ref. 7) in the projectile fragmentation region, is not shown.

$$b \equiv \frac{D_{F\text{-}B}{}^2}{D_{F\text{-}F}{}^2} = \frac{\int_{y_1}^{y_1+\Delta_1} dy \int_{y_2-\Delta}^{y_2} dy' \left[\langle N^{p\text{-}A}(y) N^{p\text{-}A}(y')\rangle - \langle N^{p\text{-}A}(y')\rangle\langle N^{p\text{-}A}(y')\rangle\right]}{\int_{y_1}^{y_1+\Delta_1} dy \int_{y_1}^{y_1+\Delta_1} dy' \left[\langle N^{p\text{-}A}(y) N^{p\text{-}A}(y')\rangle - \langle N^{p\text{-}A}(y')\rangle\langle N^{p\text{-}A}(y)\rangle\right]} . \tag{2}$$

This correlation will certainly have a long range in rapidity provided $y_1 - y_2 > \lambda$, where λ is a correlation length characteristic of the short-range correlation (typically 1.5—2 units).

Let us now compute b within the model. As explained in the Introduction, we assume SRO for the particles produced in the individual chains. This implies

$$\langle N^i(y) N^i(y')\rangle = \langle N^i(y)\rangle\langle N^i(y')\rangle, \quad |y-y'| \rangle \lambda . \tag{3}$$

We also assume that particles produced in different chains are uncorrelated, i.e., for any y and y' one has

$$\langle N^i(y) N^j(y')\rangle = \langle N^i(y)\rangle\langle N^j(y')\rangle, \quad i \neq j . \tag{4}$$

Using Eqs. (1), (3), and (4), we obtain for the numerator of b in Eq. (2),

$$D_{F\text{-}B}{}^2 = [\langle \nu^2\rangle - \langle \nu\rangle^2] \int_{y_1}^{y_1+\Delta_1} dy [N^{qq_A\text{-}q_p^s}(y) + N^{q_A^v\text{-}\bar{q}_p^s}(y)] \int_{y_2-\Delta_2}^{y_2} dy' [N^{qq_A\text{-}q_p^s}(y') + N^{q_A^v\text{-}\bar{q}_p^s}(y')] . \tag{5}$$

We see that with only short-range correlations in the individual chains, a nonvanishing LRC arises as a consequence of the fluctuation in the number of chains $(\langle \nu^2\rangle \neq \langle \nu\rangle^2)$. Its strength is proportional to the product of the sums of the average multiplicities of the two rescattering chains (qq_A-q_p^s and q_A^v-$\bar{q}_p^s$), in the rapidity intervals under consideration. [Another source of correlation is the fluctuation in the position of the chains in rapidity space. This correlation which is quite sizable up to ISR energies,[9] goes away with increasing s (4). The simplified version of the model presented in this section neglects this source of correlation, which is, however, taken into account in the numerical calculation (see Appendix A).] Therefore in order to observe a large LRC one has to choose the two rapidity intervals in such a way that the product of multiplicities of the rescattering terms in these two intervals is as large as possible. This is achieved by choosing them symmetric with respect to $y_0 \sim -0.5$ (see Fig. 1). [The first factor in Eq. (5) depends only on the nature of the projectile and target and on the incoming energy.]

In order to compute the value of b, we have, of course, to compute the denominator of Eq. (2). This requires the knowledge of the quantities

$$(D_{F\text{-}F}^i)^2 = \int_{y_1}^{y_1+\Delta} dy \int_{y_1}^{y_1+\Delta} dy' [\langle N^i(y) N^i(y')\rangle - \langle N^i(y)\rangle\langle N^i(y')\rangle]$$
$$= \langle (N_F^i)^2\rangle - \langle N_F^i\rangle^2 , \tag{6}$$

i.e., the dispersion squared of the individual chains in the appropriate interval. An ansatz, which is quite natural from the assumed SRO within a chain, is to assume an independent emission, i.e.,

$$(D^i_{F\text{-}F})^2 = k\langle N^i_F\rangle \tag{7}$$

with $k=1$ if the particles are directly produced and more generally k equal to the average number of particles in each cluster (resonance). (An alternative ansatz, which leads to very similar results, is to use the experimental e^+e^- and lp data for the dispersion of the chains.)

III. NUMERICAL RESULTS

In order to compare the results of the model with data already available,[6] we consider a proton-xenon interaction at 200 GeV/c. Our results for various nuclei are collected in Table I. First we take for the two rapidity intervals the whole forward and backward hemispheres. Clearly, our calculation can only give, in this case, a lower bound for b. Indeed, using Eq. (3) in the region $y \sim y'$ (which is now present in the numerator of b because $y_1 = y_2$), one neglects the contribution of the short-range correlations. For p-Xe interactions at 200 GeV/c we obtain $b=0.57$, a rather large value. The value of the numerator of b [which is a less model-dependent quantity since it does not require the assumption in Eq. (7)] is also given in Table I.

The experimental value is $b=1.23\pm0.07$. Such a large value is by itself a strong indication of the existence of strong LRC. It is indeed quite implausible that the short-range correlation (which gives $b\sim0.2$ in proton-proton collisions at comparable energies) can be so important here. Presumably, the difference between the experimental value and the computed value of $b_{\rm LR}$ is not entirely due to the short-range correlation but also to the effect of the intranuclear cascade in the target fragmentation region. Let us discuss this point in some detail. The physical origin of the LRC in a multiple-scattering model is the following.[3] If one observes a large fluctuation of multiplicity in some rapidity interval, one is most probably looking at a contribution consisting of several inelastic collisions—the probability of observing a large multiplicity fluctuation in a single inelastic collision being much smaller due to SRO. Therefore one is bound to observe multiplicity fluctuations far away in rapidity (LRC). The increase in the number of inelastic collisions will also produce an increase in the number of (slow) particles produced via intranuclear cascade. Such an effect is intimately associated to the LRC—and goes away when $b_{\rm LR}$ vanishes. Our purpose here is not to discuss this phenomenon quantitatively (a rather difficult task) but to compute the LRC slope and show how to observe it.

In order to eliminate the contribution of the short-range correlation, the authors of Ref. 6 have also considered the two rapidity intervals $y>0.75$ and $y'<-0.75$. The measured value of b is consistent with zero and the authors conclude that there is no LRC. However, from Eq. (5) and Fig. 1, the value of $b_{\rm LR}$ is expected to be much smaller here —since the average multiplicity of the rescattering chains is smaller than in the previous case. A numerical calculation gives $b_{\rm LR}=0.13$. Thus with this choice of intervals most of the LRC has been eliminated together with the short-range one. A somewhat larger value of the LRC is obtained when one keeps the same rapidity gap between the two intervals but, as explained in Sec. III, shifts them toward the left. Let us take the two intervals $y>0.25$ and $y'<-1.25$ which are symmetric with respect to $y_0=-0.5$. In this case we obtain $b_{\rm LR}=0.17$. If our approach is correct the measured value will presumably be somewhat larger than the calculated one due to the intranuclear cascade. The latter effect could be eliminated to a large extent by choosing the two rapidity intervals $-2.25<y'<-1.25$ and $0.25<y<1.25$. In this case the LRC slope is larger. We obtain $b_{\rm LR}=0.27$. The predictions for $p_L=1$ TeV/c are given in Table II. At this energy, the average rapidity length and the plateau height of the rescattering chains are larger than at 200 GeV/c. Therefore the predicted values of the LRC are also larger. For example, taking the rapidity intervals $0.75<y<4$ and $-4<y'<-0.75$, we obtain $b_{\rm LR}=0.61$.

TABLE I. The values of the calculated long-range correlation slope $b_{\rm LR}$ and its numerator $D_{F\text{-}B}{}^2$ [Eq. (2)] at 200 GeV/c for three different targets and four sets of rapidity intervals. The decrease of $b_{\rm LR}$ with increasing A for ($y>0.75$, $y'<-0.75$) is presumably a consequence of the attenuation effect in the projectile fragmentation region (Ref. 7).

A	$D_{F\text{-}B}{}^2$	$b_{\rm LR}$	Rapidity intervals
40	2.73	0.45	$y>0$
131	3.82	0.57	$y'<0$
268	4.36	0.61	
40	0.43	0.15	$y>0.75$
131	0.40	0.13	$y'<-0.75$
268	0.28	0.09	
40	0.66	0.14	
131	0.91	0.17	$y>0.25$
268	0.97	0.18	$y'<-1.25$
40	0.52	0.20	$0.25<y<1.25$
131	0.80	0.27	$-2.25<y'<-1.25$
268	0.97	0.30	

IV. COMPARISON WITH OTHER MODELS

The LRC slope is a parameter which contains a lot of dynamical information on the mechanism of hadron pro-

TABLE II. Same as Table I for 1 TeV and two sets of rapidity intervals.

A	$D_{F\text{-}B}{}^2$	$b_{\rm LR}$	Rapidity intervals
40	2.67	0.42	$0.75<y<4$
131	4.56	0.61	$-4<y'<-0.75$
268	5.79	0.71	
40	1.58	0.39	$0.25<y<1.25$
131	3.02	0.56	
268	4.24	0.66	$-2.25<y'<-1.25$

duction. Therefore it should be an excellent arena to test the existing models of the multihadron production in hadron-nucleus interactions. For instance, there are two different versions of the dual parton model, which are developed by the Austin[8] and Orsay[7] groups. The main difference between the two formulations is that $N^{q_A^v\text{-}\bar{q}^s}(y)\equiv 0$ in the Austin version.[9] At very high energies, where all chains develop a plateau of comparable size, one obtains from Eq. (1)

$$\frac{(dN/dy)_{\text{Orsay}}}{(dN/dy)_{\text{Austin}}}=\frac{2\langle\nu\rangle}{\langle\nu\rangle+1} .$$

At present energies the contribution of the q_A^v-$\bar{q}^s$ chain is rather small and the two approaches give comparable results for dN/dy, where the contribution of the rescattering chains is mixed with that of the two main (single-scattering) chains. On the contrary, the LRC slope b_{LR} is directly proportional to the product of multiplicities of the rescattering chains, since the contribution of the two main (single-scattering) chains cancels out in b. It is clear from Eq. (5) and Fig. 1 that the quantity

$$D_{F\text{-}B}{}^2\equiv\langle N_F^{p\text{-}A}N_B^{p\text{-}A}\rangle-\langle N_F^{p\text{-}A}\rangle\langle N_B^{p\text{-}A}\rangle ,$$

where F denotes the interval $0.25<y<1.25$ and B the interval $-2.25<y'<1.25$ will be appreciably smaller in the Austin version than in the Orsay one. Numerically, we obtain at 200 GeV/c $(D_{F\text{-}B}{}^2)_{\text{Austin}}=0.60$ and $(D_{F\text{-}B}{}^2)_{\text{Orsay}}=0.80$. (The difference between these two values increases with s, and the test becomes more relevant.)

Likewise the LRC slope should allow the testing of the dual parton model versus other types of models, such as the additive quark model (AQM). Moreover, there are also different versions of the AQM, which differ from one another by the assumptions for the multiplicity of particles produced in multiple inelastic collisions of a same constituent quark with the nucleus. Presumably they lead to rather different predictions for the value of the LRC slope.

V. CONCLUSIONS

Multiple-scattering models of hadron-nucleus interactions predict the existence of strong long-range correlations. In the dual-parton-model version of these models the long-range-correlation slope is already quite sizable at 200 GeV/c. We show how to choose the two rapidity intervals in such a way that the long-range correlation between them is maximal.

A measurement of a large long-range-correlation slope at these energies will provide a nice test of the multiple-scattering models and in particular of the dual parton model. It will also provide further evidence that the large long-range correlation found in the SPS colliders is due to standard multiple inelastic interactions, with short-range order in each individual collision.

Moreover the value of the long-range correlation slope should allow us to distinguish between different models of particle production in hadron-nucleus interactions.

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APPENDIX A

In the dual parton model the rapidity distribution in a hadron-nucleus collision is given by[7]

$$N^{p\text{-}A}(y)=\frac{1}{\sigma}\sum_{\nu=1}^{A}\sigma_\nu\int dx\,\rho(x)\int\cdots\int dx_1\cdots dx_{2\nu}\rho_\nu(x_1,x_2\cdots x_{2\nu})$$
$$\times\left\{N^{qq_A\text{-}q_P^v}(1-x,x_1;y)+N^{q_A^v\text{-}qq_P}(x,x_{2\nu};y)\right.$$
$$\left.+(\nu-1)[N^{qq_A\text{-}q_P^s}(1-x,x_2;y)+N^{q_A^v\text{-}\bar{q}_P^s}(x,x_3;y)]\right\} . \quad \text{(A1)}$$

The momentum-distribution functions ρ and the rapidity distributions $N^{\text{chain}}(x,x',y)$ of the individual chains for fixed positions of the chain ends are given in Ref. 7 [where the approximations leading from (A1) to Eq. (1) are also discussed]. We have used the following quark[4] and diquark[10] fragmentation functions:

$$zD^{q\to\pi^\pm}(z)[1.32(1-z)^{2.37}+0.03]/(1-0.5z) ,$$
$$zD^{qq\to\pi^\pm}(z)=1.35[(1-z)^2+2(1-z)^{4.5}+(1-z)^6] ,$$

and used the same exponential thresholds for the individual chains as in Ref. 4. The fixed thresholds used in Ref. 7 yield very similar results. (Notice however that with exponential thresholds the plateau height of the rescattering chains is somewhat lower than with fixed thresholds. As a consequence the A dependence is smaller in the former case and in better agreement with experiment.[6])

Using Eqs. (3) and (4) for $N^{\text{chain}}(x,x';y)$, we obtain for $|y-y'|>\lambda$,

$$
\begin{aligned}
\langle N^{p\text{-}A}(y)N^{p\text{-}A}(y')\rangle
&= \frac{1}{\sigma}\sum_{\nu=1}^{A}\sigma_\nu \int dx\,\rho(x)\int dx_1\cdots\int dx_{2\nu}\rho_\nu(x_1,\ldots,x_{2\nu}) \\
&\quad\times\{N^{qq_A\text{-}q_p^v}(1-x,x_1;y)N^{qq_A\text{-}q_p^v}(1-x,x_1;y')+N^{q_A^v\text{-}qq_p}(x,x_{2\nu};y)N^{q_A^v\text{-}qq_p}(x,x_{2\nu};y') \\
&\quad+(\nu-1)[N^{qq_A\text{-}q_p^s}(1-x,x_2;y)N^{qq_A\text{-}q_p^s}(1-x,x_2;y')+N^{q_p^v\text{-}\bar{q}_p^s}(x,x_3;y)N^{q_p^v\text{-}\bar{q}_p^s}(x,x_3,y')] \\
&\quad+[N^{qq_A\text{-}q_r^v}(1-x,x_1;y)N^{q_A^v\text{-}qq_p}(x,x_{2\nu};y')+(\nu-1)N^{qq_A\text{-}q_p^s}(1-x,x_2;y)N^{q_A^v\text{-}\bar{q}_p^s}(x,x_3;y') \\
&\quad+(\text{sym } y\leftrightarrow y')]\} \\
&\quad+\frac{(\nu-1)}{\sigma}\sum_{\nu=1}^{A}\sigma_\nu\int dx\,\rho(x)\int dx'\rho(x')\cdots\int dx_1\cdots dx_{2\nu}\rho_\nu(x_1,\ldots,x_{2\nu}) \\
&\quad\times\Big\{[N^{qq_A\text{-}q_p^v}(1-x,x_1;y)+N^{q_A^v\text{-}qq_p}(x,x_{2\nu};y)] \\
&\quad\times[N^{qq_A\text{-}q_p^s}(1-x',x_2;y')+N^{q_A^v\text{-}\bar{q}_p^s}(x',x_3;y')] \\
&\quad+(\text{sym } y\leftrightarrow y') \\
&\quad+(\nu-2)[N^{qq_A\text{-}q_p^s}(1-x,x_2;y)+N^{q_A^v\text{-}\bar{q}_p^s}(x,x_3;y)] \\
&\quad\times[N^{qq_A\text{-}q_p^s}(1-x',x_4;y')+N^{q_A^v\text{-}\bar{q}_p^s}(x'x_5;y')]\Big\}\,. \qquad \text{(A2)}
\end{aligned}
$$

The corresponding expression for $D_{F\text{-}F}{}^2$ in Eq. (2) is easily obtained by using the Poisson distribution, Eq. (7), for the individual chains. In the numerical calculations we have used $k=1$.

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