RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 29, NUMBER 10 15 MAY 1984

Chiral-symmetry breaking in three-dimensional electrodynamics

Robert D. Pisarski
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 7 February 1984; revised manuscript received 2 April 1984)

When there are many flavors of massless fermions, both three-dimensional electrodynamics, and a supersymmetric variant thereof, each spontaneously break chiral symmetry. For the latter, this occurs without breaking supersymmetry, and without a photino condensate.

While the importance of chiral-symmetry breaking has been clear for some time,[1] there remain few models in which this can be studied analytically. Four-fermion interactions provide some examples in two dimensions,[2,3] but since their broken chiral symmetry is discrete, they do not display true pions.

With this in mind, consider a three-dimensional gauge theory coupled to N flavors of massless fermions. Though in three dimensions there are no infinite renormalizations of the bare Lagrangian, the infrared behavior might be analogous to that in four dimensions. The gauge coupling e is $\sim (\text{mass})^{1/2}$, so if any mass scale is generated dynamically, it will be a pure number times e^2. For small N, such a pure number should be of order one, and so difficult to obtain analytically.[4]

I am thus led to the limit of large N, with $\alpha \equiv e^2 N$ fixed as $N \to \infty$.[5] In this limit, non-Abelian interactions are down by N^{-1}, so I might as well start with the Abelian theory, three-dimensional QED. With N fermion flavors to one photon, if a fermion mass m is dynamically generated, it should be possible to solve for it by a N^{-1} expansion.

By finding a self-consistent solution to the Schwinger-Dyson equations with $m \neq 0$, I show in this Rapid Communication that QED_3 spontaneously breaks chiral symmetry at large N.

I then turn to a supersymmetric electrodynamics[6] in three dimensions,[7] super QED_3. Expanding in a large number of massless matter fields, I show that chiral symmetry is spontaneously broken in super QED_3 as well. To the order I calculate, supersymmetry is unbroken, in accord with general arguments of Witten.[8]

Various analyses of supersymmetric gauge theories in four dimensions have indicated that the chiral limit breaks supersymmetry,[9,10] has a gluino condensate,[11] and/or no stable ground state.[12] I find no evidence for any of this in super QED_3 at large N.

I conclude with a discussion of how my results could be obtained from effective potentials.

I. LARGE-N QED_3

The Lagrangian for massless QED_3 is

$$\mathscr{L} = \bar{\psi} i \not{\mathscr{D}} \psi + \tfrac{1}{4} F_{\mu\nu}{}^2 \ , \tag{1}$$

where $\mathscr{D}_\mu = \partial_\mu + ieA_\mu$. I take ψ to be a four-component Dirac spinor; implicitly, ψ includes an index for N flavors.

Chiral symmetries are a bit unusual in three dimensions. To describe spinorial representations of the Lorentz group, two-component spinors will do. For two-component spinors, however, the flavor symmetry is the same if the fermions are massless or not, and so it is not a chiral symmetry. This is because the Pauli matrices form three 2×2 matrices γ_μ, and no other 2×2 matrix anticommutes with all of these γ_μ.

In contrast, four-component spinors have flavor symmetries which are chiral, in that massless fermions have a greater symmetry than massive ones. Simply put, in three dimensions there are two matrices which anticommute with the three 4×4 γ_μ's—γ_4 and γ_5.[13]

The flavor symmetry of Eq. (1) is actually one of $\mathrm{U}(2N)$: essentially, ψ represents $2N$ varieties of two-component spinors.[14] In four-component notation, the Lie algebra of $\mathrm{U}(2N)$ is given by that of $\mathrm{U}(N)$ flavor and combinations of the Dirac matrices 1, γ_4, γ_5, and $\gamma_{4,5} \equiv -i\gamma_4\gamma_5$.

By a $\mathrm{U}(2N)$ rotation, any mass for ψ can be written as a sum of *two* distinct terms:

$$i\bar{\psi}(m + m'\gamma_{4,5})\psi \ . \tag{2}$$

The mass m' is two-component-like, for if $m=0$ but $m' \neq 0$, the flavor symmetry remains $\mathrm{U}(2N)$. The mass m is chiral, since when $m \neq 0$, the symmetry is reduced to the $\mathrm{U}(1)\times \mathrm{U}_{4,5}(1)\times \mathrm{SU}_{(1+\gamma_{4,5})}(N)\times \mathrm{SU}_{(1-\gamma_{4,5})}(N)$ subgroup of $\mathrm{U}(2N)$. If $m \neq 0$ occurs spontaneously in the chiral limit, $2N^2$ Goldstone bosons are generated: ignoring flavor indices, there are N^2 γ_4-type pions, coupling to $\bar{\psi}\gamma_\mu\gamma_4\psi$, and N^2 γ_5-type pions, coupling to $\bar{\psi}\gamma_\mu\gamma_5\psi$.

Less obvious is that the mass m', but not that of m, is odd under the discrete space-time symmetries of parity (P) and time reversal (T).[15] Following the analogy to four dimensions, except where noted, I choose $m'=0$.

To prove chiral symmetry is broken, I obtain a self-consistent solution, with $m \neq 0$, to the Schwinger-Dyson equation for the fermion self-energy. I do so assuming that $m << \alpha$, which will be justified after the fact. I emphasize that by Eqs. (6)–(8) below, this m is a dynamical mass, the result of a vacuum condensate $\langle\bar{\psi}\psi\rangle \neq 0$, and *not* a bare mass *per se*.

Adopting the Landau gauge, in momentum space the photon propagator is

$$\Delta_{\mu\nu} = (g_{\mu\nu} - p_\mu p_\nu/p^2)/\{p^2[1+\Pi(p)]\} \ . \tag{3}$$

To leading order in N^{-1}, all radiative corrections are determined by one-loop graphs. For the photon self-energy, this gives[16]

$$\Pi(p) = \frac{\alpha}{4\pi p^2}\left[2m + \frac{(p^2-4m^2)}{p}\sin^{-1}\left(\frac{p}{(p^2+4m^2)^{1/2}}\right)\right] \ . \tag{4}$$

When $p >> m$,

$$\Pi(p) \approx \alpha/(8p) \ , \tag{5a}$$

while for $m >> p$,

$$\Pi(p) \approx \alpha/(6\pi m) \ . \tag{5b}$$

The fermion self-energy at zero momentum is

$$\Sigma(0) = -2\frac{\alpha}{N}\int \frac{im}{p^2[1+\Pi(p)](p^2+m^2)}\frac{d^3p}{(2\pi)^3} \ . \tag{6}$$

Over momenta $\alpha >> p >> m$, the photons' propagator is dominated by $\Pi(p)$ in Eq. (5a), and $\Sigma(0) \sim \int d^3p/p^3$. From Eq. (6), this logarithmic dependence is naturally cut off by α in the ultraviolet, and by m in the infrared:[17]

$$\Sigma(0) = -8im[\ln(\alpha/m) + c_0]/(\pi^2 N) \ , \tag{7}$$

where c_0 is a number $\sim O(1)$.[18] Requiring

$$\Sigma(0) = -im \tag{8}$$

gives[19]

$$m = c\alpha \exp(-\pi^2 N/8) \ . \tag{9}$$

Since $c = \exp(c_0)$, c is a positive number $\sim O(1)$.

As promised, the fermions are extremely light at large $N - m << \alpha$. The fermions turn out to be so light crucially because they interact only via gauge fields. For fermions coupled to a scalar field σ by a Yukawa interaction $\sim g\bar{\psi}\sigma\psi$, $m \sim g\langle\sigma\rangle$, and the fermions are heavy at large N, $m \sim O(1)$.[2]

The fermion-antifermion static potential, $V(r)$, is of some interest. At large N, $V(r)$ is determined by single-photon exchange. Over small distances, $r << \alpha^{-1}$, the photon propagator can be taken as the bare one, so $V(r) \sim (\alpha/N)\ln(r\alpha)$. For intermediate r, $\alpha^{-1} << r << m^{-1}$, from Eq. (5a) the potential is Coulombic, $V(r) \sim (Nr)^{-1}$. Finally, over very large distances $r >> m^{-1}$, the potential returns to a confining form, but from Eq. (5b), the many, light fermions strongly screen the bare charge by a large, finite renormalization $\sim 1/[1+\Pi(0)] \sim m/\alpha$: $V(r) \sim (m/N)\ln(rm)$.

II. LARGE-N SUPER QED$_3$

To obtain a supersymmetric electrodynamics in three dimensions, I start with the form of four-dimensional supersymmetric electrodynamics given by Wess and Zumino,[6] and then assume that all fields are independent of one spatial coordinate. By this dimensional reduction I obtain the Lagrangian

$$\begin{aligned}\mathscr{L} = {} & |\mathscr{D}_\mu S_+|^2 + |\mathscr{D}_\mu S_-|^2 + \bar{\psi} i\not{\mathscr{D}}\psi + \tfrac{1}{4}F_{\mu\nu}{}^2 + \tfrac{1}{2}\bar{\lambda} i\not{\partial}\lambda + \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}D^2 + e^2\phi^2(|S_+|^2 + |S_-|^2) + ieD(|S_+|^2 - |S_-|^2) \\ & - e\phi(\bar{\psi}\gamma_4\psi) - \sqrt{2}e(\bar{\psi}S_+\tilde{P}_+\lambda + \bar{\psi}S_-\tilde{P}_-\lambda + \text{H.c.}) \ ,\end{aligned} \tag{10}$$

where $\tilde{P}_\pm = (1 \pm \gamma_5)/2$. The matter fields are the complex scalars S_+ and S_-, and the four-component Dirac field ψ; there are N flavors of each. The gauge multiplet includes the photon A_μ, the photino λ (which is a four-component Majorana spinor), and two real scalars ϕ and D.[20] Like that in four dimensions,[9] the global flavor symmetry of Eq. (10) is $U_{(1+\gamma_5)}(N) \times U_{(1-\gamma_5)}(N) \times U_R(1)$.

The mass terms allowed for the matter fields are

$$m_s{}^2(|S_+|^2 + |S_-|^2) + i\bar{\psi}(m_f + m_f'\gamma_{4,5})\psi \ . \tag{11}$$

I ignore the possibility of a P- and T-odd mass for ψ by setting $m_f' = 0$. Anticipating my results, I take, for the time being, supersymmetric values for the masses, $m_s = m_f \equiv \tilde{m}$. A mass $\tilde{m}$ corresponds to vacuum condensates for $\langle\bar{\psi}\psi\rangle$ and $\langle|S_+|^2\rangle = \langle|S_-|^2\rangle \neq 0$; supersymmetry relates these condensates in a precise way.[21] This breaks the flavor symmetry to $U(N)$, with N^2+1 Goldstone bosons. This symmetry breaking is not a Higgs effect: composite operators have nonzero vacuum expectation values, but those for any elementary scalar field—$\langle S_+\rangle$, $\langle S_-\rangle$, $\langle\phi\rangle$, and $\langle D\rangle$—vanish to each order in N^{-1}.

To leading order in N^{-1}, the propagators for the gauge multiplet are

$$\begin{aligned}\Delta_{\mu\nu} &= (g_{\mu\nu} - p_\mu p_\nu/p^2)/\{p^2[1+\tilde{\Pi}(p)]\} \ , \\ \Delta_\lambda \not{p} &= \Delta_\phi p^2 = \Delta_D = 1/[1+\tilde{\Pi}(p)] \ ,\end{aligned} \tag{12}$$

where

$$\tilde{\Pi}(p) = \alpha \sin^{-1}[p/(p^2+4\tilde{m}^2)^{1/2}]/(2\pi p) \ . \tag{13a}$$

When $p >> \tilde{m}$,

$$\tilde{\Pi}(p) \approx \alpha/(4p) \ . \tag{13b}$$

For $p << \tilde{m}$,

$$\tilde{\Pi}(p) \approx \alpha/(4\pi\tilde{m}) \ . \tag{13c}$$

As a consequence of the supersymmetry, a single function $\tilde{\Pi}(p)$ suffices to describe charge renormalization.

One aspect of Eq. (12) is surprising, at least at first. Dimensionally, a photino mass $\sim \tilde{m}$ could occur, but does not due to the γ_5 dependence of the photino's couplings. At one-loop order, the photino self-energy arises from a virtual scalar-fermion pair, where one vertex brings in a factor of $(1+\gamma_5)$, and the other of $(1-\gamma_5)$. The result is entirely wave-function renormalization for the photino.

This can be understood generally. A photino mass reflects a vacuum condensate $\langle\bar{\lambda}\lambda\rangle \neq 0$. A discrete $Z(2)$ symmetry, which includes $\lambda \to \gamma_5\lambda$, implies that $\langle\bar{\lambda}\lambda\rangle = 0$ unless the vacuum spontaneously breaks this symmetry.[6] To *any* finite order in the N^{-1} expansion, the vacuum respects $\lambda \to \gamma_5\lambda$, and $\langle\bar{\lambda}\lambda\rangle = 0$.[22]

To establish that chiral-symmetry breaking occurs, I assume $\alpha >> m_s$ and m_f. To the order computed, only the form of $\tilde{\Pi}(p)$ for $\alpha >> p >> m_s$, m_f will matter. Since in this regime $\tilde{\Pi}(p)$ is independent of mass, Eq. (13b), the propagators of Eq. (12) can be used even if I do not take $m_s = m_f$.

To leading order in N^{-1}, at zero momentum the fermion self-energy is[23]

$$\Sigma_f(0) = -6im_f[\ln(\alpha/m_f) + O(1)]/(\pi^2 N) \ . \tag{14}$$

Setting $\Sigma_f(0) = -im_f$,

$$m_f = c_f \alpha \exp(-\pi^2 N/6) \quad . \tag{15}$$

The zero-momentum scalar self-energy is

$$\Sigma_s(0) = [8m_f^2 \ln(\alpha/m_f) - 2m_s^2 \ln(\alpha/m_s)]/(\pi^2 N) \quad , \tag{16}$$

where terms $\sim m_s^2/N$, m_f^2/N have been dropped.[18] Requiring $\Sigma_s(0) = m_s^2$ and using Eq. (15), I find

$$m_s^2 = 4m_f^2/3 - 2m_s^2 \ln(\alpha/m_s)/(\pi^2 N) \quad . \tag{17}$$

If I take $m_s >> m_f$, Eq. (17) has the solution $m_s \sim \alpha \times \exp(+\pi^2 N/2)$. This satisfies $m_s >> m_f$, but not $\alpha >> m_s$, and so must be discarded.[24] Equation (17) has no solution for $m_f >> m_s$. Hence $m_s/m_f \sim O(1)$ at large N:

$$m_s = c_s \alpha \exp(-\pi^2 N/6) \quad . \tag{18}$$

In Eqs. (15) and (18), c_f and c_s are positive numbers $\sim O(1)$. While calculations beyond leading order in N^{-1} would be needed to determine c_f and c_s,[18] I emphasize that, to the order I work, the chiral-symmetry breaking *is* supersymmetric. For a theory like super QED$_3$, one knows *a priori* merely that m_f/α and m_s/α are pure numbers. There is only one reason why the exponential dependence on N for m_f and m_s should be the same, and that is supersymmetry.

Witten[8] has argued that supersymmetry cannot be broken dynamically in theories like super QED$_3$. In four dimensions, Peskin has suggested that these arguments might be evaded in the chiral limit,[10] but at large N super QED$_3$ does not.

III. EFFECTIVE POTENTIALS

I have shown that in large-N QED$_3$, there is a solution to the Schwinger-Dyson equations with $m \neq 0$. Trivially, $m = 0$ is also a solution, so I should prove that the solution with $m \neq 0$, Eq. (9), is the stable ground state. This is easy: if the effective potential has only two extrema, one at $m = 0$, and the other for $m \neq 0$, as long as the theory has some stable ground state, then the solution with $m \neq 0$ must be it. The theory obviously has a stable ground state, since for large m, all radiative corrections are small, $\sim \alpha/m$.

With the vacuum expectation value of $\langle \bar{\psi}\psi \rangle \sim iNm^2$, the effective potential for $\langle \bar{\psi}\psi \rangle$ is, to leading order in N^{-1},[2]

$$V_{\text{eff}}(m) = -N \operatorname{tr} \ln[(\not{p} - im)/\not{p}] \quad . \tag{19}$$

After an ultraviolet renormalization,[25]

$$V_{\text{eff}}(m) = N(m^2)^{3/2}/(3\pi) \quad . \tag{20}$$

Calculating the corrections of $\sim O(1)$ in V_{eff} would be another way of showing that chiral symmetry is spontaneously broken in QED$_3$.

Defining the condensates of super QED$_3$ as $\langle \bar{\psi}\psi \rangle \sim iNm_f^2$ and $\langle |S_+|^2 \rangle = \langle |S_-|^2 \rangle \sim Nm_s$, to leading order in N^{-1} the effective potential is

$$\tilde{V}_{\text{eff}}(m_f, m_s) = N[(m_f^2)^{3/2} - (m_s^2)^{3/2}]/(3\pi) \quad . \tag{21}$$

To demonstrate that super QED$_3$ breaks chiral symmetry is simply a matter of computing terms of $\sim O(1)$ in $\tilde{V}_{\text{eff}}$, and requiring $\tilde{V}_{\text{eff}}(m_f, m_s)$ to be stationary under independent variations of m_f and m_s.

Suppose, however, that one insisted on working in a supersymmetric fashion. For $m_f = m_s = \tilde{m}$, $\tilde{V}_{\text{eff}}(\tilde{m}) = 0$—for all $\tilde{m}$! This result holds to all order in N^{-1}, as an example of a nonrenormalization theorem.[26] How, then, does the correct vacuum choose one particular value of $\tilde{m}$? I suggest that if the effective *potential* does not pick a unique vacuum, the effective *action* will. In particular, if the Green's functions were calculated for any $\tilde{m}$, due to finite renormalizations, the supersymmetric Ward identities would be obeyed for only a single value of $\tilde{m}$. By necessity, this $\tilde{m}$ must be that found from the Schwinger-Dyson equations, Eqs. (15) and (18). For now, this is a conjecture about super QED$_3$, but in the supersymmetric nonlinear σ model,[3] exactly the same thing happens.[27]

Further calculations are underway, and will be presented separately.

ACKNOWLEDGMENTS

I thank J. Harvey and S. Shenker for helpful comments. This research was supported in part by the National Science Foundation under Grant No. PHY77-27084, supplemented by funds from the National Aeronautics and Space Administration.

---

[1] Y. Nambu, Phys. Rev. Lett. 4, 380 (1960); Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[2] D. J. Gross and A. Neveu, Phys. Rev. D 10, 3235 (1974).

[3] O. Alvarez, Phys. Rev. D 17, 1123 (1978).

[4] For one approach to chiral-symmetry breaking in small-N QED$_3$ see J. M. Cornwall, Phys. Rev. D 22, 1452 (1980).

[5] Massless scalar QED$_3$ has been studied at large N by T. Appelquist and R. D. Pisarski, Phys. Rev. D 23, 2305 (1981).

[6] J. Wess and B. Zumino, Nucl. Phys. B78, 1 (1974).

[7] I. Affleck, J. Harvey, and E. Witten, Nucl. Phys. B206, 413 (1982).

[8] E. Witten, Nucl. Phys. B202, 253 (1982).

[9] S. Dimopoulos and S. Raby, Nucl. Phys. B192, 353 (1981); M. Dine, W. Fischler, and M. Srednicki, *ibid.* B189, 575 (1981).

[10] M. E. Peskin, SLAC Report No. 3061, 1983 (unpublished); F. Karsch, E. Rabinovici, G. Shore, and G. Veneziano, CERN Report No. TH.3722, 1983 (unpublished).

[11] H. P. Nilles, Phys. Lett. 112B, 455 (1982); 129B, 103 (1983); G. Veneziano and S. Yankielowicz, *ibid.* 113B, 231 (1982); T. R. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. B218, 493 (1983); V. A. Novikov, M. A. Shifman, A. I. Vainstein, and V. I. Zakarov, *ibid.* B229, 381 (1983); B229, 394 (1983); B229, 407 (1983); E. Cohen and C. Gomez, Phys. Rev. Lett. 52, 237 (1984); T. R. Taylor, Phys. Lett. 125B, 185 (1983); 128B, 403 (1983); Fermilab Report No. 84/20 (unpublished); H. S. Sharatchandra, Max Planck Institut Reports No. 54/83, No. 85/83, and No. 91/83 (unpublished).

[12] A. C. Davis, M. Dine, and N. Seiberg, Phys. Lett. 125B, 487 (1983); I. Affleck, M. Dine, and N. Seiberg, Phys. Rev. Lett. 51, 1026 (1983); Princeton Reports, 1983 (unpublished).

[13] The metric has signature $(-++)$, with γ_μ's such that $-\gamma_0^2 = \gamma_1^2 = \gamma_2^2 = 1$; $\gamma_4^2 = \gamma_5^2 = 1$. The momentum $\bar{p} = (p_0, \vec{p})$; $p \equiv (p_\mu p^\mu)^{1/2}$. Feynman integrals, such as Eq. (6), are rotated from Minkowski to Euclidean space-time without comment.

[14] All triangle diagrams are ultraviolet finite in three dimensions, so there are no anomalies. Still, the $U_{4,5}(1)$ symmetry does behave

in an unusual fashion. With $J_\mu^{4,5}=\bar{\psi}\gamma_{4,5}\gamma_\mu\psi/\sqrt{N}$ the $U_{4,5}(1)$ current, at one-loop order

$$\langle 0|J_\mu^{4,5}(\bar{p})|A_\nu(-\bar{p})\rangle \approx \epsilon_{\mu\nu\lambda}p^\lambda\sqrt{\alpha}/(2\pi) \ ,$$

$\bar{p}\rightarrow 0$. At two-loop order,

$$\langle 0|J_\mu^{4,5}(\bar{p})J_\nu^{4,5}(-\bar{p})|0\rangle \approx 3m(g_{\mu\nu}-p_\mu p_\nu/p^2)/(2\pi) \ ,$$

$\bar{p}\rightarrow 0$, where Eq. (5b), and $m'=0$, have been used. It appears as if the $U_{4,5}(1)$ symmetry is spontaneously broken, with the photon for a Goldstone boson. I believe, however, that appearances are misleading. $\langle 0|J_0^{4,5}|A_\nu\rangle \sim \epsilon_{0\nu\lambda}\bar{p}^\lambda$, so the matrix element for the $U_{4,5}(1)$ charge, given by $J_0^{4,5}$ as $\bar{p}\rightarrow 0$, vanishes. In this way, Goldstone's theorem is avoided. Nevertheless, it is peculiar to find long-range correlations in a current which is neither anomalous nor spontaneously broken: I term this a "$U_{4,5}(1)$ peculiarity." See also Sec. II, Ref. 7.

[15]The spin density is $\sim\psi^\dagger[\gamma_1,\gamma_2]\psi\sim\bar{\psi}\gamma_{4,5}\psi$. Spin is a pseudoscalar in three dimensions, and thus so is the mass m'. Similarly, a single two-component spinor has a P- and T-odd mass. A four-component spinor creates a P- and T-even mass m by pairing up two-component spinors of equal mass and opposite sign.

[16]If in Eq. (2) I set $m=0$, $m'\neq 0$, as follows from a condensate $\langle\bar{\psi}\gamma_{4,5}\psi\rangle \sim iN(m')^2$, to one-loop order the photon propagator is

$$\Delta_{\mu\nu}=\frac{g_{\mu\nu}-p_\mu p_\nu/p^2-\epsilon_{\mu\nu\lambda}p^\lambda\Pi'/p^2}{(1+\Pi)[p^2+(\Pi')^2]} \ .$$

Π is that of Eq. (4), with m replaced by m'. The value of m', found self-consistently as in Eqs. (6)–(8), equals that of Eq. (9). For $p<<\alpha$, $\Pi'\sim m'$: the P- and T-odd mass of the fermion induces one for the photon. For other discussions of these odd masses, see (a) W. Siegel, Nucl. Phys. B156, 135 (1979); (b) J. Schonfeld, *ibid.* B185, 157 (1981); S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) 140, 372 (1982); and Ref. 7.

[17]Similar logarithms $\sim N^{-1}\ln(\alpha/p)$ are found in massless scalar QED_3 at large N (Ref. 5). Unlike the present model, there these logarithms signal the appearance of critical indices $\sim N^{-1}$.

[18]A detailed analysis shows that there are two contributions to c_0. One comes from momenta $p\lesssim m$ and $p\gtrsim\alpha$ in Eq. (6), for which the exact form of $\Pi(p)$ in Eq. (4) is needed. The other comes from certain two- and three-loop graphs, which give rise to terms $\sim N^{-2}\ln(\alpha/m)\sim N^{-1}$.

[19]I have glossed over several subtleties in the derivation of Eq. (9). The first is finite wave-function renormalization for the fermion. Writing the inverse fermion propagator as

$$\Delta_\psi^{-1}=Z(p)(\not{p}-im) \ ,$$

$Z(p)\rightarrow 1$ for $p>>\alpha$. To determine $Z(0)\equiv Z$, I use the Ward identity to take $Z(e\gamma^\mu)$ as the renormalized vertex. Then $\Pi\rightarrow Z^{-1}\Pi$, and

$$\Sigma(\bar{p})=-8Z(\not{p}/3+im)\ln(\alpha/m)/(\pi^2N) \ ,$$

$\bar{p}\rightarrow 0$. With

$$\not{p}+\Sigma(\bar{p})=Z(\not{p}-im) \ ,$$

the relation which determines m is unchanged. The above is in the Landau gauge, and gives $Z=\frac{3}{4}$. Secondly, the physical mass is properly determined by the position of the pole in Δ_ψ, and not by $\Sigma(\bar{p})$ as $\bar{p}\rightarrow 0$. These two definitions coincide in the Landau gauge (at least to leading order in N^{-1}), but in covariant gauges other than the Landau, the former must be used. Z, unremarkably, is gauge dependent.

[20]ϕ represents the component of A_μ in the reduced direction. This dimensional reduction yields a complex supersymmetry in three dimensions, Refs. 16(a) and 7.

[21]M. E. Peskin, SLAC Report No. 3021, 1982, Sec. 7.2 (unpublished).

[22]While invariably $\langle\bar{\lambda}\lambda\rangle=0$, the symmetry of $\lambda\rightarrow\gamma_5\lambda$ does not prohibit a condensate $\langle\bar{\lambda}\gamma_{4,5}\lambda\rangle\neq 0$. This occurs if I take $m_s=m_f'=\tilde{m}'$, $m_f=0$, in Eq. (11). Then

$$\Delta_\lambda^{-1}=\not{p}+(\not{p}-2i\tilde{m}'\gamma_{4,5})\tilde{\Pi}(p) \ ,$$

where $\tilde{\Pi}(p)$ is that of Eq. (13), with $\tilde{m}'$ instead of $\tilde{m}$. With this replacement, Δ_ϕ and Δ_D in Eq. (12) are unchanged, but $\Delta_{\mu\nu}$ becomes that of Ref. 16, after $\Pi\rightarrow\tilde{\Pi}$, and with $\Pi'=2\tilde{m}'$ for $p<<\alpha$. This means that both the photon and the photino have a supersymmetric mass $=2\tilde{m}'$ which is P and T odd [Eq. (A.32), Ref. 7]. Clearly, a photino condensate $\langle\bar{\lambda}\gamma_{4,5}\lambda\rangle\neq 0$ is special to three dimensions, and has no counterpart in four dimensions.

[23]Following the example of QED_3 (Ref. 19) I presume that my results are unaltered by the effects of wave-function renormalization for the matter fields. To show this in super QED_3 would, for all practical purposes, require an explicitly supersymmetric form of the theory, which that of Eq. (10) is not (Ref. 6).

[24]If $m_s>>\alpha$, $\Delta_{\bar{D}}^{-1}=1$ for all p, up to terms $\sim\alpha/m_s$. Then $\Sigma_s(0)\sim\alpha m_s/N$, so $\Sigma_s(0)=m_s^2$ gives $m_s\sim\alpha/N$, which is not $>>\alpha$: $m_s>>m_f$ cannot be obtained as a self-consistent solution to the Schwinger-Dyson equations.

[25]With an ultraviolet cutoff Λ, a term $\sim Nm^2\Lambda$ needs to be subtracted from $V_{\rm eff}$. As a function of the two condensates $\langle\bar{\psi}\psi\rangle$ and $\langle\bar{\psi}\gamma_{4,5}\psi\rangle$, the effective potential $V_{\rm eff}(m,m')$ depends only on $m^2+(m')^2$. It is possible to consider $m\neq 0$, $m'=0$, as I have done, by reaching the chiral limit from one of nonzero bare mass, and adjusting the bare mass to vanish accordingly.

[26]B. Zumino, Nucl. Phys. B89, 535 (1975); D. M. Capper and M. Ramón Medrano, J. Phys. G 2, 269 (1976); P. C. West, Nucl. Phys. B106, 219 (1976). These proofs were obtained for perturbation theory in e^2, but as all that is required are the forms of the propagators in superspace, they apply as well to the N^{-1} expansion.

[27]In the notation and with the equation numbers of Ref. 3: as in super QED_3, the effective potential vanishes for any supersymmetric mass m—$\mathscr{L}_{\rm eff}$, Eq. (3.8), $=0$ if $g^2\alpha=g^2\phi^2=m^2$, $\beta=0$. Supersymmetry requires

$$\Delta_\phi^{-1}(0)=4m^2\Delta_\alpha^{-1}(0)/g^2 \ ,$$

Eq. (3.41b). From Eq. (3.37c),

$$4m^2\Delta_\alpha^{-1}(0)/g^2=g^2N/(2\pi) \ ,$$

while from Eqs. (3.18) and (3.36),

$$\Delta_\phi^{-1}(0)=g^2N/(2\pi)+1+g^2N\ln(m/M)/(2\pi) \ .$$

Only if $m=M\exp[-2\pi/(g^2N)]$ [Eqs. (3.21) and (3.25)] are the Ward identities of supersymmetry fulfilled.