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Comments

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Comment on the nonrenormalization theorem in supersymmetric gauge theories

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We discuss some subtleties connected with the nonrenormalization theorem in supersymmetric gauge theories. In particular, we reconcile the need for infinite-mass counterterms in the Wess-Zumino gauge with the existence of the theorem.

Supersymmetric theories have a number of amazing properties of which, perhaps, the most remarkable are the nonrenormalization theorems. These theorems were first proven for theories of interacting chiral superfields ϕ_i,[1] where it was shown that the only infinities in the theory reside in the wave-function renormalization constants for the chiral superfields:

$$\phi_i = Z_i^{1/2}\phi_i^{\text{ren}} \quad . \tag{1}$$

Other infinities in the theory can be absorbed by rescaling the bare-mass parameters and bare coupling constants, appropriately, with the Z_i's, to wit,

$$\begin{aligned} m_{ij}^{\text{ren}} &= (Z_i Z_j)^{1/2} m_{ij}^0 \ , \\ g_{ijk}^{\text{ren}} &= (Z_i Z_j Z_k)^{1/2} g_{ijk}^0 \ . \end{aligned} \tag{2}$$

These results were extended also to the case of supersymmetric Yang-Mills theories.[2] In particular, it was shown by Ferrara and Piguet[3] that, after the rescaling (2), no additional mass renormalization is needed for the chiral superfields, even in the presence of gauge interactions. The absence of mass renormalization demonstrated by Ferrara and Piguet[3] is rather formal. What they show is that, calculating with superpropagators, graphs involving two chiral superfields have always a negative degree of divergence, and hence require no counterterms. In contrast, graphs involving a chiral and an antichiral superfield diverge logarithmically, requiring a wave-function renormalization.

These formal results appear to be vitiated if one performs the simplest one-loop calculation in supersymmetric QED, as was indeed done by Wess and Zumino[4] in their original paper. A straightforward calculation, in the Wess-Zumino gauge,[4] gives for the fermion self-energy at one loop the expression

$$\Sigma^{\text{WZ}}(p) = ie^2 \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2[(q-p)^2+m_0^2]}\left[m_0(\xi-4)+\gamma\cdot p(\xi-2)-\gamma\cdot q\frac{p^2+m_0^2}{q^2}\xi\right] \ . \tag{3}$$

Here ξ is a gauge parameter, with $\xi=0$ corresponding to the Feynman gauge. The logarithmic infinity in Eq. (3) can be absorbed in a wave-function renormalization

$$Z^{\text{WZ}} = 1 - ie^2(\xi-2)\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2(q^2-2q\cdot p)}\bigg|_{p^2=-m_0^2} \tag{4}$$

and a mass rescaling

$$m = m_0\left[1-2ie^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2(q^2-2q\cdot p)}\right]\bigg|_{p^2=-m_0^2} \ . \tag{5}$$

However, the mass rescaling appears to have nothing to do with the gauge-dependent, wave-function renormalization constant.

The purpose of this note is to resolve this conundrum. Our results will be essentially pedagogic, and are probably already well understood by the experts. Nevertheless, we feel that they cast some more light on the nonrenormalization theorem and as such they may be of some use. We shall see that Eq. (5), in fact, is in perfect accord with the results of Ferrara and Piguet,[3] once one understands properly the meaning of the nonrenormalization theorem.

We shall be working only at one-loop order and in supersymmetric QED,[4] there being in this case no practical distinctions between Abelian and non-Abelian contributions to the chiral superfield self-energy. The relevant pieces of the Lagrangian for the theory to consider are

$$\begin{aligned} L = {} & (\bar\phi_+ e^{2eV}\phi_+ + \bar\phi_- e^{-2eV}\phi_-)|_{\theta\theta\bar\theta\bar\theta} \\ & + m_0\phi_+\phi_-|_{\theta\theta} + m_0\bar\phi_+\bar\phi_-|_{\bar\theta\bar\theta} \ . \end{aligned} \tag{6}$$

By scaling the superfields ϕ_+, ϕ_- via

$$\phi_\pm = \sqrt{Z}\,\phi_\pm^{\text{ren}} \ , \tag{7}$$

and defining a renormalized mass

$$m = Z_m m_0 \ , \tag{8}$$

we can rewrite Eq. (6) in terms of renormalized quantities,[5] plus a counterterm Lagrangian. One finds for this latter

Lagrangian

$$L_{\text{count}} = (Z-1)[(\bar{\phi}_+^{\text{ren}} e^{2e_{\text{ren}} V_{\text{ren}}} \phi_+^{\text{ren}} + \bar{\phi}_-^{\text{ren}} e^{-2e_{\text{ren}} V_{\text{ren}}} \phi_-^{\text{ren}})|_{\theta\theta\bar{\theta}\bar{\theta}}]$$
$$+ \delta m (\bar{\phi}_+^{\text{ren}} \bar{\phi}_-^{\text{ren}}|_{\bar{\theta}\bar{\theta}} + \phi_+^{\text{ren}} \phi_-^{\text{ren}}|_{\theta\theta}) \quad , \tag{9}$$

where

$$\delta m = m(Z Z_m{}^{-1} - 1) \quad . \tag{10}$$

Clearly, there will be no mass counterterm only if $Z_m = Z$. The nonrenormalization theorem of Ferrara and Piguet[3] is the statement that, with Z and Z_m appropriately defined, δm indeed vanishes. The puzzle to solve is why the explicit calculation of Wess and Zumino [cf. Eqs. (4) and (5)] gives Z different from Z_m and therefore δm infinite and gauge dependent.

The resolution of this apparent discrepancy is perhaps most easily arrived at by focusing on the mass rescaling parameter Z_m. If m is taken to be the physical mass associated with the superfields $\phi_\mp^{\text{ren}}$ and m_0 is the mass parameter in the bare Lagrangian, it is clear that Z_m must be independent of the gauge of quantization. That is, Z_m should be independent of the gauge parameter ξ associated with the gauge chosen to quantize the vector propagator and, furthermore, it should also be independent of whether one quantizes the theory in an explicit supersymmetric way or whether one chooses to quantize the theory in the Wess-Zumino gauge, in which supersymmetry is not manifestly preserved.

This assertion can, in fact, be readily checked. From Eq. (5) one has

$$Z_m = 1 - 2ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2(q^2 - 2q\cdot p)}\bigg|_{p^2 = -m_0{}^2} \quad , \tag{11}$$

which is independent of the gauge parameter ξ, when the Wess-Zumino gauge is used. This same result is also obtained by examining the results of the explicit calculation of de Wit,[6] in which the theory is quantized in an explicit supersymmetry-preserving way. If one writes for the fermion self-energy the decomposition

$$\Sigma(p) = \gamma\cdot p\, \Sigma_1(p^2) + \Sigma_2(p^2) \tag{12}$$

and one renormalizes on-shell, then it follows that

$$Z = 1 - \Sigma_1(-m^2) \quad , \tag{13}$$

$$\delta m = -\Sigma_2(-m^2) \quad . \tag{14}$$

Whence, to the order we are working

$$Z_m = 1 + \frac{\Sigma_2(-m^2)}{m} - \Sigma_1(-m^2) \quad . \tag{15}$$

A direct examination of the results of Sec. VI of Ref. 6, where Σ_1 and Σ_2 are computed, demonstrates that Z_m is free of any gauge parameters and agrees with the result (11), obtained in the Wess-Zumino gauge. A similar result follows if one calculates Σ_1 and Σ_2 in terms of supergraph techniques.

Having demonstrated that Z_m is a gauge-independent quantity—as it must be for physical cogency—it becomes clear why there can be mass counterterms in one calculation[4] and not in another.[3] Since Z is clearly gauge dependent, the counterterm δm *of Eq. (10) is itself gauge dependent.*[7] Quantizing the theory in the Wess-Zumino gauge the supersymmetry is not preserved for the wave-function renormalization constants. Indeed the Z factors are, in fact, different for the scalar and fermionic components of the superfields $\phi_\pm$.[4] It is therefore not terribly surprising that the infinite pieces in Z_m and Z do not match, so that δm, except for $\xi = 4$, is infinite.

If one quantizes the theory in an explicitly supersymmetric way the situation is slightly different. In this case, the content of the nonrenormalization theorem is that the function $\Sigma_2(p^2)$ associated with corrections to the $\phi_+\phi_-$ superpropagators is always convergent.[3] In effect this implies

$$\Sigma_2(p^2) = p^2 \hat{\Sigma}_2(p^2) \quad , \tag{16}$$

with $\hat{\Sigma}_2(p^2)$ being given by some convergent integral. Thus the mass counterterm

$$\delta m = m^2 \hat{\Sigma}_2(-m^2) \tag{17}$$

is *always finite*, but it does not necessarily vanish. In fact, by explicit calculation,[6] one finds that δm is gauge dependent, vanishing in the Feynman gauge:

$$\delta m = -4im^3 e^2 \xi \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4(q^2 - 2q\cdot p)}\bigg|_{p^2 = -m_0{}^2} \quad . \tag{18}$$

Note that although (18) is ultraviolet finite, it is infrared divergent.

A final comment is in order. If one returns to Eq. (10), and uses the fact that quantizing in a supersymmetric way δm is finite, it is clear that the infinite pieces in Z and Z_m always match, and that the only gauge dependence in Z resides in its finite contributions. By choosing an appropriate gauge (Feynman gauge) one may totally match Z with Z_m and then no mass counterterms are needed. Alternatively, one may define the renormalized mass parameter not to be the physical mass, but the value of the inverse fermion propagator at $p^2 = 0$, $\gamma\cdot p = 0$. In this case

$$\delta m = -\Sigma_2(0) = 0 \tag{19}$$

on account of Eq. (16). Finite renormalizations can always be disposed of by redefining the normalization point.

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[3]S. Ferrara and O. Piguet, Ref. 2.

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[5]Note that for the vector superfield interactions $eV = e_{\text{ren}} V_{\text{ren}}$.

[6]B. de Wit, Ref. 2.

[7]Note that δm defined in Eq. (10) *is not* equal to $m - m_0$. This latter quantity is gauge independent and is sometimes denoted by δm in the literature.