Analytic distributions of various parameters in $ee \to eex^+x^-$ under standard experimental conditions

A. Courau
Laboratoire de l'Accélérateur Linéaire, Université Paris—Sud, 91405 Orsay Cedex, France
(Received 27 June 1983)

We consider various distributions of physical parameters for relativistic-pair production in $\gamma\gamma$ collisions ($ee \to eex^+x^-$), expected in no-tag or antitag experiments, taking into account experimental constraints. Invariant-mass, visible-energy, and transverse-momentum distributions are derived analytically, with use of the double equivalent-photon approximation, in an easily computable integral form, at least whenever the Q^2 dependence of the $\gamma\gamma$ cross section can be neglected.

I. INTRODUCTION

In a previous paper,[1] we determined the transverse-momentum behavior of fermions produced in $\gamma\gamma$ reactions[2] ($e^+e^- \to e^+e^-f^+f^-$) for the case of nontagging measurements. By means of the double equivalent-photon approximation (DEPA), we have obtained for $d\sigma/dp_t^2$ a simple and transparent expression which takes account of the limited acceptance of the central detector. When applied to the production of muon pairs detected at large angles, that expression fits the experimental data of Mark J (Ref. 3) in a way at least as satisfactory as the predictions obtained from a long and hardly transparent Monte Carlo computation performed by the authors of Ref. 3 on the basis of the Vermaseren program.[4] Generalizing our procedure, we here shall determine various differential cross sections (i.e., one-parameter distributions) for pair production through $\gamma\gamma$ processes in e^+e^- collisions, assuming experimental conditions that involve a predominating contribution of $Q^2 \simeq 0$ photons. We thus limit ourselves to experiments without any tagging or with antitagging[5] (or 0° tagging[6]), and exclude the case of finite-angle tagging measurements.

No-tag measurements are, generally speaking, dominated by quasireal-photon interactions. Actually, in order to improve the signal/noise ratio, such $\gamma\gamma$ measurements require a sufficiently high beam energy since the relative contamination by other processes decreases as the beam energy is increased,[7,8] and possibly an upper cutoff of the total transverse momentum of the particles produced; both these requirements tend to make the quasireal photons predominate even more. Obviously that is also true when a maximum scattering angle of the electrons is experimentally imposed (antitagging or 0° tagging). We are then fully justified in using a factorization formula based on differential equivalent-photon spectra, as well as kinematic relations applying to photons that are treated as real and colliding along the same axis as the incident electrons. This being so, it becomes possible to write down Monte Carlo programs which are much simpler and quicker to perform than the very sophisticated ones that are often necessary in other configurations.

If (as is the general case) the cross section of $\gamma\gamma \to X$ can be treated as independent of the Q^2 values of either photon, its determination—accounting for the acceptance of the experimental apparatus in the laboratory on the one hand, and of the distribution of particles produced in the $\gamma\gamma$ center-of-mass frame on the other hand—depends only on the total invariant mass and on the velocity (or rapidity) of the $\gamma\gamma$ system in the laboratory frame. Now we have shown[7–9] that, by convoluting both equivalent-photon spectra after integrating each of them over the corresponding Q^2 values (i.e., over the corresponding angular distribution), one can define and express a differential luminosity $d^2\mathscr{L}(\gamma\gamma)$ that depends itself on nothing else than the two parameters mentioned above. However, in general, the computation of the cross section $ee \to eeX$ for multiparticle production will still require the use of a Monte Carlo program; at least it will be a very simplified one.

In the case of pair production, the simple correlation that exists between laboratory and c.m. emission angles allows one to reexpress in the $\gamma\gamma$ c.m. frame the acceptance limits of the apparatus as they are defined in the laboratory. As a result, no Monte Carlo simulation is necessary any longer, and integral expressions may be written for the various distributions considered, taking account of the given acceptance limits; in many cases these expressions can be partially or totally integrated by analytic means. The remaining formulas are easily computable even by means of a pocket computer. We shall establish such expressions for various distributions, as well in the general case as in some particular types of processes where simplified forms are obtained.

Actually we shall be led to distinguish between two types of measurements, depending on whether the Q^2 of either photon is not limited experimentally or is cut off in some way. Such a cutoff may be performed by setting a higher limit either to the electrons' scattering angles (0° tagging or antitagging) or to the total transverse momentum of the particle pair produced.

II. ASSUMPTIONS AND NOTATIONS

We shall assume that the acceptance of the central detector is given by well-defined cuts on the transverse momenta of the particles produced (such that one will only measure transverse momenta much higher than the masses of those particles) and on their emission angles. The angular acceptance will be assumed to be the same for both tracks and symmetric with respect to the e^+e^- collision axis as well as to the interaction point. We shall also consider various additional experimental cuts among the most usual ones.

We shall use the following notations: $E=\sqrt{s}/2$ is the incident-beam energy; ω_i is the energy of either photon; p_t is the transverse momentum of either particle produced, θ and θ^* its angle of emission in the laboratory frame and the $\gamma\gamma$ c.m. frame, respectively (with respect to the collision axis); W is the invariant mass of the $\gamma\gamma$ system, while E', β, and y $(=\tanh^{-1}\beta)$ are, respectively, the visible energy, the velocity, and the rapidity of that system, all defined in the laboratory frame; p_0 and θ_0 are acceptance limits in the laboratory, defined by $p_t \ge p_0$ and $|\cos\theta| \le \cos\theta_0$. Other possible experimental cutoff values considered are p_M^{tot}, W_M, E'_M, p_M, and β_0, defined by $|\sum \vec{p}_t| \le p_M^{\text{tot}}$, $W \le W_M$, $E' \le E'_M$, $p_t \le p_M$, and $\sin(\theta_1+\theta_2)/(\sin\theta_1+\sin\theta_2) \ge \beta_0$ (the latter inequality represents a convenient way of defining an acollinearity cut, as is sometimes needed in order to eliminate the cosmic-ray background).

In addition we define the following dimensionless quantities:

$$X_i=\omega_i/E,\quad Z=W/2E,\quad V=E'/2E\ ,$$
$$x_t=p_t/E,\quad u=\cos\theta^*\ ,$$

and

$$x_0=p_0/E,\quad u_0=\cos\theta_0,\quad t_0=\tan(\theta_0/2)\ ,$$
$$Z_M=W_M/2E,\quad V_M=E'_M/2E\ ,$$
$$x_M=p_M/E,\quad x_M^{\text{tot}}=p_M^{\text{tot}}/E\ ,$$
$$y_0=\tanh^{-1}u_0\equiv\cosh^{-1}(1/\sin\theta_0)$$
$$\equiv\tfrac{1}{2}\ln\frac{1+u_0}{1-u_0}\equiv\ln(1/t_0)\ .$$

III. KINEMATIC RELATIONS AND EXPERIMENTAL LIMITS

In the equivalent-photon approximation, the $\gamma\gamma$ collision axis is identified with the incident-beam axis and one has the relations

$$Z^2=X_1X_2,\quad V=(X_1+X_2)/2,\quad y=\tfrac{1}{2}\ln(X_2/X_1)\ .$$

If one limits oneself to the production of pairs of extreme-relativistic particles one gets, in addition, the relations

$$x_t=Z(1-u^2)^{1/2}$$

and

$$\cos\theta=(\beta-u)/(1-\beta u)\ .$$

First one notices that one may equivalently use various sets of parameters, since those sets $((X_1,X_2),u)$, $((Z,y),u)$, $((V,y),u)$, and (x_t,y,u) are connected by relations such as

$$X_i=Ze^{\pm y}=V(1\pm\tanh y)$$
$$=x_te^{\pm y}/(1-u^2)^{1/2}\ ,$$
$$V=Z\cosh y,\quad u=(1-x_t^2/Z^2)^{1/2}\ .$$

As a result, first, any expression given as a function of the parameters belonging to one of the sets given above can be redefined as a function of the parameters belonging to any other of those sets. One also concludes that all limitations due to phase space $(X_i\le 1)$, to the transverse momenta $(x_t \ge x_0)$, or to various other possible experimental cuts, such as $x_t\le x_M$, $Z\le Z_M$, $V\le V_M$, or

$$\sin(\theta_1+\theta_2)/(\sin\theta_1+\sin\theta_2)=\tanh^{-1}y\ge\beta_0\ ,$$

can be expressed analytically in the form of limits imposed on any set of parameters. Finally, using the relation $\cos\theta=(\beta-u)/(1-\beta u)$, one concludes that the same is true for the limitation of the angular acceptance of the central detector in the laboratory ($|\cos\theta|\le u_0$); if one uses a set of variables involving both u and y, one gets, according to whether the first integration is performed over u or y,

$$|u|\le U_y \quad\text{with}\quad |y|\le y_0$$

or

$$|y|\le Y_u \quad\text{with}\quad |u|\le u_0\ ,$$

where one defines

$$Y_u=\tanh^{-1}(u_0)-\tanh^{-1}|u|$$
$$\equiv y_0-\tanh^{-1}|u|$$
$$\equiv\tfrac{1}{2}\ln\frac{1-|u|}{1+|u|}\Big/t_0^2\ ,$$
$$U_y=\tanh(y_0-y)\ .$$

The analytic expressions of all these limits, as functions of the set of parameters chosen and of the order of integration over those parameters, are given in Tables I—VI. Let us notice that they are only meaningful when they are positive definite, and that they always define symmetric ranges in $\pm u$ and $\pm y$. We also notice that one always has $u\le\cos\theta_0<1$.

IV. EQUIVALENT-PHOTON SPECTRA (INTEGRATED OVER Q^2)

In the Williams-Weizsäcker approximation, the equivalent-photon spectrum integrated over the azimuthal angle $(\Delta\phi=2\pi)$ is written

$$d^2N(E,X_i,Q^2)=\frac{\alpha}{\pi}\left[\left[1-X_i+\frac{X_i^2}{2}\right]-(1-X_i)\frac{Q^2_{\min}}{Q^2}\right]\frac{dQ^2}{Q^2}\frac{dX_i}{X_i}\ ,$$

where $Q^2_{\min}=m_e^2X_i^2/(1-X_i)$ is the kinematic lower lim-

TABLE I. Limits of the variables Z,u,y involved in computing the invariant-mass distribution. $H(E,Z)=\int_{u'_Z}^{U'_Z}\int_{y_Z}^{Y_Z} d^2H(E,Z,u,y)$.

	$Z_{\min}=$ Highest value of:	$Z_{\max}=$ Lowest value of:	$u'_Z=$ Highest value of:	$U'_Z=$ Lowest value of:	$y_Z=$ Highest value of:	$Y_Z=$ Lowest value of:
Acceptance						
Phase space $X_i \le 1$		1	0		0	$\ln(1/Z)$
p_t trigger $x_t > x_0$	x_0		0	$[(1-x_0^2/Z^2)]^{1/2}$	0	
Angular acceptance $\lvert\cos\theta\rvert \ge u_0$			0	u_0	0	$y_0-\tanh^{-1}(u)$
Exptl. cuts						
$Z < Z_M$		Z_M	0		0	
$x_t < x_M$		$x_M/\sin\theta_0$	$(1-x_M^2/Z^2)^{1/2}$		0	
$V < V_M$		V_M	0		0	$\cosh^{-1}(V_M/Z)$
$\dfrac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2} > \beta_0$		$\left[\dfrac{1-\beta_0}{1+\beta_0}\right]^{1/2}$	0		$\tanh^{-1}\beta_0$	

it on Q^2. When one integrates over Q^2 from $Q^2{}_{\min}$ to $Q^2{}_{\max}$ or, equivalently, over the electron's scattering angle up to a maximum value θ_e^M, one obtains

$$dN(E,X_i)=\frac{\alpha}{\pi}S(E,X_i)dX_i/X_i$$

with

$$S(E,X_i)=(1-X_i+X_i^2/2)\ln(1+\Delta^2)$$
$$-(1-X_i)[1-1/(1+\Delta^2)]\ ,$$

where

$$\Delta^2=(Q^2{}_{\max}-Q^2{}_{\min})/Q^2{}_{\min}\ .$$

Since, experimentally, one always has $Q^2{}_{\max} \gg\gg Q^2{}_{\min}$ $(\Delta^2 \gg\gg 1)$ one thus gets

$$S(E,X_i)=2(1-X_i+X_i^2/2)\ln\Delta-(1-X_i)$$

with

$$\Delta=\frac{E}{m_e}\frac{1-X_i}{X_i}(2\sin\theta_e^M/2)\ .$$

One may simplify the expression of $S(E,X_i)$ as follows:

$$S(E,X_i)=2(1-X_i+X_i^2/2)(\ln\Delta-\tfrac{1}{2})\ .$$

This is in general a good approximation since the logarithmic term dominates, and since increasingly large values of X_i contribute less and less (and eventually not at all); the latter fact is due to both dynamic and kinematic reasons, i.e., to the fact that small-Z values predominate (all the more as one practically always applies an experimental cutoff in order to eliminate the large-Z values where the background will dominate) and that the limited angular acceptance tends strongly to favor values of either X_i close to that of Z (i.e., small values of y).

TABLE II. Limits of the variables Z,y,u involved in computing the invariant-mass distribution. $H(E,Z)=\int_{y'_Z}^{Y'_Z}\int_{u_Z}^{U_Z} d^2H(E,Z,u,y)$.

	$Z_{\min}=$ Highest value of:	$Z_{\max}=$ Lowest value of:	$y'_Z=$ Highest value of:	$Y'_Z=$ Lowest value of:	$u_Z=$ Highest value of:	$U_Z=$ Lowest value of:
Acceptance						
Phase space $X_i \le 1$		1	0	$\ln(1/Z)$	0	
p_t trigger $x_t > x_0$	x_0		0		0	$(1-x_0^2/Z^2)^{1/2}$
Angular acceptance $\lvert\cos\theta\rvert \ge u_0$			0	y_0	0	$\tanh(y_0-y)$
Exptl. cuts						
$Z \le Z_M$		Z_M			0	
$x_t \le x_M$		$x_M/\sin\theta_0$			$(1-x_M^2/Z^2)^{1/2}$	
$V \le V_M$		V_M		$\cosh^{-1}(V_M/Z)$	0	
$\dfrac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2} \ge \beta_0$		$\left[\dfrac{1-\beta}{1+\beta_0}\right]^{1/2}$	$\tanh^{-1}\beta_0$		0	

TABLE III. Limits of the variables V,u,y involved in computing the visible-energy distribution. $I(E,V)=\int_{u'_V}^{U'_V}\int_{y_V}^{Y_V} d^2I(E,V,u,y)$.

	$V_{\min}=$ Highest value of:	$V_{\max}=$ Lowest value of:	$u'_V=$ Highest value of:	$U'_V=$ Lowest value of:	$y_V=$ Highest value of:	$Y_V=$ Lowest value of:
Acceptance						
Phase space $X_i\leq 1$		1	0		0	$\frac{1}{2}\ln\left[\frac{1}{2V-1}\right]$
p_t trigger $x_t>x_0$	x_0		0	$(1-x_0{}^2/V^2)^{1/2}$	0	$\cosh^{-1}\left[\frac{V(1-u^2)^{1/2}}{x_0}\right]$
Angular acceptance $\lvert\cos\theta\rvert\geq u_0$			0	u_0	0	$y_0-\tanh^{-1}\lvert u\rvert$
Exptl. cuts						
$Z<Z_M$		$\begin{Bmatrix} Z_M/\sin\theta_0 \\ (1+Z_M{}^2)/2 \end{Bmatrix}$			$\cosh^{-1}(V/Z_M)$	
$x_t<x_M$		$x_M/\sin\theta_0$	$(1-x_M{}^2/V^2)^{1/2}$			
$V<V_M$		V_M				
$\frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}>\beta_0$		$1/(1+\beta_0)$			$\tanh^{-1}\beta_0$	

Let us add a remark concerning the choice of θ_e^M. Actually the "quasireality condition" is that Q^2 is negligible as compared to E^2Z^2 or (roughly) to $E^2X_i{}^2$. When that condition is satisfied, one has

$$X_i\sin\theta_\gamma=(1-X_i)\sin\theta_e\ .$$

Since $\sin\theta_\gamma\leq 1$, the quasireality condition imposes

$$\sin\theta_e\leq X_i/(1-X_i)\ .$$

In no-tag measurements one will thus take $\Delta\simeq E/m_e$. In measurements where an upper limit is experimentally imposed to the scattering angle of the electron ($\theta_e\leq\theta_e^M<1$), the Δ value becomes $\Delta=(E/m_e)\eta_i$ with $\eta_i=\min(1,(1-X_i)\theta_e^M/X_i)$.

The cutoff we are imposing on this basis obviously involves the neglect of the contribution of highly virtual photons. This is justified since the virtual-photon contribution is in fact depressed by a sort of form factor $[1/(1+Q^2/W^2)^2]$ which comes in addition to the $1/Q^2$ factor.[10] What we are doing, roughly, is replacing that form factor by a sharp cut at $Q^2\simeq W^2/4$. More precisely, such a cut performed with respect to Z instead of X would lead to $\eta_i\leq(1-X_i)^{1/2}(Z/X_i)$ which, in general, should not be very different from 1. In fact the precise value of

TABLE IV. Limits of the variables V,y,u involved in computing the visible-energy distribution. $I(E,V)=\int_{y'_V}^{Y'_V}\int_{u_V}^{U_V} d^2I(E,V,u,y)$.

	$V_{\min}=$ Highest value of:	$V_{\max}=$ Lowest value of:	$y'_V=$ Highest value of:	$Y'_V=$ Lowest value of:	$u_V=$ Highest value of:	$U_V=$ Lowest value of:
Acceptance						
Phase space $X_i\leq 1$		1	0	$\frac{1}{2}\ln\left[\frac{1}{2V-1}\right]$	0	
p_t trigger $x_t>x_0$	x_0		0	$\cosh^{-1}(V/x_0)$	0	$\left[1-\left[\frac{x_0\cosh y}{V}\right]^2\right]^{1/2}$
Angular acceptance $\lvert\cos\theta\rvert\geq u_0$			0	y_0	0	$\tanh(y_0-y)$
Exptl. cuts						
$Z\leq Z_M$		$\begin{Bmatrix} Z_M/\sin\theta \\ (1+Z_M{}^2)/2 \end{Bmatrix}$	$\cosh^{-1}(V/Z_M)$		0	
$x_t\leq x_M$		$x_M/\sin\theta_0$			$(1-x_M{}^2/V^2)^{1/2}$	
$V\leq V_M$		V_M			0	
$\frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}\geq\beta_0$		$1/(1+\beta_0)$	$\tanh^{-1}\beta_0$		0	

TABLE V. Limits of the variables x_t,u,y involved in computing the transverse-momentum distribution. $G(E,x_t)=\int_{u'_x}^{U'_x}\int_{y_x}^{Y_x} d^2G(E,x_t,u,y)$.

	$x_{t\,\min}=$ Highest value of:	$x_{t\,\max}=$ Lowest value of:	$u'_x=$ Highest value of:	$U'_x=$ Lowest value of:	$y_x=$ Highest value of:	$Y_x=$ Lowest value of:
Acceptance						
Phase space $X_i\le 1$		1	0	$(1-x_t^2)^{1/2}$	0	$\frac{1}{2}\ln\left[\frac{1-u^2}{x_t^2}\right]$
p_t trigger $x_t>x_0$	x_0		0		0	
Angular acceptance $\lvert\cos\theta\rvert\ge u_0$			0	u_0	0	$y_0-\tanh^{-1}\lvert u\rvert$
Exptl. cuts						
$Z<Z_M$		$Z_M\sin\theta_0$	0	$(1-x_t^2/Z_M^2)^{1/2}$	0	
$x_t<x_M$		x_M	0		0	
$V<V_M$		V_M	0	$(1-x_t^2/V_M^2)^{1/2}$	0	$\cosh^{-1}\left[\frac{V_M(1-u^2)^{1/2}}{x_t}\right]$
$\frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}>\beta_0$			0		$\tanh^{-1}\beta_0$	

this cut is not very relevant since the E/m_e factor predominates in the argument of $\ln\Delta$ anyway. In practice our choice, which leads to the standard equivalent-photon spectrum in the no-tag case, gives a fairly good approximation.[11]

V. GENERAL EXPRESSION OF DISTRIBUTIONS

When the $\gamma\gamma$ cross section does not depend on the Q^2 of either photon, it can be written in the following general form:

$$\frac{d\sigma(\gamma\gamma\to A^+A^-)}{du}=\frac{2\pi}{W^2}f(W,u)$$
$$\equiv\frac{\pi}{2E^2X_1X_2}f(2E\sqrt{X_1X_2},u)\ .$$

One thus obtains the differential cross section of $e^+e^-\to e^+e^-A^+A^-$ by simply factorizing this $\gamma\gamma$ cross section with both equivalent-photon spectra integrated over their acceptance in Q^2 (or in θ_e). Then setting

$$P(E,X_1,X_2)=S(E,X_1)S(E,X_2)\ ,$$

one has

$$E^2\frac{d^3\sigma}{dX_1dX_2du}=\frac{\alpha^2}{2\pi}\frac{P(E,X_1,X_2)}{(X_1X_2)^2}f(2E\sqrt{X_1X_2},u)\ .$$

One derives the expressions

$$E^2\frac{d^3\sigma}{dZ^2du\,dy}=\frac{\alpha^2}{2\pi}\frac{d^2H(E,Z,u,y)}{du\,dy}Z^{-4}\ ,$$
$$E^2\frac{d^3\sigma}{dV^2du\,dy}=\frac{\alpha^2}{2\pi}\frac{d^2I(E,V,u,y)}{du\,dy}V^{-4}\ ,$$
$$E^2\frac{d^3\sigma}{dx_t^2du\,dy}=\frac{\alpha^2}{2\pi}\frac{d^2G(E,x_t,u,y)}{du\,dy}x_t^{-4}$$

TABLE VI. Limits of the variables x_t,y,u involved in computing the transverse-momentum distribution. $G(E,x_t)=\int_{y'_x}^{Y'_Z}\int_{u_x}^{U_x} d^2G(E,x_t,u,y)$.

	$x_{t\,\min}=$ Highest value of:	$x_{t\,\max}=$ Lowest value of:	$y'_x=$ Highest value of:	$Y'_x=$ Lowest value of:	$u_x=$ Highest value of:	$U_x=$ Lowest value of:
Acceptance						
Phase space $X_i\le 1$		1	0	$\ln(1/x_t)$	0	$[1-(e^y/x_t)^2]^{1/2}$
p_t trigger $x_t>x$	x_0		0		0	
Angular acceptance $\lvert\cos\theta\rvert\ge u_0$			0	y_0	0	$\tanh(y_0-y)$
Exptl. cuts						
$Z\le Z_M$		$Z_M\sin\theta_0$	0		0	$(1-x_t^2/Z_M^2)^{1/2}$
$x_t\le x_M$		x_M	0		0	
$V\le V_M$		V_M	0	$\cosh^{-1}(V_M/x_t)$	0	$\left[1-\left[\frac{x_t\cosh y}{V_M}\right]^2\right]^{1/2}$
$\frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}\ge\beta_0$			0	$\tanh^{-1}\beta_0$	0	

with

$$d^2H(E,Z,u,y)=l(E,Z,y)f(2EZ,u)du\,dy\ ,$$

$$d^2I(E,V,u,y)=m(E,V,y)f(2EV/\cosh y,u)du\,dy\ ,$$

$$d^2G(E,x_t,u,y)=n(E,x_t,u,y)\ ,\quad \times f(2Ex_t/(1-u^2)^{1/2},u)du\,dy\ ,$$

where

$$l(E,Z,y)=P(E,X_1,X_2)\ \text{ with } X_i=Ze^{\pm y}\ ,$$

$$m(E,V,y)=\cosh^2 y\,P(E,X_1,X_2)\ \text{ with } X_i=V(1\pm\tanh y)\ ,$$

$$n(E,x_t,u,y)=(1-u^2)P(E,X_1,X_2)\quad \text{with } X_i=e^{\pm y}x_t/(1-u^2)^{1/2}\ .$$

One notices that both the integrands and the integration limits defined by the experimental conditions are symmetric with respect to $\pm u$ and $\pm y$. One may thus integrate over y and u, respectively, in either order (i.e., y first or u first) between the positive-definite lower and upper limits (shown in Tables I—VI) in order to obtain $H(E,Z)$, $I(E,V)$, or $G(E,x_t)$; one must then multiply by 4. Thus, one has

$$E^2(d\sigma/dZ^2)=\frac{2\alpha^2}{\pi}H(E,Z)Z^{-4}\ ,$$

$$E^2(d\sigma/dV^2)=\frac{2\alpha^2}{\pi}I(E,V)V^{-4}\ ,$$

$$E^2(d\sigma/dx_t^2)=\frac{2\alpha^2}{\pi}G(E,x_t)x_t^{-4}\ .$$

VI. MEASUREMENTS WITHOUT EXPERIMENTAL Q^2 LIMITATION

Let us first consider measurements where the Q^2 values of either photon are not limited by experimentally imposed cuts on the electrons' scattering angles or on the transverse momentum of the pair produced. In that case Δ can be taken as $\Delta=E/m_e$, and one thus gets

$$S(E,X_1)=2(1-X_1+X_i^2/2)(\ln E/m_e-\tfrac{1}{2})\ ,$$

$$P(E,X_1,X_2)=P_0(X_1,X_2)R_0^2(E)$$

with

$$P_0(X_1,X_2)=[2-(X_1+X_2)]^2-[(X_1+X_2)-X_1X_2]^2\ ,$$

$$R_0(E)=\ln E/m_e-\tfrac{1}{2}\ .$$

One notices that the E dependence from the photon spectra can be factorized out from the differential cross sections as follows:

$$H(E,Z)=(\ln E/m_e-\tfrac{1}{2})^2H_0(E,Z)\ ,$$

$$I(E,V)=(\ln E/m_e-\tfrac{1}{2})^2I_0(E,V)\ ,$$

$$G(E,x_t)=(\ln E/m_e-\tfrac{1}{2})^2G_0(E,x_t)\ ,$$

where $H_0(E,Z)$, $I_0(E,V)$, and $G_0(E,x_t)$ are obtained by integrating successively over y and u (or u and y), between the positive-definite lower and upper limits given in Tables I—VI, the expressions

$$d^2H_0(E,Z,u,y)=l_0(Z,y)f(2EZ,u)du\,dy\ ,$$

$$d^2I_0(E,Z,u,y)=m_0(V,y)f(2EV/\cosh y,u)du\,dy\ ,$$

$$d^2G_0(E,x_t,u,y)=n_0(x_t,u,y)f(2Ex_t/(1-u^2)^{1/2},u)du\,dy\ ,$$

with

$$l_0(Z,y)=4(1-Z\cosh y)^2+Z^2(2\cosh y-Z)^2\ ,$$

$$m_0(V,y)=4(1-V^2)\cosh^2 y+V^2(2\cosh y-V/\cosh y)^2\ ,$$

$$n_0(x_t,u,y)=(1-u^2)l_0(x_t/(1-u^2)^{1/2},y)\ .$$

We notice that in any case these integrations are very simply performed numerically. In addition, as we shall show, analytic integration can be applied in many cases at least partially. This is always the case for $H_0(E,Z)$ and $G_0(E,x_t)$, since the y dependence of the functions $d^2H_0(E,Z,u,y)$ and $d^2G_0(E,x_t,u,y)$ is only contained in the factor $l_0(Z,y)$ [or $l_0(x_t/(1-u^2)^{1/2},y)$] which is analytically integrable over y, leading to

$$L_0(Z,Y)=\int_0^Y l_0(Z,y)dy =(Z^2+2)^2Y-4Z(Z^2+2-Z\cosh Y)\sinh Y\ .$$

As a result one has

$$H_0(E,Z)=\int_{u_Z'}^{U_Z'}[L_0(Z,Y_Z)-L_0(Z,y_Z)]f(2EZ,u)du\ ,$$

$$G_0(E,x_t)=\int_{u_x'}^{U_x'}(1-u^2)[L_0(Z,Y_x)-L_0(Z,y_x)] \times f(2EZ,u)du\quad [Z=x_t/(1-u^2)^{1/2}]\ ,$$

where y_A,Y_A,u_A',U_A' for $A=Z,x_t$ are given, respectively, in Tables I and V.

On the other hand the u dependence of the functions $d^2H_0(E,Z,u,y)$ and $d^2I_0(E,V,u,y)$ is only contained in the expression of the $\gamma\gamma$ cross section. If, as is often the case, $f(W,u)$ can easily be integrated over u (at least for u strictly less than 1), so that one gets an analytic expression for

$$F(W,U)=\int_0^U f(w,u)du\ ,$$

$H_0(E,Z)$ and $I_0(E,V)$ are then as well expressed as a single integral, this time over y:[12]

$$H_0(E,Z)=\int_{y_Z'}^{Y_Z'}[F(2EZ,U_Z)-F(2EZ,u_Z)]l_0(Z,y)dy\ ,$$

$$I_0(E,V)=\int_{y_V'}^{Y_V'}[F(2EV/\cosh y,U_V) -F(2EV/\cosh y,u_V)]m_0(V,y)dy\ ,$$

where u_A,V_A,y_A',Y_A' for $A=Z,V$ are given, respectively, in Tables II and IV.

Now, if $f(W,u)$ actually depends only on u and not on W, i.e.,

$$\frac{d\sigma(\gamma\gamma\to A^+A^-)}{du}=\frac{2\pi}{W^2}f'(u)\ ,$$

it results that d^2H_0, d^2I_0, and d^2G_0 become independent of E and that the y dependence of d^2I_0 is only contained in the factor $m_0(V,y)$, which is analytically integrable, leading to

$$M_0(V,Y)=\int_0^Y m_0(v,y)dy$$
$$=(1-2V+V^2)(Y+\sinh 2Y)-4V^3Y+V^4\tanh Y$$

so that one simply has

$$H(E,Z)=(\ln E/m_e-\tfrac{1}{2})^2H_0'(Z)\ ,$$
$$I(E,V)=(\ln E/m_e-\tfrac{1}{2})^2I_0'(V)\ ,$$
$$G(E,x_t)=(\ln E/m_e-\tfrac{1}{2})^2G_0'(x_t)\ ,$$

where the various integrals over u and y remain independent of the energy and are given by

$$H_0'(Z)=\int_{u_Z'}^{U_Z'}[L_0(Z,Y_Z)-L_0(Z,y_Z)]f'(u)du\ ,$$
$$I_0'(V)=\int_{u_V'}^{U_V'}[M_0(V,Y_V)-M_0(V,y_V)]f'(u)du\ ,$$
$$G_0'(x_t)=\int_{u_x'}^{U_x'}(1-u^2)[L_0(Z,Y_x)-L_0(Z,y_x)]$$
$$\times f'(u)du\quad [Z=x_t/(1-u^2)^{1/2}]\ .$$

Since in general $f'(u)$ is easily integrable over u (for $0\le u\le U<1$) so that one is led to an analytical function $F'(U)$, one also has

$$H_0'(Z)=\int_{y_Z'}^{Y_Z'}l_0(Z,y)[F'(U_z)-F'(u_z)]dy\ ,$$
$$I_0'(V)=\int_{y_V'}^{Y_V'}m_0(V,y)[F'(U_V)-F'(u_V)]dy\ .$$

Let us notice that, in this case, the various distributions considered can even be expressed in a scale-invariant way with respect to the beam energy since, setting $F(\sqrt{s})=E/(\ln E/m_e-\frac{1}{2})$, one gets

$$F^2(\sqrt{s})(d\sigma/dZ^2)=H_0'(Z)Z^{-4}\ ,$$
$$F^2(\sqrt{s})(d\sigma/dV^2)=I_0'(V)V^{-4}\ ,$$
$$F^2(\sqrt{s})d\sigma/dx_t^2)=G_0'(x_t)x_t^{-4}\ .$$

That is, in particular, the case of the Born approximation, which is a good approximation for the production of fermions (up to higher-order corrections) and just a model for bosons, and where one has (since u remains strictly less than 1) $f'(u)=\alpha(1+u^2)/(1-u^2)$, leading to $F'(U)=\alpha(2\tanh^{-1}U-U)$, for a pair of relativistic fermions, and $f(u)=2\alpha$, leading to $F'(U)=2\alpha U$, for a pair of relativistic scalar bosons.

Now, when $f(W,u)$ actually depends on u and W, but can be factorized into a product of separate functions of u and W, i.e.,

$$\frac{d\sigma(\gamma\gamma\to A^+A^-)}{du}=\frac{2\pi}{W^2}B(W)f'(u)\ ,$$

the E dependence of d^2H_0 can again be factorized out from the integration over u and y and one gets

$$H_0(E,Z)=B(2EZ)H_0'(Z)\ ,$$

where $H_0'(Z)$ is defined exactly as before. It results that the integral evaluated for the determination of $d\sigma/dZ^2$ remains again independent of the energy.

This can be applied, in particular, to the production of a pair of relativistic particles through an isolated resonance of well-defined spin, parity, and helicity, so that the angular distribution is uniquely defined independently of the mass of the pair and one has

$$f(W,u)=(2J+1)\frac{\Gamma_{\gamma\gamma}\Gamma_{A^+A^-}}{(W-W_R)^2+\Gamma^2/4}f'(u)\ .$$

It can also be shown that, as long as one has $\Gamma\ll W_R\le 2E\sin\theta_0$ with $p_0/\sin\theta_0<W_R\pm N\Gamma<2E\times\tan(\theta_0/2)$ for $N\gg 1$, the total cross section due to such a narrow resonance is proportional, in first order, to a product of separate and well-known functions of energy and angular acceptance:

$$\sigma_R\propto\left[\ln\frac{E}{m_e}-\frac{1}{2}\right]^2\int_0^{u_0}\frac{F'(u)}{1-u^2}du\ .$$

These results should be, for instance, valid for $\pi^+\pi^-$ pairs created through an f^0 and nothing else, f^0 being produced with helicity ± 2; here we have $f'(u)=(1-u^2)^2$ and $F'(U)=U-2U^3/3+U^5/5$.

Actually the resonance may overlap (and interfere) with other processes. This seems to be occurring with the f^0, since the experimental peak appears to be shifted, and the form of the invariant-mass distribution appears to be changed. Here $f(W,u)$ must be derived from the amplitudes of interfering processes. If this function can be explicitly expressed, as for instance in Mennessier's model,[13] $H_0(E,Z)$ can be immediately reduced at least to a single integral over u, as shown in the general case.

Let us notice, by the way, that if any one of the above functions is studied for any range of values of the parameter considered (Z, V, or x_t) that are lower than the value of a possible experimental cutoff performed on any of these parameters [i.e., lower than $\min(Z_M,x_M,V_M)$], these cuts do not introduce any lower limit on u and y so that, for instance, one has

$$H_0'(Z)=\int_0^{U_Z'}L_0(Z,Y_Z)f'(u)du$$
$$=\int_0^{Y_Z'}l_0(Z,y)F'(U_Z)dy\ ,$$
$$I_0'(V)=\int_0^{U_V'}M_0(Z,Y_V)f'(u)du$$
$$=\int_0^{Y_V'}m_0(V,y)F'(U_V)dy\ ,$$
$$G_0'(x_t)=\int_0^{U_x'}(1-u^2)L_0(x_t/(1-u^2)^{1/2},Y_x)f'(u)du\ .$$

If, with the same assumptions, one introduces in addition the acollinearity cut as described above [i.e., $\sin(\theta_1+\theta_2)/(\sin\theta_1+\sin\theta_2)\geq\beta_0$], one gets (setting $\epsilon=\tanh^{-1}\beta_0$)

$$H'_0(Z)=\int_0^{U'_Z}[L_0(Z,Y_Z)-L_0(Z,\epsilon)]f'(u)du\ ,$$

$$I'_0(V)=\int_0^{U'_V}[M_0(V,Y_V)-M_0(V,\epsilon)]f'(u)du\ ,$$

$$G'_0(x_t)=\int_0^{U'_x}(1-u^2)[L_0(Z,Y_x)-L_0(Z,y_x)]\times f'(u)du\quad [Z=x_t/(1-u^2)^{1/2}]\ .$$

Let us remark that, as far as $H'_0(Z)$ and $I'_0(V)$ are concerned, this only amounts to subtracting, from their previous expressions, the functions $F'(U_z)L_0(Z,\epsilon)$ and $F'(U_V)M_0(V,\epsilon)$.

In general the various integrals written above can again be computed through analytic integration. However, the occurrence of distinct ranges, where the integration limits are different, leads to different expressions for those ranges, some of which are rather complicated. In the Appendix we shall study the determination of those ranges and the corresponding types of variations for $H'_0(Z)$, $I'_0(V)$, and $G'_0(x_t)$. In some cases we shall even give the full analytic expressions.

In practice, the computation, point by point, of these quantities and thus of the distributions looked for can actually be performed more easily, and almost as rapidly, on the basis of the single integrals that we have written above.

VII. MEASUREMENTS WITH EXPERIMENTAL Q^2 LIMITATION

Now we shall consider measurements where one limits either the electrons' scattering angles, $\theta_e<\theta_e^M$ (double tagging around 0° or antitagging at finite angle), or the total transverse momentum of the pair produced, $|\sum\vec{p}_t|\leq Ex_M^{\text{tot}}$. These two limitations may practically be considered as almost equivalent. One may indeed assume, in first approximation, that the latter corresponds to limiting the transverse momentum of one of the scattered electrons when the other one is close to zero.[11] This is expressed by setting $\theta_e^M\simeq x_M^{\text{tot}}/(1-X_i)$.

For such measurements the treatment of the photons as being quasireal (in the kinematics, and by using the DEPA) is justified *a fortiori,* since their Q^2 values are limited experimentally.

On the other hand, the upper limit of the integration over the angular spectrum of the photons is changed, and one has (see Sec. IV)

$$S(E,X_i)=2(1-X_i+X_i{}^2/2)\left[\ln\frac{E\eta_i}{m_e}-\frac{1}{2}\right]$$

$$P(E,X_1,X_2)=P_0(X_1,X_2)R_1(E,X_1,X_2)$$

with

$$R_1(E,X_1,X_2)=\left[\ln\frac{E\eta_1}{m_e}-\frac{1}{2}\right]\left[\ln\frac{E\eta_2}{m_e}-\frac{1}{2}\right]\ ,$$

where

$$\eta_i=\min\left[1,\frac{1-X_i}{X_i}\theta_e^M\right]$$

or

$$\eta_i=\min\left[1,x_M^{\text{tot}}/X_i\right]\ .$$

One notices that here the E dependence from the photon spectra cannot be factored out anymore; indeed one has

$$d^2H(E,Z,u,y)=l_1(E,Z,y)f(2EZ,u)du\,dy\ ,$$

$$d^2I(E,V,u,y)=m_1(E,V,y)f(2E,V/\cosh y,u)du\,dy\ ,$$

$$d^2G(E,x_t,u,y)=n_1(E,x_t,u,y)\times f(2E,x_t/(1-u^2)^{1/2},u)du\,dy$$

with

$$l_1(E,Z,y)=l_0(Z,y)R_1(E,X_1,X_2)\quad (X_i=Ze^{\pm y})\ ,$$

$$m_1(E,V,y)=m_0(V,y)R_1(E,X_1,X_2)\quad [X_i=V(1\pm\tanh y)]\ ,$$

$$n_1(E,x_t,u,y)=(1-u^2)l_1(E,Z,y)\quad [Z=x_t/(1-u^2)^{1/2}]\ .$$

One derives the following consequences.

First, $H(E,Z)$, $I(E,V)$, and $G(E,x_t)$ will never show the advantage that their E dependence can be factored out, and that one only has to evaluate integrals over u and y, which remain invariant with respect to the beam energy. This will be true even if $d\sigma(\gamma\gamma)/du=f(u)/W^2$.

Second, whenever we are able, for a given process, to integrate analytically $f(W,u)$ over u, $H(E,Z)$ and $I(E,V)$ will be expressed as single integrals over y in the same way as $H_0(E,Z)$ and $I_0(E,V)$, simply substituting $l_1(E,Z,y)$ and $m_1(E,V,y)$ for $l_0(Z,y)$ and $m_0(V,y)$, respectively. Thus, one has

$$H(E,Z)=\int_{y'_Z}^{Y'_Z}[F(2EZ,U_Z)-F(2EZ,u_Z)]l_1(E,Z,y)dy\ ,$$

$$I(E,Z)=\int_{y'_V}^{Y'_V}[F(2EZ,U_V)-F(2EZ,u_V)]m_1(E,V,y)dy\ .$$

On the other hand it is no longer easy to obtain, in the whole range of Z, V, or x_t, expressions that are given, after integration over y, as single integrals over u.

Nevertheless, in practice, when $Z>x_M^{\text{tot}}$ (which is generally satisfied experimentally), or when $Z>\theta_e^M$, one may use the values of Δ given by the experimental cuts and approximate them by $\eta_Z=(1-Z)\theta_e^M/Z$ or $\eta_Z=x_M^{\text{tot}}/Z$. This is, in general, a good approximation, since η_i is only contained in the argument of $\ln(\eta_iE/m_e)$ and since experimentally values of Z close to X_i are strongly favored. Thus, one has

$$l_1(E,Z,y)=l_0(Z,y)R_1{}^2(E,\gamma_Z)\ ,$$

$$L_1(E,Z,y)=L_0(Z,y)R_1{}^2(E,\gamma_Z)$$

with

$$R_1(E,\gamma_Z)=\left[\ln\frac{E\eta_Z}{m}-\frac{1}{2}\right]\ ,$$

so that one gets

$$H(E,Z)=\int_{u'_Z}^{U'_Z}[L_0(Z,Y_Z)-L_0(Z,y_Z)]\times R_1{}^2(E,\gamma_Z)f(2EZ,u)du$$

and

$$G(E,x_t)=\int_{u'_x}^{U'_x}(1-u^2)[L_0(Z,Y_x)-L_0(Z,y_x)]\times R_1{}^2(E,\gamma_Z)f(2EZ,u)du$$

with $Z=x_t/(1-u^2)^{1/2}$.

Here, it should be emphasized that, for the determination of $H(E,Z)$, $R_1{}^2(E,\gamma_Z)$ can be factorized out from the integral. It results that, in all experimental configurations considered, $d\sigma/dZ^2$ involves the computation of the same integral and one has

$$E^2\frac{d\sigma}{dZ^2}=\left[\ln\frac{E\eta_Z}{m}-\frac{1}{2}\right]^2 H_0(E,Z)Z^{-4} ,$$

where

$$\eta_Z=1$$

if we do not assume any experimental Q^2 limitation,

$$\eta_Z=(1-Z)\theta_e^M/Z$$

for Z larger than a given experimental cut on θ_e, and

$$\eta_Z=x_M^{\text{tot}}/Z$$

when an experimental cut on x_M^{tot} is imposed.

Let us recall again that $H_0(E,Z)$ involves only an integral function over u and y, which stays independent of the beam energy when the angular distribution of the process remains independent of the invariant mass.

Actually, since $X_i=Ze^{\pm y}$, it seems *a priori* that it should be more precise to use an expression of $l_1(E,Z,y)$ slightly different from above, given by

$$l_1(E,Z,y)=l_0(Z,y)[R_1{}^2(E,\gamma_Z)-y^2]$$

or even, in the antitagging case, by[14]

$$l_1(E,Z,y)=l_0(Z,y)\{[R_1(E,\theta_e^M/Z)-Z\cosh y]^2-(y+Z\sinh y)^2\} .$$

Here we notice that the same type of expression would have been obtained as well in the case without experimental Q^2 limitation, using $Q^2\leq W^2/4$ instead of $Q^2\leq EX_i{}^2$. Anyway these expressions remain easily integrable over y. In practice, however, using them would not improve the accuracy of the results.

VIII. REMARK ON THE Q^2 DEPENDENCE OF THE $\gamma\gamma$ CROSS SECTION

Let us remark that if the $\gamma\gamma$ cross section depends on the Q^2 of either photon, one can nevertheless still treat these photons, for the type of measurements considered, as quasireal in the sense that one may still apply the DEPA as well as—in principle—the kinematic relations for real photons. In that case, however, one must use, in the factorization formula corresponding to the DEPA, the equivalent-photon spectra in their differential form with respect to Q^2. Here again it will be possible to use a simplified Monte Carlo simulation.

In addition, if the Q^2 dependence of $d\sigma(\gamma\gamma)$ is factorizable,

$$\frac{d\sigma(\gamma\gamma)}{du}=\frac{1}{W^2}f(W,u)f_1(Q_1{}^2)f_2(Q^2) ,$$

as for instance in the usual vector-dominance model, one shall again obtain analytic expressions for pair production provided one replaces the factors $S(E,X_i)$ by

$$S(E,X_i)=\int_{Q^2_{\min}}^{Q^2_{\max}}dS(Q^2,X_i)f_i(Q^2)\frac{dQ^2}{Q^2}$$

with

$$dS(Q^2,X_i)=(1-X_i+X_i{}^2/2)-(1-X_i)Q^2_{\min}/Q^2 ,$$

$Q^2_{\min}$ and $Q^2_{\max}$ being defined as previously.

ACKNOWLEDGMENTS

I wish to express my gratitude to Professor S. C. C. Ting and Dr. J. P. Revol for having drawn my attention to this type of problem. I also thank Professor P. Kessler, and Professor J. Field, and Dr. C. Carimalo for their interest and for very useful discussions.

APPENDIX

The various ranges of different behavior of the functions considered are studied in some detail in an internal report.[15] Let us just recall here the main consequences involved in the change of the limits of integration over u and y, given either by phase space or by the detector acceptances ($u\leq u_0$, $x_t\geq x_0$).

Looking first at the invariant-mass distribution and assuming—as is experimentally realistic—that $x_0<2\sin^2(\theta_0/2)$, one notices that there are three ranges of values of Z for which the expressions of the upper limits and thus of $H(E,Z)$ are different. Considering, for instance, the general case where the E dependence can be factorized out, one concludes the following.

For $x_0\leq Z\leq x_0/\sin\theta_0$, the behavior of $H'_0(Z)$ is essentially determined by the limit $u\leq(1-x_0{}^2/Z^2)^{1/2}$ imposed by the transverse-momentum acceptance ($x_t\geq x_0$), so that $H'_0(Z)$ increases with Z.

For $x_0/\sin\theta_0\leq Z\leq\tan(\theta_0/2)$, the upper limits over u and y are defined, independently of Z, by the angular acceptance. Thus, the behavior of $H'_0(Z)$ is only determined by the smooth variation of the integrands [due to $l_0(Z,y)$], so that $H'_0(Z)$ only slowly decreases with Z increasing and tends to become flat as Z (more precisely $Z/\sin\theta_0$) tends to small values.

For $\tan(\theta_0/2)\leq Z\leq 1$, the behavior of $H'_0(Z)$ is essentially determined by the limit $|y|\leq\ln(1/Z)$, imposed by phase space, so that $H'_0(Z)$ strongly decreases with Z increasing.

These considerations can be visualized in our X_1X_2 diagram, where a logarithmic scale is used for the orthogonal coordinates X_1 and X_2. Such a plot appears generally helpful for showing the main features of a $\gamma\gamma$ process. On the one hand, since $d(\ln X_i)=dX_i/X_i$, any observed area gives directly the integral, over the corresponding X_1X_2 ranges, of the predominant term in the $\gamma\gamma$ luminosity $[(dX_1/X_1)(dX_2/X_2)]$. On the other hand, the physical parameters of the $\gamma\gamma$ system itself, i.e., Z^2 $(=X_1X_2)$ and y $[=\frac{1}{2}\ln(X_2/X_1)]$, also appear as orthogonal coordinates (first and second bisectrix), the first one being defined again on a logarithmic scale.

Moreover, from the relation $x_t{}^2=(1-u^2)Z^2$, one gets $\ln x_t{}^2-\ln Z^2=\ln(1-u^2)$. It results that if we plot the values of $x_t{}^2$ on the same logarithmic axis as those of Z^2, the observed distance between Z^2 and $x_t{}^2$ on this axis corresponds to a given value of u. Thus, definite values of u and y, as well as a given (u,y) area, are obtained for different values of Z by simply performing a translation along the Z^2 axis (first bisectrix). As long as, in such a translation, the integration area over u and y, as defined by the angular acceptance $(|y|\le\tanh^{-1}u_0-\tanh^{-1}u)$, remains inside the limits given by the phase space $(|y|\le\ln 1/Z)$ or by the transverse-momentum acceptance $(x_t\ge x_0)$, $\int\int f'(u)du\,dy$ stays constant. Thus, $H_0'(Z)$ will vary only slightly as a result of the smooth variation of the factor $l_0(Z,y)=S_0(X_1)S_0(X_2)$. Otherwise, when Z decreases below $x_0/\sin\theta_0$ down to x_0, or increases from $\tan(\theta_0/2)$ up to 1, the allowed area of integration becomes more and more limited, so that $H_0'(Z)$ strongly decreases. All these effects show up in a particularly obvious way in Fig. 1.

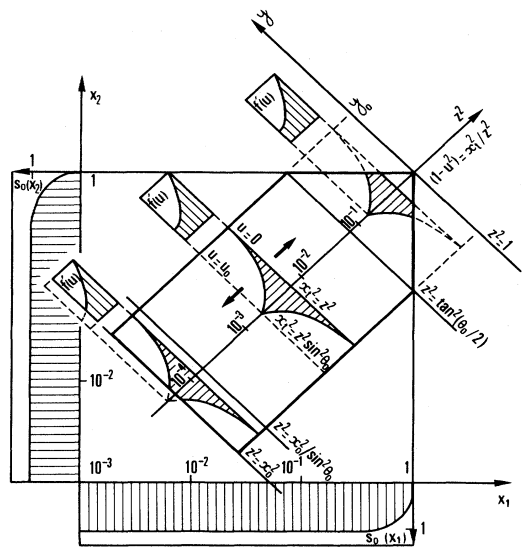

FIG. 1. Visualization of the variation of $H_0'(Z)$ for an angular-acceptance range $30°<\theta<150°$ [see explanation in the text; the shape of $f'(u)$ is arbitrary].

Now we notice that $H_0'(Z)$ can be generally expressed in a completely analytical way. Then, if one sets

$$h_1(Z,U)=\int_0^U L_0(Z,Y_U)f'(u)du \quad (Y_U=y_0-y_u\equiv\tanh^{-1}u_0-\tanh^{-1}u)\ ,$$

$$h_2(Z,Y)=\int_0^Y l_0(Z,y)F'(U_y)dy \quad [U_y=\tanh(y_0-y)]\ ,$$

one has

$$H_0'(Z)=h_1(Z,U_1) \text{ with } U_1{}^2=1-x_0{}^2/Z^2 \text{ for } x_0\le Z\le x_0/\sin\theta_0\ ,$$

$$H_0'(Z)=h_1(Z,u_0)=h_2(Z,y_0) \text{ for } x_0/\sin\theta_0\le Z\le\tan(\theta_0/2)\ ,$$

$$H_0'(Z)=h_2(Z,Y_1) \text{ with } Y_1=\ln(1/Z) \text{ for } \tan(\theta_0/2)\le Z\le 1$$

with

$$h_1(Z,U)=(Z^2+2)^2I_1(U)-(4Z/\sin\theta_0)(Z^2+2)I_2(U)+(2Z/\sin\theta_0)^2I_3(U)\ ,$$

$$h_2(Z,Y)=(Z^4+4)J_1(Y)-4Z(Z^2+2)J_2(Y)+4Z^2J_3(Y)\ ,$$

where, in general, the integrals $I_i(U)$ and $J_i(Y)$ can be evaluated analytically. Then, for instance, for the production of a relativistic pair of fermions or quarks one gets (dropping the coefficient α)

$$I_1(U)=y_Uy_0+(y_0-y_U)U+\tfrac{1}{2}\ln(1-U^2)\ ,$$

$$I_2(U)=3-(3-U^2)/(1-U^2)^{1/2}+u_0[2U/(1-U^2)^{1/2}-\sin^{-1}U]\ ,$$

$$I_3(U)=u_0[U(3-U^2)/(1-U^2)^{1/2}-2y_U]-(1-u_0{}^2)[U^2/(1-U^2)+\tfrac{1}{2}\ln(1-U^2)]\ ,$$

$$J_1(Y)=Y(2y_0-Y)+\ln[\sin\theta_0\cosh(y_0-Y)]\ ,$$

$$J_2(Y)=3(\cosh Y-1)+2(y_0-Y)\sinh Y+\tan^{-1}[\sinh(y_0-Y)-(\pi/2-\theta_0)]/\tan\theta_0\ ,$$

$$J_3(Y)=2\sinh^2Y+(y_0-Y)\sinh(2Y)-Y\sinh(2y_0)+(2y_0-Y)Y-2\ln[\cosh(y_0-Y)/\cosh y_0]/\tan\theta_0\ .$$

Let us notice that for $x_0/\sin\theta_0 < Z < \tan(\theta_0/2)$, where $H_0'(Z)=h_1(Z,u_0)=h_2(Z,y_0)$, one simply gets

$$I_1(u_0)=J_1(y_0)=y_0^2+\ln(\sin\theta_0)\ ,$$

$$I_2(u_0)=J_2(y_0)\sin\theta_0$$
$$=3(1-\sin\theta_0)-(\pi/2-\theta_0)\cos\theta_0\ ,$$

$$I_3(u_0)=2\cos\theta_0(y_0+\cos\theta_0)-(1+\cos^2\theta_0)\ln(\sin\theta_0)\ .$$

The same type of expression can also be derived for the pair production of relativistic scalar bosons in the Born approximation where, in particular, one gets, for $x_0/\sin\theta_0 < Z < \tan(\theta_0/2)$ (dropping again the coefficient α),

$$I_1(u_0)=4\ln(1/\sin\theta_0)\ ,$$

$$I_2(u_0)=4(\pi/2-\theta_0)\cos\theta_0-(1-\sin\theta_0)\ ,$$

$$I_3(u_0)=6(\pi/2-\theta_0)\cos\theta_0$$
$$+2\sin\theta_0(2+\cos^2\theta_0)-4(1+\cos^2\theta_0)\ .$$

Let us now consider the ranges of different behavior of the visible energy, assuming $x_0 < \tan(\theta_0/2)$, which is experimentally realistic. Thus, one gets four different ranges of values of V for which the expression of the integration limits over u and y, and consequently the expression and behavior of $I(E,V)$, are different.

For $x_0 \le V \le x_0(1+\sin\theta_0)/(2\sin\theta_0)$, the two upper limits over u and y are defined by the transverse-momentum acceptance as increasing functions of V/x_0. Thus, $I_0'(V)$ strongly increases with V.

For $x_0(1+\sin\theta_0)/(2\sin\theta_0) \le V \le x_0/\sin\theta_0$, the upper limit $Y_V'=\cosh^{-1}(V/x_0)$ remains an increasing function of V, but the upper limit on u becomes independent of V in a y range increasing with V. Thus, $I_0'(V)$ remains an increasing function of V, but increases less and less as V becomes larger.

For $x_0/\sin\theta_0 \le V \le 1/(1+\cos\theta_0)$, the upper limits are now defined by the angular acceptance independently of V, so that $I_0'(V)$ decreases, but only slowly, due to the smooth variation of $m_0(V,y)$ with V increasing.

For $1/(1+\cos\theta_0) \le V \le 1$, one has $Y_V'=\frac{1}{2}\ln[1/(2V-1)]$ and $U_V'=\tanh(y_0-y)$. The behavior of $I_0(V)$ is then essentially determined by the limitation on y due to the space phase, so that it strongly decreases with Z increasing.

In general $I(E,V)$ should have the same type of behavior as that described for $I_0'(V)$. This is obviously not true if there is a resonance within the range considered.

Finally, considering the transverse-momentum distribution, one notices that there are three ranges of values of x_t for which the expression of the upper limits on u and y, and consequently the behavior of $G(E,x_t^2)$, are different:

For $x_0 \le x_t \le \tan(\theta_0/2)$, the upper limits are only determined by the angular acceptance, independently of x_t. It results that $G_0'(x_t)$ decreases with x_t increasing, but only slowly, due to the smooth variation of the integrand inside the corresponding integration range. Let us notice that $G_0'(x_t)$ tends to become flat as x_t (or more precisely $x_t/\sin\theta_0$) tends to zero, leading for instance to $d\sigma/dx_t^2 \to x_t^{-4}$ in the Born approximation. ($d\sigma/dx_t^2 \to (8\alpha^4/3\pi)[4\ln(1/\sin^2\theta_0)-\cos^2\theta_0]x_t^{-4}$ for the production of a pair of relativistic fermions.)

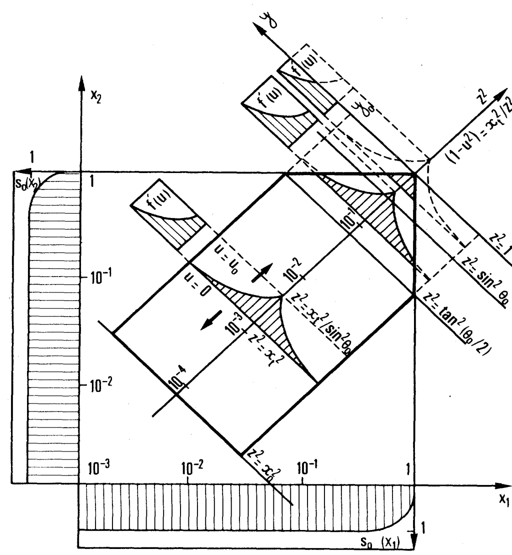

FIG. 2. Visualization of the variation of $G_0'(x_t)$ for an angular-acceptance range $30° < \theta < 150°$ [see explanation in the text; the shape of $f'(u)$ is arbitrary].

For $\tan\theta_0/2 \le x_t \le \sin\theta_0$, the upper limit on u remains defined independently by x_t by the angular acceptance, but the upper limit on y becomes limited by the phase space, all the more as x_t increases. It results that $G_0(x_t)$ decreases with x_t increasing.

For $\sin\theta_0 \le x_t \le 1$, the upper limit on u becomes also limited by the phase space, so that $G_0'(x_t)$ definitely tends to strongly decrease with x_t increasing.

All these considerations can again be visualized in the same way as has been shown for $H_0'(Z)$, using the same X_1X_2 diagram and just exchanging the roles of Z and x_t. One then has $\ln x_t^2-\ln Z^2=-\ln(1-u^2)$ for a given value of x_t, instead of $\ln Z^2-\ln x_t^2=\ln(1-u^2)$ for given values of Z [let us note that $\ln(1-u^2)$ is negative since $1-u^2=\sin^2\theta^* < 1$] (see Fig. 2).

The influence of additional experimental cuts is studied in Ref. 15. Let us only notice that $H(E,Z)$ $I(E,V)$, and $G(E,x_t)$ remain unmodified by cuts like $Z < Z_M$, $V < V_M$, and $x_t < x_M$, as long as one, respectively, has $Z < (Z_M, V_M/\sin\theta_0, x_M)$, $V < (Z_M, V_M, x_M)$, and $x_t < (Z_M\sin\theta_0, V_M, x_M)$.

Let us also remark that as far as $H(E,Z)$ and $G(E,x_t)$ are concerned, the effects of the various experimental cuts can again be easily visualized by plotting them on the X_1X_2 diagram and looking at the limitation they involve.

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