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Hamiltonian variational study of SU(2) lattice gauge theory

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The Hamiltonian variational method is applied to the SU(2) lattice gauge theory in $d+1$ dimensions using a plaquette-independent ansatz. The calculations in the equivalent model have been performed using a mean-field approach in plaquette variables. We obtain only one confining phase. Possible generalizations are also discussed.

I. INTRODUCTION

Gauge theories on the lattice are a powerful tool for dealing with nonperturbative aspects in QCD. Recently, Monte Carlo simulations have led to numerical estimations of the hadronic spectrum,[1] glueball masses,[2] string tension,[3] deconfinement temperature,[4] and other physical magnitudes. It is, however, convenient to develop analytical methods which allow us to obtain a qualitative picture of the main features of the theory, i.e., confinement and asymptotic freedom. Among the analytic and semianalytic techniques that have been particularly used are series expansions,[5] renormalization group,[6] mean field with radiative corrections,[7] finite-lattice approach,[8] and variational approximations in the Lagrangian[9] or Hamiltonian[10] formulation.

In this paper we have applied the variational technique to the Hamiltonian SU(2) gauge theory on the lattice in $d+1$ dimensions by means of a plaquette-independent *Ansatz* and its generalizations. The calculations in the equivalent model [Lagrangian SU(2) gauge theory in d dimensions] have been performed using a mean-field approach in plaquette variables which give very good results at low dimensionality ($d=3$). We obtain a confining phase for every value of the coupling constant. We do not take this result as strong evidence for confinement because the used trial state strongly favors this property. In fact, the main objective of our work is to exhibit the possibility of analytical calculations for realistic theories and dimensions using nontrivial trial states without relying on Monte Carlo simulations in the equivalent model. We consider this approach as the first step towards a generalization of the state with several plaquettes in interaction. This might allow us to obtain information of the continuum through a "scaling window" for the crossover region.

The organization of the paper is as follows.

In Sec. II we describe the variational method and the trial states used in the work. Section III deals with the application of the method to the SU(2) model in 2 + 1 and 3 + 1 dimensions explaining the "mean-plaquette" technique. In Sec. IV we discuss the results and analyze possible generalizations.

II. INDEPENDENT-PLAQUETTE METHOD

The Hamiltonian variational method is based on the inequality

$$\epsilon_0 \le \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} , \tag{1}$$

where ϵ_0 is the ground-state energy and $|\psi\rangle$ is a trial function depending on parameters determined by minimization. The Hamiltonian for a gauge theory in $d+1$ dimensions, invariant under transformations belonging to a compact group G, is[11]

$$H=\frac{g^2}{2}\left[\sum_{\text{links}} {E_l}^2-\frac{\lambda}{2}\sum_{\text{plaq}} \text{tr}_F(U_p+\text{H.c.})\right] , \tag{2}$$

where $\lambda=4/g^4$ and $\text{tr}_F(U_p)$ is the trace of the product of the link variables U_l in the fundamental representation of G along a plaquette of the d-dimensional lattice. The electric field operator is defined on each link by means of

$$[E^\alpha, U_{ij}]=T^\alpha_{ik}U_{kj} , \tag{3}$$

where α is the color index and T^α the corresponding generator of infinitesimal transformations [$\alpha=1,2,3$; $T^\alpha=\sigma^\alpha/2$ for $G=\text{SU}(2)$].

Owing to the absence of nodes in the gauge-invariant ground state[12] we propose

$$|\psi\rangle=\exp[S(\{U_l\},\{\beta_j\})/2]\,|0\rangle_E , \tag{4}$$

where S depends on closed-loop combinations of link variables U_l and on the set of variational parameters $\{\beta_j\}$. The state $|0\rangle_E$ satisfies ${E_l}^2|0\rangle_E=0$.

The norm of $|\psi\rangle$,

$$\langle\psi|\psi\rangle=\int \prod_l dU_l e^S , \tag{5}$$

is equivalent to the partition function of a Lagrangian model in d dimensions with action S which we shall call the "equivalent model." The calculation of $\langle H\rangle_\psi$ is therefore based on the evaluation of statistical averages in this equivalent model.

The choice of the function S is the fundamental step of the method. It must be flexible enough to reproduce the features of the ground state in the whole range $0\le\lambda<\infty$. The analytic complication of S is limited by the corresponding difficulty in the evaluation of the statistical averages, which is particularly crucial in 3 + 1 dimensions.

For the SU(2) gauge theory the most frequently used variational state[13–19] is the so-called independent-plaquette one:

$$|\psi\rangle=\exp\left[\frac{\beta}{2}\sum_{\text{plaq}}\text{tr}_{1/2}(U_p)\right]|0\rangle_E\ , \tag{6}$$

which, since the plaquettes involved in Eq. (6) are only spatial, corresponds to magnetic fluctuations for each point in the continuum limit:

$$|\psi\rangle \underset{\text{continuum}}{\longrightarrow} \exp\left[-\frac{\beta}{2}\int d^dx\,B^2(x)\right]|0\rangle_E\ . \tag{7}$$

The trial state of Eq. (6) corresponds to the first-order perturbative expansion[20] of the ground state in the strong-coupling limit which ensures its correct behavior in this region.

For $2+1$ dimensions qualitative arguments[21] indicate that Eq. (7) is a good approximation for any coupling. It is easy to show[22] that this state confines for all values of g^2 and, moreover, the equivalent model for this case is a gauge theory in two dimensions which may be exactly solved.

For spin or chiral models (variables $U_x\in G$ on the lattice sites) the state analogous to Eq. (6) is an independent-link state,

$$|\psi\rangle=\exp\left\{\frac{\beta}{4}\sum_{\text{link}}[(UU')_l+\text{H.c.}]\right\}|0\rangle_E\ , \tag{8}$$

which has been used for Abelian[19,23] and non-Abelian[24] groups where it is known as a Jastrow[25] state because of its original application to many-body problems.

III. SU(2) GAUGE THEORY

We must calculate the expectation value of the Hamiltonian Eq. (2) using the state of Eq. (6). The mean value of the operator E_l^2 is proportional to that of the magnetic part of Eq. (2) by means of the identity[14]

$$\int\prod_l dU_l f(\{U_l\})E_l^2 f(\{U_l\}) =\tfrac{1}{2}\int\prod_l dU_l f^2(\{U_l\})E_l^2\ln f(\{U_l\})\ . \tag{9}$$

Choosing

$$f(\{U_l\})=\exp\left[\frac{\beta}{2}\sum_{\text{plaq}}\text{tr}_{1/2}(U_p)\right] \tag{10}$$

and since $E^2U_{1/2}=\frac{3}{4}U_{1/2}$, one obtains

$$\left\langle\sum_{\text{link}}E_l^2\right\rangle_\psi=\frac{3\beta}{4}\left\langle\sum_{\text{plaq}}\text{tr}_{1/2}(U_p)\right\rangle\ . \tag{11}$$

The expression which must be minimized is therefore

$$\frac{\langle H\rangle_\psi}{N_p}=\frac{g^2}{2}\left[\frac{3\beta}{4}-\lambda\right]\langle\text{tr}_{1/2}(U_p)\rangle(\beta)\ , \tag{12}$$

where N_p is the number of plaquettes. This result is valid for any dimensionality.

The most general independent-plaquette state may be written summing in the action of the equivalent model over all the irreducible representations of the group, i.e.,

$$|\tilde\psi\rangle=\exp\left[\sum_{\text{plaq}}\sum_{j=\frac{1}{2},1,\ldots}\frac{\beta_j}{2}\text{tr}_j(U_p)\right]|0\rangle_E\ , \tag{13}$$

where β_j are the variational parameters.

The generalization of Eq. (12) is now

$$\frac{\langle H\rangle_{\tilde\psi}}{N_p}=\frac{g^2}{2}\left[\sum_j j(j+1)\beta_j\langle\text{tr}_j(U_p)\rangle(\beta) -\lambda\langle\text{tr}_{1/2}(U_p)\rangle(\beta)\right]\ , \tag{14}$$

where the statistical averages must be evaluated with the action

$$S=\sum_{\text{plaq}}\sum_j\beta_j\text{tr}_j(U_p)\ . \tag{15}$$

Recalling that

$$\text{tr}_j(U)=U_{2j}[\text{tr}_{1/2}(U)]\ , \tag{16}$$

where $U_{2j}(x)$ is the Chebychev polynomial of second kind and order $2j$, the only magnitudes which must be calculated are $\langle\text{tr}_{1/2}{}^n(U_p)\rangle$, $n=1,2,\ldots$, with the Boltzmann factor given by Eq. (15).

We consider the following particular cases.

A. 2 + 1 dimensions

The averages in the equivalent model may be exactly calculated[14–17] and, keeping in Eq. (15) only the fundamental representation, one obtains

$$\langle\text{tr}_{1/2}(U_p)\rangle=2\frac{I_2(2\beta)}{I_1(2\beta)}\ , \tag{17}$$

where $I_\nu(x)$ is the modified Bessel function of order ν.

The variational energy as a function of λ is shown in Fig. 1, and one observes no discontinuities either in the function or in the derivative indicating absence of phase transitions as expected. The string-tension calculations[14,16] are trivial and show a confining phase for any λ.

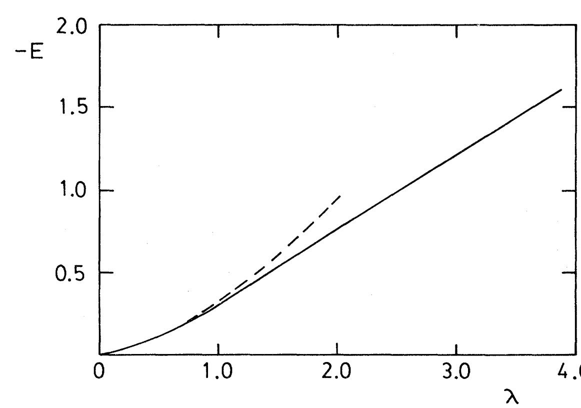

FIG. 1. Variational energy in $2+1$ dimensions using state Eq. (6). Dashed line represents the first order in a perturbative expansion ($E=-\lambda^{3/2}/3$).

TABLE I. (a) Variational energy in 2 + 1 dimensions using state Eq. (13) with different numbers of variational parameters ($\lambda=4$ and 8 as examples). (b) Values of the parameters β_j corresponding to the last column in (a).

Maximum representation considered in Eq. (13)				
		(a)		
λ	$\frac{1}{2}$	1	$\frac{3}{2}$	2
4	−3.3194	−3.3329	−3.3334	−3.3335
8	−8.8248	−8.8408	−8.8412	−8.8412
		(b)		
λ	$\beta_{1/2}$	β_1	$\beta_{3/2}$	β_2
4	2.421	−0.232	0.040	−0.005
8	3.238	−0.285	0.029	0.008

Keeping in Eq. (14) up to four variational parameters, the results for $\lambda=4$ and 8, as examples, may be seen in Table I. The improvement of the variational energy due to the inclusion of the adjoint term is $\sim 0.4\%$ whereas the influence of higher representations is negligible ($\sim 0.01\%$) because the corresponding parameters remain very small.

B. 3 + 1 dimensions

The equivalent model is a gauge theory in three dimensions and the evaluation of the statistical average must rely on approximate methods. The Monte Carlo simulations[26] would give the most precise results but at the expense of losing the analytical control and simplicity of calculations. In this work we shall use the mean-plaquette method[27,28] based on a change of variables link→plaquette. With this procedure a Bianchi identity appears for each lattice cube:

$$\delta(B_c-1)=\delta(U_{12}(\vec r)U_{23}(\vec r)U_{13}(\vec r+\hat y)U_{12}^\dagger(\vec r+\hat z) \times U_{23}^\dagger(\vec r+\hat x)U_{13}^\dagger(\vec r)-1)\ , \qquad (18)$$

where each plaquette variable has been denoted by two directions and one site index. Performing this change of variables one obtains

$$\langle \mathrm{tr}_{1/2}U_p\rangle=\frac{1}{Z}\int\prod_p dU_p\,\mathrm{tr}_{1/2}(U_p)\exp\left[\beta\sum_p \mathrm{tr}_{1/2}(U_p)\right]\prod_{\text{cubes}}\delta(B_c-1)\ . \qquad (19)$$

We remark that in two dimensions no constraint appears in the corresponding change of variables.

The numerical treatment of Eq. (19) consists of fixing all the plaquette variables, except for a finite number of them, to their mean value $\langle U\rangle$ which is evaluated by self-consistency. The use of plaquette variables automatically ensures the gauge invariance.

Considering only one plaquette exactly in Eq. (19) the self-consistency equation is

$$\langle U\rangle=\frac{1}{Z}\sum_{j_1,j_2=0,\frac{1}{2},1,\ldots}(2j_1+1)^{-4}(2j_2+1)^{-4}U_{2j_1}{}^5\left[\frac{\langle U\rangle}{2}\right]U_{2j_2}{}^5\left[\frac{\langle U\rangle}{2}\right] \times\int dU\,e^{\beta\,\mathrm{tr}U}U_{2j_1}\left[\frac{\mathrm{tr}U}{2}\right]U_{2j_2}\left[\frac{\mathrm{tr}U}{2}\right]\mathrm{tr}_{1/2}U\ , \qquad (20)$$

where j_1 and j_2 are variables associated with the cubes to which the exactly treated plaquette belongs, and Z differs from the numerator only in the absence of the factor $\mathrm{tr}_{1/2}U$. The proof of Eq. (20) is completely analogous to that given in Ref. 27 for the four-dimensional case. Even though Eq. (20) presents several roots, we must keep only the one which satisfies the limits

$$\langle U\rangle_{\beta=0}=0,\quad \langle U\rangle \underset{\beta\to\infty}{\longrightarrow} 1\ . \qquad (21)$$

TABLE II. $\langle \mathrm{tr}_{1/2}U\rangle$ from Eq. (20) for different values of j_{max} and $\beta=8$ and 15 as examples. The last column is a fit of the Monte Carlo (MC) data (Ref. 26) given by $\langle \mathrm{tr}_{1/2}U\rangle=2-1/\beta-0.35/2\beta^2$.

	j_{max}				
β	$\frac{1}{2}$	1	$\frac{3}{2}$	2	MC data
8	0.847	0.872	0.872	0.871	0.870
15	0.911	0.920	0.927	0.931	0.932

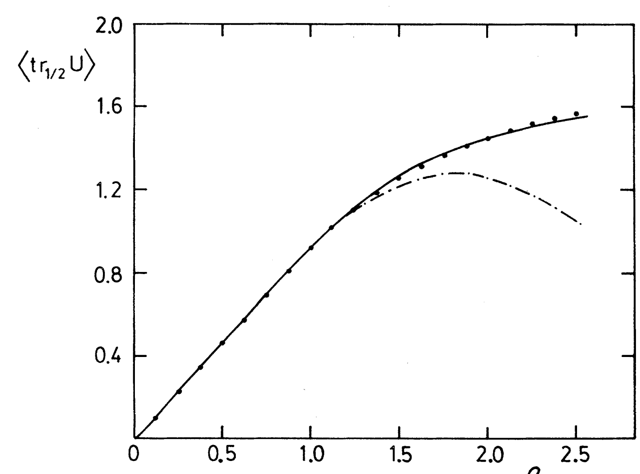

FIG. 2. $\langle \mathrm{tr}_{1/2}U\rangle$ given by the mean-plaquette method in a Lagrangian SU(2) gauge theory (continuous line) in three dimensions [with $j_{max}=\frac{1}{2}$ in Eq. (20)] in comparison with Monte Carlo results (●) taken from Ref. 26 and series expansions (dot-dashed line) from Ref. 29.

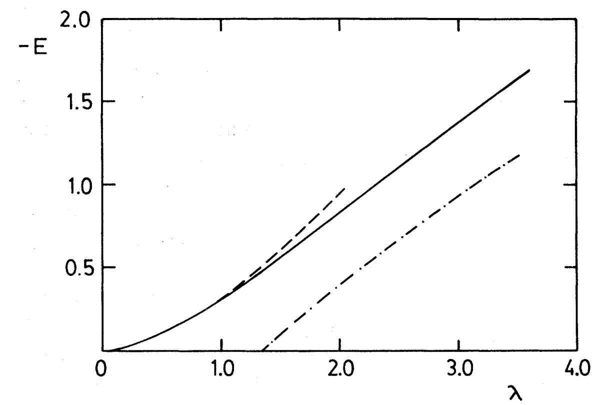

FIG. 3. Variational energy in $3+1$ dimensions using state Eq. (6) and $j_{\max}=\frac{1}{2}$ in Eq. (20). Dashed line represents the first order in a perturbative expansion ($E=-\lambda^{3/2}/3$) while dot-dashed line is the variational energy using the link non—gauge-invariant state Eq. (22).

The numerical solution of Eq. (20) with maximum j_1 and j_2 equal to $\frac{1}{2}$ is compared in Fig. 2 with Monte Carlo data[26] and series expansions.[29] The agreement is excellent with a maximal discrepancy of 0.5% for $\beta=4$. Table II indicates that the inclusion of higher representations improves the results (differences of 0.1% for $\beta=15$ with j_1, j_2 up to 2). The Bethe-Peierls[18] approximation allows a better precision but, because of its slow convergence, it has not been used in the present work.

We may examine the possible analytical alternatives to the mean-plaquette method. Among them the Lagrangian variational method[9] has been applied to SU(2) in three dimensions with results[30] comparable to ours. The renormalization group in the manner of Migdal and Kadanoff was used[31] in four dimensions with differences up to 10%

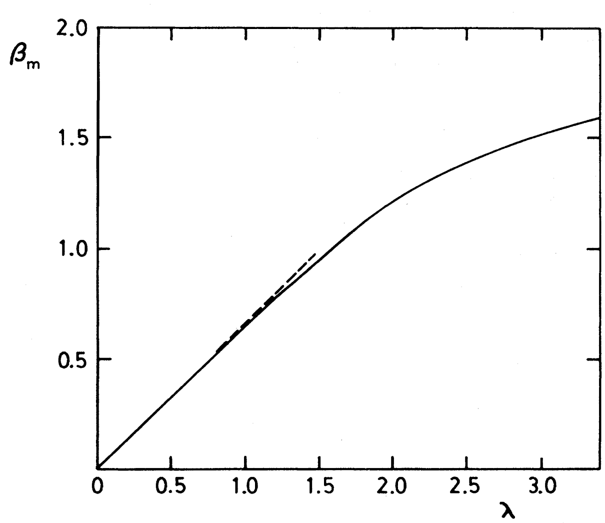

FIG. 4. Evolution of the variational parameter (β_m) which minimizes the energy in $3+1$ dimensions using state Eq. (6). Dashed line represents the first order in a perturbative expansion ($\beta_m=2\lambda/3$).

with respect to the Monte Carlo method. The mean-field (MF) approach with radiative corrections[32] [the non—gauge-invariant MF (Ref. 33) gives a spurious phase transition] being equivalent to a $1/d$ expansion gives results not too accurate for low dimension (3). All these techniques are more cumbersome than the mean-plaquette method. Moreover, the latter can be easily generalized for states which involve more than one plaquette.

We go now to the results obtained with the state of Eq. (6). Figure 3 shows that the variational energy as a function of λ is qualitatively similar to the $(2+1)$-dimensional case.

Looking at the value β_m of the variational parameter (Fig. 4) which minimizes $\langle H\rangle_\psi$ one sees that, for small λ, $\beta_m \simeq 2\lambda/3$ as for $d=2$. The variational energy is also equal to that of the model for $d=2$, i.e., $E\simeq-\lambda^{3/2}/3$ with $\lambda \ll 1$. The fact that the first term of the perturbative expansion of the ground-state energy is correctly reproduced by Eq. (6) is valid for any group and dimension.

For completeness, Fig. 3 also shows the energy corresponding to a mean-field,[15] or independent-link, state

$$|\phi\rangle=\exp\left[\frac{\beta}{2}\sum_{\text{link}}\operatorname{tr}_{1/2}(U_l)\right]|0\rangle_E\ , \qquad (22)$$

which turns out to be

$$\frac{\langle\phi|H|\phi\rangle}{N_p\langle\phi|\phi\rangle}=\frac{g^2}{2}\left\{\frac{3\beta}{8}\frac{I_2(2\beta)}{I_1(2\beta)}-2\lambda\left[\frac{I_2(2\beta)}{I_1(2\beta)}\right]^4\right\}. \qquad (23)$$

This state exhibits a phase transition for $\lambda_c\simeq1.30$ which is incorrect. Figure 3 shows that the independent-plaquette state is better than the independent-link one over all the range of λ. One notes that the state of Eq. (22) is not gauge invariant but its symmetrization leads to mathematical expressions of difficult treatment.[34] If the trend of the Abelian theories (where the gauge symmetrization[35] gives way to the equivalent XY spin model) is maintained, the equivalent model for the SU(2) gauge theory would be the chiral SU(2)×SU(2) which perhaps may be studied by means of Monte Carlo simulation.

The comparison of the energy for the states of Eqs. (6) and (22) is not useless since for the U(1) case they cross each other.[15] This fact, though not interpretable as a phase transition, indicates that the true state for $\lambda \gg 1$ is more similar to a MF than to an independent-plaquette one.

To end this section we mention that the inclusion of the variational parameter corresponding to the adjoint representation in Eq. (13) involves a self-consistent equation similar to Eq. (20) for $\langle\operatorname{tr}_{1/2}{}^2(U_p)\rangle$ with an improvement in the variational energy relatively similar to the $(2+1)$-dimensional case (Table I).

IV. CONCLUSIONS

The goal of this work has been the description of the Hamiltonian variational method in $3+1$ dimensions for the SU(2) gauge theory using independent-plaquette states. It is remarkable that the calculations in the equivalent model may be performed analytically with enough precision by means of the mean-plaquette technique. We must, however, note that the state used in Eq. (6) or Eq. (13) tends to enhance the "disorder" or confinement. For example, it has been shown[18] that the critical parameter for the $Z(2)$ gauge theory in $2+1$ dimensions is higher than what is obtained with other methods. Moreover in the $Z(N)$ gauge theory[19] only the Coulomb-Higgs transition, where the critical coupling increases with N, is reproduced. Therefore, the U(1) gauge theory in $3+1$ dimensions would present no phase transitions, which is wrong. However, the inclusion of several plaquettes[18] in the states for $2+1$ dimensions improves the result. States with interactions among several plaquettes are unavoidable for the study of spatial Wilson loops. With the inclusion of the next perturbative order of Ref. 20 the strong-coupling phase would be satisfactorily treated. Calculations of this type for $3+1$ dimensions seem possible, the only difficulty being represented by the evaluation of multiple integrals when the mean-plaquette method is used. It would be interesting to see whether the renormalization group or the Lagrangian variational method can be adapted to these calculations.[36]

Other *Ansätze* used in the literature[37] containing interactions of pairs of long-distance plaquettes, though convenient for QED in $2+1$ dimensions, seem inappropriate for the SU(2) theory as was noticed in Ref. 34.

The only possibility of obtaining continuum results seems to take large but finite clusters of interacting variables and look for a scaling window to the continuum in the beginning of the weak-coupling region. This means that one should reproduce correctly the crossover region. If this program may be fulfilled we would obtain for the first time, analytical results for the continuum without using Monte Carlo simulations or series expansions extrapolated by Padé approximants.

After completion of this work we received a preprint[38] where results similar to ours are obtained by means of the state of Eq. (6) calculating the equivalent-model averages with Monte Carlo simulations. In particular their Fig. 1 practically coincides with our Fig. 3. Moreover, in this paper a detailed study of the symmetrized independent-link state is performed, with the chiral theory as equivalent model, giving variational energies similar to those of the independent-plaquette state.

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