Experimental tests of dynamical state-vector reduction

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We consider theories in which the Schrödinger equation is modified so that the reduction of the state vector becomes a dynamical process characterized by a mean reduction time τ_R. It is generally believed that such a theory and quantum theory differently predict the outcome of any two experiments performed in rapid enough succession (separated by a time interval $\lesssim \tau_R$), such as Papaliolios's successive measurements of a photon's polarization. It is shown that the predictions of a plausible class of dynamical reduction theories (for which the ensemble average of $|\langle \phi_n | \psi \rangle|^2$ remains constant during the reduction process, where $|\psi\rangle$ is the state vector, and $|\phi_n\rangle$ is any one of the basis states to which it may reduce) and quantum theory do not differ for any Papaliolios-type experiment. However, there are deviations from the predictions of quantum theory if the second experiment measures interference between the superposed states created by the first measurement. As an example, the result of a recent two-slit neutron interference experiment by Zeilinger *et al.* is applied to the theory of Pearle, placing an upper limit $\tau_R \gtrsim 5$ sec on the neutron self-reduction time.

I. INTRODUCTION

According to quantum theory, after a measurement which is completed at time $t=0$, the state vector (which describes the measured system and the apparatus which measures it) can be written as

$$|\psi,t\rangle = \sum_n a_n(t)\,|\phi_n(t)\rangle\ ,\quad t\geq 0\ . \tag{1}$$

Each state $|\phi_n(t)\rangle$ describes one of the possible outcomes of the measurement. These states depend upon t, since the system and the apparatus continue to evolve after the measurement. However, each squared amplitude $|a_n(t)|^2$, which represents the probability of the nth outcome, does not change with time for $t\geq 0$.

In a dynamical reduction theory, the Schrödinger equation is modified so that the amplitudes $a_n(t)$ continue to evolve after the measurement. Indeed, all of the amplitudes eventually vanish except a single amplitude whose magnitude reaches the value one. In this way the theory purports to describe a single system in nature (not an ensemble of systems) and to solve the so-called "problem of the theory of measurement": why we observe a system in the state $|\phi_n(t)\rangle$, and not in a superposition of such states.

Because the outcomes of a repeated experiment vary uncontrollably among the permitted values, a dynamical reduction theory must have a mechanism whereby the reduced state vectors that are calculated likewise vary among the permitted $|\phi_n(t)\rangle$. In the theory of Bohm and Bub[1–4] the reduced state vector is determined by certain hidden variables (first introduced by Wiener and Siegal[5]) which remain fixed during the reduction process, as well as by the initial state vector (1). Each physical system is supposed to possess definite values of these hidden variables, and the distribution of these hidden variables among otherwise identical systems gives rise to the different experimental outcomes. In the theory of Pearle,[6–9] the reduced state vector is determined by certain randomly fluctuating matrix elements, as well as by the initial state vector (1). The different random fluctuations that a physical system may undergo give rise to the different experimental outcomes.

In order that the predictions of a dynamical reduction theory agree with the predictions of quantum theory for the measurement described by (1), the state vector must reduce to $|\phi_n(t)\rangle$ for a fraction $|a_n(0)|^2$ of the identical physical systems upon which the experiment is performed. In the following, we shall refer to this as "the fundamental property of dynamical reduction theories." It is interesting that the necessary and sufficient condition for this to be achieved is simply

$$\langle\, |a_n(T)|^2 \rangle = |a_n(0)|^2\ , \tag{2}$$

where the average is over the ensemble of identical physical systems, and where T is the time at which the reduction is completed for all physical systems.

To prove this assertion we note that, for each system, $|a_n(T)|^2$ either equals 1 or 0 since that is the final result of the reduction process. Therefore, $\langle\, |a_n(T)|^2\rangle$ equals the number of systems for which $|a_n(T)|^2=1$ divided by the total number of systems, i.e., $\langle\, |a_n(T)|^2\rangle$ is precisely the fraction of systems for which the state vector reduces to $|\phi_n(t)\rangle$.

Now, Eq. (2) can also be written more symmetrically as

$$\langle\, |a_n(T)|^2 \rangle = \langle\, |a_n(0)|^2 \rangle\ , \tag{2$'$}$$

because all systems start with the same initial values $|a_n(0)|^2$ immediately following the measurement, before the reduction process has begun. Since Eq.(2′) states that the expectation values of the squared amplitudes are initially and finally the same, it is natural to ask whether there is any experimental difference between theories for which these expectation values are also constant *during* the reduction process,

$$\frac{d}{dt}\langle |a_n(t)|^2\rangle = 0 , \quad 0 \le t \le T , \tag{3}$$

and theories which do not satisfy (3). In the former class is the theory of Pearle; in the latter class is the theory of Bohm and Bub.

We will now show that satisfaction of (3) is the condition that there be no deviation from the predictions of quantum theory for two successive experiments separated by a time interval less than T, provided the second experiment does not measure interference between the states produced by the first experiment.

II. ANALYSIS OF TWO SUCCESSIVE EXPERIMENTS

Let us suppose that the first experiment is completed at time 0, and the second experiment takes place at time t_1 (we are assuming that the duration of this experiment is negligible[10]). Immediately after the first experiment, at time 0, the state vector is, according to Eq. (1),

$$|\psi,0\rangle = \sum_n a_n(0)\,|\phi_n(0)\rangle . \tag{4}$$

Thereafter, until the second experiment at time t_1, quantum theory predicts the state vector will be of the form (1), and that

$$|a_n(t)|^2 = |a_n(0)|^2 , \quad 0 \le t \le t_1 . \tag{5}$$

On the other hand, while a dynamical reduction theory also predicts the state vector is of the form (1) during the time interval $(0,t_1)$, the squared amplitudes $|a_n(t)|^2$ do not remain constant during this time interval. Nonetheless, if the theory obeys (3), we can assert

$$\langle |a_n(t)|^2\rangle = |a_n(0)|^2 , \quad 0 \le t \le t_1 \tag{6}$$

where the average is taken over the ensemble of state vectors describing the ensemble of such experiments.

When the second experiment is performed at time t_1, the state $|\phi_n(t)\rangle$ undergoes an evolution:

$$|\phi_n(t_1)\rangle \to \sum_m a_{nm}\,|\phi_{nm}\rangle \tag{7}$$

(the index m labels the outcome of the second experiment). Therefore, according to quantum theory or according to a dynamical reduction theory, the state vector immediately after the second experiment is

$$|\psi,t_1\rangle = \sum_{n,m} a_n(t_1)a_{nm}\,|\phi_{nm}\rangle . \tag{8}$$

Let us first assume that the set of states $\{|\phi_{nm}\rangle\}$ are all orthogonal. This is overwhelmingly the common situation. For example, this is true for all situations in which the first experimental results are ineradicably recorded.

According to quantum theory, the squared amplitude multiplying each state $|\phi_{nm}\rangle$ in Eq. (8) does not change for $t > t_1$. Thus, the predicted probability that the measured sequence is n,m is

$$P^Q_{nm} = |a_n(t_1)a_{nm}|^2 = |a_n(0)|^2\,|a_{nm}|^2 , \tag{9}$$

where the final result follows from Eq. (5).

According to a dynamical reduction theory, the squared amplitude multiplying each state $|\phi_{nm}\rangle$ in Eq. (8) evolves to 0 or 1 for $t > t_1$ as the reduction proceeds unimpeded to completion. The fraction of systems which start at time t_1 with a particular amplitude $a_n(t_1)a_{nm}$ and which end up reduced to the state $|\phi_{nm}\rangle$ is $|a_n(t_1)a_{nm}|^2$, by the fundamental property of dynamical reduction theories. The total fraction of systems which end up reduced to the state $|\phi_{nm}\rangle$ is therefore obtained by multiplying $|a_n(t_1)a_{nm}|^2$ by the fraction of systems which possess this particular squared amplitude, and summing over all such squared amplitudes, so it is

$$P^R_{nm} = \langle |a_n(t_1)a_{nm}|^2\rangle = \langle |a_n(t_1)|^2\rangle\,|a_{nm}|^2 . \tag{10}$$

By comparing Eq. (9) and Eq. (10) [using Eq. (6)] we see that if and only if condition (3) is satisfied, the probabilities P^Q_{nm} predicted by quantum theory and the probabilities P^R_{nm} predicted by a dynamical reduction theory will be identical, for a sequence of two experiments separated by an arbitrary interval t_1. This is the most important point in this paper.[11] [A subsidiary result that will later be of interest is that the conditional probabilities are equal:

$$P^Q_{nm} \Big/ \sum_m P^Q_{nm} = P^R_{nm} \Big/ \sum_m P^R_{nm} = |a_{nm}|^2 \tag{11}$$

whether or not condition (3) is satisfied.]

III. DISTINGUISHING QUANTUM THEORY FROM A DYNAMICAL REDUCTION THEORY

How, then, might one experimentally distinguish a dynamical reduction theory satisfying condition (3) from quantum theory? One obvious way is to perform two successive experiments such that the final states $|\phi_{nm}\rangle$ are not orthogonal.

For example, the first experiment might separate a wave packet describing a particle into spatially separated packets (e.g., by means of a scattering, a Stern-Gerlach, or a diffraction apparatus), but might not actually measure where the particle is. Now, it is a tenet of dynamical reduction theories that the reduction proceeds when the states in the superposition (1) become "macroscopically distinguishable." This may be taken to mean that the states $|\phi_n(t)\rangle$ describe a reasonably massive object (e.g., a "pointer") that can occupy macroscopically distinct positions (labeled by n). So, if the first experiment separates the wave packets by a "macroscopic" distance, and the particle is massive enough, one might suppose that the reduction process might proceed, although perhaps at a slow rate. One might say that the wave packet in such circumstances begins to "spontaneously reduce." If we define any processing of a system that brings on a reduction as an "experiment," then such a splitting of a wave packet may be called an experiment, even though a measuring apparatus is not involved. The second experiment, which should take place after as long a time as possible in order to let the effects of the slow reduction reach a measurable magnitude, would involve recombining the wave packets and measuring their interference.

With this motivation, we will consider two successive experiments with the maximum possible interference, i.e., $|\phi_{nm}\rangle = |\phi_{n'm}\rangle$ for all n,n'. The quantum-theory prediction of the mth outcome is

$$P_m^Q = \left| \sum_n a_n(t_1) a_{nm} \right|^2 = \sum_n |a_n(0)|^2 |a_{nm}|^2 + \sum_{n\neq n'} a_n(0) a_{n'}^*(0) a_{nm} a_{n'm}^* . \quad (12)$$

[In writing (12) we have used the condition $a_n(t)=a_n(0)$ which is stronger than $|a_n(t)|^2=|a_n(0)|^2$, but permissible since any time-varying phases can be absorbed in the $|\phi_n(t)\rangle$, and will ultimately alter the phases of the a_{nm}].

The dynamical-reduction-theory prediction, by reasoning parallel to that preceding Eq. (10), is

$$P_m^R = \left\langle \left| \sum_n a_n(t_1) a_{nm} \right|^2 \right\rangle . \quad (13)$$

Because the reduction rate is presumed slow following the first "experiment," it is useful to write $a_n(t) = \sqrt{x_n(t)} \exp[i\theta_n(t)]$, and expand $a_n(t)$ about $a_n(0)$, obtaining

$$a_n(t) = a_n(0) \left[1 + i\Delta\theta_n - \tfrac{1}{2}(\Delta\theta_n)^2 + i\Delta\theta_n \frac{\Delta x_n}{2x_n(0)} + \frac{\Delta x_n}{2x_n(0)} - \frac{(\Delta x_n)^2}{8x_n(0)^2} + \cdots \right] \quad (14)$$

up to second order in $\Delta\theta_n \equiv \theta_n(t) - \theta_n(0)$, $\Delta x_n \equiv x_n(t) - x_n(0)$. Equation (14) may be substituted into Eq. (13). The result is simplified if we assume the following properties of the dynamical reduction theory (all satisfied by the theory of Pearle[7]). First, $\langle \Delta x_n \rangle = 0$ [this follows from condition (3)]. Second, the phase angles also have no drift, so $\langle \Delta\theta_n \rangle = 0$. Third, the squared amplitudes and the phase angles are uncorrelated, so $\langle \Delta x_n \Delta\theta_n \rangle = \langle \Delta x_n \rangle \langle \Delta\theta_n \rangle = 0$. Then Eq. (13) becomes

$$P_m^R = \sum_n |a_n(0)|^2 |a_{nm}|^2 + \sum_{n\neq n'} a_n(0) a_{n'}^*(0) a_{nm} a_{n'm}^* \left[1 - \tfrac{1}{2}\langle (\Delta\theta_n - \Delta\theta_{n'})^2 \rangle - \frac{1}{8} \left\langle \left[\frac{\Delta x_n}{x_n(0)} - \frac{\Delta x_{n'}}{x_{n'}(0)} \right]^2 \right\rangle \right] . \quad (15)$$

Comparison of Eq. (15) with Eq. (12) shows that the interference terms are slightly "washed out" in the dynamical reduction theory compared to quantum theory. In the theory of Pearle,[7,9] the phase angles and squared amplitudes obey a diffusion equation which for short times is ordinary Gaussian diffusion with zero drift and $\sim 1/\tau_R$ variance (τ_R characterizes the reduction time), so Eq. (15) may be written

$$P_m^R = \sum_n |a_n(0)|^2 |a_{nm}|^2 + \sum_{n\neq n'} a_n(0) a_{n'}^*(0) a_{nm} a_{n'm}^* \left[1 - C \frac{t_1}{\tau_R} \right] , \quad (16)$$

where C is a positive constant of order of magnitude 1.

IV. EXPERIMENTAL TESTS

Have there been any experiments performed that can be used to distinguish between quantum theory and dynamical reduction theories? Shortly after the Bohm-Bub theory appeared, Papaliolios[12,3] performed an experiment in which linearly polarized light (polarized at 80° to the vertical) successively passed through two closely spaced polaroids (the first polarized vertically, the second polarized at a variable angle θ to the vertical) 7.5×10^{-14} sec of photon travel time apart and into a counter. He varied the angle θ of the second polaroid, obtaining the $\cos^2\theta$ dependence of the transmitted photon intensity (Malus's law) to $\approx 1\%$ accuracy.

This experiment was performed to test a different variant of the Bohm-Bub theory than the one we are considering here. Bohm, Bub, and Papaliolios hypothesized that the first reduction occurred essentially *instantaneously* within the first polaroid, but that the hidden variables (for which they did *not* give a dynamical evolution equation), which were appropriately distributed for the first experiment, did not relax to their correct distribution for the second experiment, even after the photon hit the second polaroid. As a consequence, the reduction following the second measurement was guided by a "biased" (incorrectly distributed) set of hidden variables and did not obey the fundamental property of dynamical reduction theories, thereby producing deviations from the predictions of quantum theory. Thus, this experiment was regarded as placing an upper limit of order 10^{-14} sec on the *relaxation* time of the hidden variables,[13] and was not regarded as a test of the *reduction* time of the dynamical reduction theory.

What we have in mind when we consider the Bohm-Bub theory is a different dynamics of the hidden variables, namely, that they instantaneously relax to their uniform distribution immediately following each experiment, before the reduction commences. In this case, the correct hidden-variable distribution always guides each interval of reduction, and the deviation from quantum theory is solely due to nonsatisfaction of condition (3) by the Bohm-Bub dynamical equations. To summarize, following each experiment, instead of an instantaneous reduction followed by a slow relaxation, we hypothesize an instantaneous relaxation followed by a slow reduction. This latter hypothesis may be regarded as more natural than the former because the dynamics of reduction was described by Bohm and Bub, but the dynamics of relaxation was not.

To analyze Papaliolios's experiment, we suppose that the first experiment is performed at the first polaroid, after which the reduction commences between two states (labeled "absorbed at first polaroid" and "transmitted by first polaroid"). We further suppose that the second experiment takes place at the second polaroid, where the transmitted photon state splits into two further states (labeled "absorbed at second polaroid" and "transmitted by both polaroids"), so that finally three states compete in the reduction "game."[8] All three states are orthogonal, so by our previous discussion, the intensities measured with any orientations of the polaroids are predicted identically by quantum theory and by a dynamical reduction theory satisfying (3) (such as Pearle's theory). However, there will be some disagreement with the predictions of a dynamical reduction theory not satisfying (3) (such as the Bohm-Bub theory).

Papaliolios did not vary the orientation of the first polaroid,[14] but only the orientation of the second polaroid. As can be seen from Eq. (11), the detected intensity is proportional to the conditional probability $|a_{nm}|^2 \equiv |a_{1\theta}|^2 = \cos^2\theta$ whether or not condition (3) is obeyed. Thus, from this point of view, Papaliolios's experiment, as performed, did not provide any limit for the reduction time of any dynamical reduction theory. But a rotation of the first polaroid provides a test of the Bohm-Bub theory.

To see this, let $x(t) = |a_1(t)|^2$ be the squared amplitude for the photon state transmitted by the first polaroid. For $t > 0$, x evolves with time according to the Bohm-Bub dynamical equation

$$\frac{dx}{dt} = \frac{1}{\tau_R} x(1-x)\left[\frac{x}{z} - \frac{(1-x)}{(1-z)}\right], \tag{17}$$

where z, the hidden variable, can take on any value between zero and one with equal likelihood. The solution of this equation at time t_1 is

$$\frac{x^{1-z}(1-x)^z}{x-z} = \frac{(\sin^2\epsilon)^{1-z}(\cos^2\epsilon)^z}{\sin^2\epsilon - z} e^{-t_1/\tau_R}, \tag{18}$$

where $x(0) = \sin^2\epsilon$ (ϵ is the angle the incident photon polarization makes with the horizontal, and had the fixed value 10° in Papaliolios's experiment). From Eq. (18) one can see that $x(t_1, z)$ moves from $\sin^2\epsilon$ at $t_1 = 0$ toward 1 (toward 0) as $t \to \infty$, for $z < \sin^2\epsilon$ (for $z > \sin^2\epsilon$).

We note, according to Eq. (10), that the detected intensity is proportional to $P = \langle x(t_1)\rangle\cos^2\theta$. A computer can be used to solve Eq. (18) for $x(t_1, z)$, and to calculate $\langle x(t_1)\rangle \equiv \int_0^1 dz\, x(t_1, z)$. A graph of $\langle x(t_1)\rangle/\sin^2\epsilon$ vs t_1/τ_R for selected angles ϵ appears in Fig. 1. For angles ϵ between 0° and 45° (between 45° and 90°) the theory predicts an anomalously low (high) photon flux at the detector: For example, at $\epsilon = 10°$, the flux is more than 4% below the quantum theory prediction for $2.7\tau_R > t_1 > 0.01\ \tau_R$, and is as much as 24% below for $t_1 \approx 0.4\tau_R$. Moreover, if the angle ϵ is varied (keeping θ fixed), the theory predicts a deviation in intensity from the expected $\sin^2\epsilon$ dependence (Malus's law).

Since agreement with quantum theory is obtained for long reduction times (quantum theory is the limit $\tau_R \to \infty$, or no reduction) or for short reduction times compared to t_1, any such experiment can be used to place a lower and

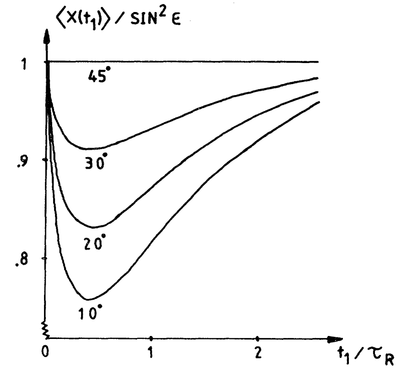

FIG. 1. A graph of $\langle x(t_1)\rangle/\sin^2\epsilon$ vs t_1/τ_R, for angles $\epsilon = 10°$, 20°, 30°, 45°. For angles $90° > \epsilon > 45°$, the curves can be found from $\langle x(t_1)\rangle(\epsilon) = 1 - \langle x(t_1)\rangle((\pi/2) - \epsilon)$, and so lie above the horizontal line at 1.

an upper limit on τ_R. If one expects the reduction time to be short, then it would be appropriate to use Papaliolios's apparatus to place an upper limit of order 10^{-14} sec on the reduction time τ_R. However, the two states immediately following the first experiment ("photon absorbed" and "photon transmitted") can hardly be thought of as being macroscopically distinguishable, differing as they do only in the state of a single photon and an atom that did (or did not) absorb the photon. Accordingly, we might expect the reduction time to be long. This suggests that the polaroids be placed as far apart as possible instead of as close together as possible as they were in Papaliolios's experiment. Moreover, suppose the atom which absorbs the photon initiates a sufficiently large disturbance which spreads through the polaroid with the speed of sound. Then the two states, "absorbed" and "transmitted," might soon be regarded as "macroscopically distinguishable," with a consequent lowering of τ_R. From this point of view, the numerous experimental evidence supporting Malus's law at moderate distances could be regarded as refuting dynamical reduction theories such as that of Bohm and Bub which do not satisfy condition (3).

V. TWO-SLIT NEUTRON INTERFERENCE EXPERIMENT

For an experiment that does test dynamical reduction theories satisfying condition (3), consider a recent two-slit neutron interference pattern obtained by Zeilinger *et al.*[15] The slit separation was $a \approx 126\ \mu$m, the slit width was $b \approx 22\ \mu$m, the slit-detector separation was $L \approx 5$ m, and the neutron wavelength was $2\pi/k \approx 20$ A. The data was fit to $\approx 1\%$ accuracy by adjusting three parameters: the overall intensity, the center position of the pattern, and the slit-width parameter. The latter was necessary because the two slits were formed by inserting a 104-μm-thick Boron wire into the gap of a 148-μm single slit, and the interference pattern constituted the best measurement of the degree of centering the wire: one slit was $\approx 5\%$ wider than the other.[16] The solution of the Schrödinger equation with which the data was compared was obtained by solving a Fresnel-Kirchhoff—type integral by computer, assuming a plane wave coherent over the plane of the slits.

A lower limit can be placed on the spontaneous reduction time for the neutron in this experiment, according to the discussion in Sec. III.[17] For illustrative purposes, we will assume that the slits were exactly of equal width, and that the distance L was large enough for the Fraunhofer diffraction approximation of the Fresnel-Kirchhoff integral to be valid (both assumptions were not quite correct for the actual experiment). Then $a_1(0)=a_2(0)=1/\sqrt{2}$ are the amplitudes of each wave packet as they leave their respective slits, and

$$a_{r,m} \sim x_m^{-1} \exp[(-1)^{r+1}(ikx_m a/2L)] \sin(kx_m b/2L) , \tag{19}$$

where x_m is the position of a detector in the plane a distance L from the slit screen ($x_m=0$ being equidistant from both slits). According to Eq. (16), if a slight reduction of the wave packets takes place on their way from the slit screen to the detector, but the reduction dynamics obeys condition (3), then

$$P_m^R \sim \tfrac{1}{2} x_m^{-2} \sin^2(kx_m b/2L) \times [1+\cos(kx_m a/2L)(1-Ct_1/\tau_R)] . \tag{20}$$

A 1% agreement of experiment with the quantum theory prediction [the standard two-slit interference expression, Eq. (20) with $\tau_R=\infty$] leads to the inequality

$$t_1/\tau_R \lesssim 0.01 . \tag{21}$$

Since the neutron velocity in this experiment was $v=\hbar k/M \approx 10^4$ cm/sec, the neutron traveled the distance $L \sim 5$ m between the diffraction slits and the detector in $t_1 \approx 0.05$ sec. (It is the possibility of making this time long by making v small and L large which makes this experiment an effective test of the dynamical reduction theory.) Thus, we obtain from (21) an approximate lower bound on the reduction time:

$$\tau_R \gtrsim 5 \text{ sec} . \tag{22}$$

This rather crude analysis nonetheless illustrates the utility of interference experiments involving macroscopic distances and/or macroscopic masses[18] in investigating dynamical reduction theories satisfying condition (3).

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[1] D. Bohm and J. Bub, Rev. Mod. Phys. 38, 453 (1966).

[2] J. H. Tutsch, Phys. Rev. 183, 1116 (1969); J. Math. Phys. 12, 1711 (1971).

[3] F. Belinfante, *A Survey of Hidden Variable Theories* (Pergamon, London, 1973).

[4] N. Gisin and C. Piron, Lett. Math. Phys. A5, 379 (1981) contains a set of dynamical reducing equations whose reduction mechanism is similar to that of the Bohm-Bub equations, but they are more complicated equations and less general.

[5] N. Wiener and A. Siegal, Phys. Rev. 101, 429 (1956).

[6] P. M. Pearle, Phys. Rev. D 13, 857 (1976).

[7] P. M. Pearle, Int. J. Theor. Phys. 18, 489 (1979).

[8] P. M. Pearle, Found. Phys. 12, 249 (1982).

[9] P. M. Pearle, in *The Wave-Particle Dualism—A Tribute to Louis deBroglie on his 90th Birthday,* edited by S. Diner, D. Fargue, G. Lochau, and F. Selleri (Reidel, Dordrecht, 1984).

[10] More precisely, the duration of this experiment should be small compared to the reduction time. In this case, the evolution of the amplitudes $a_n(t)$ during the second experiment is essentially governed by the ordinary Schrödinger equation, and so the $a_n(t)$ do not change appreciably due to the reduction dynamics. Deviations from the predictions of quantum theory may arise in dynamical reduction theories where the Schrödinger dynamics and the reduction dynamics take place simultaneously and have the same rate. Such experiments are not discussed in this paper. Calculations of such effects will be worthwhile when the details of dynamical reduction theories are better defined than at present.

[11] It is not hard to extend the argument to three or more successive experiments.

[12] C. Papaliolios, Phys. Rev. Lett. 18, 622 (1967).

[13] The theory predicts, instead of the transmitted intensity $I \sim \cos^2\theta$, that $I=0$ for $90° \geq \theta \geq 50°$ ($50°=45°+\frac{1}{2}\epsilon$, where ϵ is the angle the incident polarized light makes with the horizontal; $\epsilon=10°$ in this experiment),

$$I \sim 1-\pi^{-1}\cos^{-1}(-\alpha\cot\epsilon)+\alpha[\sin^2\epsilon(1+\alpha^2)^{1/2}]^{-1} \times \{-\tfrac{1}{2}+(1/\pi)\tan^{-1}[\cot\epsilon(1+\alpha^2)^{1/2}(1-\alpha^2\cot^2\epsilon)^{-1/2}]\}$$
$$[\alpha \equiv \tfrac{1}{2}(\tan\theta-\cot\theta)]$$

for $50° \geq \theta \geq 45°$ [this formula, given here for the first time, but numerically calculated by Belinfante (Ref. 3), represents a precipitate rise from 0 to 0.5], and the graph of I vs θ is symmetrical by rotation through 180° about the center point $I=0.5$, $\theta=45°$ giving I for $45° \geq \theta \geq 0$. This result assumed a two-state quantum system corresponding to the photon's two orthogonal polarization states. However, if one considers the states of the photon and apparatus, it is more appropriate to think of this as a three-state quantum system (photon absorbed by first polaroid, and photon transmitted or absorbed by second polaroid). We have found (analysis unpublished) that in this case $I \sim \frac{1}{2}[1-\alpha(\sin^2\epsilon+\alpha^2)^{-1/2}]$ for $90° \geq \theta \geq 0$. This much simpler formula still makes the I vs θ curve look more like an inverted integral sign than $I \sim \cos^2\theta$, and does not change Papaliolios's conclusion.

[14] Belinfante (Ref. 3) pointed out that this procedure would also have produced a useful test of the relaxation time of the hidden variables, the aspect of the Bohm-Bub theory tested by Papaliolios.

[15] A. Zeilinger, R. Gaehler, C. G. Shull, and W. Treimer, in *Neutron Scattering—1981 (Argonne),* proceedings of the Conference on Neutron Scattering, edited by J. Faber, Jr. (AIP, New York, 1982).

[16] The off-centering of the wire resulted in a slightly asymmetrical interference pattern. This asymmetrical pattern cannot be attributed to the dynamical reduction behavior here for a centered wire since, according to Eq. (20), that produces a

symmetrical pattern. However, a dynamical reduction theory in which the phase angles have nonzero drift could produce an asymmetrical interference pattern.

[17]The conditions of "spontaneous reduction" described in Sec. III are more uncomplicatedly applicable to, say, a Michaelson interferometer situation (where the two wave packets are clearly separated during most of the experiment by a macroscopic distance) than a two-slit interference situation (where the two wave packets start to overlap sufficiently far from the slits). We are assuming that the reduction, once begun, is not inhibited by the subsequent wave-packet overlap.

[18]A. J. Leggett, Suppl. Prog. Theor. Phys. 69, 80 (1980) has pointed out that quantum theory has not been experimentally tested for interference experiments involving appreciably many-particle states. He has suggested coherent flux tunneling in a superconducting ring containing a Josephson junction as a technological possibility. Such an experiment might be employed as a test of dynamical reduction theories.