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Time development of Higgs field at finite temperature

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We derive the equation of motion of the thermodynamical average of the Higgs field. The friction term in the equation is explicitly given in simple models. The equation obtained may be relevant to the cosmological phase transitions.

I. INTRODUCTION

On the basis of grand unified theories (GUT's) of strong and electroweak interactions, an ambitious scenario of the very early Universe has been proposed for simultaneous resolution of the horizon, flatness, and monopole problems.[1,2] According to the scenario called the inflationary Universe by Guth, the Universe has experienced a period of exponential expansion at the time of $\sim 10^{-36}$ sec after the beginning of the Universe. However, the original version which assumes a first-order phase transition has the serious difficulty that the phase transition will never be completed. This difficulty has been cured by the new version of the inflationary Universe;[3] the whole Universe in sight at present was inside a single bubble. The completion of the phase transition is not necessary. The new scenario qualitatively explains the desirable spectrum of density perturbations which eventually show up as galaxies.[4] People working in this field of cosmology have tried to construct newer versions which can quantitatively fit the numbers.[5] It seems to us, however, that the basic idea of the new inflationary scenario—Coleman-Weinberg-type effective potential—will survive.

In the new inflationary-Universe scenario, the Universe is supposed to pass through the rolling-down phase in the Coleman-Weinberg-type potential for quite a long time. (See Fig. 1). The Higgs field Φ starts at a point a little bit away from the top and then rolls down along the long almost flat slope to the minimum of the effective potential. The oscillation around the bottom eventually dies out because of friction. Or if the friction force is strong enough, there will be no oscillation and Φ simply slowly rolls down to the bottom. In the literature,[6] the motion of the thermodynamical average has been analyzed numerically, where the friction force is phenomenologically introduced.

In this paper, we are going to derive the equation of motion of the thermodynamical average of the Higgs field and compute the friction coefficient in simple models. We note that the friction is responsible for the heat-up of the Universe in the final stage of the rolling-down phase transition. The estimate of the strength of the friction seems very important. Although we are motivated by the GUT's cosmology, we are primarily concerned with the nonequilibrium statistical problems of the Higgs system. In order to understand the basic thermodynamical aspects of the rolling-down phase, we will work mainly in flat space-time. Generalization to the Friedmann background will be briefly discussed in the last section. Detailed analysis of the equation of motion in the inflationary background will be published elsewhere.

A sound physical intuition clearly tells us that our equation for Φ has the form

$$\Box\Phi + V'_{\text{eff}}(\Phi) + \eta(\Phi)\dot{\Phi} = 0 \ . \tag{1.1}$$

V'_{eff} means the derivative of the effective potential with respect to Φ and the last term represents the friction force. Our primary aim of this paper is the derivation of Eq. (1.1) and the computation of $\eta(\Phi)$. Unfortunately no reliable theory of a system far from equilibrium has yet been available. We have to content ourselves to work in the hydrodynamical regime, i.e., slight-off-equilibrium state. Hence our derivation is legitimate only if the mac-

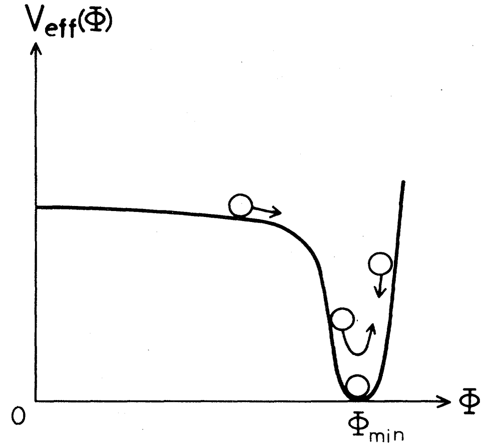

FIG. 1. Schematic description of the Coleman-Weinberg-type effective potential. Small circles represent the expectation value of the Higgs field which rolls down to the bottom of the effective potential.

roscopic order parameter Φ varies slowly in comparison with the microscopic time scale of collisions, τ_{coll}, i.e.,

$$\tau_{\text{coll}} \ll \left| \frac{\Phi}{\dot{\Phi}} \right| . \tag{1.2}$$

This limitation will be very important to the application to the inflationary Universe.

The plan of this paper is as follows. In Sec. II we present a heuristic derivation of Eq. (1.1) on the basis of kinetic theory in the relaxation-time approximation, after a brief introduction to the elementary evaluation of transport coefficients.[7] We reproduce the same equation by using Zubarev's method of a nonequilibrium statistical operator in Sec. III. Here we obtain a closed form for the friction coefficient $\eta(\Phi)$, which is valid beyond perturbation theory. In Sec. IV we demonstrate the increase of entropy and discuss the release of energy originally in the form of Higgs potential energy to radiation (Higgs quanta). We evaluate the friction coefficient at high temperature in Sec. V. Section VI is devoted to summary, discussions, and brief remarks on the application to cosmology.

II. KINETIC THEORY DERIVATION

A. Transport coefficients

In order to familiarize ourselves with the transport theory, it is very instructive to recall the elementary computation of heat conductivity.[7] Consider the case in which the temperature has a gradient, say, in the x direction. (See Fig. 2.) Let us compute the energy flow caused by the temperature gradient in a system of interacting scalar bosons. We have

$$\langle T_{0x}(x) \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{\omega(p)p_x}{\omega(p)} n(p,x) . \tag{2.1}$$

Here $\omega(p) = (p^2+m^2)^{1/2}$ and $n(p,x)d^3p/(2\pi)^3$ are the energy and the number density of particles with momentum p, respectively. Here $n(p,x)$ is a slight deviation from the local equilibrium distribution $n_0(p,x)$,

$$n(p,x) = n_0(p,x) + \delta n , \tag{2.2}$$

$$n_0(p,x) = 1/(e^{\beta(x)\omega(p)} - 1) , \tag{2.3}$$

with $\beta^{-1}(x) = T(x)$ being the local temperature. A simple evaluation of the deviation δn is given by the relaxation-time approximation of the kinetic equation:[7]

$$\partial_t n + \frac{\vec{p}}{\omega(p)} \cdot \vec{\nabla} n = -\frac{1}{\tau(p)} \delta n , \tag{2.4}$$

where $\tau(p)$ is the relaxation time. In the lowest-order approximation, we obtain the heat current as

$$\langle T_{0x} \rangle \simeq \int \frac{d^3p}{(2\pi)^3} \frac{\omega(p)p_x}{\omega(p)} (-\tau) \frac{p_x}{\omega(p)} \partial_x n_0 . \tag{2.5}$$

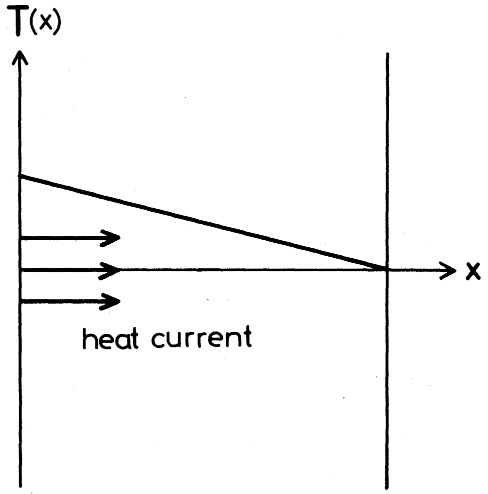

FIG. 2. Temperature T has a gradient in the x direction. This gradient induces the heat current.

With n_0 being the local equilibrium distribution (2.3) we have

$$\partial_x n_0 = -\partial_x \beta(x) \omega(p) n_0 (1+n_0) . \tag{2.6}$$

The definition of the heat conductivity κ,

$$\langle T_{0x} \rangle = -\kappa \frac{\partial_x T}{T} = \kappa \frac{\partial_x \beta}{\beta} , \tag{2.7}$$

and Eqs. (2.5) and (2.6) directly lead to

$$\begin{aligned} \kappa &= \beta \int \frac{d^3p}{(2\pi)^3} p_x{}^2 \tau n_0 (1+n_0) \\ &= \frac{\beta}{3} \int \frac{d^3p}{(2\pi)^3} \vec{p}\,^2 \tau n_0 (1+n_0) . \end{aligned} \tag{2.8}$$

Similarly we can obtain the shear viscosity and the bulk viscosity. We just quote the results. The shear viscosity and bulk viscosity are defined by

$$\langle T_{xy} \rangle = \eta(\partial_x u_y + \partial_y u_x) , \tag{2.9}$$

$$\langle T_{xx} + T_{yy} + T_{zz} \rangle = \langle p \rangle - \zeta \vec{\nabla} \cdot \vec{u} , \tag{2.10}$$

where $\vec{u}$ is the velocity field. They are obtained as

$$\eta = \frac{\beta}{15} \int \frac{d^3p}{(2\pi)^3} \frac{(\vec{p}\,^2)^2}{\omega(p)^2} \tau(p) n_0(p) [1+n_0(p)] , \tag{2.11}$$

$$\begin{aligned} \zeta = \frac{\beta}{3} \int \frac{d^3p}{(2\pi)^3} & \frac{(\frac{1}{3}\vec{p}\,^2 - c_s{}^2\omega^2)\frac{1}{3}\vec{p}\,^2}{\omega(p)^2} \tau(p) n_0(p) \\ & \times [1+n_0(p)] . \end{aligned} \tag{2.12}$$

The expressions agree with the lowest-order evaluation of the Kubo-type formulas as shown in Ref. 8.

The relaxation time is given by a standard collision integral

$$\tau^{-1}(p)=\frac{1}{2\omega(p)}\int\frac{d^3p_1}{(2\pi)^3 2\omega(p_1)}\frac{d^3p_2}{(2\pi)^3 2\omega(p_2)}\frac{d^3p_3}{(2\pi)^3 2\omega(p_3)}(2\pi)^4\delta^4(p+p_1-p_2-p_3)$$
$$\times\frac{\lambda^2}{4}\{n_0(p_1)[1+n_0(p_2)][1+n_0(p_3)]-[1+n_0(p_1)]n_0(p_2)n_0(p_3)\}\ , \tag{2.13}$$

in the lowest-order approximation in the quartic coupling constant in the $\lambda\phi^4$ theory.

B. Time development of order parameter

Now we are going to derive the equation of motion of the thermodynamical average of the Higgs field ϕ:

$$\Phi=\langle\phi\rangle=\mathrm{tr}(\rho\phi) \tag{2.14}$$

for a suitable nonequilibrium statistical operator ρ. In the next section we shall explicitly construct the statistical operator ρ. In this section, however, what we need is the simple property that the statistical operator ρ in the Heisenberg representation is time independent:

$$\frac{d\rho}{dt}=0\ . \tag{2.15}$$

To make the presentation as simple as possible, we consider the case of a single scalar field ϕ with a potential $V(\phi)$ which is assumed to be convex everywhere. Moreover, the effective potential $V_{\mathrm{eff}}(\Phi)$, which includes the finite-temperature effect as well as quantum corrections, has a shape depicted in Fig. 3(b). As we can subsequently see, it is straightforward to generalize our method to include gauge fields. Hence our hypothetical effective potential is actually realized in the Coleman-Weinberg potential.

Now let us start with the field equation for the Heisenberg field ϕ:

$$\Box\phi+V'(\phi)=0\ . \tag{2.16}$$

Take the thermodynamical average of Eq. (2.16), noting that the derivatives can be taken out of the average operation $\langle\ \rangle$ because of the time independence of the statistical operator. We have

$$\Box\Phi+\langle V'(\phi)\rangle=0\ . \tag{2.17}$$

Then we split the Heisenberg field as

$$\phi=\Phi+\varphi \tag{2.18}$$

where φ is the fluctuation field around the thermodynamical average value Φ. Expand the potential term in Eq. (2.17) as

$$\langle V'(\Phi+\varphi)\rangle=V'(\Phi)+V''(\Phi)\langle\varphi\rangle+\frac{V^{(3)}(\Phi)}{2!}\langle\varphi^2\rangle+\cdots\ . \tag{2.19}$$

Then we arrive at a key formula:

$$\Box\Phi+V'(\Phi)+\frac{V^{(3)}(\Phi)}{2!}\langle\varphi^2\rangle+\cdots=0\ . \tag{2.20}$$

Here we have used the identity

$$\langle\varphi\rangle=0\ .$$

We shall also use Eq. (2.20) as a starting point in the subsequent sections.

Let us compute $\langle\varphi^2\rangle$ in Eq. (2.20) using the relaxation-time approximation of the kinetic equation in a parallel way to the previous subsection. We proceed,

$$\langle\varphi^2\rangle=\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega(p)}n(p)$$
$$=\langle\varphi^2\rangle_0+\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega(p)}\delta n\ , \tag{2.21}$$

$$\langle\varphi^2\rangle_0=\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega(p)}n_0(p)\ , \tag{2.22}$$

where $n_0(p)$ is the local equilibrium distribution:

$$n_0(p)=1/(e^{\beta\omega(p)}-1)\ . \tag{2.23}$$

In order to elucidate the essential mechanism of friction, let us assume that the temperature increase and its inhomogeneity by the time development of the Higgs field can be ignored. Contrary to the previous case of thermal conductivity, the deviation δn comes from the time dependence of the "mass^2" of the fluctuation quanta, which is

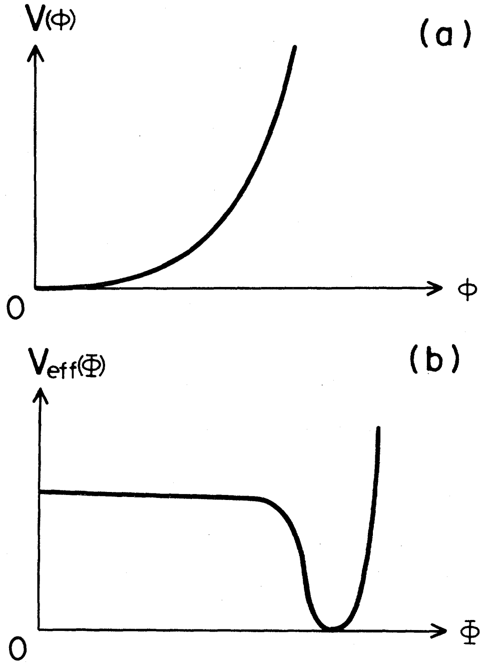

FIG. 3. Shapes of (a) the potential $V(\Phi)$ in the tree level and (b) the effective potential $V_{\mathrm{eff}}(\Phi)$ which contains the finite-temperature effect and quantum corrections.

given by the second derivative of the potential $V(\Phi)$. Namely, the energy of fluctuation quanta with momentum $\vec{p}$ is

$$\omega(p)=[\vec{p}^{\,2}+V''(\Phi)]^{1/2} \, .$$

Hence the relaxation-time approximation of the kinetic equation gives

$$\begin{aligned}\delta n &= -\tau\partial_t n_0 \\ &= \tau\beta n_0(1+n_0)\frac{\partial[\vec{p}^{\,2}+V''(\Phi)]^{1/2}}{\partial t} \\ &= \tau\beta n_0(1+n_0)\frac{1}{2\omega(p)}V^{(3)}(\Phi)\dot{\Phi} \, . \end{aligned} \tag{2.24}$$

Putting Eqs. (2.20)—(2.24) together, we finally obtain

$$\Box\Phi+V'_{\rm eff}(\Phi)+\eta(\Phi)\dot{\Phi}=0 \, . \tag{2.25}$$

Here we have used the fact that the first derivative of the effective potential can be written as

$$V'_{\rm eff}(\Phi)=V'(\Phi)+\frac{V^{(3)}(\Phi)}{2!}\langle\varphi^2\rangle_0+\cdots \, . \tag{2.26}$$

The friction coefficient $\eta(\Phi)$ is obtained as

$$\eta(\Phi)=\beta\int\frac{d^3p}{(2\pi)^3}\frac{\tau}{2\omega(p)^2}n_0(1+n_0)\left[\frac{V^{(3)}(\Phi)}{2}\right]^2+\cdots \, , \tag{2.27}$$

where τ is the relaxation time of fluctuation particles and the local equilibrium distribution n_0 is given in Eq. (2.23).

We did not evaluate $\langle\varphi^3\rangle,\langle\varphi^4\rangle,\ldots$, which should appear in Eq. (2.20) though not explicitly written. It is clear, however, that those terms are higher order in coupling constants. Perhaps the intuitive methods based on the relaxation-time approximation cannot be pushed further to the higher-order calculations. In the next section we present a more systematic method.

C. Generalization

The extensions to more general field theories are almost straightforward. We illustrate the procedure in the case of the Yukawa interaction. The Heisenberg equations of motion are

$$\Box\phi+V'(\phi)+g\bar{\psi}\psi=0 \, , \tag{2.28}$$

$$i\partial\psi-g\phi\psi=0 \, . \tag{2.29}$$

Exactly in the same way as we did in Sec. II B, we write down the expectation value of the equation of motion (2.28) to obtain

$$\Box\Phi+V'(\Phi)+\frac{V^{(3)}(\Phi)}{2}\langle\varphi^2\rangle+\cdots+g\langle\bar{\psi}\psi\rangle=0 \, . \tag{2.30}$$

The expectation values $\langle\varphi^2\rangle$ and $\langle\bar{\psi}\psi\rangle$ are evaluated by the relaxation-time approximation as

$$\langle\varphi^2\rangle=\int\frac{d^3p}{(2\pi)^3}\frac{n_{B0}}{\omega(p)}+\int\frac{d^3p}{(2\pi)^3}\frac{\delta n_B}{\omega(p)} \, , \tag{2.31}$$

$$\langle\bar{\psi}\psi\rangle=\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}n_{F0}+\int\frac{d^3p}{(2\pi)^3}\frac{M}{E_p}\delta n_F \, , \tag{2.32}$$

$$n_{B0}=1/(e^{\beta\omega(p)}-1) \;\; [\omega(p)=(\vec{p}^{\,2}+m^2)^{1/2},m^2=V''(\Phi)] \, , \tag{2.33}$$

$$n_{F0}=1/(e^{\beta E(p)}+1) \;\; [E(p)=(\vec{p}^{\,2}+M^2)^{1/2},M^2=g^2\Phi^2] \, , \tag{2.34}$$

$$\delta n_B\cong-\tau_B\dot{n}_{B0}=\tau_B n_{B0}(1+n_{B0})\frac{V^{(3)}(\Phi)}{2}\frac{\dot{\Phi}}{\omega(p)} \, , \tag{2.35}$$

$$\delta n_F\cong-\tau_F\dot{n}_{F0}=\tau_F n_{F0}(1-n_{F0})g^3\Phi\frac{\dot{\Phi}}{E(p)} \, . \tag{2.36}$$

Substitution of Eqs. (2.31) and (2.32) with (2.33)—(2.36) into Eq. (2.30) gives

$$\Box\Phi+V'_{\rm eff}(\Phi)+\eta(\Phi)\dot{\Phi}=0 \, , \tag{2.37}$$

where

$$\begin{aligned}V'_{\rm eff}(\Phi)&=V'(\Phi)+\frac{V^{(3)}(\Phi)}{2}\int\frac{d^3p}{(2\pi)^3}\frac{n_{B0}}{\omega(p)} \\ &\quad+g^2\Phi\int\frac{d^3p}{(2\pi)^3}\frac{n_{F0}}{E(p)}+\cdots \, , \end{aligned} \tag{2.38}$$

$$\begin{aligned}\eta(\Phi)&=\frac{[V^{(3)}(\Phi)]^2}{4}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega(p)^2}\tau_B n_{B0}(1+n_{B0}) \\ &\quad+g^4\Phi^2\int\frac{d^3p}{(2\pi)^3}\frac{1}{E(p)^2}\tau_F n_{F0}(1-n_{F0}) \, . \end{aligned} \tag{2.39}$$

III. KUBO-TYPE FORMULA

So far we have not used the precise form of the nonequilibrium statistical operator ρ. In this section we explicitly construct ρ following the spirit of Zubarev's method[8,9] and derive the fluctuation-dissipation Kubo-type formula for the friction coefficient $\eta(\Phi)$ in the linear-response approximation [Eq. (3.21)]. As shown in the Appendix, in the lowest order in perturbation theory, the resultant expression for $\eta(\Phi)$ agrees with the one obtained in Sec. II from the relaxation-time approximation. Nevertheless the closed form for the friction coefficient is valuable both in the higher-order computation and in possible nonperturbative evaluations.

We divide the derivation into the following five steps and the illustration is made in the simple scalar theory discussed in the previous section.

A. Hamiltonian density

In the original Lagrangian,

$$L=\tfrac{1}{2}(\partial_\mu\phi)^2-V(\phi) \tag{3.1}$$

we put

$$\phi=\Phi+\varphi \tag{3.2}$$

and regard the fluctuation field φ around the c-number function Φ as a dynamical variable. We have

$$L=\tfrac{1}{2}\dot{\Phi}^2+\dot{\Phi}\dot{\varphi}+\tfrac{1}{2}\dot{\varphi}^2-\tfrac{1}{2}[\partial_i(\Phi+\varphi)]^2-V(\Phi+\varphi) \, . \tag{3.3}$$

It is very convenient to change the Lagrangian density (3.3) by a total time derivative so that the interference term $\dot{\Phi}\dot{\varphi}$ is eliminated:

$$L'=L-\frac{d}{dt}\left(\dot{\Phi}\varphi+\int^{t}\dot{\Phi}^2dt'\right)$$

$$=-\tfrac{1}{2}\dot{\Phi}^2+\tfrac{1}{2}\dot{\varphi}^2-\tfrac{1}{2}[\partial_i(\Phi+\varphi)]^2-V(\Phi+\varphi)-\ddot{\Phi}\varphi\ . \qquad (3.4)$$

Then the standard canonical formalism gives the Hamiltonian (we omit the prime on H) as

$$H=H_\Phi+H_\varphi+H_1\ , \qquad (3.5a)$$

$$H_\Phi=\tfrac{1}{2}\dot{\Phi}^2+\tfrac{1}{2}(\partial_i\Phi)^2+V(\Phi)\ ,$$

$$H_\varphi=\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\partial_i\varphi)^2+\frac{V''(\Phi)}{2!}\varphi^2+\frac{V^{(3)}(\Phi)}{3!}\varphi^3+\cdots\ , \qquad (3.5b)$$

$$H_1=\varphi[\Box\Phi+V'(\Phi)]\ . \qquad (3.5c)$$

We keep the linear term (3.5c) in the fluctuation field φ, since Φ is chosen as the thermodynamical average of the original Heisenberg operator ϕ rather than a classical solution.

In effect we have chosen a suitable Hamiltonian in which the influences of the time-dependent order parameter Φ on the fluctuation are minimized.

Our canonically transformed Hamiltonian is time dependent due to the c-number background field and is related to the original conserved Hamiltonian $\overline{H}$ as

$$H=\overline{H}+\varphi\ddot{\Phi}-\pi\dot{\Phi}\ . \qquad (3.6)$$

For readers who are still uneasy about our change of the Hamiltonian, we can say that in the subsequent construction of the local equilibrium operator our choice (3.6) is most economical, though the original $\overline{H}$ leads to the same local equilibrium operator and hence to the nonequilibrium statistical operator.

B. Local statistical operator

Throughout this paper we always assume that in a small region of space-time, local thermal equilibrium holds. Roughly speaking, the local equilibrium statistical operator is the product of the statistical operator of the Gibbs ensemble at each spatial point. Following the spirit of Zubarev's method, we propose

$$\rho_l=Q_l{}^{-1}\exp\left\{-\int d^3x\beta(xt)[H(xt)+j(xt)\varphi(xt)]\right\}\ , \qquad (3.7)$$

$$\mathrm{tr}\rho_l=1\ .$$

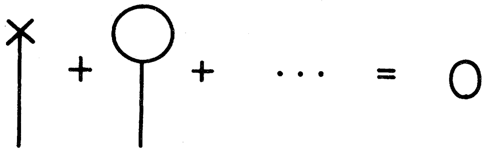

FIG. 4. Cancellation of the tadpole contribution by the source term.

(To simplify our presentation, we have not included the velocity field here.) The source term $j\varphi$ is inserted to ensure the important property of the local statistical operator

$$\langle\phi\rangle_0=\mathrm{tr}(\rho_l\phi)=\langle\phi\rangle=\Phi\ . \qquad (3.8)$$

The thermodynamical parameter j is internally fixed by the condition exactly in the same way that the temperature β^{-1} is internally determined by the condition

$$\langle H\rangle_0=\mathrm{tr}(\rho_l H)=\langle H\rangle\ . \qquad (3.9)$$

[If we employ $\overline{H}$ instead of H, we would need an extra source term $k(xt)\pi(xt)$ and the further condition

$$\langle\pi\rangle_0\equiv\mathrm{tr}(\rho_l\pi)=\langle\pi\rangle$$

and will end up with the same expression for ρ_l.] Absorbing the c-number part of the Hamiltonian into the normalization and the linear term H_1 in the Hamiltonian into the source term, we have

$$\rho_l=Q'_l{}^{-1}\exp\left\{-\int d^3x\,\beta(xt)[H_\varphi(xt)+F(xt)\varphi(xt)]\right\}\ , \qquad (3.10)$$

$$F=j+\Box\Phi+V'(\Phi)\ .$$

Let us see how the requirement (3.8) determines F (or j). With $\phi=\Phi+\varphi$ [Eq. (3.2)], Eq. (3.8) becomes

$$\langle\varphi\rangle_0=0\ . \qquad (3.11)$$

This is nothing but the condition that the tadpole contribution cancels the source term (Fig. 4). In the one-loop approximation, we have

$$F=-\frac{V^{(3)}(\Phi)}{2!}\langle\varphi^2\rangle_0\ . \qquad (3.12)$$

This completes the construction of the local statistical operator.

C. Nonequilibrium statistical operator

Zubarev himself did not mention any possible application of his method of the nonequilibrium statistical operator to the time development of the order parameter in his textbook.[9] A natural extension of Zubarev's method gives the following nonequilibrium statistical operator for our problem as

$$\rho=Q'^{-1}\exp\left\{-\epsilon\int_{-\infty}^{t}dt'e^{\epsilon(t'-t)}\int d^3x'\beta(x't')[H_\phi(x't')+F(x't')\varphi(x't')]\right\}\ (\epsilon\rightarrow+0)\ , \qquad (3.13)$$

where Q' is the normalization factor so that $\mathrm{tr}\rho=1$. The exponent of the nonequilibrium operator is a sort of long time average

$$\epsilon\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\cdots$$

of the exponent of the local equilibrium operator (3.10).

It is easy to see the time independence of ρ, which we have used in the derivation of the equation for the thermodynamical average of the Higgs field in the previous section. The time derivative of the exponent is

$$\epsilon\left[-\int d^3x'\beta(x't)[H_\varphi(x't)+F(x't)\varphi(x't)]+\epsilon\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\int d^3x'\beta(x't')[H_\varphi(x't')+F(x't')\varphi(x't')]\right].$$

The first term in the large parentheses is the exponent of the local statistical operator and the second one is that of the nonequilibrium operator. They are assumed to be finite. Because of the extra ϵ in front of the parentheses, the above quantity vanishes in the limit $\epsilon\to+0$.

Writing

$$\epsilon\, e^{\epsilon(t'-t)}=\frac{d}{dt'}(e^{\epsilon(t'-t)})$$

in the expression (3.13) and integrating by parts we have

$$\begin{aligned}\rho=Q'^{-1}\exp\Bigg[&-\int d^3x\,\beta(xt)[H_\varphi(xt)+F(xt)\varphi(xt)]\\ &+\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\int d^3x'\beta(x't')\left[c\text{-number terms}-\dot{\varphi}[\Box\Phi+V'_{\text{eff}}(\Phi)]+\dot{F}\varphi+\frac{V^{(3)}(\Phi)}{2!}\dot{\Phi}\varphi^2+\cdots\right]\\ &+\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\int d^3x'\dot{\beta}(x't')[H_\varphi(x't')+F(x't')\varphi(x't')]\Bigg].\end{aligned}\tag{3.14}$$

Here we have used the field equation for φ and the fact

$$\begin{aligned}V'_{\text{eff}}(\Phi)&=V'(\Phi)+\frac{V^{(3)}(\Phi)}{2}\langle\varphi^2\rangle_0+\cdots\\ &=V'(\Phi)-F,\end{aligned}\tag{3.15}$$

in the manipulation of the second term in the exponent in Eq. (3.14). The first term in the exponent represents the local equilibrium part. The rest causes nonequilibrium effects.

D. Linear-response approximation

In order to see the essential point of our problem let us assume for the moment that the temperature increase and inhomogeneity by the space-time development of our order parameter are negligible. Such a case may be realized if a huge heat reservoir of particles is present without being affected by the order parameter. The expression which includes the temperature and velocity change in space-time will be given in Sec. VI.

Let us assume that the time development of the Higgs field is so slow that we can expand the nonequilibrium statistical operator ρ around the local equilibrium operator ρ_l and keep the linear term in $\dot{\Phi}$. Write

$$\begin{aligned}A&=\beta\int d^3x(H_\varphi+F\varphi),\\ B&=\beta\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\left[c\text{-number terms }-\dot{\varphi}[\Box\Phi+V'_{\text{eff}}(\Phi)]+\dot{F}\varphi+\frac{V^{(3)}(\Phi)}{2}\dot{\Phi}\varphi^2+\cdots\right].\end{aligned}\tag{3.16}$$

We use the expansion formula

$$e^{-A+B}=e^{-A}+\int_0^1 d\tau\, e^{A\tau}B\,e^{-A\tau}e^{-A}+\cdots$$

to obtain an approximation

$$\rho\simeq\left[1+\int_0^1 d\tau(e^{A\tau}B\,e^{-A\tau}-\langle B\rangle_0)\right]p_l\,,\tag{3.17}$$

where

$$\langle B\rangle_0=\mathrm{tr}(\rho_l B)\,.$$

Then we obtain

$$\langle\varphi^2\rangle\simeq\langle\varphi^2\rangle_0+\beta\int_{-\infty}^{t}dt'e^{\epsilon(t'-t)}\Bigg[(\varphi^2(x,t),\varphi^2(x't'))\frac{V^{(3)}(\Phi)}{2!}\dot{\Phi}(x't')-(\varphi^2(xt),\dot{\varphi}(x't'))[\Box\Phi+V'_{\text{eff}}(\Phi)]$$
$$+(\varphi^2(xt),\varphi(x't'))\dot{F}\Bigg]\ . \tag{3.18}$$

The parentheses (,) are defined by

$$(X,Y)=\int_0^1 d\tau\langle X(e^{-A\tau}Ye^{A\tau}-\langle Y\rangle_0)\rangle_0\ , \tag{3.19}$$

which is called a relaxation function or a correlation function of X and Y.

Note that the relaxation function has a short time correlation:

$$(\varphi^2(xt),\varphi^2(x't'))\sim\exp\left[-\frac{|t-t'|}{\tau}\right],\ \text{etc.}\ ,$$

with τ being a relaxation time (see Appendix). As we always assume, Φ varies slowly and does not change significantly during the collision time of constituent particles. We may take the factors $V^{(3)}(\Phi)\dot{\Phi}$, etc., out of the integration. Substitute the expression (3.18) for $\langle\varphi^2\rangle$ into the equation for Φ

$$\Box\Phi+V'(\Phi)+\frac{V^{(3)}(\Phi)}{2!}\langle\varphi^2\rangle+\cdots=0$$

obtained in the previous section on the very general ground. Then we end up with the equation

$$\Box\Phi+V'_{\text{eff}}(\Phi)+\eta(\Phi)\dot{\Phi}=0\ , \tag{3.20}$$

with the friction coefficient being

$$\eta(\Phi)=\beta\int d^3x'\int_{-\infty}^{t}dt'e^{\epsilon(t'-t)}\Bigg[\frac{\partial}{\partial\Phi}H_\varphi(xt),\frac{\partial}{\partial\Phi}H_\varphi(x't')$$
$$+\frac{\partial F}{\partial\Phi}\varphi+\eta(\Phi)\dot{\varphi}\Bigg]\ . \tag{3.21}$$

Equation (3.21) is a Kubo-type formula and a manifestation of the fluctuation-dissipation theorem.

E. Evaluation of the Kubo formula

The evaluation of the correlation function in perturbation theory has been done in Ref. 8. In the Appendix, we shall give a brief sketch of its procedure. The only term which does not vanish in the lowest-order approximation in perturbation theory is

$$\left[\frac{V^{(3)}(\Phi)}{2!}\right]^2\beta\int d^3x'\int_{-\infty}^{t}dt'e^{\epsilon(t'-t)}(\varphi^2(xt),\varphi^2(x't'))\ . \tag{3.22}$$

In the lowest order in the skeleton expansion (Fig. 5) for the expression (3.22), we get the same expression for $\eta(\Phi)$ derived by the relaxation-time approximation of the kinetic equation. The obvious merit of the fluctuation-dissipation formula is that we can compute the higher-order corrections in perturbation theory and maybe in a nonperturbative way.

IV. ENTROPY PRODUCTION

In this section we show the increase of entropy due to the friction force. The generation of entropy partially supports the validity of our approach to nonequilibrium processes. It is indispensable for the application to the inflationary Universe. Following Zubarev,[9] we define entropy as

$$S=-\langle\ln\rho_l\rangle\ , \tag{4.1}$$

where ρ_l is the local equilibrium statistical operator (3.10). Here we cannot choose the quantity $-\langle\ln\rho\rangle$ as entropy, since it is time independent from Eq. (2.15). Equation (4.1) can be intuitively interpreted as follows. In the hydrodynamical regime thermal equilibrium is locally realized on a scale of mean free path and entropy is locally defined at each domain. So entropy of the whole system is given by summing up the local entropy. From Eq. (3.10), the explicit form of entropy is

$$S=\ln Q_l'+\int d^3x\,\beta(xt)\langle H_\varphi(xt)+F(xt)\varphi(xt)\rangle$$
$$\equiv\int d^3x\,S(x)\ , \tag{4.2}$$

where

$$Q_l'=\mathrm{tr}\left[\exp\left[-\int d^3x\,\beta(xt)[H_\varphi(xt)+F(xt)\varphi(xt)]\right]\right], \tag{4.3}$$

and $S(x)$ is entropy density. Differentiating Q_l' with respect to β, F, and Φ, we obtain the thermodynamical relations

$$\frac{\delta\ln Q_l'}{\delta\beta(xt)}=-\langle H_\varphi(xt)\rangle_0=-\langle H_\varphi(xt)\rangle\ , \tag{4.4a}$$

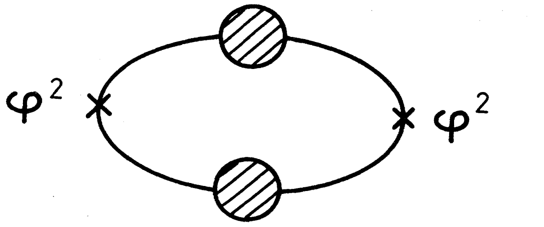

FIG. 5. Feynman diagram of the lowest-order skeleton expansion for the correlation function in Eqs. (3.12) and (A1). Hatched parts denote the self-energy.

$$\frac{\delta \ln Q_l'}{\delta F(xt)} = -\beta(xt)\langle \varphi(xt)\rangle_0$$

$$= -\beta\langle \varphi(xt)\rangle = 0 , \tag{4.4b}$$

$$\frac{\delta \ln Q_l'}{\delta \Phi(xt)} = -\beta(xt)[\tfrac{1}{2}V'''(\Phi)\langle \varphi^2(xt)\rangle_0 + \cdots] . \tag{4.4c}$$

The second and third equalities of Eqs. (4.4a) and (4.4b) are derived from Eqs. (3.8), (3.9), and (3.11).

Now it is easy to show that the friction force generates entropy. Differentiating the entropy (4.2) with respect to t and using relations (4.4), we obtain

$$\dot{S}(x) = \beta(xt)\dot{\Phi}(xt)[\tfrac{1}{2}V'''(\Phi)(\langle \varphi^2\rangle - \langle \varphi^2\rangle_0) + \cdots] . \tag{4.5}$$

Since the terms in the brackets are identical to the friction force, the change of entropy can be expressed as

$$\dot{S}(x) = \beta(xt)\eta(\Phi)[\dot{\Phi}(xt)]^2 . \tag{4.6}$$

The rest is to prove that the friction coefficient is always positive. The full expression for the friction coefficient is given by

$$\eta(\Phi) = \beta \int d^3x' \times \int_{-\infty}^{t} dt' e^{\epsilon(t'-t)} \left(\frac{\partial}{\partial \Phi} H_\varphi(xt), \frac{\partial}{\partial \Phi} H_\varphi(x't') + \frac{\partial F}{\partial \Phi}\varphi + \eta(\Phi)\dot{\varphi} \right) . \tag{4.7}$$

Here we have used the relation $j(xt) = \eta(\Phi)\dot{\Phi}(xt)$ which is derived from Eqs. (3.10), (3.15), and (3.20). Furthermore, Eq. (4.7) becomes

$$\eta(\Phi) = \beta \int d^3x' \int_{-\infty}^{t} dt' e^{\epsilon(t-t')}(C(x), C(x't')) , \tag{4.7'}$$

where

$$C(xt) = \frac{\partial}{\partial \Phi}[H_\varphi(xt) + F(xt)\varphi(xt)] + \eta(\Phi)\dot{\varphi}(xt) , \tag{4.8}$$

since $\langle \varphi \rangle = \langle \varphi \rangle_0 = \langle \dot{\varphi} \rangle = \langle \dot{\varphi} \rangle_0 = 0$. As shown in the Appendix, the friction coefficient can be expressed in terms of the retarded Green's function:

$$\eta(\Phi) = i \frac{d}{dp_0} \widetilde{G}_R(p_0, \vec{p}) \bigg|_{p_0=0,\, \vec{p}=0} , \tag{4.9}$$

where

$$\widetilde{G}_R(p_0, \vec{p}) = \int d^4x\, e^{ip(x-x')}\{ -i\theta(t-t')\langle [C(xt), C(x't')]\rangle \} . \tag{4.10}$$

We use the spectral representation of the retarded Green's function:

$$\widetilde{G}_R(p_0, \vec{p}) = \int d\omega \frac{J(\omega, \vec{p})(1 - e^{-\beta\omega})}{p_0 - \omega + i\epsilon} . \tag{4.11}$$

Here the spectral function takes the form

$$J(p) = Q_l'^{\,-1} \sum_{n,m} |\langle n \,|\, C(0) \,|\, m \rangle|^2 e^{-\beta E_n} \delta^4(p_m - p_n - p) . \tag{4.12}$$

Substituting the spectral representation (4.11) into the expression of the friction coefficient (4.9), we obtain

$$\eta(\Phi) = 2\pi\beta J(0) . \tag{4.13}$$

Since Eq. (4.12) ensures the positivity of the spectral function, we conclude that the entropy certainly increases.

The entropy production can be intuitively explained as follows. As the c-number part $\Phi(xt)$ varies in time, it disturbs the fluctuation $\varphi(xt)$ around itself. The disturbances generate entropy and have a tendency to resist the motion of $\Phi(xt)$. Using the equation for Φ in (1.1), we obtain

$$\dot{S}(x) = -\beta(xt)\frac{d}{dt}\{ \tfrac{1}{2}\dot{\Phi}^2(xt) + \tfrac{1}{2}[\partial_k \Phi(xt)]^2 + V_{\text{eff}}(\Phi)\}. \tag{4.14}$$

This equation means that entropy is generated by the release of the energy stored in the c-number part of Φ. Although our Hamiltonian H, Eq. (3.5), explicitly depends on time, its average value $\langle H \rangle = \text{tr}(\rho H)$ is conserved. This can be written as

$$\frac{d}{dt}(\langle H_\varphi \rangle - \langle H_\varphi \rangle_0) = -\frac{d}{dt}[\tfrac{1}{2}\dot{\Phi}^2 + \tfrac{1}{2}(\partial_k \Phi)^2 + V_{\text{eff}}(\Phi)] . \tag{4.15}$$

From Eq. (4.15), we can see that we have to choose $\langle H_\varphi \rangle - \langle H_\varphi \rangle_0$ rather than $\langle H_\varphi \rangle$ as the internal energy of radiation. The local equilibrium average $\langle H_\varphi \rangle_0$, which we have subtracted from $\langle H_\varphi \rangle$, corresponds to a contribution of radiation to the effective potential. Thus we replace the Hamiltonian H_φ by $H_\varphi - \langle H_\varphi \rangle_0$ in the statistical operators ρ and ρ_0. For example, the local statistical operator becomes

$$\rho_l = Q'''_l{}^{-1} \exp \left[-\int d^3x\, \beta(xt)\{[H_\varphi(xt) - \langle H_\varphi(xt)\rangle_0] + F(xt)\varphi(xt)\} \right] . \tag{4.16}$$

Since this change of Hamiltonian is just a c number, it does not alter the results we have previously obtained. Introducing the local free energy $\mathscr{F}(xt)$ and the internal energy $u(xt)$ as

$$\int d^3x\, \beta(xt)\mathscr{F}(xt) = -\ln Q_l'' , \quad u(xt) = \langle H_\varphi \rangle - \langle H_\varphi \rangle_0 ,$$

we obtain familiar relations of thermodynamics:

$$\mathscr{F}(xt) = u(xt) - \beta^{-1}(xt)S(xt) , \tag{4.17}$$

$$du(xt)=\beta^{-1}(xt)dS(xt)\ . \tag{4.18}$$

Equations (4.14) and (4.18) imply the release of the energy of the c-number part Φ to the radiation.

V. EVALUATION OF FRICTION COEFFICIENT AT HIGH TEMPERATURE

In order to evaluate the expression for the friction coefficient [Eq. (2.27)] we need the relaxation time τ as a function of momentum $\vec{p}$. In the scalar model we have

$$\tau^{-1}(p)=\frac{1}{2\omega(p)}\int\frac{d^3p_1}{(2\pi)^3 2\omega(p_1)}\frac{d^3p_2}{(2\pi)^3 2\omega(p_2)}\frac{d^3p_3}{(2\pi)^3 2\omega(p_3)}(2\pi)^4\delta^4(p+p_1-p_2-p_3)$$
$$\times\frac{\lambda^2}{4}\{n_0(p_1)[1+n_0(p_2)][1+n_0(p_3)]-[1+n_0(p_1)]n_0(p_2)n_0(p_3)\}\ , \tag{5.1}$$

where

$$\lambda=V^{(4)},\quad \omega(p)=(m^2+\vec{p}^{\,2})^{1/2},\quad \text{and } m^2=V''(\Phi)\ .$$

We have taken account of the four-point contact interaction only for simplicity and do not consider the nonrenormalizable interactions (i.e., $V^{(n)}=0,\ n\geq 5$). The Born diagrams which come from the cubic interaction become higher-order corrections at high temperature and in relativistic regions. We can carry out the eight integrations of Eq. (5.1) to obtain

$$\tau^{-1}(p)=\frac{\lambda^2}{2^7\pi^3}\frac{1}{\beta\omega p}\left[\int_m^\omega d\omega'(\omega'^2-m^2)^{1/2}e^{\beta\omega'}\left[\frac{1}{e^{\beta\omega'}-1}-\frac{1}{e^{\beta\omega'}-e^{-\beta\omega}}\right]\ln\left[\frac{\sinh\frac{1}{2}\beta\omega}{\sinh\frac{1}{2}\beta\omega'}\right]\right.$$
$$+\int_m^\omega d\omega'(\omega'^2-m^2)^{1/2}e^{\beta\omega'}\left[\frac{1}{e^{\beta\omega'}-1}+\frac{1}{e^{\beta\omega}-e^{\beta\omega'}}\right]\ln\left[\frac{1-e^{-\beta\omega}}{1-e^{-\beta\omega'}}\right]$$
$$\left.+p\int_\omega^\infty d\omega' e^{\beta\omega'}\left[\frac{1}{e^{\beta\omega'}-e^{\beta\omega}}-\frac{1}{e^{\beta\omega'}-1}\right]\ln\left[\frac{e^{\beta\omega'}-e^{\beta(\omega-\omega')}}{e^{\beta\omega'}-1}\right]\right]\ . \tag{5.2}$$

In the high-temperature limit $T\gg m$, we have

$$\tau^{-1}\approx\frac{\lambda^2}{2^9 3\pi}\frac{T^2}{\omega} \tag{5.3}$$

for $p\gg m$. It is worthwhile to point out that $\tau^{-1}\omega$ is constant in the high-temperature and ultrarelativistic limits.

Now let us evaluate the friction coefficient at high temperature. Owing to the Bose factor in the integrand, the most important contribution comes from the region $T\sim p\gg m$ and we obtain

$$\eta(\Phi)\approx\frac{[V^{(3)}(\Phi)]^2}{\lambda^2}\frac{2^5 3}{\pi}\frac{1}{T}\ln\left[\frac{T}{m}\right]\ . \tag{5.4}$$

VI. SUMMARY AND DISCUSSIONS

We have derived the equation of motion for the thermodynamical average of the Higgs field [Eq. (1.1)] with the friction term both by the kinetic theory and by Zubarev's method of nonequilibrium statistical operator. The friction coefficient is expressed by the Kubo-type formula which agrees with the result of the relaxation-time approximation of kinetic theory in the lowest order in perturbation theory. It is shown that the total entropy increases when the Higgs field rolls down and that the potential energy of the classical Higgs field turns into the radiation of Higgs quanta. The high-temperature behavior of the friction coefficient is roughly

$$\eta(\Phi)\sim\frac{(V^{(3)})^2}{\lambda^2}\frac{1}{T}\ln\left[\frac{T}{m}\right]\ .$$

We have a few remarks on the possible applications to the cosmology, particularly to the inflationary stage of the early Universe. In the kinetic theory approach we can easily go over to Friedmann space-time, if we neglect the Hawking radiation by the background gravitational field.[10] We just quote the final result in the spatially flat case, which also includes the effect of the temperature increase by friction:

$$\ddot{\Phi}+\frac{3\dot{a}}{a}\dot{\Phi}-a^{-2}\Delta\Phi+V'_{\text{eff}}(\Phi)+\eta(\Phi)\dot{\Phi}=-\gamma(\Phi)\partial_i u_i\ . \tag{6.1}$$

Here we have used the ideal-gas relation

$$\dot{\beta}=\beta c_s{}^2\partial_i u_i \tag{6.2}$$

and

$$\gamma=\frac{\beta}{2}V^{(3)}(\Phi)\int\frac{d^3p}{(2\pi)^3}\frac{\tau}{2\omega^2}\left[c_s{}^2\omega^2-\frac{\vec{p}^{\,2}}{3}\right]n_0(1+n_0)\ , \tag{6.3}$$

with c_s and u_i being the sound velocity and the velocity field, respectively. $a(t)$ is the scale factor of the Robertson-Walker metric:

$$ds^2=dt^2-a^2(t)d\vec{x}^2 , \tag{6.4}$$

and the temperature in V'_{eff}, η, and γ is the red-shifted temperature

$$T=T_0a^{-1}(t) . \tag{6.5}$$

However as we have emphasized in the Introduction, Eq. (6.1) is valid only if the hydrodynamical condition is satisfied:

$$\tau \ll |a/\dot{a}|, |\Phi/\dot{\Phi}| .$$

One has to be careful about the region in which Eq. (6.1) is applicable. We also have to study the low-temperature behavior of the friction coefficient. Detailed analysis is in progress.

On the other hand it is not straightforward to more rigorously derive the equation for the order parameter in curved space-time on the basis of Zubarev's method.

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APPENDIX

In this appendix, we give a brief sketch of the evaluation of the friction coefficient in perturbation theory. A similar procedure has been explained in detail in Ref. 8.

Let us express the friction coefficient in a more convenient form for practical calculations. With some manipulations,[8] Eq. (3.21) can be rewritten as

$$\begin{aligned}\eta(\Phi)=&-\frac{[V^{(3)}(\Phi)]^2}{4}\\&\times\int d^3x'\int_{-\infty}^{t}dt_1e^{\epsilon(t_1-t)}\\&\times\int_{-\infty}^{t_1}dt'G_R(x-x',t-t') ,\end{aligned} \tag{A1}$$

where

$$G_R(x-x',t-t')=-i\theta(t-t')\langle[\varphi^2(xt),\varphi^2(x',t')]\rangle_0$$

is the retarded Green's function and $\langle\cdots\rangle_0$ denotes the average over the thermal equilibrium distribution. Integrations over space-time lead to an expression for $\eta(\Phi)$:

$$\eta(\Phi)=\frac{i}{4}[V^{(3)}(\Phi)]^2\frac{d}{dk_0}G_R(k)\Bigg|_{\substack{k_0=0\\ \vec{k}=0}} . \tag{A2}$$

In order to calculate the retarded Green's function, we make use of the temperature Green's function

$$G_\beta(x-x',\tau-\tau')=-i\langle T_\tau[\varphi^2(x\tau)\varphi^2(x'\tau')]\rangle_0 , \tag{A3}$$

where $\tau=it$ is imaginary time, since the Feynman diagrammatic method is available for the computation of this function.[11] First we evaluate the temperature Green's function (A3) in perturbation theory, then analytically continue it into the retarded Green's function by means of the relation[12]

$$G_R(k_0,\vec{k})=G_\beta(\omega_n,\vec{k})\,|_{i\omega_n\to k_0+i\epsilon} . \tag{A4}$$

In the lowest order in the skeleton expansion (Fig. 5) the temperature Green's function (A3) is given by

$$\begin{aligned}iG_\beta(\tau_1-\tau_2,\vec{k})&=2\int\frac{d^3p}{(2\pi)^3}i\tilde{D}_\beta(\tau_1-\tau_2,\vec{k}+\vec{p})i\tilde{D}_\beta(\tau_2-\tau_1,\vec{p})\\&=2\int\frac{d^3p}{(2\pi)^3}\int d\eta_1d\eta_2n_0(\eta_1)n_0(\eta_2)\rho(\eta_1,\vec{k}+\vec{p})\rho(\eta_2,\vec{k})e^{(\eta_1+\eta_2)|\tau_1-\tau_2|} .\end{aligned} \tag{A5}$$

Here $\tilde{D}$ represents the full propagator of the scalar field φ and admits the spectral representation

$$\tilde{D}_\beta(\omega_n,\vec{k})=\int d\omega\frac{\rho(\omega,\vec{k})}{i\omega_n-\omega} , \tag{A6}$$

or its Fourier inverse transform[13]

$$i\tilde{D}_\beta(\tau,\vec{k})=\int_{-\infty}^{\infty}d\eta\,\rho(\eta,\vec{k})n_0(\eta)e^{\eta|\tau|} , \tag{A7}$$

where n_0 is the boson factor $n_0(\eta)=(e^{\beta\eta}-1)^{-1}$.

If we used the naive perturbation expansion rather than the skeleton expansion, we would have divergent results. We perform the double Fourier transformations of (A5) with respect to τ_1 and τ_2:

$$iG_\beta(\omega_1,\omega_2,\vec{k})=-\int_0^\beta d\tau_1\int_0^\beta d\tau_2e^{i(\omega_1\tau_1-\omega_2\tau_2)}iG_\beta(\tau_1-\tau_2,\vec{k}) .$$

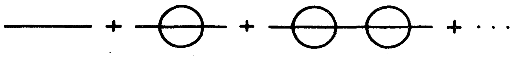

FIG. 6. Lowest-order skeleton expansion for the full propagator $\tilde{D}_R(k)$.

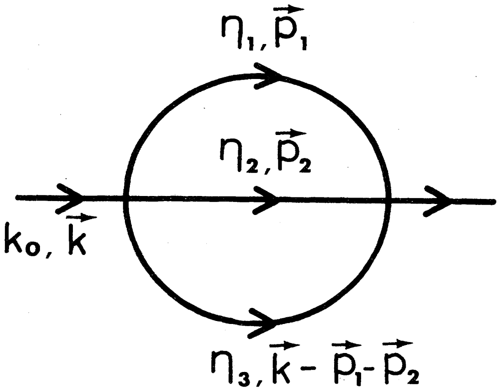

FIG. 7. Feynman diagram for the self-energy $\Sigma_R(k)$.

The integrations over τ_1 and τ_2 give two kinds of terms, one proportional to $\delta_{\omega_1,\omega_2}$ and the other not proportional. The second kind of term should be discarded from the fact that the Green's functions should be functions of the difference between the coordinates only.[13] Performing the analytic continuation (A4), we obtain

$$G_R(k)=-2\int\frac{d^3p}{(2\pi)^3}\int d\eta_1 d\eta_2 n_0(\eta_1)n_0(\eta_2)\rho(\eta_1,\vec{k}+\vec{p})\rho(\eta_2,\vec{p})\left[\frac{1}{k_0-\eta_1-\eta_2+i\epsilon}-\frac{1}{k_0+\eta_1+\eta_2+i\epsilon}\right]. \tag{A8}$$

Now we evaluate the spectral function ρ which is given by the imaginary part of the full propagator. In the lowest-order expansion in Fig. 6, the full propagator has the form

$$D_R(k)=\frac{Z_2}{(k_0+i\epsilon)^2-\vec{k}^2-m^2-\Sigma_R(k_0,\vec{k})}. \tag{A9}$$

Here the divergence of the real part of Σ_R is removed by wave-function and mass renormalizations and the temperature-dependent contribution to the mass is ignored. The self-energy Σ_R shown in Fig. 7 can be evaluated in the same manner as Eq. (A8):

$$\Sigma_R(k_0,\vec{k})=-\frac{[V^{(4)}(\Phi)]^2}{6}\int\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\int d\eta_1 d\eta_2 d\eta_3 n_0(\eta_1)n_0(\eta_2)n_0(\eta_3)\rho_0(\eta_1,\vec{p}_1)\rho_0(\eta_2,\vec{p}_2)\rho_0(\eta_3,\vec{p}_3)$$
$$\times\left[\frac{1}{k_0+i\epsilon-\eta_1-\eta_2-\eta_3}-\frac{1}{k_0+i\epsilon+\eta_1+\eta_2+\eta_3}\right], \tag{A10}$$

where ρ_0 is the spectral function for the free propagator

$$\rho_0(\eta,\vec{p})=\mathrm{sgn}(\eta)\delta(\eta^2-\vec{p}^{\,2}).$$

Putting $\mathrm{Im}\Sigma_R(k_0,k)=-2k_0\Gamma(\vec{k})$, we obtain the full propagator

$$\tilde{D}_R(k)=\frac{1}{(k_0+i\Gamma)^2-\omega^2(k)} \tag{A11}$$

with

$$\omega(k)=(\vec{k}^2+m^2)^{1/2}.$$

The retarded Green's function can be evaluated by means of the spectral function $\rho(k_0,\vec{k})=-(1/\pi)\mathrm{Im}\tilde{D}_R(k_0,\vec{k})$. For $\omega\gg\Gamma(k)$, we finally obtain the expression for $\eta(\Phi)$ in the lowest-order skeleton expansion in Fig. 5 as

$$\eta(\Phi)=\beta\left[\frac{V^{(3)}(\Phi)}{2}\right]^2\int\frac{d^3p}{(2\pi)^3 2\omega^2(p)}\tau n_0(1+n_0), \tag{A12}$$

$$\tau^{-1}=\frac{1}{2\omega(p)}\int\frac{d^3p_1}{(2\pi)^3 2\omega(p_1)}\int\frac{d^3p_2}{(2\pi)^3 2\omega(p_2)}\int\frac{d^3p_3}{(2\pi)^3 2\omega(p_3)}(2\pi)^4\delta^4(p+p_1-p_2-p_3)\frac{\lambda^2}{4}$$
$$\times\{n_0(p_1)[1+n_0(p_2)][1+n_0(p_3)]-[1+n_0(p_1)]n_0(p_2)n_0(p_3)\}. \tag{A13}$$

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