Cosmological perturbations in inflationary-universe models

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(Received 3 October 1983)

We study the growth of energy-density perturbations in inflationary-universe models, applying an extension of Bardeen's gauge-invariant framework derived in a previous paper. The complete analysis is exemplified in the case of the "new inflationary universe" of Linde and Albrecht and Steinhardt. For this model we obtain the following result: the amplitude of energy-density fluctuations at horizon crossing is of order 50, far too large to match with the usual pictures of galaxy formation. Our result agrees with other published analyses. We conclude that the amplification of energy-density perturbations is determined by the change in the equation of state between initial and final Hubble radius crossing. It is, however, independent of the phase structure between the two crossings. In particular, it is independent of the reheating mechanism. We also derive a simple formula which describes the complete evolution of perturbations outside the horizon whenever entropy and anisotropic stress perturbations are unimportant. Finally, we comment on previous published methods and discuss their limitations.

I. INTRODUCTION

Two years ago Guth[1] proposed a new variant of the standard big-bang cosmology, which could solve many of the problems that arise in the usual formulation (e.g., the horizon, flatness, and monopole problems) without destroying the successes (e.g., nucleosynthesis). The main idea was to consider matter in the very early universe as described by a grand unified field theory with a metastable ground state corresponding to unbroken gauge symmetry, and a stable ground state (the state we are in today) with broken symmetry. In its originally proposed form,[1] the "old" inflationary universe did not "gracefully exit" to a spatially homogeneous, or even approximately homogeneous, final state. It thus disagreed with cosmological observation.

The problem of graceful exit was solved in a modified variant, the "new" inflationary universe, independently proposed by Linde[2] and by Albrecht and Steinhardt.[3] The crucial point was to require that the potential of the Higgs field responsible for symmetry breaking be of Coleman-Weinberg[4] type, i.e., that the mass term be fine tuned to zero. In this case the effective potential of the Higgs field ϕ has the shape given in Fig. 1.

At high temperatures $T \gg \sigma$, where σ is the scale of grand unified symmetry breaking and is of order 10^{14} GeV in models such as the minimal SU(5) Georgi-Glashow model,[5] the symmetric ground state is rendered stable by finite-temperature corrections to the effective potential.[6] When T drops below σ, $\phi=0$ becomes unstable. Although the exact evolution of the system immediately after the onset of metastability is not well understood (see, e.g., the review in Ref. 7), it is generally assumed that the time evolution can be described semiclassically almost immediately. Due to quantum fluctuations (see below) the expectation value $\langle\phi\rangle$ of the Higgs field ϕ starts with a value of order H, and with an initial velocity of order H^2, and then begins to move towards the minimum at σ according to the classical equations of motion for the effective potential. Here, H is the Hubble constant given by

$$H^2 = \frac{8\pi G}{3}\rho(0) \simeq \left[\frac{\sigma}{m_{\text{Planck}}}\right]^2 \sigma^2 . \tag{1.1}$$

For the minimal SU(5) model, $H \sim 10^9$ GeV.

That the initial fluctuation in ϕ is set in scale by the value H can be justified independently on thermal, quantum-mechanical, and gravitational grounds. First, the location of the relative maximum of the finite-temperature effective potential is of order H. Second, the exit point of a quantum-mechanical tunneling event through the finite-temperature potential barrier is of this order. Third, Hawking radiation in the de Sitter phase[8,9] induces quantum fluctuations in ϕ of order H. In a certain sense, H is "the only scale around" available to the initial fluctuations: while σ determines the height of the effective potential, only H appears as a characteristic parameter in its shape near the origin. Likewise, the expansion of the background spacetime knows only about H, through Eq. (1.1).

Since the slope of the effective potential near the origin

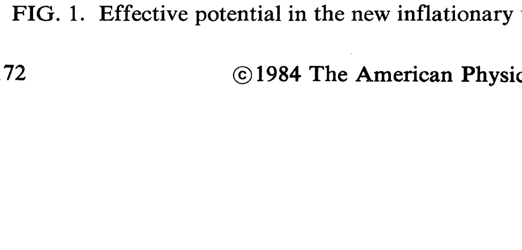

FIG. 1. Effective potential in the new inflationary universe.

is only of order H^2, the expectation value ϕ (we henceforth drop the expectation-value brackets) can remain near the origin for a period set in scale by H^{-1} times some nondimensional factor. As we will see below, this factor can arguably be on the order of $(2\pi)^2$ or larger. During this period the potential energy will remain virtually constant. Hence integration of the Friedmann-Robertson-Walker (FRW) equation

$$\left[\frac{\dot{a}}{a}\right]^2=\frac{8\pi G}{3}\rho(t) \tag{1.2}$$

shows the universe to be in a "de Sitter phase" characterized by exponential growth (inflation) of the scale factor $a(t)$. After this period, ϕ finds the true minimum of the potential, whose height and slope are both set in scale by σ, whose inverse is the characteristic time, much shorter than one inverse Hubble time, for the next stage of evolution: The value of ϕ oscillates about the true minimum with an amplitude which is rapidly damped due to terms in the effective action coupling ϕ to other quantum fields, in particular fermions.[10] The damping corresponds to particle production and reheating of the universe. Since the reheating period is very short (order σ^{-1}) compared to the expansion period H^{-1}, the red-shift of the energy is negligible and the universe will reheat to a temperature of order σ. In the ensuing radiation-dominated FRW period, baryogenesis and nucleosynthesis proceed as in the usual big-bang cosmology.

Recently, it was recognized[11–14] that the new inflationary universe provides a mechanism which might generate the primordial energy-density perturbations necessary for galaxy formation, from first principles and without postulating special initial conditions. Hawking radiation in the de Sitter phase[8,9] will produce quantum field fluctuations on all scales λ. The fluctuations cross the de Sitter horizon at times $t_{\lambda i}$ (initial horizon crossing of the scale λ). As first discussed by Mukhanov and Chibisov[15] and by Lukash,[16] one of the two physical modes of induced gravitational perturbations does not decay while it is outside the horizon. The fluctuation reenters the FRW horizon at a final horizon crossing time $t_{\lambda f}$ with an amplitude that has increased by a factor that is independent of λ. We will show in this paper that this increase can be viewed as being due to the change in the equation of state during and immediately before reheating, but that an approximate conservation law guarantees that the increase is independent of the details of the reheating.

De Sitter space is time-translational invariant. Therefore any perturbations, such as Hawking radiation, that are intrinsic to the spacetime will cross their respective horizons with some invariant amplitude. Perturbations which cross later correspond to smaller comoving spatial scales λ; these perturbations will reenter their FRW horizon *earlier*. If the growth factor between the initial and final crossing times $t_{\lambda i}$ and $t_{\lambda f}$ is indeed independent of λ, then the amplitudes at times $t_{\lambda f}$ will be independent of λ, even though $t_{\lambda f}$ does depend on λ. This is the so-called Zel'dovich[17] spectrum of perturbations, whose single free parameter is conventionally denoted ϵ, where

$$\epsilon \equiv \frac{\delta\rho}{\rho}(t_{\lambda f}) \; . \tag{1.3}$$

Plausible, although not definitively accepted, models of galaxy formation require ϵ to be in the range of $10^{-3\pm1}$. A plausible theory of the early universe should produce a value of ϵ in this range.

It is a significant success of the new inflationary universe that it produces a Zel'dovich spectrum at all. Unfortunately, as the authors of Refs. 11–14 discovered, the predicted value of ϵ is much too large. While Refs. 11–14 all obtain roughly the same value for ϵ, they do so by very different, and in some ways incomplete, methods. One purpose of this paper is to give a careful and complete calculation of ϵ for the new inflationary universe; we would not want to reject so elegant a model until we are certain that its ϵ is being calculated correctly. Doing so, we obtain approximately the same answer as previous calculations.

A second purpose of this paper is to reconcile the different calculational methods used in Refs. 11–14. We will show that the different calculational methods are *not* equivalent. There are two mathematically different effects which enter into the growth of the fluctuations. Each of Refs. 11–14 compute one or the other of these effects, but none compute both. For the case of the new inflationary universe, the effects are both of the same order, so comparable answers were obtained by previous authors. We will show, however, that there are inflationary models (characterized by different forms of the effective potential) for which the two effects are not comparable. For these examples, the application of previous methods in the literature would give incorrect results. In at least one further case,[18] an incorrect approximation during reheating has led to a wrong result being obtained for the new inflationary universe, one in agreement with neither Refs. 11–14 nor with this paper.

The third purpose of this paper is to exemplify how one may carefully calculate the growth of fluctuations, not only in the new inflationary universe, but also in a variety of different inflationary models.

The outline of this paper is as follows: In Sec. II, we briefly review the gauge-invariant framework developed in Ref. 19 (hereafter cited as paper I), and describe the approximation scheme which is justified for inflationary models. We develop an approximate conservation law which can easily be applied to any particular model. In Sec. III, we apply this scheme specifically to the new inflationary universe in considerable detail, with some technical points relegated to the appendices. In Sec. IV, we consider several "toy" models, and use them to show how the growth of the perturbations does not depend on the details of the particle-physics model while the perturbations are outside of their horizon. Also in the context of these toy models, we discuss the inadequacies of previous calculational methods. Section V summarizes our results.

A few words about our notation. Greek indices run from 0 to 3, Latin ones only over the space indices. We use the Einstein summation convention; m_{Planck} stands for the Planck mass and the equation of state of the background matter is given by

$$w=\frac{p}{\rho} \; , \tag{1.4}$$

$$c_s{}^2 = \frac{\dot{p}}{\dot{\rho}} \,. \tag{1.5}$$

We expand the metric about a FRW background:

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2(t), a^2(t), a^2(t)) + a^2(t) g^{(1)}_{\mu\nu} \,. \tag{1.6}$$

$a(t)$ is the scale factor, $g^{(1)}_{0\mu} = 0$ in the synchronous gauge, and

$$g^{(1)}_{ij} = A\delta_{ij} + B_{,ij} \,. \tag{1.7}$$

Similarly, the energy-momentum tensor can be expanded into a background piece and a perturbation $T_{\mu\nu}$. Finally, the gauge-invariant metric potential Φ_H is in the synchronous gauge given by

$$\Phi_H = \tfrac{1}{2}(A - a\dot{a}\dot{B}) \,. \tag{1.8}$$

II. OUTLINE OF THE METHOD

In Ref. 19 we derived a complete dynamical system of equations of motion describing cosmological perturbations of a FRW universe. The system consists of three components: the background metric given by its scale factor $a(t)$, the metric perturbations which can be described by the gauge-invariant metric potential $\Phi_H(x,t)$, and third the matter fields, in our case a single scalar field $\phi(\vec{x},t)$.

All three components are coupled. The FRW equations relate the scale factor to the space-averaged matter energy-momentum tensor

$$\left[\frac{\dot{a}}{a}\right]^2 \equiv H^2 = \frac{8\pi G}{3}\rho \quad \text{with } \rho \equiv -\langle T^0_0 \rangle \,, \tag{2.1}$$

$$2\frac{\ddot{a}}{a} + \left[\frac{\dot{a}}{a}\right]^2 = -8\pi G p \quad \text{with } p \equiv \tfrac{1}{3}\langle T^i_i \rangle \,. \tag{2.2}$$

Formally, averaging is to be performed with respect to the FRW coordinates. In practice we can average over constant time hypersurfaces in any gauge in which the metric perturbations are small (of order Δ). The correction terms in the gravitational perturbation equations will then be of order Δ^2 and hence negligible.

The gauge-invariant equation of motion for Φ_H is

$$\ddot{\Phi}_H + (4 + 3c_s{}^2)H\dot{\Phi}_H + 3H^2(c_s{}^2 - w)\Phi_H = I \,, \tag{2.3}$$

$I(t,\vec{x})$ is a combination of matter source terms. Its precise form will be discussed below. Both the matter and the background metric couple to Φ_H. The former determines the equation of state of the system, i.e., the functions w and $c_s{}^2$, and also the matter source terms $I(t,\vec{x})$. The latter determines $H(t)$. Finally, the metric fluctuations couple back to matter: they enter into the equations of motion for matter, e.g., for a single scalar field $\phi(\vec{x},t)$ in the synchronous gauge

$$\left[-\partial_t{}^2 - 3\frac{\dot{a}}{a}\partial_t + a^{-2}\nabla^2 + D^{(1)}\right]\phi = V'(\phi) \,. \tag{2.4}$$

The term $D^{(1)}$ is the correction term linear in metric perturbations:

$$D^{(1)}(\phi) \equiv -a^{-2}g^{ij(1)}\phi_{,ij} - a^{-2}g^{ij(1)}_{,i}\phi_{,j} - \tfrac{1}{2}\dot{h}\phi_{,t} + \tfrac{1}{2}a^{-2}h_{,i}\phi_{,i} \,, \tag{2.5}$$

$$h \equiv g^{(1)}_{ii} = 3A + \nabla^2 B \,. \tag{2.6}$$

The necessary initial conditions for the above dynamical system follow from Hawking radiation. Curved spacetime induces fluctuations in the quantum field. By the Einstein constraint equations

$$\Phi_H = 4\pi G a^2 \nabla^{-2}(T^{0(1)}_0 - 3a\dot{a}\nabla^{-2}T^{k(1)}_{0,k}) \,, \tag{2.7}$$

$$\dot{\Phi}_H = 4\pi G[a^2\nabla^{-2}(T^{0(1)}_0 - 3a\dot{a}\nabla^{-2}T^{k(1)}_{0,k})]_{,t} \,, \tag{2.8}$$

the matter perturbations are inextricably linked to metric inhomogeneities.

The gravitational side of the dynamical system is described in terms of gauge-invariant quantities. Not so the matter side. Unless the background solution $\phi_0(t)$ is time independent, the deviation $\delta\phi(\vec{x},t)$ of the scalar function $\phi(\vec{x},t)$ from its background value will not be gauge independent. The only way to construct a gauge-invariant function from $\delta\phi(\vec{x},t)$ is to combine it with a metric perturbation with opposite gauge transformation properties.[18] We choose to stick with $\delta\phi(\vec{x},t)$. In order to evolve the matter field we must therefore pick a gauge and in that gauge determine $g^{(1)}_{\mu\nu}$ in terms of Φ_H, the matter source terms $\mathscr{P}_i$ (see below) and any initial conditions required to totally fix the gauge. For example, in the synchronous gauge $A(t)$ and $B(t)$ are determined by integrating (I 3.23) and (I 3.7),

$$\dot{A} = -8\pi G a^2 \nabla^{-2}\mathscr{P}_3 \,, \tag{2.9}$$

$$\dot{B} = (a\dot{a})^{-1}(A - 2\Phi_H) \,, \tag{2.10}$$

and specifying $A(t_i)$ and $B(t_i)$ at some initial time t_i.

We now turn to the reformulation of the dynamical equation of motion for Φ_H as an approximate conservation law.

Bardeen, Steinhardt, and Turner[14] noticed that for perturbations with wavelength greater than the Hubble radius H^{-1}, a metric potential ζ is approximately conserved. The term ζ was defined in terms of the uniform Hubble-constant metric perturbation h_u,

$$\zeta = h_u \left[1 + \frac{k^2}{a^2 12\pi G(\rho + p)}\right] \,, \tag{2.11}$$

h_u is the coefficient of the longitudinal spatial harmonic for the metric perturbation $g^{(1)}_{\mu\nu}$. As such it is proportional to what we should denote $3A - k^2B$, but in the uniform Hubble-constant gauge. The statement $\dot{\zeta} = 0$ can be derived from the equations of motion for metric perturbations in the uniform Hubble-constant gauge [see formulas (2.13) and (2.14) in Ref. 14] provided entropy and anisotropic stress terms can be neglected, and provided we are outside the Hubble radius (such that terms with an additional factor k^2/H^2a^2 are irrelevant).

In terms of our gauge-invariant variables, ζ can be written as

$$\zeta=\frac{2}{3}\frac{\Phi_H+H^{-1}\dot{\Phi}_H}{1+w}+\Phi_H\left[1+\frac{2}{9}\left[\frac{k}{aH}\right]^2\frac{1}{1+w}\right]. \tag{2.12}$$

For the purposes of this paper, Eq. (2.12) may be regarded as the definition of ζ. Readers familiar with the uniform Hubble-constant gauge analysis[14] will easily recognize the equivalence of (2.11) and (2.12) using formula (2.43) of Ref. 14.

We claim that ζ is constant for perturbations outside the Hubble radius provided entropy and anisotropic stress source terms are negligible. The proof is simple: using the continuity equation to substitute for $\dot{w}$ and the FRW equations to determine $\dot{H}$, we derive

$$\tfrac{3}{2}\dot{\zeta}H(1+w)=\ddot{\Phi}_H+(4+3c_s{}^2)H\dot{\Phi}_H+3(c_s{}^2-w)H^2\Phi_H \tag{2.13}$$

[up to terms with an additional factor $(k/aH)^2$]. Thus for the growing mode $\dot{\zeta}=0$ is equivalent to the homogeneous equation of motion (2.3) for Φ_H. (The decaying mode can be of the same order as the subdominant source terms. Hence for it the conservation law is not a good approximation.) In the absence of matter source terms ζ is a conserved quantity outside the horizon.

The above analysis also shows that the descriptions of the homogeneous growth of perturbations in comoving and uniform Hubble-constant variables are in fact identical.

For perturbations outside the Hubble radius the constant of motion is

$$\frac{\Phi_H+H^{-1}\dot{\Phi}_H}{1+w}+\Phi_H\,. \tag{2.14}$$

In a de Sitter-type phase the second term is negligible. Comparing initial and final horizon crossings t_i and t_f in inflationary-universe models [which have $1+w(t_f)=\frac{4}{3}$ and $\dot{\Phi}_H(t_f)=0$], we obtain

$$\Phi_H(t_f)=\frac{2}{5}\left[\frac{\frac{4}{3}\Phi_H(t_i)+H^{-1}\dot{\Phi}_H(t_i)}{1+w(t_i)}\right]+\tfrac{3}{5}\Phi_H(t_i)\,. \tag{2.15}$$

At horizon crossing Φ_H is simply related to the relative energy-density perturbation in the comoving gauge. By the constraint equation (2.7),

$$\Phi_H(t_i)=\frac{3}{2}\frac{\delta\rho}{\rho}(t_i) \tag{2.16}$$

(and analogously at t_f). Hence the physical interpretation of our conservation law is that the ratio of the energy-density perturbation over the total nonvacuum energy is the same at initial and final horizon crossings.

Our method may now be summarized as follows. We first consider the complete dynamical system, matter, and geometry, and analyze its evolution in each phase of the cosmological model. In particular, we determine the magnitude of the matter source terms in Eq. (2.3). If they are negligible, then the amplification of perturbations can only depend on the change in the equation of state, since it enters only the left-hand side of Eq. (2.3). In this case we can apply Eq. (2.15) to obtain the magnitude of energy-density fluctuations at final horizon crossing. Checking the magnitude of matter source terms must however, be an integral part of any application.

We analyze the effect of matter source terms in Eq. (2.3) using the Green's-function method. We set

$$\Phi_H=\Phi_H^h+\Phi_H^p\,. \tag{2.17}$$

The term Φ_H^h is the solution of the homogeneous equation [i.e., Eq. (2.3) with $I(t)=0$] with the given initial conditions. It represents the increase in Φ_H due to gravitational effects alone. Φ_H^p is the particular solution of the inhomogeneous equation (2.3) with vanishing initial conditions for both Φ_H and $\dot{\Phi}_H$. Physically it represents the additional perturbations caused by the matter perturbations which were initially set in motion.

As an ordinary second-order differential equation (2.3) admits two eigenmodes $f_1(t)$ and $f_2(t)$. Thus

$$\Phi_H^h(t)=c_1f_1(t)+c_2f_2(t)\,, \tag{2.18}$$

where c_1 and c_2 are determined by matching initial conditions. By the Green's-function method

$$\Phi_H^p(t)=-f_1(t)\int_{t_0}^{t}I(t')\epsilon(t')f_2(t')dt'+f_2(t)\int_{t_0}^{t}I(t')\epsilon(t')f_1(t')dt'\,, \tag{2.19}$$

where t_0 is the initial time and

$$\epsilon(t)=[f_2(t)\dot{f}_1(t)-\dot{f}_2(t)f_1(t)]^{-1}\,. \tag{2.20}$$

In order to determine the effect of matter source terms, we need the eigenmodes $f_1(t)$ and $f_2(t)$. Thus we need to solve the complete dynamical system (2.1)–(2.10). In general it will be impossible to find an exact analytic solution. We propose the following approximation scheme: we will cut the link between metric perturbations and matter, i.e., we will evolve the scalar field $\phi(\vec{x},t)$ in the unperturbed background metric. This approximation will only be good if we evolve matter in a gauge in which metric perturbations are small (or at least remain small long enough). In a constant-ϕ gauge this will not be true for typical inflationary-universe models. As we will show in Sec. III, Hawking radiation in a de Sitter phase of an inflationary universe generates large quantum field perturbations. The surfaces of constant ϕ will therefore have a large extrinsic curvature perturbation. In Appendix A we prove that the induced relative metric perturbations are of order 1, much too large for perturbation theory to be valid. In the synchronous gauge on the other hand, the metric fluctuations remain small up to reheating.

A second approximation is to determine the equation of state $w(t)$ and $c_s{}^2(t)$ by considering only the spatially homogeneous evolution of matter. In the case of the new inflationary universe this leads to a rapid transition from a de Sitter-type equation of state to a radiation-dominated FRW phase, rapid in the sense that the transition period τ is much shorter than H^{-1}. Thus the Hubble parameter $H(t)$ is constant to an excellent approximation.

With these approximations, the calculation of energy-density fluctuations in inflationary-universe models proceeds in two steps. Given $H(t)$, we first determine the evolution of the scalar field in the synchronous gauge. This determines $w(t)$ and $c_s^2(t)$ and thus the crucial deviations in the equation of state from that of an exact de Sitter phase. We also obtain the matter source terms. Next, we solve the equation of motion for Φ_H using the method of (2.17)–(2.20). We thus obtain the time development of metric perturbations. The term Φ_H at final horizon crossing is related to the quantity we want to calculate, the energy-density perturbation $\delta\rho/\rho$ in comoving coordinates, by (2.16).

Both to obtain a better understanding of the physics and to facilitate the mathematical analysis, we analyze the dynamical system separately in the various phases of the model being considered. A phase is a time interval with some characteristic time evolution of the equation of state. How we pick the phases will depend on the particular particle-physics model we consider.

For our detailed analysis of the new inflationary universe in Sec. III, it will be convenient to slightly rewrite the source terms. In Ref. 19 they were given in the form

$$I=4\pi G[-\mathscr{P}_1-3c_s^2Ha^2\nabla^{-2}\mathscr{P}_3 +3H^2(\tfrac{2}{3}-w+c_s^2)a^2\mathscr{P}_2+a^2H\dot{\mathscr{P}}_2] . \tag{2.21}$$

In terms of the matter perturbations $T^{\mu\nu(1)}$,

$$\begin{aligned}\mathscr{P}_1&=a^2\nabla^{-2}T^{ij(1)}_{,ij} ,\\ \mathscr{P}_2&=a^2\nabla^{-2}(\delta_{ij}T^{ij(1)}-3\nabla^{-2}T^{ij(1)}_{,ij}) ,\\ \mathscr{P}_3&=T^{0i(1)}_{,i} .\end{aligned} \tag{2.22}$$

$\mathscr{P}_2$ is the anisotropic stress and is individually gauge invariant, $\mathscr{P}_1$ is the pressure perturbation, and $\mathscr{P}_3$ a matter flux term. The combination of $\mathscr{P}_1$ and $\mathscr{P}_3$ in (2.21) is gauge invariant. For a scalar field ϕ as matter source (I 4.5) through (I 4.7) give

$$\begin{aligned}\mathscr{P}_1&=a^{-2}\nabla^{-2}(\phi_{,i}\phi_{,j})_{,ij}+\tfrac{1}{2}\dot\phi^2-\tfrac{1}{2}a^{-2}(\nabla\phi)^2\\ &\quad -V(\phi)-\tfrac{1}{2}\langle\dot\phi^2\rangle\\ &\quad +\tfrac{1}{6}a^{-2}\langle(\nabla\phi)^2\rangle+\langle V(\phi)\rangle ,\\ \mathscr{P}_2&=a^{-2}\nabla^{-2}(\phi_{,i}\phi_{,i})-3a^{-2}\nabla^{-4}(\phi_{,i}\phi_{,j})_{,ij} ,\\ \mathscr{P}_3&=-a^{-2}(\dot\phi\phi_{,i})_{,i} .\end{aligned} \tag{2.23}$$

We first extract a gauge-invariant piece from $\mathscr{P}_1$:

$$\mathscr{P}_1=\tfrac{1}{3}a^2\delta_{ij}T^{ij(1)}-\tfrac{1}{3}\nabla^2\mathscr{P}_2 . \tag{2.24}$$

Next we observe that

$$\dot{\mathscr{P}}_2=-2H\mathscr{P}_2+a^{-2}[\nabla^{-2}a^4(\delta_{ij}T^{ij(1)}-3\nabla^{-2}T^{ij(1)}_{,ij})]_{,t} . \tag{2.25}$$

The first piece cancels one of the terms proportional to $\mathscr{P}_2$ in (2.21). Thus (2.3) can be rewritten as

$$\ddot\Phi_H+(4+3c_s^2)H\dot\Phi_H+3H^2(c_s^2-w)\Phi_H=I , \tag{2.26}$$

$$I=I_{11}+I_{12}+I_{\dot 2}+I_2+I_3 . \tag{2.27}$$

Expressions I_{12}, $I_{\dot 2}$, and I_2 are individually gauge invariant, I_{11} and I_3 only jointly. In general terms

$$\begin{aligned}I_{11}&=-4\pi G\tfrac{1}{3}a^2\delta_{ij}T^{ij(1)} ,\\ I_{12}&=4\pi Ga^2(\delta_{ij}T^{ij(1)}-3\nabla^{-2}T^{ij(1)}_{,ij})\tfrac{1}{3} ,\\ I_{\dot 2}&=4\pi GH[a^4\nabla^{-2}(\delta_{ij}T^{ij(1)}-3\nabla^{-2}T^{ij(1)}_{,ij})]_{,t} ,\\ I_2&=12\pi GH^2a^4\nabla^{-2}(\delta_{ij}T^{ij(1)}-3\nabla^{-2}T^{ij(1)}_{,ij})(c_s^2-w) ,\\ I_3&=-12\pi Gc_s^2Ha^2\nabla^{-2}T^{0i(1)}_{,i} .\end{aligned} \tag{2.28}$$

For a scalar field it follows by (2.23) that

$$\begin{aligned}I_{11}&=-4\pi G[\tfrac{1}{2}\dot\phi^2-\tfrac{1}{6}a^{-2}(\nabla\phi)^2-V(\phi)-\tfrac{1}{2}\langle\dot\phi^2\rangle\\ &\qquad +\tfrac{1}{6}a^{-2}\langle(\nabla\phi)^2\rangle+\langle V(\phi)\rangle] ,\\ I_{12}&=4\pi Ga^{-2}[\tfrac{1}{3}(\nabla\phi)^2-\nabla^{-2}(\phi_{,ij})_{,ij}] ,\\ I_{\dot 2}&=4\pi GH[\nabla^{-2}(\nabla\phi)^2-3\nabla^{-4}(\phi_{,i}\phi_{,j})_{,ij}]_{,t} ,\\ I_2&=12\pi GH^2(c_s^2-w)[\nabla^{-2}(\nabla\phi)^2-3\nabla^{-4}(\phi_{,i}\phi_{,j})_{,ij}] ,\\ I_3&=12\pi GHc_s^2\nabla^{-2}(\dot\phi\phi_{,i})_{,i} .\end{aligned} \tag{2.29}$$

Compared to $I_{\dot 2}$ and I_2, I_{12} is suppressed by a factor $k^2a^{-2}H^{-2}$. Outside the horizon this factor is much smaller than one ($k^{-1}a=H^{-1}$ is precisely the horizon-crossing condition). Thus for our application I_{12} is negligible. Except near t_R the order of magnitude of the other source terms can be roughly estimated in the following way:

$$\begin{aligned}I_{11}&\sim(m_{\text{Planck}})^{-2}\,|\Delta p|\ ,\\ I_{\dot 2}&\sim(m_{\text{Planck}})^{-2}H^2\,|\phi|^2 ,\\ I_2&\sim(m_{\text{Planck}})^{-2}H^2(c_s^2-w)\,|\phi|^2 ,\\ I_3&\sim(m_{\text{Planck}})^{-2}H^2c_s^2\,|\phi|^2 .\end{aligned} \tag{2.30}$$

The term $|\phi|$ is the magnitude of the Higgs field, $|\Delta p|$ is the amplitude of the pressure oscillation on the constant time hypersurfaces, and time derivatives have been replaced by the inverse expansion period H.

Since the quantum field perturbations are large, at least in the case of the new inflationary universe, it is incorrect to *a priori* neglect I_2 and $I_{\dot 2}$ compared to the other matter source terms. Our analysis (see Sec. III and Appendix C) proves however that in the essential period near reheating the pressure perturbation and flux terms I_{11} and I_3 dominate.

III. THE NEW INFLATIONARY UNIVERSE

We will describe the application of our method in considerable detail for the case of greatest physical interest, the new inflationary universe of Linde[2] and Albrecht and Steinhardt.[3] These calculations can then be straightforwardly extended to other particle physics models. We will omit many details in our discussion of toy models in Sec. IV.

The one-loop effective potential in the minimal SU(5) Georgi-Glashow model with Coleman-Weinberg potential is

$$V(\phi)=\tfrac{1}{2}B\sigma^4+B\phi^4\left[\ln\frac{\phi^2}{\sigma^2}-\frac{1}{2}\right] . \tag{3.1}$$

The constant B is given, e.g., in Ref. 20:

$$B=\frac{5625}{1024}\frac{g^4}{\pi^2} , \tag{3.2}$$

where g is the SU(5) coupling constant.

We will first apply the conservation law (2.15) to obtain the value of energy-density fluctuations at horizon crossing under the assumption that the matter source terms are negligible. In a second step we will go back and carefully investigate the time development of Φ_H keeping track of the simultaneous growth of matter perturbations at each instant.

A. Application of the conservation law

In order to be able to apply the conservation law (2.15) we must first investigate the equation of motion of the background homogeneous scalar field $\phi(t)$ and of the scalar field perturbations $\delta\phi(\vec{x},t)$ for times t_i at which scales of astronomical interest cross the de Sitter-Hubble radius. We need this analysis to determine the initial metric perturbations.

The new inflationary universe is based on the assumption that after some initial period in which gravitational effects dominate[21] or after a phase transition, either homogeneous in space[22] or via the formation of bubbles of the new phase in a surrounding sea of the old symmetric phase,[23] the quantum field can be described semiclassically. The expectation value of the quantum field will start moving homogeneously in space (or at least homogeneously compared to wavelengths within the present horizon) towards the value $\phi=\sigma$ according to the classical equation of motion for ϕ in the effective potential $V(\phi)$. The initial value for ϕ at time t_0, the beginning of the rolling phase, is of order H.

We will first focus on the $k=0$ mode $\phi_0(t)$.

The first approximation in solving the equation of motion for ϕ is to replace the logarithm in (3.1) by a negative constant of order $\ln H^2/\sigma^2$ in the de Sitter phase. This is a good approximation, since during most of the rolling phase the value of ϕ will be of order H and hence the logarithm will change only very slowly. Equation (3.1) can be rewritten as

$$V(\phi)=V(0)-\tfrac{1}{4}\lambda\phi^4 , \tag{3.3}$$

λ is of order unity since the SU(5) coupling constant at the grand unification scale (our renormalization point) is of order 0.5. Following Ref. 11, we will use $\lambda=0.5$ in any numerical evaluations. The approximation will break down when ϕ_0 becomes of order σ and the positive curvature of the effective potential near the absolute minimum becomes non-negligible. The time t_R when this occurs signals the end of the de Sitter phase. For the sake of definiteness we define t_R by

$$\phi_0(t_R)\equiv\tfrac{3}{4}\sigma . \tag{3.4}$$

Since the slope of the effective potential near the origin is very small, the initial field acceleration $\ddot{\phi}_0$ will be negligible. The second approximation in solving the equation of motion

$$\ddot{\phi}_0+3H\dot{\phi}_0=-V'(\phi_0) \tag{3.5}$$

for ϕ_0 is therefore to neglect the second time derivative. We obtain the very simple equation

$$3H\dot{\phi}_0(t)=\lambda\phi_0{}^3(t) , \tag{3.6}$$

which has the solution

$$\phi_0(t)=\left[\frac{3H}{2\lambda}\right]^{1/2}\frac{1}{(t^*-t)^{1/2}} , \tag{3.7}$$

$$t^*=\frac{3H}{2\lambda\phi_0{}^2(0)} , \tag{3.8}$$

t^* is the time when the approximate solution $\phi_0(t)$ shoots off to infinity. Since this will occur at about the time ϕ_0 reaches the point where the approximation (3.3) for the effective potential breaks down, t^* is a good measure of the duration of the de Sitter phase.

In order to obtain sufficient inflation to solve the homogeneity and flatness problems, the period of inflation must satisfy[1,2]

$$t^*\geq 65H^{-1} . \tag{3.9}$$

This requirement constrains the initial value of ϕ_0 at $t=0$, the beginning of the rolling phase. If we set $\phi_0(0)=\gamma H$, then by (3.8) and (3.9)

$$\gamma\leq\left[\frac{3}{130\lambda}\right]^{1/2}\simeq 0.2 \tag{3.10}$$

By dimensional analysis γ should be of order 1. Our value is at the lower end of the admissible range. This is the fine-tuning problem in the new inflationary universe, a problem emphasized by Starobinskii.[13] It is distinct from the mass fine tuning needed to obtain a Coleman-Weinberg potential. As we will show below, (3.10) is perfectly acceptable if the initial value is generated exclusively by quantum fluctuations in curved spacetime.

We consider fluctuations of galactic scales, i.e., with physical wavelength

$$l\sim 10^6\,\text{lt.yr}\sim 10^{38}\ \text{GeV}^{-1} \tag{3.11}$$

and comoving wave number

$$\begin{aligned} k&=a(t_R)\frac{a(3^0K)}{a(t_R)}l^{-1}\\ &\simeq a(t_R)\times 10^{-11}\ \text{GeV} . \end{aligned} \tag{3.12}$$

The horizon-crossing condition is

$$H^{-1}=k^{-1}a , \tag{3.13}$$

where

$$a(t)=\exp[H(t-t_R)]a(t_R) .$$

Hence on galactic scales

$$(t_R - t_i) \simeq 50H^{-1} . \tag{3.14}$$

Thus at horizon crossing t_i the scalar field and its velocity are

$$\phi_0(t_i) \simeq 10^{-1}H, \;\; \dot{\phi}_0(t_i) \simeq 10^{-3}H^2 . \tag{3.15}$$

Next we will analyze scalar field fluctuations. The de Sitter phase plays two important roles in the evolution of matter fluctuations. All initial classical perfect-fluid-type matter fluctuations and their associated metric perturbations will decay exponentially in the supercooling phase prior to rolling. There is no Jeans instability for de Sitter space.[24] This is easy to verify in our formalism: as demonstrated in Sec. IV, Φ_H decays exponentially when the corrections to the de Sitter equation of state are determined by a perfect fluid. On the other hand, all quantum fields in de Sitter space[8] or in a de Sitter phase of a FRW universe[9] experience the Hawking effect: Hawking radiation induces quantum field fluctuations on all scales inside the Hubble radius $H^{-1}(t)$ (outside the Hubble radius no microphysical effects can act coherently).

We will consider perturbations characterized by a comoving wave number k. Outside the horizon the fluctuations will evolve classically. The initial perturbation $\delta\phi(k)$ is determined by Hawking radiation at horizon crossing t_i. The term $\delta\phi(k)$ is the average deviation of $\phi(\vec{x},t)$ from the mean value $\phi_0(t)$ on a scale k. By standard statistics (see, e.g., Ref. 25) it is related to the correlation function for which we use the two-point function of the quantum field:

$$\delta\phi(\vec{k},t_i) = \left[\frac{k^3}{(2\pi)^3} \int d^3\vec{x}\, e^{i\vec{k}\cdot\vec{x}} \langle \phi(\vec{x},t_i)\phi(\vec{0},t_i) \rangle \right]^{1/2} . \tag{3.16}$$

Since early in the de Sitter phase the potential at $\phi_0(t)$ is almost flat, the two-point function can be taken to be that of a scalar field in de Sitter space, i.e. (see Ref. 12),

$$\langle \phi(\vec{x},t_i)\phi(\vec{0},t_i) \rangle = -\frac{H^2}{\pi^2}\ln(H\,|\,\vec{x}\,|\,) . \tag{3.17}$$

Hence

$$\delta\phi(\vec{k},t_i) = \left[\frac{H^2}{4\pi^3} \right]^{1/2} \simeq 10^{-1}H . \tag{3.18}$$

$\delta\dot{\phi}(\vec{k},t_i)$ is obtained by solving the linearized field equation of motion (see Appendix B). From the known values of ϕ_0 and $\delta\phi$ at t_i it follows immediately that

$$\delta\dot{\phi}(\vec{k},t_i) \simeq 10^{-3}H . \tag{3.19}$$

Equations (3.15), (3.18), and (3.19) are the initial values required to compute $w(t_i)$ and the metric perturbations at t_i.

Pressure and energy are given by the homogeneous mode $\phi_0(t)$:

$$\rho(t) = \tfrac{1}{2}\dot{\phi}_0{}^2(t) + V(\phi_0(t)) , \tag{3.20}$$

$$p(t) = \tfrac{1}{2}\dot{\phi}_0{}^2(t) - V(\phi_0(t)) . \tag{3.21}$$

Hence

$$w(t) = \frac{p(t)}{\rho(t)} = -1 + \frac{\dot{\phi}_0{}^2}{\rho(t)} . \tag{3.22}$$

In particular, at t_i

$$1 + w(t_i) \simeq 10^{-4} \left[\frac{H}{\sigma} \right]^4 . \tag{3.23}$$

In Appendix B we verify that

$$\Phi_H(t_i) \simeq H^{-1}\dot{\Phi}_H(t_i) \simeq 10^{-3} \left[\frac{H}{m_{\text{Planck}}} \right]^2 . \tag{3.24}$$

Applying our conservation law (2.15) and using

$$\frac{H}{\sigma} \simeq \frac{\sigma}{m_{\text{Planck}}} \tag{3.25}$$

[see (2.1)], we immediately conclude

$$\Phi_H(t_f) \simeq 10. \tag{3.26}$$

This value for $\delta\rho/\rho$ is four or five orders of magnitude larger than the acceptable experimental value.

So far we neglected the matter source terms in Eq. (2.3). In order to take them into account, we must explicitly integrate (2.3) phase by phase. This detailed analysis will also shed more light on the causes of the increase in Φ_H.

B. Equation of state

In the new inflationary universe we distinguish three phases, the de Sitter phase (rolling phase) during which the $k=0$ mode $\phi_0(t)$ is evolving towards the minimum of the effective potential at $\phi = \sigma$, the reheating phase at time t_R (of length σ^{-1}), during which the vacuum energy of the scalar field is rapidly converted into thermal energy of baryons, and the ensuing FRW phase up to the time t_f at which the perturbations reenter the horizon.

It will be convenient to separate the de Sitter phase into two periods. In the first, the field value $\phi_0(t)$ is increasing very slowly. $\phi_0(t)$ is of the order H and thus the deviations in the equation of state from that of an exact de Sitter space are negligible. Equation (2.6) is a good approximation to the equation of motion. Once

$$\ddot{\phi}(t) = 3H\dot{\phi}_0(t) , \tag{3.27}$$

the approximation (2.6) breaks down. The time t_B at which this breakdown occurs signals the end of the first de Sitter period. By (3.7) and (3.27),

$$(t^* - t_B) = 2H^{-1} . \tag{3.28}$$

The value of ϕ_0 at the end of this first period is

$$\phi_0(t_B) = \left[\frac{3}{\lambda} \right]^{1/2} H . \tag{3.29}$$

In the second period of the de Sitter phase lasting from t_B to t_R we must solve (3.5) in a different approximation. Since beginning at t_B the $\ddot{\phi}_0$ term dominates over the

Hubble damping term, we can neglect the latter. Equation (3.5) becomes

$$\ddot{\phi}_0(t)=\lambda\phi_0{}^3(t)\ . \tag{3.30}$$

Equivalently, we can solve either of the two first-order differential equations

$$\dot{\phi}_0(t)=\pm\left[\frac{\lambda}{2}\right]^{1/2}\phi_0{}^2\ , \tag{3.31}$$

which can be explicitly integrated. The solution of the equation with the + sign gives a growing mode, the other a decaying mode. The general solution is

$$\phi_0(t)=c_1\left[\frac{2}{\lambda}\right]^{1/2}[\alpha-(t-t_B)]^{-1}+c_2\left[\frac{2}{\lambda}\right]^{1/2}[\alpha+(t-t_B)]^{-1} \tag{3.32}$$

with

$$\alpha=\left[\frac{2}{\lambda}\right]^{1/2}\phi_0{}^{-1}(t_B)=(\tfrac{2}{3})^{1/2}H^{-1}\ , \tag{3.33}$$

where c_1 and c_2 are determined by matching the initial conditions at t_B. We can easily check that $0<c_2<c_1$. Hence we can drop the decaying mode entirely and write

$$\phi_0(t)=\left[\frac{2}{\lambda}\right]^{1/2}[\alpha-(t-t_B)]^{-1}\ . \tag{3.34}$$

It is easy to check our method for self-consistency. As t increases from t_B to t_R, both $\ddot{\phi}_0$ and $\phi_0{}^3$ increase from $\sim H^3$ to $\sim\sigma^3$, while $3H\dot{\phi}_0$ only grows from $\sim H^3$ to $\sim H\sigma^2$. Dropping the Hubble damping term is therefore justified. The physical reason is that the second period of the de Sitter phase is short compared to the Hubble expansion time H^{-1}: Since the end of the de Sitter phase is determined by $\phi_0(t_R)=\frac{3}{4}\sigma$ (3.34) gives

$$(t_R-t_B)\simeq\alpha=(\tfrac{2}{3})^{1/2}H^{-1}\ . \tag{3.35}$$

Hence in this period the kinematics of the scalar field ϕ dominates over Hubble red-shift.

We can now determine the equation of state in the de Sitter phase. Since $\dot{\phi}(t)\simeq H^2$ in the entire first period of the de Sitter phase, Eq. (3.22) shows that $w=-1$ is an excellent approximation. In period 2,

$$-1\le w<\tfrac{1}{3}\ . \tag{3.36}$$

A significant deviation of w from the value $w=-1$ only starts in the final interval of length σ^{-1}. Since both $c_s{}^2$ and the source terms in the equation of motion for Φ_H vary by many orders of magnitude in this time interval, setting $w=-1$ in the entire de Sitter phase is a good approximation.

$c_s{}^2$ can be reexpressed using the continuity equation

$$c_s{}^2=\frac{\dot{p}}{\dot{\rho}}=-\frac{\dot{p}}{3(p+\rho)H}\equiv-1+\gamma \tag{3.37}$$

with

$$\gamma=-\frac{2\ddot{\phi}_0}{3H\dot{\phi}_0}\equiv-\tfrac{4}{3}f\ . \tag{3.38}$$

From (3.7) it follows that in the first de Sitter period γ will be small. As t increases from t_i to t_B, $-\gamma$ will grow from $\sim10^2$ to ~1. Hence $c_s=-1$ is a good approximation. We have thus verified that the deviations from the equation of state

$$w=c_s{}^2=-1 \tag{3.39}$$

of a perfect de Sitter space are negligible.

In the second de Sitter period, however, by (3.34),

$$f=[\alpha-(t-t_B)]^{-1}H^{-1}\ . \tag{3.40}$$

f and hence $|c_s{}^2|$ will increase in time from a value of order 1 to one of order 10^5 (see Fig. 2)

$$f\!:\ \mathcal{O}(1)\rightarrow\mathcal{O}(1)\frac{\sigma}{H}\ \text{as}\ t\!:\ t_B\rightarrow t_R\ . \tag{3.41}$$

This time dependence of the deviation in the equation of state from that of an exact de Sitter space is typical for the new inflationary universe or for any model in which the deviation is produced by a homogeneously evolving and accelerating scalar field. In Sec. IV, we will show that perfect fluid matter red-shifting away in the presence of a large cosmological constant leads to an entirely different equation of state. We can explain the curves of Fig. 2 in the following way. Shortly before reheating the pressure increases due to the growing scalar-field kinetic energy. At the same time the energy density only decreases insignificantly (it is red-shifting away, but $\sigma^{-1}\ll H^{-1}$). Thus $\dot{\rho}$ is minute, leading to the very large value of $c_s{}^2$ [by (3.37)].

Next we turn to the equation of state during reheating. Once the scalar field $\phi_0(t)$ reaches $\phi=\frac{3}{4}\sigma$ at time t_R, the approximation (3.3) for the effective potential breaks down. The curvature of the potential at its minimum σ begins to dominate. Thus the Higgs field starts rapidly oscillating about $\phi=\sigma$. Via Yukawa coupling terms in the gauge field Hamiltonian this leads to the production

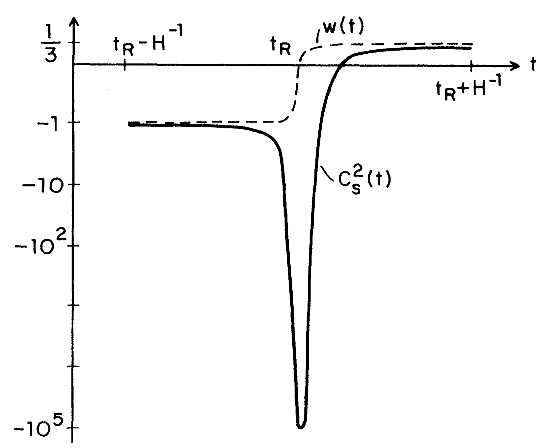

FIG. 2. Equation of state near reheating. $c_s{}^2$ is graphed linearly for $c_s{}^2>-1$, but logarithmically for $c_s{}^2<-1$.

of baryons whose equation of state is that of a relativistic gas. This process is discussed in detail in Ref. 10. The consequence of the evolution of the scalar field is an effective damping term $\Gamma\dot{\phi}$ in the equation of motion for ϕ. The energy ρ_f in the scalar field hence decreases exponentially,

$$\rho_f(t)=e^{-\Gamma(t-t_R)}\rho_f(t_R)=e^{-\Gamma(t-t_R)}\rho(t_R) \tag{3.42}$$

and is transformed into thermal energy $\rho_m(t)$. Since $\Gamma\simeq\sigma \gg H$, the Hubble red-shift is negligible in determining $w(t)$.

The equation of state is given by

$$\rho(t)=\rho_f(t)+\rho_m(t)\simeq\rho(t_R)\ , \tag{3.43}$$

$$p(t)=p_f(t)+p_m(t)\ . \tag{3.44}$$

The pressure $p_f(t)$ from the scalar-field component of matter can be calculated using the quantum-field equation of motion (see Appendix D). Averaging over one oscillation period gives

$$p_f(t)=0\ . \tag{3.45}$$

We interpret this as the pressure of a coherent state of scalar particles at rest. The thermal pressure obviously is

$$p_m(t)=\tfrac{1}{3}\rho_m(t)\ . \tag{3.46}$$

Hence

$$w(t)=\tfrac{1}{3}(1-e^{-\Gamma(t-t_R)})\ . \tag{3.47}$$

To obtain $c_s{}^2(t)$ we must keep track of the Hubble expansion: $p_m(t)$ and $\rho(t)$ red-shift. Hence

$$c_s{}^2(t)\simeq w(t)-\frac{\Gamma}{3H}e^{-\Gamma(t-t_R)}\ . \tag{3.48}$$

As sketched in Fig. 2, both $w(t)$ and $c_s{}^2(t)$ rapidly relax towards their FRW values $\frac{1}{3}$ during reheating.

C. Matter source terms

As we showed in Sec. III A, Hawking radiation produces initial matter inhomogeneities at horizon crossing. We will propagate these perturbations up to reheating according to the scalar field equation and use the result to determine the matter source terms.

An important point to note is that the linear approximation for

$$\delta\phi(\vec{x},t)=\phi(\vec{x},t)-\phi_0(t) \tag{3.49}$$

breaks down as soon as $\delta\phi(\vec{x},t)\simeq\phi_0(t)$. Using Eq. (B1), we can show that this happens almost immediately after horizon crossing. In Appendix B we demonstrate that during the entire de Sitter phase the scalar field perturbations can be encoded in the form

$$\phi(\vec{x},t)=\phi_0(t-\delta\tau(\vec{x}\,)) \tag{3.50}$$

as a space-dependent time lag

$$\delta\tau(\vec{x}\,)=e^{i\vec{k}\cdot\vec{x}}\delta\tau(\vec{k}\,)\simeq\frac{\delta\phi(\vec{x},t_i)}{\dot{\phi}(t_i)}\ , \tag{3.51}$$

whose amplitude is time independent and by (3.50), (3.51), and (3.7) is given by

$$\delta\tau(k)=\left[\frac{2\lambda}{3\pi^3}\right]^{1/2}H^{-1}\ln^{3/2}\left[\frac{H}{k}a(t^*)\right]\ . \tag{3.52}$$

By (3.12), for galactic scale perturbations,

$$\delta\tau\simeq 10H^{-1}\ . \tag{3.53}$$

It is this large value of the time lag which is ultimately responsible for the excessive magnitude of the inhomogeneities in the new inflationary universe.

It is important to remark at this point that in many particle physics models an asymptotic time lag $\delta\tau(\vec{x}\,)$ can be defined by (3.50) in the limit $t \gg t_i$. The value obtained may or may not agree with the initial "time lag" (3.51). The logarithmic potential model of Sec. IV is an example for which the two expressions do not agree.

Returning for a moment to our approximation scheme outlined in Sec. II: If we had chosen to determine $w(t)$ and $c_s{}^2(t)$ from the space-averaged values of $p(t)$ and $\rho(t)$, the transition in the equation of state would have been slow. Over a period $\delta\tau$ the universe changes from being all in a de Sitter phase to being all in a FRW phase. Expressions $w(t)$ and $c_s{}^2(t)$ will thus slowly change over this period; $H(t)$ can no longer be set constant up to reheating. We were not able to find a good analytic approximation. Hence the entire analysis would have to be numerical. Since the source terms do not dominate, the conservation law (2.15) applies and guarantees that the analysis will be independent of this approximation.

The magnitude and time dependence of the matter source terms can now be calculated by inserting into Eq. (2.29). As sketched in more detail in Appendix B, but is already intuitively clear from (2.30), I_{11} and I_3 are the dominant terms in the entire de Sitter phase. In the first period

$$I(t)\simeq I_0(t^*-t)^{-2} \tag{3.54}$$

with

$$I_0\simeq\left[\frac{H}{m_{\text{Planck}}}\right]^2\ . \tag{3.55}$$

In the second period, Δp space as $\phi_0{}^4$. Hence by (3.34) and (3.40)

$$I(t)\simeq I_0H^2f^4\ . \tag{3.56}$$

At reheating the scalar-field inhomogeneities get transformed into radiation fluid inhomogeneities. Their amplitude during reheating is equal to their amplitude at the end of the de Sitter phase. In the FRW phase, however, the matter source terms will red-shift away as the background fluid itself. This approximation is consistent with our approximation scheme. Taking geometric perturbations into account for the matter evolution would lessen the decrease (a reflection of the Jeans instability of a FRW universe with a relativistic fluid). Note that it would be incorrect to turn the matter sources off after reheating. The T_{ij} perturbations are freely specifiable, but not the flux terms. They are related to the geometric perturbations via the constraint equations.

From (3.54), (3.55), and the above discussion we therefore conclude that in the FRW phase

$$I(t)\simeq H^2\left[\frac{a(t)}{a(t_R)}\right]^{-4}. \tag{3.57}$$

D. Evolution of Φ_H

We have now determined all the quantities which enter into the equation of motion (2.3) for Φ_H and can begin a phase-by-phase analysis of the growth of Φ_H.

In the first de Sitter period $t\epsilon[t_i,t_B]$ it is justified to work with the following approximation to (2.3):

$$\ddot{\Phi}_H+H\dot{\Phi}_H=I_0(t^*-t)^{-2}. \tag{3.58}$$

As in every spacetime with $c_s{}^2=w$, one eigenmode is constant while the other decays,

$$f_1(t)=1, \qquad f_2(t)=e^{-H(t-t_i)}, \tag{3.59}$$

$$\epsilon(t)=H^{-1}e^{H(t-t_i)}. \tag{3.60}$$

By matching initial conditions we obtain

$$\Phi_H^h(t)=\left[d_1+d_2e^{-H(t-t_i)}\right]\left[\frac{H}{m_{\text{Planck}}}\right]^2 \tag{3.61}$$

with d_1 and d_2 both constants of order 10^{-3}. On the other hand, by the Green's function formula (2.19),

$$\Phi_H^p(t)\simeq I_0H^{-1}[(t^*-t)^{-1}-(t^*-t_i)^{-1}]. \tag{3.62}$$

Hence at t_B the particular solution dominates:

$$\Phi_H(t_B)\simeq\Phi_H^p(t_B)\simeq I_0, \qquad \dot{\Phi}_H(t_B)\simeq\dot{\Phi}_H^p(t_B)\simeq I_0H. \tag{3.63}$$

We conclude that in the first de Sitter period immediately after horizon crossing matter source terms are non-negligible. They contribute by the same order of magnitude to Φ_H, and the factors of order unity are in fact slightly larger.

In the second period of the de Sitter phase $w=-1$ is the only justifiable approximation. The equation of motion (2.3) for Φ_H becomes

$$\ddot{\Phi}_H+(1-4f)H\dot{\Phi}_H-4H^2f\Phi_H=I(t)\simeq I_0H^2f^4. \tag{3.64}$$

Since $|c_s{}^2|$ is rapidly increasing, the constant mode of the first de Sitter phase becomes a growing mode. Its exact form was first given by Bardeen, Steinhardt, and Turner in Ref. 14. Introducing the abbreviation

$$E(t)=\rho(t)+p(t), \tag{3.65}$$

the modes can be written as

$$f_1(t)=H\int_{t_B}^{t}\exp[H(t'-t)]\frac{E(t')}{E(t_B)}dt', \qquad f_2(t)=e^{-H(t-t_B)}. \tag{3.66}$$

(More details on the growing mode are given in Ref. 26.)

The homogeneous solution $\Phi_H^h(t)$ is given by (2.18). By matching initial conditions at t_B with (3.63), we find

$$c_1\simeq c_2\simeq\left[\frac{H}{m_{\text{Planck}}}\right]^2. \tag{3.67}$$

In Appendix C, we verify that the particular solution, i.e., the effect of matter source terms, is not dominant. We also show that Φ_H and $\dot{\Phi}_H$ increase by three and four powers of σ/H, respectively,

$$\Phi_H(t_R)=\left[\frac{\sigma}{H}\right]^3c_1\simeq\frac{H}{\sigma}, \tag{3.68}$$

$$\dot{\Phi}_H(t_R)=\left[\frac{\sigma}{H}\right]^4c_1H\simeq H. \tag{3.69}$$

The large increase in Φ_H in the second de Sitter period is mainly a consequence of the homogeneous evolution of Φ_H, the self-gravitation of initial perturbations in a universe in which $|c_s{}^2|$ is rapidly increasing.

The matter perturbations grow concurrently with the geometric fluctuations. Equation (3.56) gives the exact time dependence. This time scaling could also be obtained using the constraint equations. Although $I(t)$ turns out to be unimportant in the second de Sitter period, it is important to keep track of its magnitude for the analysis in the FRW period.

The upshot of the analysis of Eq. (2.3) during reheating is that the reheating period Γ^{-1} is too short for any order of magnitude change in Φ_H or $\dot{\Phi}_H$ to occur. Since no energy scale larger than σ arises in any of the coefficient functions in (2.3), it is fairly obvious neither Φ_H nor $\dot{\Phi}_H$ can change by more than a factor of order unity. We check this claim rigorously in Appendix D.

By integrating the FRW equation and matching with the initial conditions after reheating we obtain

$$a(t)=a(t_R)[1+2H(t-t_R)]^{1/2}, \tag{3.70}$$

$$H(t)=H[1+2H(t-t_R)]^{-1} \tag{3.71}$$

during the FRW period. The term H is the de Sitter-phase Hubble constant. Since $c_s{}^2=w=\frac{1}{3}$, one of the eigenmodes of the equation of motion

$$\ddot{\Phi}_H+5H(t)\dot{\Phi}_H=I(t) \tag{3.72}$$

is constant. The second decays as

$$f_2(t)=[1+2H(t-t_R)]^{-3/2}. \tag{3.73}$$

The homogeneous solution of (3.72) is

$$\Phi_H^h(t)=c_1+c_2[1+2H(t-t_R)]^{-3/2}. \tag{3.74}$$

If it were not for the large initial value (3.69) of $\dot{\Phi}_H$, Φ_H would remain constant. Since

$$c_2\simeq\dot{\Phi}_H(t_R)H^{-1}, \tag{3.75}$$

both c_2 and c_1 must be of order one. Hence

$$\Phi_H^h(t_f)\simeq c_1\sim 1. \tag{3.76}$$

It is easy to check that $\Phi_H^p(t)$ is dominated by

$$\Phi_H^P(t) \simeq -\int_{t_R}^{t} I(t')\epsilon(t')f_2(t')dt'$$
$$\simeq \ln[1+2H(t-t_R)] \ . \tag{3.77}$$

In particular, at horizon crossing $\Phi_H^P(t)$ dominates and

$$\Phi_H(t_f) \simeq 50 \ , \tag{3.78}$$

in agreement with the results of previous analysis.[11–14] To obtain (3.78), recall that at horizon crossing

$$[1+2H(t_f-t_R)] = \left[\frac{Ha(t_R)}{k}\right]^2 , \tag{3.79}$$

which for galactic scale perturbations is of order 10^{40}. We obtain a spectrum which is scale invariant up to logarithmic corrections.

One final comment is appropriate. We agree with Ref. 14 on the magnitude of the increase of Φ_H during the FRW period. In Ref. 14 the growth stems from the homogeneous evolution alone. Nevertheless there is no disagreement. Our respective equations differ by a term

$$a^{-2}\nabla^2 c_s{}^2 \Phi_H \tag{3.80}$$

added to both sides of (2.3). Thus part of our dominant source term appears as one of the coefficients of Φ_H in Ref. 14.

E. Conclusions

The time evolution of Φ_H is sketched in Fig. 3. The amplification of Φ_H is mainly due to the rapid change in the equation of state prior to reheating. We keep track of the time development of the matter source terms. In contrast to previous claims we conclude that these source terms are not negligible. Keeping track of them does not, however, change the order of magnitude of the final result, but at most the prefactor of order unity.

Our answer for the amplitude of energy-density fluctuations at horizon crossing agrees with the previous analyses:[11–14] the spectrum is nearly scale invariant, i.e., it depends only logarithmically on k, but the amplitude is too large.

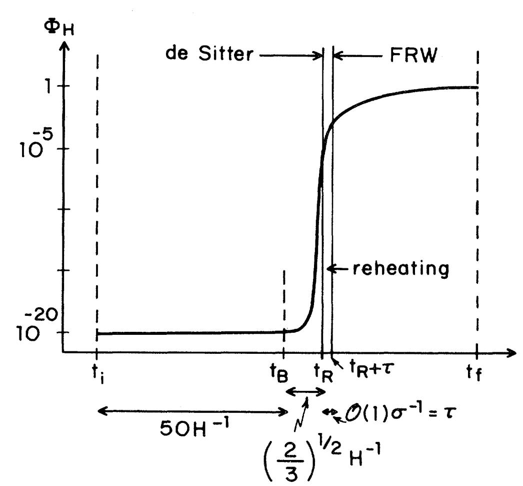

FIG. 3. Phases of the evolution of the new inflationary universe and growth of Φ_H.

An estimate of the accuracy of the various approximations is given in Ref. 26.

IV. OTHER MODELS

A. General considerations

The first aim of this section is to present toy models which demonstrate that the final magnitude of energy-density fluctuations at horizon crossing is independent of the details of the phase structure of the particle physics model between the times when scales of interest leave the Hubble radius and when they reenter. In particular, the amplification factor of perturbations is independent of the specific reheating mechanism and of the details of the quantum-field equation of motion. To this end we will consider a model with an additional period of inflation at a lower energy scale, e.g., associated with the Weinberg-Salam phase transition (Sec. IV B), and a model with inefficient reheating (Sec. IV C).

The conservation law (2.15) renders these claims fairly obvious. We must check however that the matter source terms are, in fact, unimportant.

A second aim is to compare the previous methods in the literature[11–14] and discuss their limitations.

One of the proposed methods[11–13] is based on evolving matter perturbations (as scalar field perturbations) in the unperturbed de Sitter metric up to reheating. At that point a coordinate transformation to constant ϕ gauge is performed. By this gauge transformation the perturbations shift to the metric. They are then evolved in time during the FRW phase according to one of the usual frameworks for analyzing cosmological perturbations.[27,28]

The main drawback of this method is that it neglects metric fluctuations in the de Sitter phase and focuses exclusively on the pressure perturbations induced at reheating by the time lag in the scalar field. In addition, reheating is considered to be an instantaneous process.

The authors[11–13] derive the following "magic" formula:

$$\frac{\delta\rho}{\rho}(t_f) \simeq H\delta\tau \ . \tag{4.1}$$

$\delta\tau$ is the time lag in the evolution of the quantum field. Most papers naively compute $\delta\tau$ at initial horizon crossing by

$$\delta\tau = \frac{\delta\phi(t_i)}{\dot{\phi}_0(t_i)} \ . \tag{4.2}$$

As stressed by Guth,[29] the asymptotic value of $\delta\tau$ just before reheating must be used. Such an asymptotic value might be difficult to compute. It might also be unreliable, since close to reheating the linear approximation for $\delta\phi$ will break down.

We present a model for which the naive application of (4.2) fails, but where inserting the asymptotic value for $\delta\tau$ in (4.1) does give the correct result (the logarithmic potential model of Sec. IV D).

The second method in the literature[14] applies Bardeen's gauge-invariant analysis of cosmological perturbations[30] to the new inflationary universe, consistently in all the periods. Our method is but a modest extension: we include the effects of matter source terms by keeping track of the time evolution of scalar field fluctuations. Reference 14 focuses exclusively on the self-gravitational increase in the amplitude of fluctuations.

There are two mechanisms which cause initial energy-density perturbations to grow. The first mechanism, which we will call "self-gravitation" of initial inhomogeneities, is due to the attractive nature of the gravitational force. Overdense regions tend to clump further. The expansion of the universe counteracts this tendency. The net balance is self-gravitation. In a radiation dominated FRW universe this process produces the well-known Jeans instability (see, e.g., Ref. 31).

The second source of amplification are pressure perturbations which arise during the evolution of the system. As is well known (see, e.g., Ref. 27), in a radiation-dominated FRW universe pressure perturbations of amplitude A acting over a time interval H^{-1} will produce energy-density fluctuations of amplitude A at horizon crossing.

The separation into these two mechanisms is not gauge invariant. In the new inflationary universe, for example, pressure perturbations at reheating act only during a period σ^{-1}, while in synchronous gauge such perturbations are of order unity during a period $\delta\tau \simeq H^{-1}$. In the first gauge the effect is unimportant, in the second crucial. We maintain, however, that neglecting metric fluctuations in the de Sitter phase in method 1 and neglecting matter source terms for method 2 means neglecting the component of one of the two mechanisms in the respective gauge. The best proof of this claim is to present models for which one of the methods fails. We do this in Sec. IV D.

When do formulas (4.1) and (4.2) reproduce the entire homogeneous growth of fluctuations according to our conservation law (2.15)? We derive a simple criterion under which this is true. To this end we estimate the order of magnitude of the results for both methods.

To evaluate (2.15) we apply the constraint equations (2.7) and (2.8). Linearizing in $\delta\phi$ (thus, in particular, dropping subdominant spatial gradient terms) and using the equations of motion for $\phi_0(t)$ and $\delta\phi(t)$, we obtain

$$\tfrac{4}{3}\Phi_H(t_i)+H^{-1}\dot{\Phi}_H(t_i) = \frac{4\pi G}{3}a^2\nabla^{-2}(-\dot{\phi}_0\delta\dot{\phi}+\ddot{\phi}_0\delta\phi)(t_i)\ . \qquad (4.3)$$

If the "slow-rolling" approximation is valid, i.e., if $\ddot{\phi}_0$ is negligible, then the first term dominates and using the equation of motion for $\delta\phi(t)$ once more we obtain [by (2.15)]

$$\Phi_H(t_f)\simeq\frac{V''(\phi_0(t_i))\delta\phi(t_i)}{\dot{\phi}_0(t_i)H}\ . \qquad (4.4)$$

If the slow-rolling approximation is not valid, then the second term in (4.3) dominates and

$$\Phi_H(t_f)\simeq\frac{V'(\phi_0(t_i))\delta\phi(t_i)}{\dot{\phi}_0^{\,2}(t_i)}\ . \qquad (4.5)$$

In physical terms if the scale which determines the curvature of the potential at the point $\phi(t_i)$ when the perturbations leave the horizon is H, then the "naive" application of (4.1) and (4.2) will give the correct result.

If, in addition to linearizing in $\delta\phi$ we can write $\delta\phi(\vec{x},t)=\dot{\phi}_0(t)\delta\tau(\vec{x})$, then the two terms on the right-hand side of Eq. (4.3) cancel. If this Taylor expansion is valid—and inside the Hubble radius there is no reason this should be so—then $\Phi_H(t_i)$ is dominated by terms of order $\delta\phi^2$, in particular by the spatial gradient terms.

B. A model with a second period of inflation

In grand unified theories the full gauge symmetry is usually broken to the observed symmetry group in two or more stages. For example, in minimal SU(5),[5] SU(5) is broken to SU(3)$\times$SU(2)$\times$U(1) at the grand unification scale $\sigma\simeq10^{14}$ GeV. In a second stage, the Weinberg-Salam phase transition at $\sigma'\simeq100$ GeV, the electroweak symmetry group SU(2)$\times$U(1) is further reduced to the observed U(1) of electromagnetism. We will consider a toy model in which the Weinberg-Salam transition is a second-order phase transition mediated by a Coleman-Weinberg potential. The transition will produce an additional period of inflation.

The phases in our model are sketched in Fig. 4. The evolution up to just before t_A is as in the new-inflationary-universe model of Sec. III: the de Sitter phase of the grand unified symmetry breaking from t_0 to t_{R1} is followed by reheating at t_{R1}, and a first FRW phase. Once the universe cools below σ', it gets trapped in the symmetric metastable vacuum state of the Higgs field mediating the Weinberg-Salam symmetry breaking and supercools during the interval from t_A to t_B. This phase is followed by the second rolling phase lasting from t_B until reheating at t_{R2}.

We consider scales which leave the horizon in the first rolling phase and reenter in the final FRW phase. Energy-density fluctuations on these scales are unaffected by the new phases. The easiest way to see this is to consider (2.15). $\Phi_H(t_f)$ depends only on the initial fluctuations $\Phi_H(t_i)$ and $\dot{\Phi}_H(t_i)$ and on the initial and final equations of state [$w(t_i)$ and implicitly $w(t_f)$].

To appreciate the power of the conservation law (2.15), we will sketch the nontrivial derivation of this result by

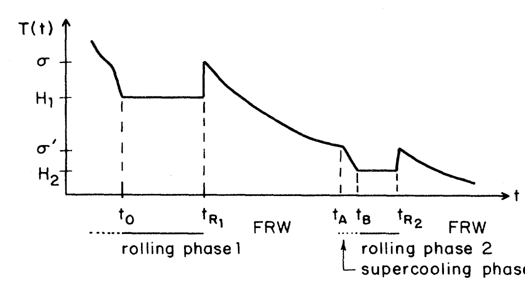

FIG. 4. Phases in the model with a second period of inflation.

integrating the differential equation (2.3) phase by phase. Up to t_A the analysis is as in Sec. III:

$$\Phi_H(t_A)\simeq\frac{E(t_{R_1})}{E(t_i)}\Phi_H(t_i)\ . \tag{4.6}$$

In the supercooling phase the equation of state is given by

$$\begin{aligned}\rho(t)&=\rho_f(t)+\rho_m(t)\ ,\\ p(t)&=-\rho_f(t)+\tfrac{1}{3}\rho_m(t)\ .\end{aligned} \tag{4.7}$$

The expression $\rho_f(t)\simeq\sigma'^4$ is the vacuum energy of the metastable ground. The deviations from a pure de Sitter phase are given by relativistic matter with an energy density $\rho_m(t)$ which is red-shifting away. Since $\rho_m(t)\ll\rho_f(t)$, we have

$$\begin{aligned}w&=-1\ ,\\ c_s{}^2&=\tfrac{1}{3}\ .\end{aligned} \tag{4.8}$$

The equation of motion (2.3) for Φ_H becomes (with $H\equiv H_2$)

$$\ddot{\Phi}_H+5H\dot{\Phi}_H+4H^2\Phi_H=I(t)\ . \tag{4.9}$$

The two modes of the homogeneous equation are

$$\begin{aligned}f_1(t)&=e^{-Ht}\ ,\\ f_2(t)&=e^{-4Ht}\ .\end{aligned} \tag{4.10}$$

Using $\dot{\Phi}_H(t_A)=0$, we obtain

$$\begin{aligned}\Phi_H(t)&=d_1e^{-H(t-t_A)}+d_2e^{-4H(t-t_A)}\\ &=\Phi_H(t_A)[\tfrac{4}{3}e^{-H(t-t_A)}-\tfrac{1}{3}e^{-4H(t-t_A)}]\ .\end{aligned} \tag{4.11}$$

In the following rolling phase from t_B to t_{R2}, Φ_H is given by (3.66)

$$\Phi_H(t)=c_1H\int_{t_B}^{t}\exp[H(t'-t)]\frac{E(t')}{E(t_B)}dt' +c_2e^{-H(t-t_B)}\ . \tag{4.12}$$

The boundary matching conditions at t_B show that the growing mode of the rolling phase couples only to the decaying mode of the supercooling phase,

$$c_1=-3d_2e^{-4H(t_B-t_A)}=\Phi_H(t_A)e^{-4H(t_B-t_A)}\ . \tag{4.13}$$

The analysis of the behavior of the growing mode through reheating gives

$$\Phi_H(t_f)\simeq\frac{E(t_{R_2})}{E(t_B)}c_1\ . \tag{4.14}$$

The expression $E(t_{R2})\simeq E(t_A)$ is the value after reheating. Since the nonvacuum part of the energy density red-shifts away during the supercooling phase, we have

$$E(t_B)=e^{-4H(t_B-t_A)}E(t_A)\ . \tag{4.15}$$

Combining (4.6) and (4.13)—(4.15) gives

$$\Phi_H(t_f)\simeq\frac{E(t_{R1})}{E(t_i)}\Phi_H(t_i)\simeq1\ , \tag{4.16}$$

the same result as in the case of the new inflationary universe of Sec. III. In Appendix E we verify that the source terms in the supercooling phase are unimportant.

An important corollary of our investigation is the observation that the evolution of Φ_H in an approximate de Sitter phase depends crucially on the equation of state of the small deviation. Red-shifting matter is a decreasing deviation and leads to a decreasing Φ_H, an increasing scalar field perturbation on the other hand yields an increasing Φ_H.

C. Slow-reheating model

The previous example demonstrated that the amplitude of energy-density fluctuations is independent of the phase structure between when the scales of interest leave and reenter the Hubble radius. It was convenient but not vital to use the recast form (2.15) of the equation of motion for Φ_H. In this subsection we present a model which shows that the reheating mechanism is irrelevant for $\delta\rho/\rho(t_f)$. But this time it is crucial to use (2.15). Trying to integrate (2.3) step by step through all the phases would turn into a computational nightmare.

The potential of our toy model is sketched in Fig. 5. Near the origin it is a quartic with a negative coefficient, as in the case of the new inflationary-universe model. For larger values of ϕ, the potential is quadratic about the absolute minimum at $\phi=M$ with curvature of order H^2.

We consider perturbations which leave the Hubble radius in the quartic part of the potential, i.e., for $H<\phi<\phi_c$. Thus the initial perturbations and the initial equation of state are as in the example of Sec. III. Hence by (4.5), as in the previous example,

$$\frac{\delta\rho}{\rho}(t_f)\simeq1 \tag{4.17}$$

[in agreement with (4.1) and (4.2)]. Equation (4.17) follows from the homogeneous growth of Φ_H alone. Trying to derive the result by integrating (2.3) quickly leads to problems. Since $\dot{\phi}$ is constant in the initial period of the evolution, Φ_H will be constant. In the slow-rolling phase, $H=$const is an invalid approximation. We are left with a complicated set of coupled differential equations to determine the input functions $w(t)$, $c_s{}^2(t)$, and $H(t)$ for (2.3). Without additional information about the precise values of the parameters in the model it is impossible even to find reasonable approximations.

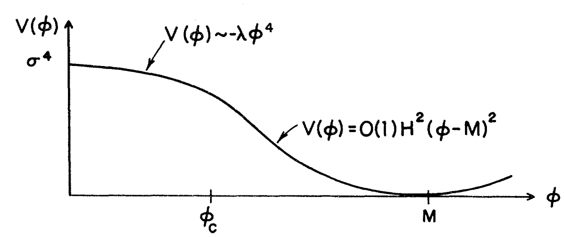

FIG. 5. Effective potential for the slow-reheating model.

Another way to understand (4.17) is to consider the evolution of Φ_H using synchronous time slicing. The period τ_R of reheating is given by the square root of the curvature of the effective potential at its minimum (see Ref. 10). The reason is the following: The decay constant of the oscillation of the scalar field is of the same order of magnitude as the physical mass of the Higgs particle which in turn is given by the square root of the curvature. Thus in our model

$$\tau_R \simeq H^{-1} . \tag{4.18}$$

The time lag $\delta\tau$ of the scalar field is of order $10H^{-1}$, as in the new inflationary universe of Sec. III. To be specific we shall assume $2\tau_R = \delta\tau$. Consider a point $\vec{x}_0$ with maximal time for lag. For an interval $\delta\tau/4$, $\vec{x}_0$ will be in the de Sitter phase while the majority of space will already be in the FRW phase (see Fig. 6). Thus there will be a relative pressure perturbation of order 1 over this period. It is well known (see, e.g., Ref. 27) that these pressure perturbations will develop into energy-density perturbations of the same order of magnitude at horizon crossing. This also follows immediately from our Green's-function formalism (inhomogeneous growth of Φ_H). We consider the short period $\delta\tau/2$ during which the background equation of state is FRW. Over this short period H can be taken to be constant. Thus the two modes of the homogeneous equation for Φ_H are

$$\begin{aligned} f_1(t) &= 1 , \\ f_2(t) &= e^{-5tH} . \end{aligned} \tag{4.19}$$

Therefore by (3.46)

$$\begin{aligned} \Phi_H^p(t_R+\delta\tau/2) &\simeq (5H)^{-1} \int_0^{\delta\tau/2} I(t')dt' \\ &\simeq \frac{4\pi}{5H(m_{\text{Planck}})^2} \, |\, \Delta p \,|\, \frac{\delta\tau}{2} . \end{aligned} \tag{4.20}$$

Since the magnitude of $|\,\Delta p\,|$ is σ^4, we obtain

$$\Phi_H^p(t_R+\delta\tau/2) \simeq H\delta\tau . \tag{4.21}$$

D. Logarithmic-potential model

We will now present a model for which the "naive" application of the magic formulas (4.1) and (4.2) gives the wrong result, but where the asymptotic value of $\delta\tau$ works. Equation (4.2) fails because the scale which determines the curvature of the potential at the exit point $\phi(t_i)$ is not H.

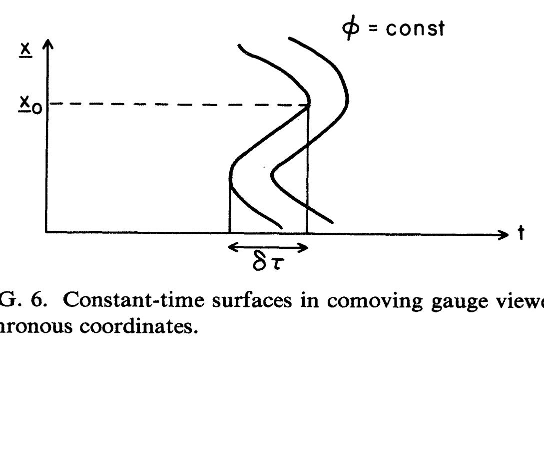

FIG. 6. Constant-time surfaces in comoving gauge viewed in synchronous coordinates.

For $\mu < \phi < m_{\text{Planck}}$ the potential is given by

$$V(\phi) = c_1\sigma^4 - c_2\mu^4 \ln\frac{\phi}{m_{\text{Planck}}} , \tag{4.22}$$

where $\sigma > \mu$. Equation (4.22) is a modification of the Higgs potential one obtains in the reverse-hierarchy supersymmetric models (see Ref. 32). In particular, in the geometric-hierarchy model,[33] $\sigma = \mu$ and a natural choice of the constants gives

$$\frac{c_1}{c_2} \gtrsim 10^2 . \tag{4.23}$$

In this case there is enough inflation while the Higgs field is in the flat part of the potential (see Fig. 7) for large values of ϕ. As discussed in Refs. 14, 34, and 35, fluctuations on scales of physical interest leave the horizon in this period and grow according to (2.15) and (4.1) and (4.2). The term Φ_H at horizon crossing is consistent with observational requirements. The drawback of the model is that without postulating special supergravity effects which generate a sharp dip in the potential at $\phi = m_{\text{Planck}}$ there is insufficient reheating. Decoupling theorems (see Refs. 33 and 35) pose additional problems.

We consider perturbations which leave the horizon at $\phi \sim \mu$. (Thus this analysis does not apply to models for which scales of interest leave the Hubble radius in the flat part of the potential.) To ensure that this first period is an approximate de Sitter phase, we require $\sigma > \mu$. The main difference between perturbations which cross the horizon at $\phi \sim \mu$ and those which cross in the flat period is the energy scale which characterizes the slope of the potential at the exit point. For $\phi \sim \mu$ the slope is μ^3, for large ϕ it is of the order $\mu^2 H$.

The naive application of the magic formula (4.10) gives

$$H\delta\tau \simeq \left[\frac{H}{\mu} \right]^2 , \tag{4.24}$$

which does not agree with the correct evolution of Φ_H according to (2.15).

Initially the damping terms in the equations of motion for $\phi_0(t)$ and $\delta\phi(t)$ are subdominant. Hence the natural initial conditions for matter at horizon crossing t_i are

$$\begin{aligned} \phi_0(t_i) &\simeq \mu , \\ \dot\phi_0(t_i) &\simeq \mu^2 , \\ \delta\phi(t_i) &\simeq H , \\ \delta\dot\phi(t_i) &\simeq H\mu . \end{aligned} \tag{4.25}$$

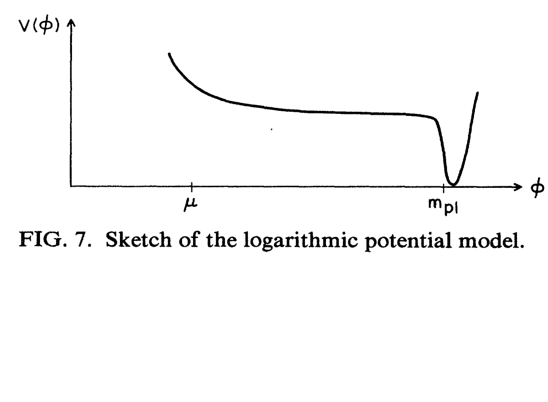

FIG. 7. Sketch of the logarithmic potential model.

Both $\dot{\phi}_0 \delta\phi$ and $V'(\phi_0)\delta\phi$ are thus of order $H\mu^3$ and hence by (4.3)

$$\Phi_H(t_i)+H^{-1}\dot{\Phi}_H(t_i)\simeq\left[\frac{H}{m_{\text{Planck}}}\right]^2\left[\frac{\mu}{H}\right]^3 . \quad (4.26)$$

Since

$$1+w(t_i)\simeq\left[\frac{\mu}{\sigma}\right]^4 , \quad (4.27)$$

Eq. (2.15) gives

$$\frac{\delta\rho}{\rho}(t_f)\simeq\frac{H}{\mu} . \quad (4.28)$$

It is easy to check that the source terms are subdominant in the de Sitter phase. Numerically we can verify that $\dot{\phi}_0$ remains of order μ^2 and $\delta\dot{\phi}$ of order $H\mu$ up to the point at which the damping terms in the matter equations of motion become important. Hence

$$I(t)\lesssim(m_{\text{Planck}})^{-2}\mu^3 H \quad (4.29)$$

and the Green's-function method gives

$$\Phi_H^p(t)\lesssim\left[\frac{\mu}{\sigma}\right]^4\frac{H}{\mu} . \quad (4.30)$$

In this model it is not hard to compute the asymptotic value of $\delta\tau$. We will now demonstrate that its use in (4.1) gives the correct result (4.28). The damping terms are unimportant for

$$\mu<\phi_0(t)\lesssim\frac{\mu^2}{H} , \quad (4.31)$$

i.e., for a time interval $\tau_1\simeq H^{-1}$. Afterwards the Hubble damping terms dominate in the scalar field orientations. The valid approximation is

$$3H\dot{\phi}_0=\mu^4\phi_0^{-1} . \quad (4.32)$$

The solution is

$$\phi_0(t)=\left[\frac{2}{3H}\right]^{1/2}\mu^2 t^{1/2} . \quad (4.33)$$

Similarly, the linearized perturbation equation for $\delta\phi$ has an approximate solution

$$\delta\phi(t)=\delta\phi(t_0)\left[\frac{t}{t_0}\right]^{-1/2} . \quad (4.34)$$

$t_0\simeq H^{-1}$ characterizes the beginning of the second period. Using $\delta\phi(t_0)\simeq\mu$, we see that $\delta\tau$ takes on an asymptotic limit

$$\delta\tau=\frac{\delta\phi(t)}{\dot{\phi}_0(t)}\simeq\mu^{-1} . \quad (4.35)$$

Hence (4.1) gives the correct result.

E. Models which "solve" the fluctuation problem

We have already mentioned one particle-physics model which could potentially solve the fluctuation problem: the geometric-hierarchy supersymmetric model.[33] Due to decoupling theorems and insufficient curvature at the absolute minimum of the potential though it seems impossible to generate sufficient reheating.[35]

Softly broken supersymmetry[36] has the same drawbacks. As noticed in Ref. 37, it trivially solves the fluctuation problem, but there is *a priori* insufficient vacuum energy to reheat the universe. Both statements follow from the simple observation that this model only differs from the standard new inflationary universe by rescaling.

We will denote quantities in the nonsupersymmetric model without a tilde, in the supersymmetric one with a tilde. The expression m_s is the scale of supersymmetry breaking. Since the one-loop corrections to the effective potential have opposite signs for bosons and fermions, there is a reduction in the height of the effective potential.

$$\tilde{B}(\tilde{\lambda})=\left[\frac{m_s}{\sigma}\right]^2 B(\lambda) . \quad (4.36)$$

This leads to the following rescalings:

$$\tilde{H}=\left[\frac{m_s}{\sigma}\right]H , \quad (4.37)$$

$$\tilde{t}^*=\left[\frac{\sigma}{m_s}\right]^3 t^* , \quad (4.38)$$

$$(\tilde{t}^*-t_i)\simeq\left[\frac{m_s}{\sigma}\right]^{-1}(t^*-t_i) . \quad (4.39)$$

Hence by (3.37) and (4.1)

$$\left.\frac{\delta\tilde{\rho}}{\tilde{\rho}}\right|_{\tilde{H}}=\left[\frac{m_s}{\sigma}\right]\left.\frac{\delta\rho}{\rho}\right|_H . \quad (4.40)$$

The fluctuation problem disappears, but since $\tilde{V}(0)$ is suppressed by $(m_s/\sigma)^2$ there is insufficient reheating.

Other particle-physics models which have been suggested in order to solve the fluctuation problem include supersymmetric primordial inflation,[38] primordial inflation in supergravity,[39] and grand unified models with more than one scale of symmetry breaking.[40] These investigations all use the magic formula (4.2). We verified that the condition (4.5) under which (4.2) can be applied is, indeed, satisfied in the models of Refs. 38 and 39. We do not comment here on the phenomenology of these models.

Another attempt to solve the fluctuation problem without abandoning inflation is to investigate in more detail the initial evolution of the system after the onset of metastability.

Mottola and Lapedes,[41] based on previous work by Hawking and Moss,[22] claim that in a potential with nonzero bare mass of order H the phase transition proceeds by a quantum tunneling process which is homogeneous in space. Inflation then takes place before symmetry breaking without generating the homogeneity problems which afflicted the old inflationary universe. At the exit point from tunneling the slope of the potential is larger than in the inflationary universe. Therefore fluctuations will be smaller.[42]

A completely different approach described recently by

Hawking and Moss[43] and Vilenkin[44] is to study the initial evolution of $\langle \phi^2 \rangle$ taking quantum corrections in curved spacetime into account. Both report reduced values for $\delta\rho/\rho$, although the conclusions have been (in our opinion correctly) questioned by Guth.[29]

Finally, Linde has proposed a model of chaotic inflation.[45] In large regions of space, in particular in one which contains the entire presently observed universe, the Higgs field ϕ is supposed to start out at some initial time at some large value $\phi > 3m_{\text{Planck}}$. In this case sufficient inflation occurs while ϕ is beginning to roll towards the absolute minimum of the potential. Provided the coupling constant λ is sufficiently small, inhomogeneities will be small enough.

V. CONCLUSIONS

We analyze the growth of energy-density fluctuations in inflationary universe models, paying particular attention to the mechanisms of amplification. We apply an extension of Bardeen's gauge-invariant framework derived in an earlier paper. In particular, we keep track of the effects of additional matter source terms in the equations for gravitational perturbations.

We exemplify our method in detail for the example of the new inflationary universe. In agreement with previous investigations we obtain the following result: the spectrum at horizon crossing is nearly scale invariant but has an amplitude which exceeds the required value for consistency with galaxy-formation pictures by about five orders of magnitude,

$$\left.\frac{\delta\rho}{\rho}\right|_H \simeq 50 \tag{5.1}$$

for galactic scales.

By examining the detailed evolution of the gauge-invariant metric potential Φ_H we conclude that most of the amplification takes place as a consequence of the rapid change in the equation of state near reheating.

We then recast the differential equation for Φ_H in terms of a conservation law and conclude that the magnitude of energy-density fluctuations at horizon crossing depends only on the equation of state when the perturbations leave the horizon (keeping track of the deviation from an exact de Sitter space is crucial), the initial values of perturbations at that point (determined by Hawking radiation and the Einstein constraint equations) and the final equation of state. In particular, the final answer is independent of the reheating mechanism or any other details concerning the phase structure of the model between initial and final horizon crossing.

Finally, we discuss the limitations of previous methods and present toy models in which often-quoted magic formulas for $\delta\rho/\rho$ fail.

Note added. After submission of this manuscript we received a report (Ref. 46) by Frieman and Turner in which similar methods are applied to study the evolution of density perturbations through cosmological phase transitions.

ACKNOWLEDGMENTS

Foremost we thank William Press for constant encouragement and for many hours of fruitful discussions. One of us (R.B.) acknowledges the hospitality of the Aspen Center for Physics, 1983, where part of the research was completed, and in particular thanks Alan Guth, Michael Turner, Ethan Vishniac, and Jennie Traschen for helpful conversations. We thank Joshua Frieman and Michael Turner for pointing out an error in one of the toy-model computations in a preliminary version of the paper.

This work was supported in part by NSF grants PHY-80-0735 and PHY-82-03669.

APPENDIX A: MAGNITUDE OF METRIC PERTURBATIONS IN DIFFERENT GAUGES

In this appendix we show that in the de Sitter phase of the new inflationary universe the relative metric perturbations are of order unity in comoving coordinates, but negligibly small in synchronous gauge.

Consider first comoving coordinates. Until reheating the constant-time hypersurfaces in comoving gauge are constant ϕ surfaces. The situation is sketched in Fig. 8.

We consider the physical distances Δs_1 and Δs_2 corresponding to a fixed coordinate distance Δx about two points $\vec{x}_1$ and $\vec{x}_2$ on the same $\phi = \text{const}$ hypersurface with minimal and maximal time lag, respectively (in synchronous coordinates, in which the metric fluctuations are negligible),

$$\begin{aligned}\Delta s_2 &= \Delta x\, e^{H(t(\vec{x}_1)+\Delta\tau)} \\ &= \Delta s_1 + \Delta x\,(e^{H(t(\vec{x}_1)+\Delta\tau)} - e^{H(t(\vec{x}_1))}) \;.\end{aligned} \tag{A1}$$

Since $\Delta\tau$ is of order H^{-1} in the new inflationary universe,

$$\Delta s_2 \sim e\,\Delta s_1 \;. \tag{A2}$$

For the metric perturbation $g_{ij}^{(1)}$ this immediately implies

$$\frac{g_{ij}^{(1)}(\vec{x}_2)}{g_{ij}^{(0)}(\vec{x}_2)} \simeq 1 \;. \tag{A3}$$

Next we verify that in synchronous gauge the metric perturbations are indeed negligible. The procedure is simple: the geometric perturbations are obtained by integrating (2.9) and (2.10). We totally fix the gauge by demand-

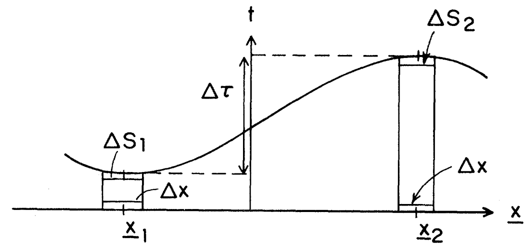

FIG. 8. Constant ϕ surfaces viewed from synchronous-gauge coordinates. Δx is a fixed coordinate distance. ΔS_1 and ΔS_2 are the corresponding physical distances at the extremal points of the constant ϕ surface.

ing $A(t_i)=B(t_i)=0$.

First consider $A(t)$. $\mathscr{P}_3$ is expressed in terms of $I_3(t)$, which scales as $I(t)$. Thus

$$A(t) \sim H^{-1} \int_{t_i}^{t} c_s^{-2}(t') I(t') dt' . \tag{A4}$$

It is easy to check that up to t_R

$$|A(t)| \lesssim 10^{-10} . \tag{A5}$$

Thus in Eq. (2.10), the term linear in Φ_H dominates,

$$\begin{aligned} B(t) &\sim \int_{t_i}^{t} \frac{k^2}{a^2 H^2} \Phi_H(t') dt' \frac{H}{k^2} \\ &\sim \frac{H}{k^2} \int_{t_i}^{t} e^{-H(t'-t_i)} \Phi_H(t') dt' . \end{aligned} \tag{A6}$$

Again it follows that up to t_R

$$|B(t)| \lesssim 10^{-10} k^{-2} . \tag{A7}$$

The time derivatives can be estimated similarly (we get an extra factor 10^5). Since outside the horizon the geometric fluctuations enter into the equation of motion for the scalar field in the combination

$$\dot{h} = 3\dot{A} + \nabla^2 \dot{B} \tag{A8}$$

[see (2.5)], we see they are indeed negligible.

APPENDIX B: ANALYSIS OF SCALAR FIELD FLUCTUATIONS

In this appendix we will prove some of the statements made in Sec. III concerning scalar field perturbations.

First we show that in the entire de Sitter phase, the scalar field perturbations may be encoded as a space-dependent time lag $\delta\tau(\vec{x})$. Immediately after horizon crossing $\delta\phi(k)$ is small compared to $\phi_0(t)$ [by (3.10) and (3.18)]. Therefore the time evolution of $\delta\phi(k)$ can be determined by linearizing (3.6) in $\delta\phi(k) \equiv \delta\phi$,

$$3H\delta\dot{\phi}(t) = 3\lambda \phi_0^2(t) \delta\phi(t) . \tag{B1}$$

Explicit integration gives

$$\delta\phi(t) = \delta\phi(t_i) \left[\frac{t^* - t_i}{t^* - t} \right]^{3/2} . \tag{B2}$$

Hence $\delta\phi$ has the same time dependence as $\dot{\phi}_0(t)$. Hence by Taylor expansion we immediately obtain (3.50) and (3.51).

Far outside the horizon the spatial gradient terms in the equation of motion for ϕ are completely negligible. Hence for any solution $\phi_0(t)$, $\phi_0(t-\delta\tau(\vec{x}))$ will also be a solution. It is only one of the two independent perturbative modes. But it is easy to verify it is the dominant one. In the second period of the de Sitter phase the perturbative modes are the solutions of

$$\begin{aligned} \delta\ddot{\phi}(\vec{x},t) &= 3\lambda\phi_0^2(t)\delta\phi(\vec{x},t) \\ &= 6[\alpha-(t-t_B)]^{-2}\delta\phi(\vec{x},t) . \end{aligned} \tag{B3}$$

The general solution of (B3) is

$$\delta\phi(\vec{x},t) = c_1[\alpha-(t-t_B)]^{-1} + c_2[\alpha-(t-t_B)]^3 . \tag{B4}$$

The first term corresponds to our mode (3.50), the second is a decaying mode and hence irrelevant for our investigation.

Next we analyze the initial conditions for Φ_H and $\dot{\Phi}_H$, given by the constraint Eqs. (2.7) and (2.8) in terms of the quantum field perturbations. As discussed in Sec. II, we evaluate the matter terms in the synchronous gauge. Linearizing in the perturbations and using (B1), we obtain

$$\begin{aligned} T_0^{0(1)} &\simeq -\dot{\phi}_0 \delta\dot{\phi} - V'(\phi_0)\delta\phi \\ &= \left[\frac{\lambda}{3H^2}\phi_0^2 - 1 \right] V'(\phi_0)\delta\phi \\ &\simeq \phi_0^3 \dot{\phi}_0 \delta\tau . \end{aligned} \tag{B5}$$

By (3.7) and (3.14), $\phi_0(t_i) \simeq 10^{-1} H$ and $\dot{\phi}_0(t_i) \simeq 10^{-3} H^2$. It is easy to check that the flux term in (2.7) is negligible in comparison with (B5). Hence

$$\Phi_H(t_i) \simeq 10^{-3} \left[\frac{H}{m_{\text{Planck}}} \right]^2 . \tag{B6}$$

The derivative of Φ_H contains two types of terms: first terms in which the derivative acts on $a(t)$, second those in which the scalar field is differentiated. The latter are suppressed with respect to the former by the factor

$$\frac{1}{t^* - t_i} H^{-1} \simeq 10^{-2}$$

[this follows from the explicit form of $\phi_0(t)$, i.e., from (3.7)]. Hence

$$\begin{aligned} \dot{\Phi}_H(t_i) = -4\pi G H^{-2} \Big[& 2HT_0^{0(1)} + \dot{T}_0^{0(1)} \\ & + 3H^{-2} \frac{(\dot{a}a^3)_{,t}}{a^4} T_{0,k}^{k(1)} \\ & + 3H^{-1} \dot{T}_{0,k}^{k} \Big] (t_i) \end{aligned} \tag{B7}$$

is dominated by the first term and thus

$$\dot{\Phi}_H(t_i) \simeq 10^{-3} \left[\frac{H}{m_{\text{Planck}}} \right]^2 H . \tag{B8}$$

Finally, we sketch the estimates for the matter source terms in both periods of the de Sitter phase. An important point to realize is that the linear approximation in the matter perturbations breaks down soon after horizon crossing. The estimates must therefore be based on the representation (3.50),

$$\phi(\vec{x},t) = \phi_0(t - \delta\tau(\vec{x})) = \phi_0(t) + \phi_1(t,\vec{x}) , \tag{B9}$$

where ϕ_1 has the same order of magnitude as ϕ_0.

The order-of-magnitude estimates are based on formulas (2.29). Outside the horizon I_{11} contains two important terms, the deviation of $\frac{1}{2}\dot{\phi}^2$ from its mean and the mean deviation of the potential energy $V(\phi)$. The former scales as $\dot{\phi}_0^2$, the latter as ϕ_0^4. It is easy to check, using the explicit formulas for $\phi_0(t)$, that in both periods of the de Sitter phase the potential term dominates. Hence in the first period we have

$$I_{11}(t) \simeq I_0 \frac{1}{(t^* - t)^2} \quad \text{(B10)}$$

with

$$I_0 \simeq \left[\frac{H}{m_{\text{Planck}}} \right]^2 . \quad \text{(B11)}$$

In the second period

$$I_{11}(t) \simeq \left[\frac{H}{m_{\text{Planck}}} \right]^2 H^2 f^4(t) . \quad \text{(B12)}$$

Both I_3 and I_{11} scale identically with time. This follows immediately from (2.29) and the known time dependence of ϕ, $\dot{\phi}$, and $c_s{}^2$. We should not be surprised. After all, I_3 and I_{11} are the only matter source terms which are not individually gauge invariant. Since they are jointly gauge invariant, it is reasonable to expect they will scale identically.

Expression I_2 is negligible in the first period of the de Sitter phase, since $c_s{}^2 - w = 0$. In the second period it is subdominant since it scales only as $f^3(t)$, as does $I_{\dot{2}}$. In the first period, $I_{\dot{2}}$ scales as $(t^* - t)^{-2}$, i.e., as $I_{11}(t)$, but its amplitude is smaller, due to the fact that $\dot{\phi}_0 H^{-1} < \phi_0$. Thus we have demonstrated that the total source $I(t)$ scales according to (B10)–(B12).

APPENDIX C: GREEN'S-FUNCTION METHOD IN THE DE SITTER PHASE

Here we fill in some details on the evolution of Φ_H in the second period of the de Sitter phase. The homogeneous part Φ_H^h, for late times is dominated by the growing mode

$$\Phi_H^h(t_R) \simeq c_1 H \int_{t_B}^{t_R} \exp[H(t' - t)] \frac{E(t')}{E(t_B)} dt' . \quad \text{(C1)}$$

Since $t_R - t_B$ is smaller than H^{-1}, the exponential in (C1) can be replaced by 1. We know that $E(t')/E(t_B)$ scales as $\dot{\phi}_0(t')/\dot{\phi}_0(t_B)$ and thus as $f^4(t)$. Since, in this period

$$\frac{d}{dt} f(t) = H f^2(t) , \quad \text{(C2)}$$

we get

$$f_1(t_R) \simeq f^3(t_R) \quad \text{(C3)}$$

and hence

$$\Phi_H^h(t_R) \simeq c_1 \left[\frac{\sigma}{H} \right]^3 . \quad \text{(C4)}$$

Since the dominant term in $\dot{f}_1(t)$ is

$$H \frac{E(t)}{E(t_B)} \sim H f^4(t) , \quad \text{(C5)}$$

we immediately conclude

$$\dot{\Phi}_H^h(t_R) \simeq c_1 \left[\frac{\sigma}{H} \right]^4 H . \quad \text{(C6)}$$

Using the explicit forms of $f_1(t)$ and $f_2(t)$ we get

$$\epsilon(t) = H^{-1} \frac{E(t_B)}{E(t)} e^{H(t - t_B)} . \quad \text{(C7)}$$

The particular solution $\Phi_H^p(t)$ is dominated by the first term in (2.19), the growing mode contribution

$$\Phi_H^p(t) \simeq -f_1(t) \int_{t_B}^{t} I(t') \epsilon(t') f_2(t') dt' . \quad \text{(C8)}$$

The integrand is constant. Thus

$$\Phi_H^p(t_R) \simeq -f_1(t_R) I_0 \simeq c_1 \left[\frac{\sigma}{H} \right]^3 . \quad \text{(C9)}$$

It is of the same order of magnitude as the homogeneous part of Φ_H. Although it is thus not crucial to carry along the matter source terms in the de Sitter phase, it is wrong to *a priori* neglect them as in Ref. 14.

APPENDIX D: DETAILS ON THE REHEATING PERIOD

We first discuss the scalar-field equation of motion. In terms of the shifted field $\chi = \phi - \sigma$, the potential close to the absolute minimum can be approximated by a Gaussian,

$$V(\chi) = 4B\sigma^2 \chi^2 . \quad \text{(D1)}$$

The initial conditions for χ at t_R are

$$\chi(t_R) = -\tfrac{1}{4}\sigma , \quad \text{(D2)}$$

$$\dot{\chi}(t_R) = \frac{9}{16} \left[\frac{\lambda}{2} \right]^{1/2} \sigma^2 . \quad \text{(D3)}$$

The equation of motion for $\chi(t)$ is

$$\ddot{\chi}(t) + \Gamma \dot{\chi}(t) = -8B\sigma^2 \chi . \quad \text{(D4)}$$

Its solution is a damped harmonic oscillation

$$\chi(t) = c\, e^{-\Gamma t/2} \sigma \cos[\omega_0(t - \gamma)] , \quad \text{(D5)}$$

where ω_0 is of order σ and c and γ are constants of order 1 determined by the initial conditions.

Next we focus on the equation of motion for Φ_H. The only change in the homogeneous equation is the fact that $w(t)$ is no longer exactly -1. Since it is always of order unity and since the other coefficient functions in Eq. (2.26) are much larger than order unity, the correction $w \neq -1$ is negligible. Hence the solutions (C1) for the homogeneous component Φ_H^h and (C8) for the contribution of matter sources can be continued through reheating. In particular, the extra combination of the sources during reheating is

$$\begin{aligned} \Delta\Phi_H^p &= f_1(t_R) \int_{t_R}^{t_R + \tau_R} I(t') \epsilon(t') f_2(t') dt' \\ &\simeq \tau_R H^2 f_1(t_R) \end{aligned} \quad \text{(D6)}$$

and is completely negligible since $\tau_R \simeq \sigma^{-1}$. Thus the values for Φ_H and $\dot{\Phi}_H$ immediately after reheating are given by (3.68) and (3.69), respectively.

APPENDIX E: COMMENTS ON THE TOY MODEL OF SEC. IV B

We must verify that the source terms are unimportant during the supercooling phase. We use the Green's-function method (see Sec. III C).

Since the matter source terms red-shift away, they can be bounded by

$$|I(t)| \leq I_0 e^{-4tH} , \tag{E1}$$

$$I_0 \simeq (m_{\text{Planck}})^{-2}\sigma'^4 . \tag{E2}$$

From (4.10) and (2.20)

$$\epsilon(t) = \tfrac{1}{3}H^{-1}e^{5tH} . \tag{E3}$$

Thus the f_1 term in (2.19) becomes

$$\begin{aligned}\Phi_H^P(t) &\simeq e^{-tH}\int_0^t I(t')\epsilon(t')f_2(t')dt' \\ &\simeq \tfrac{1}{9}H^{-2}I_0 e^{-tH}(1-e^{-3tH}) \\ &\simeq 10^{-1}e^{-tH} .\end{aligned} \tag{E4}$$

This is to be compared with

$$\Phi_H^h(t) \simeq e^{-tH} . \tag{E5}$$

Thus the source terms do not dominate.

At this point it is important to note that Hawking radiation in the supercooling phase cannot produce any coherent inhomogeneities on scales we are considering, scales which during that phase are far outside the Hubble radius such that no microphysical effects can act coherently.

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