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Ground-state baryon mass splittings from unitarity

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We discuss the hadronic mass shifts of the lightest baryons generated by coupled-channel effects. The contributions from different thresholds are assumed to be related by $SU(6)_W$ symmetry. Using very general arguments, we derive mass formulas for the relative splittings of Δ-N, Σ^*-Σ-Λ, and Ξ^*-Ξ, which are satisfied by experiment to within 20%.

It has been well known for a long time that unitarity effects are important in understanding the hadronic mass spectrum (see, e.g., Ref. 1). The difficulty to estimate their actual magnitude reliably has, however, always been an obstacle. In general, one must evaluate a large number of loop diagrams generated by unitarity and sum over many intermediate states. The most important contributions are expected to come from two-hadron intermediate states (Fig. 1). Within the QCD framework this means that in addition to gluonic exchanges one must include contributions from quark loops.

For the meson sector using an explicit model, the unitarized quark model,[2,3] one of us has been able to understand many mass splittings and mixing effects among the light mesons as well as among the heavy $c\bar{c}$ and $b\bar{b}$ states. The success of this approach suggests that hadronic-loop effects are important also for the baryons.

Several authors[4] have tried to estimate the contribution to the Δ-N mass splitting coming from mainly πN and $\pi\Delta$ intermediate states. It is generally agreed that this contribution has the right sign and a magnitude varying from ~ 50 MeV to the whole mass splitting of 293 MeV. The uncertainty in the magnitude stems from the fact that it is very sensitive to the hadron size assumed and to the number of thresholds included. The correct sign follows because hadronic shifts are negative and the nucleon is shifted more than the delta because of the stronger nearby thresholds. We shall not estimate here the actual size of the mass shifts which will be done later in an actual model calculation.[5] Instead we derive below some predictions for the ratios of mass splittings which should be essentially model independent if the unitarity effects are dominant. Under reasonable assumptions of analyticity and proper behavior of the spectral function ρ the shift of the mass of baryon B due to the process shown in Fig. 1 may be written[1,2] as a dispersion integral:

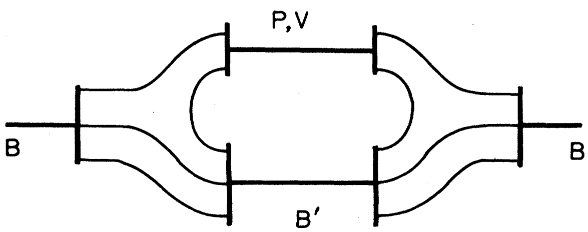

FIG. 1. The unitarity loop diagram under consideration.

$$m_B{}^2-(m_B^0)^2=\sum_i W_i^B \int_{s_{\rm thr}^{(i)}} \frac{\rho(s,s_{\rm thr}^{(i)})}{m_B{}^2-s}\,ds$$
$$=\sum_i W_i^B f(m_B{}^2,s_{\rm thr}^{(i)}) \ , \tag{1}$$

where the sum runs over all (open and closed) decay channels. In writing Eq. (1), we assumed only that the function ρ is universal and therefore that apart from the numerical constant (weight W_i^B) the contributions from all channels become identical in the symmetry limit $s_{\rm thr}^{(i)}$ = independent of i. The weights W_i^B may be calculated if a symmetry relating the different channels i is assumed. As the intermediate states i we shall consider all (allowed by quantum-number conservation) combinations of baryon (B' in Fig. 1) belonging to the SU(6) ground state and a pseudoscalar (P) or vector (V) meson. We assume $SU(6)_W$ symmetry for all $BB'M$ ($M=P,V$) vertices. The weights W_i^B can then be calculated using formulas of Ref. 5 or Ref. 6, and we have tabulated them in Table I. Since our results are not very sensitive to the exact value of the η-η' and ω-ϕ mixing we assume for simplicity in Table I that the singlet-octet mixing angles are $\theta_P=0°$ for pseudoscalar (i.e., $\eta=\eta_8$, $\eta'=\eta_1$) and $\theta_V=35.2°$ (i.e., ideal mixing) for vector mesons. From Table I it can be easily checked that for any baryon B

$$\sum_i W_i^B=48 \ . \tag{2}$$

Thus in the symmetry limit ($s_{\rm thr}^{(i)}$ independent of i) the shift in mass is the same for all members of the 56-plet and consequently no splitting is generated. However, if the positions of different thresholds do not match, the resulting output baryon masses are split.

As in all self-consistent unitarization procedures we shall start with an input degenerate baryon mass spectrum. The differences between the positions of various thresholds arise then only from differences in meson masses. We assume that a solution to the problem of meson mass splitting has already been achieved (cf., e.g., Ref. 3). Consequently for the meson masses we take their experimental values. Furthermore, let the zeroth-order splitting of input baryon masses be such that

$$\begin{aligned} m_N^0 &= m_\Delta^0 = m_0^0 \ , \\ m_\Sigma^0 &= m_\Lambda^0 = m_{\Sigma^*}^0 = m_1^0 \ , \\ m_\Xi^0 &= m_{\Xi^*}^0 = m_2^0 \ , \\ m_\Omega^0 &= m_3^0 \ . \end{aligned} \tag{3}$$

TABLE I. Relative weights with which different SU(6)-related thresholds enter the unitarity loop contribution of Fig. 1, for all members of the ground-state multiplet 56 of SU(6). The singlet-octet mixing angles are here put $\theta_p = 0^\circ$ for pseudoscalar (thus $\eta = \eta_8$, $\eta' = \eta_1$) and $\theta_V = 35.2^\circ$ (ideal mixing) for vector mesons.

B \ $B'P$	$N\pi$	$\Delta\pi$	ΣK	ΛK	$\Sigma^* K$	$N\eta$	$N\eta'$	$\Delta\eta$	$\Delta\eta'$
N	$\frac{25}{6}$	$\frac{16}{3}$	$\frac{1}{6}$	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	0	0
Δ	$\frac{4}{3}$	$\frac{25}{6}$	$\frac{4}{3}$	0	$\frac{5}{3}$	0	0	$\frac{5}{6}$	$\frac{5}{3}$

B \ $B'V$	$N\rho$	$\Delta\rho$	ΣK^*	ΛK^*	$\Sigma^* K^*$	$N\omega$	$N\phi$	$\Delta\omega$	$\Delta\phi$
N	$\frac{59}{6}$	$\frac{32}{3}$	$\frac{11}{6}$	$\frac{9}{2}$	$\frac{8}{3}$	$\frac{11}{2}$	0	0	0
Δ	$\frac{8}{3}$	$\frac{95}{6}$	$\frac{8}{3}$	0	$\frac{19}{3}$	0	0	$\frac{19}{2}$	0

B \ $B'P$	NK	ΔK	$\Sigma\pi$	$\Lambda\pi$	$\Sigma^*\pi$	ΞK	$\Xi^* K$	$\Sigma\eta$	$\Sigma\eta'$	$\Lambda\eta$	$\Lambda\eta'$	$\Sigma^*\eta$	$\Sigma^*\eta'$
Σ	$\frac{1}{9}$	$\frac{32}{9}$	$\frac{16}{9}$	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{25}{9}$	$\frac{8}{9}$	$\frac{2}{3}$	$\frac{1}{3}$	0	0	$\frac{4}{3}$	0
Λ	3	0	2	0	4	$\frac{1}{3}$	$\frac{8}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0
Σ*	$\frac{4}{9}$	$\frac{20}{9}$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{20}{9}$	$\frac{4}{9}$	$\frac{20}{9}$	$\frac{2}{3}$	0	0	0	0	$\frac{5}{3}$

B \ $B'V$	NK^*	ΔK^*	$\Sigma\rho$	$\Lambda\rho$	$\Sigma^*\rho$	ΞK^*	$\Xi^* K^*$	$\Sigma\omega$	$\Sigma\phi$	$\Lambda\omega$	$\Lambda\phi$	$\Sigma^*\omega$	$\Sigma^*\phi$
Σ	$\frac{11}{9}$	$\frac{64}{9}$	$\frac{68}{9}$	$\frac{4}{3}$	$\frac{16}{9}$	$\frac{59}{9}$	$\frac{16}{9}$	$\frac{34}{9}$	$\frac{11}{9}$	0	0	$\frac{8}{9}$	$\frac{16}{9}$
Λ	9	0	4	0	8	$\frac{11}{3}$	$\frac{16}{3}$	0	0	2	3	0	0
Σ*	$\frac{8}{9}$	$\frac{76}{9}$	$\frac{8}{9}$	$\frac{4}{3}$	$\frac{76}{9}$	$\frac{8}{9}$	$\frac{76}{9}$	$\frac{4}{9}$	$\frac{8}{9}$	0	0	$\frac{38}{9}$	$\frac{19}{9}$

B \ $B'P$	ΣK	$\Sigma^* K$	ΛK	$\Xi\pi$	$\Xi^*\pi$	ΩK	$\Xi\eta$	$\Xi\eta'$	$\Xi^*\eta$	$\Xi^*\eta'$
Ξ	$\frac{25}{6}$	$\frac{4}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{3}{2}$	$\frac{1}{3}$	$\frac{4}{3}$	0
Ξ*	$\frac{2}{3}$	$\frac{10}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{5}{3}$	$\frac{2}{3}$	0	$\frac{5}{6}$	$\frac{5}{3}$

B \ $B'V$	ΣK^*	$\Sigma^* K^*$	ΛK^*	$\Xi\rho$	$\Xi^*\rho$	ΩK^*	$\Xi\omega$	$\Xi\phi$	$\Xi^*\omega$	$\Xi^*\phi$
Ξ	$\frac{59}{6}$	$\frac{8}{3}$	$\frac{11}{6}$	$\frac{11}{6}$	$\frac{8}{3}$	$\frac{16}{3}$	$\frac{11}{18}$	$\frac{68}{9}$	$\frac{8}{9}$	$\frac{16}{9}$
Ξ*	$\frac{4}{3}$	$\frac{38}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{19}{6}$	$\frac{19}{3}$	$\frac{4}{9}$	$\frac{8}{9}$	$\frac{19}{18}$	$\frac{76}{9}$

B \ $B'P$	ΞK	$\Xi^* K$	$\Omega\eta$	$\Omega\eta'$	B \ $B'V$	ΞK^*	$\Xi^* K^*$	$\Omega\omega$	$\Omega\phi$
Ω	$\frac{8}{3}$	$\frac{10}{3}$	$\frac{10}{3}$	$\frac{5}{3}$	Ω	$\frac{16}{3}$	$\frac{38}{3}$	0	19

We adopt a similar notation for output baryon masses (with meson mass differences zero), i.e., $m_\Delta = m_N = m_0$, etc. The first-order splitting of output baryon masses (say Δ and N) is then given by

$$m_\Delta{}^2 - m_N{}^2 = \sum_i [W_i^\Delta f(m_0{}^2, s_{\rm thr}^{(i)}) - W_i^N f(m_0{}^2, s_{\rm thr}^{(i)})] \ . \tag{4}$$

If one expands this expression in the threshold positions in the vicinity of some average threshold $\langle s_{\rm thr,0}\rangle$, the constant terms cancel due to the sum rule (2). To get the leading term we keep terms linear in meson masses.

Denoting

$$C_k = \frac{1}{m_k} \frac{\partial f(m_k{}^2, s_{\rm thr})}{\partial(\sqrt{s_{\rm thr}})}\Bigg|_{s_{\rm thr} = \langle s_{{\rm thr},k}\rangle} , \tag{5}$$

and using Table I and assumption 3 we obtain from Eq. (4) (with $m_\rho = m_\omega$)

$$m_\Delta - m_N = C_0(m_\rho - 2m_\pi + \tfrac{1}{3}m_\eta + \tfrac{2}{3}m_{\eta'}) \ . \tag{6a}$$

From Eq. (6a) it is clearly seen that large m_Δ-m_N splitting is driven mainly by the smallness of the pion mass compared to the other members of the pseudoscalar and vector nonets. The sign of C_k is positive on the very general grounds that the farther the threshold the smaller is the absolute value of the negative shift. Thus the signs of the Δ-N and ρ-π splittings are related.

Formulas similar to (6a) may be easily derived for other splittings:

$$m_{\Sigma^*} - m_\Sigma = C_1[m_{K^*} - m_K + \tfrac{2}{3}(m_{\eta'} - m_\eta)] \ , \tag{6b}$$

$$m_{\Sigma^*} - m_\Lambda = C_1[\tfrac{1}{3}(m_{K^*} - m_K) + \tfrac{2}{3}(m_\rho + m_{\eta'} - 2m_\pi)] \ , \tag{6c}$$

$$m_{\Xi^*} - m_\Xi = C_2[m_{K^*} - m_K + \tfrac{2}{3}(m_{\eta'} - m_\eta)] \ . \tag{6d}$$

The constants C_i are equal in the SU(6) limit and this equality should still be a good approximation for broken SU(6). We thus find the following remarkable relations:

$$\frac{m_\Delta - m_N}{m_{\Sigma^*} - m_\Sigma} = \frac{m_\rho - 2m_\pi + \frac{2}{3}m_{\eta'} + \frac{1}{3}m_\eta}{m_{K^*} - m_K + \frac{2}{3}m_{\eta'} - \frac{2}{3}m_\eta} , \tag{7a}$$

$$\frac{m_{\Sigma^*} - m_\Lambda}{m_{\Sigma^*} - m_\Sigma} = \frac{1}{3}\frac{m_{K^*} - m_K + 2m_\rho + 2m_{\eta'} - 4m_\pi}{m_{K^*} - m_K + \frac{2}{3}m_{\eta'} - \frac{2}{3}m_\eta} , \tag{7b}$$

$$\frac{m_{\Xi^*} - m_\Xi}{m_{\Sigma^*} - m_\Sigma} = 1 \ . \tag{7c}$$

Because of the approximations made in deriving these relations we expect them to hold to about 20%. Inserting experimental masses in Eqs. (7) we obtain the first two columns of Table II.

From Eqs. (7a) and (7b) we find the relation between Σ-Λ and Δ-N splitting:

$$\frac{m_\Sigma - m_\Lambda}{m_\Delta - m_N} = \frac{2}{3}\frac{m_\rho - m_\eta - 2m_\pi + m_K - m_{K^*}}{m_\rho - 2m_\pi + \frac{2}{3}m_{\eta'} + \frac{1}{3}m_\eta} , \tag{8}$$

with the left-hand side (LHS) $=0.26$ and the right-hand side (RHS) $=0.33$. Within the framework of QCD-inspired calculations of the baryon mass spectrum,[7] there exists a well-known explanation of the sign and size of Σ-Λ splitting relative to that of Δ-N. Equation (8) provides a similar relationship between those two splittings if the unitarity effects are dominant. Within the dual unitarization framework the Σ-Λ splitting was explained by one of the authors in the so-called linear baryon model[8] which was very successful in describing separately natural and unnatural leading baryon Regge trajectories. There was however no possibility to relate it to Δ-N splitting since the latter two particles have opposite naturalities.

The deviations from equalities (7) may be qualitatively understood as second-order effects as follows.

Equation (6a) overestimates the relative size of the Δ-N splitting since there is a kind of "negative feedback": once Δ and N are split the Δ gets a larger second-order shift down in mass due to the presence of the nearby $\Delta\pi$ threshold (see Table I). For Σ^*-Σ splitting (6b), this effect is smaller since the contribution to Σ^* shift from the $\Sigma^*\pi$ channel is approximately half compared to the $\Delta-\Delta\pi$ case (see Table I). Therefore the right-hand side of Eq. (7a) is actually an overestimate, in agreement with experiment.

The origin of the deviation from equality (7b) is similar. Namely, there is a second-order effect which shifts down in mass the heavier Σ more than the lighter Λ. Thus the RHS of (7b) is also an overestimate.

The deviation of the LHS of Eq. (7c) from 1 may be understood as a result of the imperfect cancellation of the C_1 and C_2 factors from Eqs. (6b) and (6d). In fact when unfolding unitarity effects the effective strange-nonstrange quark mass difference generally decreases.[3] In other words, one expects a larger shift for the $k=2$ case as compared to the $k=1$ one and, consequently, the RHS of Eq. (7c) $=C_2/C_1 > 1$. A similar effect is also at work in relation (7a) where it acts in the same direction as the Δ-$\Delta\pi$ coupling effect discussed above.

Another way to improve the prediction, which includes some of the above-mentioned effects, is to average over the actual physical threshold positions, whereby both meson and

TABLE II. Comparison of the mass ratios of Eq. (7) with the predictions of Eqs. (7) and (9).

Ratio	Experiment LHS Eq. (7)	Prediction RHS Eq. (7)	Prediction Eq. (9)
$\frac{\Delta-N}{\Sigma^*-\Sigma}$	1.54	1.96	1.55
$\frac{\Sigma^*-\Lambda}{\Sigma^*-\Sigma}$	1.41	1.64	1.32
$\frac{\Xi^*-\Xi}{\Sigma^*-\Sigma}$	1.13	1	1.01

baryon masses are used in a self-consistent scheme. Then the predicted mass differences satisfy

$$m_{B_1} - m_{B_2} \propto \sum_i (W_i^{B_1} - W_i^{B_2}) (s_{\rm thr}^{(i)})^{1/2} \quad . \tag{9}$$

Using this formula, and in addition taking into account the more accurate meson mixing angles $\theta_P = -11°$ and $\theta_V = 38.8°$, one finds the numbers given in the third column of Table II. The agreement with experiment is now better than 12%.

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[1]G. F. Chew, *The Analytic S-Matrix* (Benjamin, New York, 1966); R. P. Feynman, *Photon Hadron Interactions* (Benjamin, New York, 1972), pp. 58 and 95.

[2]N. A. Törnqvist, Ann. Phys. (N.Y.) 123, 1 (1979); Nucl. Phys. B203, 268 (1982); Phys. Rev. Lett. 49, 624 (1982); S. Ono and N. A. Törnqvist, Z. Phys. C (to be published).

[3]M. Roos and N. A. Törnqvist, Z. Phys. C 5, 205 (1980).

[4]W. N. Cottingham, K. Tsu, and J. M. Richard, Nucl. Phys. B179, 541 (1981); A. K. A. Maciel and J. E. Paton, *ibid.* B181, 277 (1981); F. Myhrer, G. E. Brown, and Z. Xu, *ibid.* A362, 317 (1981); A. W. Thomas, S. Théberge, and G. A. Miller Phys. Rev. D 24, 216 (1981).

[5]P. Żenczykowski, Jagellonian Univ. report, 1984 (unpublished).

[6]R. Carlitz and M. Kislinger Phys. Rev. D 2, 336 (1970).

[7]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1978).

[8]P. Żenczykowski, Z. Phys. C 5, 319 (1980).