Limit on W_L-W_R mixing in the $SU(2)_L \times SU(2)_R$ model

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An upper limit of 0.005 on the W_L-W_R mixing angle in the $SU(2)_L \times SU(2)_R \times U(1)$ model is derived using semileptonic-decay data. This limit is possible because of the recent data on B decays, provided we assume only three generations.

The possibility of extending the standard $SU(2) \times U(1)$ model to the left-right-symmetric $SU(2)_L \times SU(2)_R \times U(1)$ group has been discussed in many papers.[1] Empirical constraints may be placed on the mass M_2 and the mixing angle ζ defined by

$$W_1 = \cos\zeta \, W_L + \sin\zeta \, W_R \ ,$$
$$W_2 = -\sin\zeta \, W_L + \cos\zeta \, W_R \ , \tag{1}$$

where W_1 and W_2 are the mass eigenstates with masses M_1 and M_2. Here we present a new limit on ζ. We consider only the symmetric model in which the hadronic mixing angles are the same for left and right couplings:[2] specifically the Kobayashi-Maskawa (KM) mixing matrices are related by $U_R = U$, where U is the usual KM matrix.

The most direct constraints might seem to come from muon decay, where a recent experiment[3] gives very strong limits on right-handed currents. However, a very attractive model based on $SU(2)_L \times SU(2)_R \times U(1)$ has a large Majorana mass for the right-handed neutrino.[4] Thus, neglecting the small mixing of the usual neutrinos with these heavy ones, purely leptonic processes provide no constraints at all.

The most stringent published constraints are based on nonleptonic processes. An analysis of the K_L-K_S mass difference gives a lower limit on M_2 of about $20M_1$.[5] A detailed analysis of strange-particle nonleptonic decays using current algebra[6] provides the independent limit

$$\zeta < 0.004 \ . \tag{2}$$

Because we do not have a good quantitative theory of nonleptonic processes these limits may be suspect and have unknown uncertainties.

For semileptonic processes, again assuming the right-handed neutrino can be ignored, and setting $U_R = U$, the effective weak Hamiltonian is

$$H = \frac{G}{\sqrt{2}} \sum_\alpha [\bar{\nu}_\alpha \gamma_\lambda (1-\gamma_5) l_\alpha]^\dagger \sum_{ij} U_{ij} \times \bar{U}_j [\gamma^\lambda (1+\tan\zeta) - \gamma^\lambda \gamma_5 (1-\tan\zeta)] D_i \ , \tag{3}$$

where i,j are quark generation indices. In trying to limit $\tan\zeta$ we face the problem of going from the quarks to the hadrons. How sure can we be that the quarks really have a $(V-A)$ coupling when for the simplest hadron, the nucleon, the effective coupling is $V-1.25A$? One way is to use current-algebra (or charge-algebra) relations which do involve the relative normalization of the V and A constants. The success of the Adler-Weisberger relation clearly limits $\tan\zeta$; if the relation is believed to be valid within 15% then

$$\tan\zeta < 0.05 \ . \tag{4}$$

The recent measurement of the B lifetime[7] together with the branching ratio[8] for $b \to u$ relative to $b \to c$ leads to the limit

$$U_{ub} < 0.01 \ . \tag{5a}$$

This limit is independent of ζ since the b decay depends on $g_V^2 + g_A^2$. The standard analysis of allowed pure Fermi transitions yields the result for the vector coupling,[9]

$$U_{ud}(1+\tan\zeta) = 0.974 \pm 0.0025 \ . \tag{5b}$$

The determination of the strangeness-changing vector coupling comes from the application of SU(3) to $K \to \pi e \nu$ and to hyperon decays, yielding the result[9]

$$U_{us}(1+\tan\zeta) = 0.219 \pm 0.002 \ . \tag{5c}$$

Combining these three equations and using the unitarity of U, assuming only three quark generations, gives

$$(1+\tan\zeta)^2 = 0.996 \pm 0.005 \ . \tag{6}$$

Assuming ζ is not near $\pi/2$ this gives

$$\zeta < 0.0045 \ . \tag{7a}$$

This is about the same as Eq. (2) but is based on the much more reliable semileptonic-decay analysis. Even here the uncertainty of the SU(3) analysis is probably not properly measured by the error in Eq. (5c). However, increasing the error in Eq. (5c) by a factor 5 only changes the result a little, giving

$$\zeta < 0.0055 \ . \tag{7b}$$

We have assumed $M_2 >> M_1$; more generally, ζ in Eqs. (7) should be replaced by $\zeta[1-(M_1/M_2)^2]$.

The alternative solution $\tan\zeta = -2$ corresponds to perversely asssuming that the quark currents are $V+3A$ rather than $V-A$. Even a crude quark-model calculation of g_A can convince us that this is not correct.

So far we have neglected any CP violation. In the case of CP violation we must replace $\sin\zeta$ in Eq. (1) by $\sin\zeta \, e^{i\alpha}$ and one must allow for complex quark-mixing matrices. In this case the reasonable form for providing left-right symmetry is the so-called "pseudomanifest" realization[2] for which, with a suitable phase convention, $U_L = U_R^*$. As a result we must replace $\tan\zeta$ in Eq. (3) by

$$e^{i(\alpha-2\delta_{ij})}\tan\zeta \ ,$$

where δ_{ij} is the phase of U_{ij}. The phases δ_{ij} cannot be removed by a new phase convention since they represent relative phases of the left- and right-handed mixings.[10] As a result Eq. (7a) becomes

$$\zeta[\cos(\alpha-2\delta_{ud})(0.95)+\cos(\alpha-2\delta_{us})(0.05)] < 0.0045 \ . \tag{8}$$

A nonzero value for the phase $(\alpha-2\delta_{ud})$ leads to time-reversal violation in β decay.[10] Limits on the T-violating correlation parameter D in the β decay of the neutron and ^{19}Ne lead to the result[11] that the relative phase of g_A and g_V is $(0.11 \pm 0.17)°$. This translates into the limit

$$\zeta\sin(\alpha-2\delta_{ud}) = \tfrac{1}{2}(0.002 \pm 0.003) < 0.003 \ . \tag{9}$$

Combining Eqs. (8) and (9) once again gives the result (7b).

In the model we are discussing[4] there will be some mixing of the left-handed neutrinos with the heavy "right-handed neutrinos." This effect is adequately illustrated by neglecting generational mixing in which case

$$\begin{aligned} \nu_e &= \cos\alpha_e \nu_1 + \sin\alpha_e N_1 \ , \\ \nu_\mu &= \cos\alpha_\mu \nu_2 + \sin\alpha_\mu N_2 \ , \end{aligned} \tag{10}$$

where ν_1,ν_2 are almost massless neutrinos and N_1,N_2 are heavy mass eigenstates. Since we assume N is too heavy to be produced in decays the major effect of the mixing is to replace ν_a in Eq. (3) and in the muon-decay Hamiltonian by $\nu_a\cos\alpha_a$. Since the input to our result Eqs. (5) are based on a comparison of semileptonic β decays with muon decay (used to normalized G) a nonzero α_e does not affect our conclusions. However, a nonzero α_μ leads to a larger value for G so that Eq. (6) becomes

$$(1+\tan\zeta)^2 = (0.996 \pm 0.005)\cos^2\beta \ . \tag{11}$$

In the theory[4] we expect β to be of the order $m_\mu/m(N_2)$ so that if $m(N_2) > 10$ GeV, $\beta < 10^{-2}$ and $\cos^2\beta$ can be set equal to unity without affecting the accuracy of our conclusion. Other effects of mixing associated with the exchange of W_R are even smaller.

In conclusion, we obtain an upper limit on the W_L-W_R mixing angle ζ of 0.005. This limit depends on the quark-lepton universality that is built into the gauge model. The recently measured B lifetime plays an essential role since it is no longer possible to imagine that the quarks have a larger vector coupling than the leptons which is compensated by smaller values for the mixings U_{ud} and U_{us}. Our results depend on the assumption that there are only three quark generations or that, if there is a fourth-generation b', the mixing $U_{ub'}$ is small. Our analysis also depends on the left-right symmetry of the mixing matrices, for arbitrary U_R no limit is found. Because this limit is based on semileptonic physics we believe it is much more reliable than that of Ref. 6 and represents the best limit now available.

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