PHYSICAL REVIEW D VOLUME 29, NUMBER 9 1 MAY 1984

Classical radiation zeros in gauge-theory amplitudes. II. Spin-dependent null zone

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(Received 20 September 1983)

Further aspects of tree-amplitude zeros, occurring only for certain helicity configurations and representative of the complementary radiation theorem, are considered. These polarization-dependent zeros are destroyed by nongauge interactions and correspond, in the low-frequency limit, to the classical result where there is no radiation polarized perpendicular to the scattering plane. The zeros found by Hellmund and Ranft and by Cortés, Hagiwara, and Herzog are discussed and are contrasted with the zeros of Gaemers and Gounaris. Additional experiments are suggested.

It has been proven[1–3] that all single-photon tree amplitudes possess spin-independent zeros, if the various couplings have local gauge symmetry.[4] Consequently, such amplitudes can be written in the form[5]

$$M=\sum \frac{Q_i I_i}{p_i\cdot q}\ , \tag{1}$$

for all external particles i (charge Q_i, momentum p_i, spin ≤ 1) and photon momentum q, where the currents I_i satisfy the gauge-Poincaré sum rule

$$\sum I_i = 0 \tag{2}$$

for locally gauge-invariant interactions independently of the spin states. Evidently M vanishes in the (charge-dependent but spin-independent) *charge* null zone defined by[6]

$$\frac{Q_i}{p_i\cdot q}=\text{same},\quad \text{all } i\ . \tag{3}$$

This forms the principal focus of Refs. 1 and 2.[7]

The complementary theorem[1–3] for zeros in such tree amplitudes is also evident in (1). From charge conservation,

$$\sum \delta_i Q_i = 0\ , \tag{4}$$

we see that $M=0$ in the (spin-dependent but charge-independent) *current* null zone defined by

$$\delta_i \frac{I_i}{p_i\cdot q}=\text{same},\quad \text{all } i\ , \tag{5}$$

with $\delta_i \equiv +1\ (-1)$ for outgoing (incoming) particles.[8] From momentum conservation,

$$\sum \delta_i p_i\cdot q = 0\ , \tag{6}$$

we also see that (2) is required in order that (5) hold.[9] Hence the current null zone, like the charge null zone, is spoiled by nongauge interactions, the litmus test of both types of radiation zeros.

In this paper, we amplify the discussion of the current null zone, analyzing mechanisms whereby (5) can be satisfied. In particular, a recently discovered[10,11] spin-dependent amplitude zero is described in terms of the complementary radiation theorem.

I. CONVECTION-CURRENT NULL ZONE

We study first the convection currents in the infrared factor for a general radiation amplitude,

$$A_{\text{IR}}=\sum \delta_i \frac{Q_i p_i\cdot\epsilon}{p_i\cdot q}\ , \tag{7}$$

which has both charge and current null zones.[12] In particular, the infrared limit of (5) is

$$\frac{p_i\cdot\epsilon}{p_i\cdot q}=\text{same},\quad \text{all } i\ , \tag{8}$$

which is Lorentz and gauge invariant.[13] Both types of null zones are explicit in the radiation representation:[1–3]

$$A_{\text{IR}}=\sum \left[\frac{Q_i}{p_i\cdot q}-\frac{Q_j}{p_j\cdot q}\right]\delta_i p_i\cdot q\left[\frac{p_i\cdot\epsilon}{p_i\cdot q}-\frac{p_k\cdot\epsilon}{p_k\cdot q}\right]\ , \tag{9}$$

with arbitrary j,k. While radiation symmetry implies general constraints on tree amplitudes, such as the form (9), in what follows we consider only whether the current null zone overlaps with the physical region.

We have the following general result.

Lemma. The convection-current null zone (8) requires that $\vec{p}_i\cdot\vec{\epsilon}=0$, all i, in the c.m. frame. Therefore all particles are restricted to the plane (a line) perpendicular to $\vec{\epsilon}$ for a linearly (elliptically) polarized photon.

Proof. We may use the transverse gauge $\epsilon^\mu=(0,\vec{\epsilon})$, $\vec{q}\cdot\vec{\epsilon}=0$. The restrictions (8) force the $\vec{p}_i\cdot\vec{\epsilon}$ to have the same sign for all i (real and imaginary parts separately) since $p_i\cdot q\geq 0$. By momentum conservation, this is impossible in the c.m., so the $p_i\cdot\epsilon$ in fact all vanish.

Therefore, in the physical region when the conclusion about transverse $\vec{\epsilon}$ refers specifically to the c.m. frame, (8) is satisfied only when all the convection currents vanish (to within a gauge transformation). This is in itself not very interesting but serves both as an introduction to more general tree amplitudes and as a connection with the familiar classical results, where there can be no electric dipole radiation that is perpendicularly polarized to the scattering plane.[14] The current null zone thus has a classical-particle basis just as the charge null zone does, the latter reducing to the vanishing of electric dipole radiation for equal charge/mass ratios.[1,2]

We generally are addressing an arbitrary set of charges subject only to charge conservation at each vertex. However, the convection current is irrelevant for each neutral particle r ($Q_r=0$) and the set of equations represented by (8) is reduced. Nevertheless, for a set of neutral particles in the initial *or* in the final state, we can show[15] that the lemma still applies to the nonzero charges.

II. LONGITUDINALLY POLARIZED VECTOR BOSONS

We proceed to examples with spin currents. Consider the source tree graph for fermion annihilation, $f\bar{f}'\to B$, where B is a scalar or vector boson with a nonderivative Yukawa coupling matrix Γ. Attaching a photon in all three ways, the radiation tree amplitude for $f\bar{f}'\to\gamma B$ is

$$\begin{aligned} M&=\bar{v}(p_2)\sum_{i=1}^{3}\delta_i Q_i\frac{J_i}{p_i\cdot q}u(p_1)\ ,\\ J_1&=\Gamma(p_1\cdot\epsilon+\tfrac{1}{2}\not\epsilon\not q)\ ,\\ J_2&=(p_2\cdot\epsilon-\tfrac{1}{2}\not\epsilon\not q)\Gamma\ ,\\ J_3&=\Gamma\Gamma_3 \end{aligned}\tag{10}$$

with Γ_3 denoting the remaining propagator and photon-coupling factor for the B particle. Leaving the spinors understood, $M=0$ if

$$\frac{J_1}{p_1\cdot q}=\frac{J_2}{p_2\cdot q}\tag{11}$$

and if we have radiation symmetry ($J_3/p_3\cdot q$ will then also be the same by virtue of $\sum\delta_iJ_i=0$).[16] Although (11) is gauge invariant, the convection terms in the J_i are not separately gauge invariant. Gauge-dependent and frame-dependent quantities will be evaluated in the transverse gauge and in the c.m. frame.

In the absence of a cancellation among the convection and spin currents (but see some later remarks), we require that the convection currents satisfy (11) separately. If $\bar{v}\Gamma u\neq 0$, then by the lemma all $p_i\cdot\epsilon=0$. For an arbitrary c.m. angle, the photon must be linearly polarized, perpendicular to the scattering plane.

In the scalar-boson case ($\Gamma=a+b\gamma_5$), we find $J_1=-J_2$ so that the spin term in each must also vanish for (11) to hold. The magnitude of $\bar{v}(p_2)\not\epsilon\not q\Gamma u(p_1)$, however, is proportional to $(p_1\cdot q\,p_2\cdot q)^{1/2}$ and its zeros are canceled by one or the other of the higher-order propagator poles, $(p_i\cdot q)^{-1}$.

The vector-boson case [$\Gamma=\not\eta(a+b\gamma_5)$ with polarization vector η] is richer in content. If B is longitudinally polarized, *a conserved-current coupling provides a mechanism for a physical current null zone.*[17] To see this, we point out that the c.m. longitudinal polarization vector may be written in terms of the four-momenta of B and the photon,

$$\eta_{\rm Lo}=\alpha p_3+\beta q\ .\tag{12}$$

The spin contributions to J_1 and J_2 from the q term in (12) vanish while those from the p_3 term are readily computed noting $p_3=p_1+p_2-q$. With the convection currents set equal to zero, we find (11) satisfied if the fermions are massless or, in the case of vector B currents, $m_1=m_2$. This is simply based on the fact that the conserved current coupled to B leads to a zero if η is replaced by p_3.

It is important that the B current T_μ in $M=\eta\cdot T$ is "naturally" conserved, $p_3\cdot T=0$. (Note that nonconserved p_3^μ terms are eliminated through $\eta\cdot p_3=0$.) Although we may impose current conservation on any amplitude $\mathcal{M}=\zeta\cdot\tau$ ($p\cdot\tau\neq 0$, $\zeta\cdot p=0$) by the projection $\tau'_\mu=(g_{\mu\nu}-p_\mu p_\nu/p^2)\tau^\nu$, where $p\cdot\tau'=0$, $\zeta\cdot\tau=\zeta\cdot\tau'$, we cannot generally eliminate the βq term in (12). For the zero, the B current must be conserved without the singular $p_\mu p_\nu$ construction.

Cortés, Hagiwara, and Herzog[10] have pointed out that the tree amplitude for quark-antiquark annihilation into a (zero-width) weak boson and a photon,

$$q\bar{q}'\to\gamma W\ ,\tag{13}$$

vanishes for all angles when the W is longitudinally polarized and the photon is polarized transverse to the scattering plane,[18] provided that $\kappa=1$ for the W magnetic-moment parameter. This is also seen in the earlier, comprehensive calculations of Hellmann and Ranft,[11] along with a similar zero for

$$q\bar{q}\to\gamma Z^0\ .\tag{14}$$

From the preceding paragraph, these zeros are thus describable in terms of the complementary radiation theorem.[19] The despoilment by a nongauge coupling ($\kappa\neq 1$), showing that angular-momentum conservation is not directly responsible,[20] is understood by the fact that the W is then no longer coupled to a conserved current.[21] (In other words, $\sum\delta_iJ_i$ is no longer zero.) Our identification of the role of the vector-boson conserved current also tells us that the zero requires massless quarks in (13) and (14) if the current includes an axial-vector component but only equal masses for the quarks if the current is purely vector.

We can describe a generalization of this particular kind of zero for longitudinally polarized vector bosons.

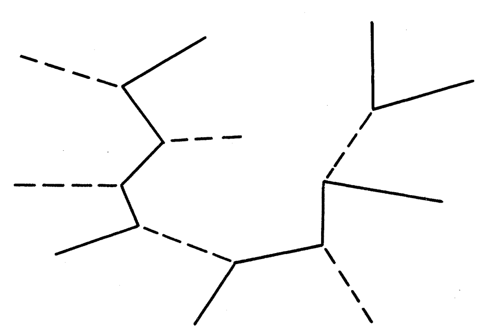

FIG. 1. A representative tree graph for spinors (solid lines) and vectors (dashed lines) and Yukawa couplings. Graphs with scalars, other couplings, and closed loops lie outside this class.

(i) It is a simple fact that all single-photon amplitudes for scalars and longitudinally polarized vectors, with no parity-violating interactions, vanish to all orders if the particles are coplanar and the photon is linearly polarized perpendicular to that plane. (Then the only nonzero invariant that could be constructed linear in ϵ is a pseudoscalar.) Such zeros go beyond tree graphs and radiation symmetry.

(ii) In order to add spinors, however, we must find a cancellation of terms, such as the $\bar{v}\not{\epsilon}u$ found in the analysis of $f\bar{f}'\to\gamma V$. For a tree source graph of the class defined in Fig. 1, let us focus attention on a given spinor-spinor-vector vertex. If the photon (polarized transverse to the vertex plane) and the vector boson (longitudinally polarized) are collinear (parallel or antiparallel), single-photon attachments to the vertex, defined by the radiative vertex expansion in Refs. 1 and 2 vanish according to the previous argument. [The elimination of the p_3 term in (12) requires a conserved current so that a gauge-invariant *set* of tree source graphs may be needed.] Hence, if all (external and internal) vector particles are collinear with the photon *and* longitudinally polarized, the tree amplitudes for the radiative scattering in a plane perpendicular to the photon polarization vanish. Related experimental remarks are given later.

(iii) In the general case where more than one source graph is needed for a conserved current, it is perhaps more natural to describe the class of longitudinal zeros in (ii) as a "massive-radiation" current null zone. (See further remarks in Sec. IV below.) The development represented by Eqs. (1)–(6) can be repeated, *mutatis mutandis,* with analogous charges and currents defined.[22] [A caveat: The analog of Eq. (6) is satisfied only in special cases. For example, if f massless fermions and any number of photons constitute the other external legs in the emission of a W boson, then the analogous sum of the inverse propagators vanishes only for $f=2$.] Then the cancellation resulting from a substitution such as (12) more naturally follows the description given at the outset of this paper. The reactions (13) and (14), however, represent a simple degeneracy with only one source graph, and either description can be used.

III. 90° ZEROS

Hellmund and Ranft[11] have also found zeros in (13) and (14) at 90° (c.m. scattering angle θ) when the photon and the vector boson are linearly polarized in collinear directions (a) perpendicular to the scattering plane (amplitude M_{22}) or (b) along the quark beams (amplitude M_{11}). Indeed, examining the $q\bar{q}'\to\gamma W$ expressions in Cortés, Hagiwara, and Herzog[10] we find these amplitudes $\propto\cot\theta$, provided that $\kappa=1$. The zero moves away from 90° for $\kappa\neq1$ with a position that then also depends on the charges.

The 90° zeros are also current null zones. The signature for these is that the presence and positions of the zeros are independent of charge for $\kappa=1$ and not directly related to angular-momentum conservation.[20] Referring back to Eqs. (10) and (11), we note that $p_1\cdot q=p_2\cdot q$ at $\theta=\pi/2$ (and $m_1=m_2$). The convection currents in J_1 and J_2 are zero in case (a) because $p_i\cdot\epsilon=0$ and in case (b) because $\bar{v}\Gamma u=0$ (requiring $m_i=0$ for vector currents). The spin currents are equal since $\not{\eta}\not{\epsilon}\not{q}=-\not{\epsilon}\not{q}\not{\eta}$ in both cases ($\eta\propto\epsilon$). Thus (11) is satisfied.

IV. GAEMERS-GOUNARIS ZEROS

To better understand the current null zones let us contrast them with zeros due to cancellations *within* a Feynman diagram that otherwise dominates a particular polarization amplitude. We note that such cancellations can be seen to be responsible for structure found in the angular distributions presented by Gaemers and Gounaris[23] for

$$e^+e^-\to W^+W^- . \tag{15}$$

Discounting certain forward/backward suppressions that we impute to helicity nonconservation, other zeros are mainly due to the vanishing of the dominant neutrino-exchange graph, with small corrections due to the γ,Z-exchange diagrams. We may describe this as a cancellation between the convection and spin currents for massive radiation in the dominant graph.

It is quite interesting that the reaction[24]

$$e^+e^-\to Z^0Z^0 \tag{16}$$

exhibits[23] 90° zeros in the amplitudes M_{11},M_{22},M_{33}. An equation analogous to (11) applies.[22] Although the massive version of (6) is not satisfied, the corresponding "charge" Q_3 is zero in reaction (16).

V. REMARKS AND CONCLUSIONS

(1) The appearance of charge-independent single-photon amplitude zeros can be expected when the photon polarization is perpendicular to every other vector in the problem. However, nontrivial current null zones can also be found where the cancellation follows the complementary radiation theorem argument, which in turn is closely patterned after that seen in the check of gauge invariance. For example, the zero appearing in (13) can be deduced by making a gauge transformation on the W (longitudinal) polarization vector, after which it can be seen that the first-order Poincaré transformations[1–3] associated with the photon couplings are zero for each particle, if $\eta^\mu\propto q^\mu$.[25]

(2) Although special spin configurations are necessary for current null zones, these zones are independent of the charges which are constrained only by conservation laws at each vertex. In contrast to charge null zones, *both signs of the charges can be present.* This makes it possible to consider e^+e^- annihilation (see below) as well as *non-Abelian processes* (where the zeros need not be washed out by averaging/summing over the "color" charges). For example, the gluon process, $q\bar{q}\to gg$, has a 90° current null zone similar to $f\bar{f}'\to\gamma V$. [Only Γ and Γ_3 are changed in (10) and (11).]

(3) We contrast the charge null zone and the current null zone for the example $q\bar{q}'\to\gamma W$, $\kappa=1$. In the first case all spin amplitudes vanish at a given angle determined by the charges,[26] while in the second case certain spin amplitudes vanish at a given angle for all charges and one spin amplitude vanishes for all angles and all charges. A characteristic of both types of null zones is that they are ruined in general, by non-gauge-theoretic interactions[27] and by closed loops.[1–3]

(4) If tree amplitudes are linear in conserved generalized

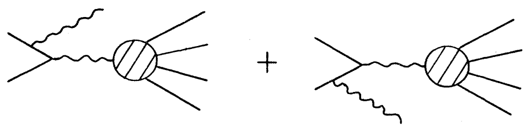

FIG. 2. Radiation by the leptons in the annihilation reaction $e^+e^-\to$hadrons. The longitudinal-internal-photon contribution vanishes in the c.m. when the radiated-photon polarization is transverse to the lepton-photon plane.

charges C_i and have zeros that are independent of these charges, the zeros must be of the form $K\sum C_i=0$ for some K. This converse radiation theorem[8] is an important characteristic of current null zones.

(5) Some experiments involving radiation in e^+e^- annihilation[28] (Fig. 2) or inelastic-electron scattering suggest themselves as a possible means of isolating the transverse cross sections in hadron production. When the virtual photon is collinear with the radiated photon[29] there is no contribution from the longitudinal part of the photon propagator if the polarization of the real photon is transverse to the scattering plane. The dominance of the electron radiation may require only that the electron-scattering plane be perpendicular and may also lead to the separation of the transverse virtual cross section at a single energy. The contributions of virtual-gluon helicities can be similarly isolated in quark reactions.

(6) We can also consider W radiative decay as well as processes where the γ is in the initial state.

(7) Condition (8) is identical to the charge null zone for circularly polarized graviton emission. For example, the external-leg Dirac current[30] is given by

$$(G\, p_i\cdot\epsilon_\pm/p_i\cdot q)\delta_i(p_i\cdot\epsilon_\pm+\tfrac{1}{2}\not\epsilon_\pm\not q)\ .$$

(A physical null zone is even more restricted in view of the circular polarization.) We see that this current involves the same Poincaré transformation encountered in the photon case;[1-3] the universality of these transformations in the gravitational null zones also leads to a gravitational-radiation theorem.

ACKNOWLEDGMENT

The research of both authors is supported by the National Science Foundation.

[1] S. J. Brodsky and R. W. Brown, Phys. Rev. Lett. 49, 966 (1982).

[2] R. W. Brown, K. L. Kowalski, and S. J. Brodsky, Phys. Rev. D 28, 624 (1983). We refer to this paper as I.

[3] A review and a discussion of "radiation symmetry" is found in R. W. Brown, in Proceedings of the Europhysics Study of Electroweak Effects at High Energies, Erice, Italy, 1983 (unpublished).

[4] We allow the following kinds of (covariant) derivatives: None for Dirac, single for scalar, single for vector and double (Higgs-type) for scalar as arise in Yang-Mills trilinear form, and products of these forms. In particular, this defines the photon couplings to be of the familiar minimal form. The condition $g=2$ alone does not suffice; in the vector case, the two free parameters (magnetic dipole and electric quadrupole) can be adjusted to keep $g=2$ even if radiation symmetry (and renormalizability) do not hold.

[5] This form refers only to external charges and simplifies the introduction of the radiation zeros but is more awkward than the gauge-invariant vertex expansion (Refs. 1 and 2) when internal lines are present.

[6] The constraint (3) has been studied in Refs. 1 and 2, and in M. A. Samuel, Phys. Rev. D 27, 2724 (1983); G. Passarino, Nucl. Phys. B224, 265 (1983); M. A. Samuel, A. Sen, G. S. Sylvester, and M. L. Laursen, Phys. Rev. D 29, 994 (1984); S. G. Naculich, *ibid.* 28, 2297 (1983).

[7] Most of the remarks about radiation zeros in these references, particularly those in Sec. VIII C of Ref. 2, refer to charge null zones.

[8] The converse of each radiation theorem is easily proven: If the tree radiation amplitude (1) vanishes for arbitrary Q_i $[I_i]$ subject only to (4) [(2)], then equal ratios (5) [(3)] follow. Arbitrariness in I_i can be related to arbitrariness in spin.

[9] One of the ratios (5) can be written in terms of the others, from (2), (6), and identity (4.3) in Ref. 2 such that one equation in (5) is automatically satisfied if the remaining equations hold.

[10] J. Cortés, K. Hagiwara, and F. Herzog, Phys. Rev. D 28, 2311 (1983). This paper provided a large part of the stimulus for the present work.

[11] M. Hellmund and G. Ranft, Z. Phys. C 12, 333 (1982). A finite W width is employed in these calculations. This introduces $O(\Gamma_W/M_W)$ corrections to the amplitude zeros.

[12] Equation (7) is also the single-photon amplitude for scalar particles scattering at a point with constant coupling. We leave complex conjugation on final-state polarization vectors understood.

[13] Thus ϵ may be regarded as a four-vector in (8).

[14] This is true even for relativistic formulas. As long as the particle velocities $\vec{v}_i$ (and accelerations) remain in a plane, then $\vec{v}_i\cdot\vec{\epsilon}=0$. See the formulas in Sec. III of Ref. 2.

[15] Let P be the timelike total four-momentum of the neutral particles. Then $P^2\geq 0$ and $P\cdot q\geq 0$. The ratio identity of I implies that $P\cdot\epsilon/P\cdot q=p_i\cdot\epsilon/p_i\cdot q$ if (8) holds for the charged particles i, and so $\vec{P}\cdot\vec{\epsilon}=0$ in the c.m. frame. For neutral particles in both initial and final states, however, P is the (possibly spacelike) four-momentum transfer.

[16] The zero in (10) can be manifested, as in (9), using the radiation representation. A particularly nice form for this three-vertex source graph can be found from the factorization formula of C. J. Goebel, F. Halzen, and J. P. Leveille, Phys. Rev. D 23, 2682 (1981). See also Zhu Dongpei, Phys. Rev. D 22, 2266 (1980) and Ref. 2.

[17] In this regard, note that electromagnetic gauge invariance corresponds to the fact that (11) is satisfied when ϵ is replaced by q. The demonstration of the complementary radiation theorem closely parallels the demonstration that a current is conserved, both resting on charge conservation.

[18] In the notation of Refs. 10 and 11, $M_{23}=0$. $M_{ss'}$ refers to the polarizations of $\gamma(s)$ and $W(s')$ in a basis where $s=1$ (2) corresponds to transverse polarization vectors in (perpendicular to) the scattering plane and the longitudinal polarization is denoted by $s=3$.

[19] The fact that the W and Z^0 are coupled to conserved currents in (13) and (14) for massless fermions is used by C. L. Bilchak, R. W. Brown, and J. D. Stroughair, Phys. Rev. D 29, 375 (1984) in the construction of a covariant polarization basis.

The zero in question corresponds to the result found there that the longitudinal W, Z^0 amplitudes are independent of the photon helicity.

[20]We keep in mind, however, that Poincaré invariance is crucial to the radiation theorem.

[21]In a gauge theory we expect $M_{s3}=0$ for $M_W=0$. (Indeed we observe that the M_{13} amplitude in Ref. 10 vanishes in this limit for $\kappa=1$.) The interesting point is that $M_{23}=0$ for $M_W\neq 0$ as well.

[22]But see Sec. X B of I. In that section, the plus sign should be replaced by $\mp$, the remark about the relation of the number of particles to (6.2c) should be deleted, and Eq. (10.10) can read simply $p \rightarrow p \pm \frac{1}{2}q$.

[23]K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C 1, 259 (1979). The rectangular polarization basis used in Refs. 10 and 11 follows this reference.

[24]R. W. Brown and K. O. Mikaelian, Phys. Rev. D 19, 922 (1979).

[25]With the displacement $d \propto \epsilon$ and the Lorentz generator $\omega^{\mu\nu}=q^\mu\epsilon^\nu-\epsilon^\mu q^\nu$, then $p_i\cdot d=0$, all i, and $\bar{v}\not{\epsilon}(a+b\gamma_5)u$ is invariant under the Lorentz transformation of any of the three wave functions.

[26]This angular zero, which we may call the original radiation zero, was first pointed out by K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. 43, 746 (1979). See also R. W. Brown, D. Sahdev, and K. O. Mikaelian, Phys. Rev. D 20, 1164 (1979).

[27]Not all anomalous interactions contribute to a given spin amplitude. For example, with the magnetic-moment interaction of Ref. 10 the M_{12} and M_{21} tree amplitudes for (13) have no κ dependence and retain the charge null zone even when $\kappa\neq 1$.

[28]The current null zones in e^+e^- reactions such as (16), and $e^+e^-\rightarrow\gamma Z^0$, $\gamma Z^0 Z^0$ do not involve internal vector particles.

[29]In the example of Fig. 2 the virtual-photon momenta k_e and k_0 corresponding to γ attachments on the electrons or other particles, respectively, are related to one another by $k_e+q=k_0$ and so if in some frame q is collinear with one it is collinear with the other. This is relevant, for example, to $e^+e^-\rightarrow\gamma\mu^+\mu^-$ where the longitudinal internal photon in all diagrams can be completely suppressed.

[30]The currents corresponding to the attachment of a graviton to scalar or Dirac-particle lines are easily calculated using the rules in G. Papini and S. R. Valluri, Phys. Rep. 33, 53 (1977). The infrared-divergent part of any tree graph for gravitation emission contains the terms $(G\,p_i\cdot\epsilon_\pm/p_i\cdot q)\,(\delta_i p_i\cdot\epsilon_\pm)$ for each external leg i, where G is the gravitational coupling and $\epsilon_\pm=\epsilon_\pm(q)$ is the graviton circular-polarization vector with complex conjugation understood [cf. S. Weinberg, Phys. Rev. 135, B1049 (1964)]. Thus for scalars the counterparts of Q_i and I_i are $G\,p_i\cdot\epsilon_\pm$ and $\delta_i p_i\cdot\epsilon_\pm$, respectively.