Decay constants and SU(2) mass splittings of pseudoscalar mesons

Vishnu S. Mathur
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

Mieko T. Yamawaki
Department of Physics, City College, City University of New York, New York, New York 10031
(Received 10 November 1983)

Borel-transformed QCD sum rules are applied to pseudoscalar mesons containing a light antiquark and a light or heavy quark. We determine the decay constants and the SU(2) mass splittings of these mesons. A suitable method of extracting the hadronic parameters from the sum rules is discussed in detail. The relationship of these sum rules to the current algebra sum rules is emphasized. Our results are in good agreement with the experimental results, where available.

I. INTRODUCTION

The sum-rule technique in quantum chromodynamics (QCD), introduced by Shifman, Vainshtein, and Zakharov,[1,2] has been successfully applied to a variety of problems. In particular, extensive work[1-3] has been done on quarkonia systems involving both heavy or light quarks. For heavy-quark systems, Shifman, Vainshtein, and Zakharov have advocated using finite-moment sum rules at zero momentum transfer. On the other hand, in systems involving light quarks only, they suggest using infinite-moment sum rules in a special limit when the momentum transfer tends to infinity; these are the Borel-transformed sum rules.

For systems involving both heavy and light quarks, Novikov *et al.*[2,4] originally chose to use the finite-moment sum rules. Subsequently, it has been found[5] that the best sum rules in this case also are the Borel-transformed sum rules. In this paper we discuss these sum rules in detail. We deal with pseudoscalar bound states of a light antiquark with another quark which could be heavy or light. In particular, we use the sum rules to determine the decay constants and the SU(2) mass splittings of pseudoscalar mesons. Some aspects of this work have been discussed in recent literature.[5-9] However, in this paper, we emphasize the method of extracting the hadronic parameters and discuss in detail the stability of the results against variations in the parameter characterizing the Borel-transformed sum rules. Another aspect we would like to emphasize here is the relationship of the QCD sum rules to the current-algebra sum rules. For systems containing light quarks, such as the pion, the QCD sum rule reduces to the current-algebra result in the appropriate limit. From this point of view the QCD sum rules for pseudoscalar mesons can be viewed as generalizing the current-algebra results. Thus, the study of pseudoscalar mesons through QCD sum rules is of special significance. For heavy pseudoscalar mesons, where the soft-meson limit is inappropriate, the QCD-sum-rule approach leads to new results.

The study of the decay constants of pseudoscalar mesons is of importance because of the role they play in many calculations. Furthermore, the decay constants are accessible to experiments through the leptonic decay modes. The electromagnetic mass difference of pions has been successfully calculated[10] by the current-algebra technique, but this technique cannot be applied to heavy pseudoscalar mesons. The QCD sum rules, in contrast, allow one to evaluate in a single framework the hadronic component of the SU(2) mass splittings of any arbitrary pseudoscalar meson.

The paper is organized as follows. In Sec. II, we briefly review the QCD-sum-rule technique, and discuss the Borel-transformed sum rule for pseudoscalar mesons. The determination of the decay constants is discussed in Sec. III, where special emphasis is placed on the stability of the sum rule. In Sec. IV, we discuss the problem of SU(2) mass splittings.

II. QCD SUM RULE FOR PSEUDOSCALAR MESON

In attempting to obtain low-energy parameters of hadrons from QCD, there are two essential problems: (a) how do we treat the nonperturbative effects of QCD and (b) how do we extract information on hadrons from a theory that deals with quark and gluon degrees of freedom?

In response to question (a), an important way nonperturbative effects are presumed to manifest themselves is through the structure of the QCD vacuum via the formation of quark-antiquark and gluon condensates. The formation of the condensates is a difficult dynamical problem. So, at present, it is more useful to treat the condensates as parameters, to be fitted from phenomenological considerations. The problem (b) is solved by a sum-rule approach: the same quantity (for example, a two-point function of currents) is evaluated in QCD, incorporating the condensates discussed above, as well as in terms of hadronic parameters using an approach based on dispersion relations.

To describe pseudoscalar mesons this way, consider the covariant two-point function of axial-vector currents,

$$\pi_{\mu\nu}(q)=i\int d^4x\, e^{iq\cdot x}\langle 0\,|\,T^*(j_\mu(x)j_\nu(0))\,|\,0\rangle\ , \qquad (1)$$

which, from Lorentz invariance, can be written in terms of transverse and longitudinal invariant functions $\pi_{t,l}$

$$\pi_{\mu\nu}(q)=\left[\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right]\pi_t(Q^2)+\frac{q_\mu q_\nu}{q^2}\pi_l(Q^2)\ , \tag{2}$$

where $Q^2=-q^2$. The invariant functions, generically called $\pi(Q^2)$, can be represented in two different ways.

(i) Dispersion representation (DR):

$$\pi(Q^2)=\frac{1}{\pi}\int\frac{\text{Im}\pi(s)}{s+Q^2}ds\ . \tag{3}$$

From unitarity, the absorptive part $\text{Im}\pi(s)$ will get contributions from hadronic states. The question of a subtraction in the DR is ignored here. This is because, as we shall see, the form (3) itself is never used but only its moment in a suitably high order.

(ii) Operator-product expansion (OPE) in QCD:

$$\pi(Q^2)=\sum_k C^k(Q^2)\langle 0\,|\,O_k\,|\,0\rangle\ . \tag{4}$$

Here O_k are local QCD operators, and $C^k(Q^2)$ are the coefficients in the expansion. If the vacuum state is chosen to be the perturbative vacuum, the only nonvanishing vacuum expectation value is when O_k is the unit operator. However, as discussed before, for the true QCD vacuum, we expect all vacuum expectation values to be nonzero. For O_k other than the unit operator, these are the condensates representing the nonperturbative effects.[11] The importance of the OPE is that at short distances, only a few low-dimensional operators need be considered. Also, at short distances, because of asymptotic freedom of QCD, the effective coupling constant is small, so that the coefficient functions $C^k(Q^2)$ may be calculated using perturbation theory.

The direct sum rule

$$\text{DR}=\text{OPE} \tag{5}$$

is generally not useful. This is because the DR is useful only for low Q^2, when $\pi(Q^2)$ can be dominated by a few low-lying hadronic contributions. On the other hand, as remarked, the OPE is useful only at short distances. However, for currents j_μ involving only heavy quarks $(c,b\ldots)$, of mass M, the quark propagator $1/(Q^2+M^2)$ connects small distances even at $Q^2=0$. Thus in this case, one might expect the direct sum rule to be useful. But, in practice, even in this case, finite-moment sum rules derived from the direct sum rule are found to be much more useful.[1]

For currents involving only light quarks (u,d,s), clearly the OPE is useful only for large Q^2. But, if we consider the nth-order moment of the DR

$$\pi_n(Q^2)\equiv\frac{1}{n!}\left[-\frac{d}{dQ^2}\right]^n\pi(Q^2)=\frac{1}{\pi}\int\frac{\text{Im}\pi(s)}{(s+Q^2)^{n+1}}ds\ , \tag{6}$$

it is clear that for a given Q^2, saturation by low-lying states is better for larger n. These considerations suggest that for large Q^2, when the OPE is useful, the DR could also be usefully employed if we consider its nth-order moment with large n. If we consider the limit

$$Q^2\to\infty,\ n\to\infty \tag{7}$$

such that

$$Q^2/n\equiv M^2 \tag{8}$$

is fixed, we obtain, starting from Eq. (5), the Borel-transformed sum rule.[1]

In the case when the currents involve a heavy and a light quark, recent investigation[5] has revealed that the best sum rules are again the Borel-transformed sum rules, rather than the finite-moment sum rules. This result is not surprising since when both heavy and light quarks are involved, one expects the light quarks to play an important role in the dynamics.

Before writing down the Borel-transformed sum rule, the coefficients of a few low-dimensional operator terms in the expansion (4) have to be determined. Generally, the OPE in QCD can be written as

$$i\int d^4x\, e^{iq\cdot x}T^*(j_\mu(x)j_\nu(0)) =C^I_{\mu\nu}(Q^2)1+\sum_i C^{m_i}_{\mu\nu}(Q^2)\bar{q}_i m_i q_i +C^G_{\mu\nu}(Q^2)G^a_{\rho\sigma}G^a_{\rho\sigma}+\cdots\ , \tag{9}$$

where the axial-vector current $j_\mu(x)$ involves the quark flavors q_1 and q_2 with masses m_1 and m_2:

$$j_\mu(x)=\bar{q}_2\gamma_\mu\gamma_5 q_1 \tag{10}$$

and $G^a_{\rho\sigma}$ represents the gluon field tensor with the SU(3) color index a $(a=1,\ldots,8)$. The coefficient functions $C_{\mu\nu}(Q^2)$ can be expressed in terms of the transverse and longitudinal components

$$C_{\mu\nu}(Q^2)=\left[\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right]C_t(Q^2)+\frac{q_\mu q_\nu}{q^2}C_l(Q^2)\ . \tag{11}$$

The expectation value of Eq. (9) in the true QCD vacuum then leads to Eq. (4). Since we are interested in describing pseudoscalar mesons, we confine our attention only to the longitudinal components. These coefficient functions $C_l(Q^2)$ may be calculated[5,12] using perturbative QCD.

To calculate $C^I_l(Q^2)$, we consider the expectation value of Eq. (9) in the perturbative vacuum state, when all matrix elements on the right-hand side of Eq. (9) vanish, except the expectation value of the unit operator. Then, in the lowest order in QCD coupling, the coefficient $C^I_l(Q^2)$ can be obtained from the diagram of Fig. 1(a). This gives

$$C^I_l(Q^2)=\text{const}+\frac{3}{4\pi^2}(m_1+m_2)\times\int_0^1 dx[(1-x)m_1+xm_2]\times\ln\left[Q^2+\frac{{m_1}^2}{x}+\frac{{m_2}^2}{1-x}\right]\ , \tag{12}$$

where the constant, independent of Q^2, is really a divergent term. However, since we are eventually going to take the derivatives of $C_l^I(Q^2)$ with respect to Q^2, this term will drop out. To calculate $C_l^{m_i}(Q^2)$, we consider the expectation value of Eq. (9) in the quark state $|q_i\rangle$, and average over the spin and momentum of the quark. In lowest-order QCD, we get contributions to $C_l^{m_1}(Q^2)$ and $C_l^{m_2}(Q^2)$ from diagrams of the type of Fig. 1(b). After some algebra, we get

$$m_1{}^2 C_l^{m_1} = -\frac{m_1(m_1+m_2)}{Q^2} - \frac{1}{2}\left[\frac{m_1{}^2-m_2{}^2}{Q^2}\right]^2 + \frac{(m_1+m_2)^2}{8m_1{}^2}(1-v)^2\frac{\bar{Q}^4}{Q^4}\ , \tag{13}$$

where

$$\bar{Q}^2 = Q^2+(m_1-m_2)^2\ , \qquad v^2 = 1+\frac{4m_1m_2}{\bar{Q}^2}\ , \tag{14}$$

$m_2{}^2C_l^{m_2}$ can be obtained from Eq. (13) by interchanging $m_1 \leftrightarrow m_2$. Expressed as a power series in $1/Q^2$, Eq. (13) can be written as

$$m_1 C_l^{m_1} = -\frac{m_1+m_2}{Q^2} - \frac{1}{2Q^4}(m_1{}^3-3m_1m_2{}^2-2m_2{}^3) + \cdots \tag{15}$$

with a similar expansion for $m_2C_l^{m_2}$.

The calculation of C^G is more complicated. The expectation value of Eq. (9) in one gluon state leads to a contribution proportional to the QCD coupling α_s on the left-hand side [see Fig. 1(c)]. However, the gluon expectation value of the right-hand side of Eq. (9) does not single out the operator term $G^a_{\rho\sigma}G^a_{\rho\sigma}$. In fact the operators $\bar{q}_i m_i q_i$ contribute[1] to the gluon expectation values to the same order in α_s [see Fig. 1(c′)]. To lowest order in QCD coupling, we may then express $C^G = C^{G_1}+C^{G_2}$, where C^{G_1} arises from Fig. 1(c) and C^{G_2} from Fig. 1(c′). For the longitudinal component, we get

$$C_l^{G_1} = -\frac{\alpha_s}{48\pi}\frac{(m_1-m_2)^2}{4m_1m_2}\frac{1}{\bar{Q}^2} \times \left[\frac{3(3v^2+1)(1-v^2)^2}{v^4}\frac{1}{2v}\ln\frac{1+v}{1-v} - \frac{9v^4+4v^2+3}{v^4}\right] \tag{16}$$

and

$$C_l^{G_2} = \frac{\alpha_s}{12\pi}(C_l^{m_1}+C_l^{m_2})\ . \tag{17}$$

Expanding in $1/Q^2$, we find

$$C_l^{G_1} = \frac{\alpha_s}{12\pi}\frac{(m_1+m_2)^2}{m_1m_2}\frac{1}{Q^2}+\cdots\ , \tag{18}$$

$$C_l^{G_2} = -\frac{\alpha_s}{12\pi}\frac{(m_1+m_2)^2}{m_1m_2}\frac{1}{Q^2}+\cdots\ , \tag{19}$$

so that

$$C_l^G = C_l^{G_2}+C_l^{G_2} = O\left[\frac{1}{Q^4}\right]\ . \tag{20}$$

It is important to note that to order $1/Q^2$, the contribution to C_l^G cancels out.

Taking the expectation value of Eq. (9) in the true QCD vacuum state, we get

$$\pi_l(Q^2) = C_l^I(Q^2) + \sum_i C_l^{m_i}(Q^2)\langle 0|\bar{q}_i m_i q_i|0\rangle + C_l^G(Q^2)\langle 0|G^a_{\rho\sigma}G^a_{\rho\sigma}|0\rangle + \cdots\ . \tag{21}$$

If we ignore the higher-dimensional operator terms in Eq. (21), we may also ignore the $O(1/Q^4)$ terms from the coefficient functions (15) and (20). Thus, to this order, the gluon condensate will not contribute[13] to $\pi_l(Q^2)$. We are now ready to write down the Borel-transformed sum rule. For this purpose, we operate on both sides of Eq. (21), by the operator

$$L_M = \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!}(Q^2)^n\left[-\frac{d}{dQ^2}\right]^n\ . \tag{22}$$

Using the dispersion representation (3) for $\pi_l(Q^2)$ and the identities

$$L_M\frac{1}{Q^2+s} = \frac{1}{M^2}\exp(-s/M^2)\ , \qquad L_M\frac{1}{(Q^2)^k} = \frac{1}{(k-1)!}\frac{1}{(M^2)^k}\ , \tag{23}$$

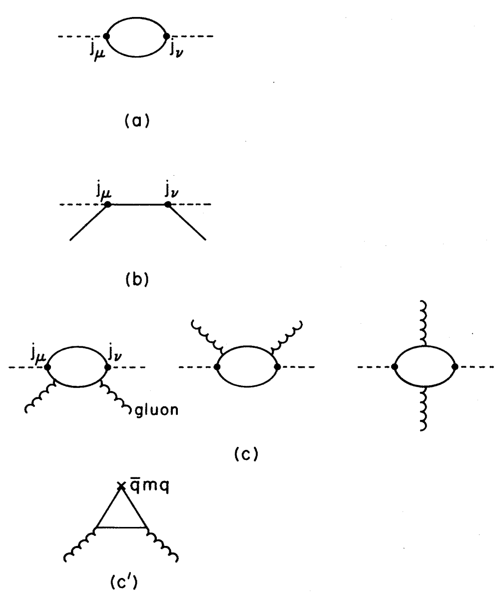

FIG. 1. Lowest-order diagrams for the coefficient functions in the OPE.

we obtain the Borel-transformed sum rule

$$\frac{1}{\pi}\int \mathrm{Im}\pi_l(s)e^{-s/M^2}ds = C_l^I(M^2)-(m_1+m_2)\langle 0|\bar{q}_1q_1+\bar{q}_2q_2|0\rangle + O\left[\frac{1}{M^2}\right], \quad (24)$$

where

$$C_l^I(M^2)=M^2L_MC_l^I(Q^2) = \frac{3}{4\pi^2}M^2(m_1+m_2)\int_0^1 dx[m_1(1-x)+m_2x] \times e^{-\xi(x)/M^2} \quad (25)$$

and

$$\xi(x)=\frac{m_1^2}{x}+\frac{m_2^2}{1-x}. \quad (26)$$

An alternative and perhaps simpler way of deriving Eq. (24) is to start with the Ward identity relating π_l to the pseudoscalar two-point function:

$$i\int d^4x\, e^{iq\cdot x}\langle 0|T(\partial_\mu j_\mu(x)\partial_\nu j_\nu(0))|0\rangle = -(m_1+m_2)\langle 0|\bar{q}_1q_1+\bar{q}_2q_2|0\rangle+q^2\pi_l .$$

This approach also shows that the cancellation of the gluon-condensate contribution to π_l to order $1/Q^2$ is not accidental. Since $\partial_\mu j_\mu(x)=(m_1+m_2)P(x)$, where $P(x)$ is the pseudoscalar density, the gluon contribution to the left-hand side of the Ward identity will be proportional to

$$\frac{(m_1+m_2)^2}{Q^2}\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle ,$$

from dimensional considerations.

In the sum rule (24), M^2 is a parameter, which can take any arbitrary value subject only to the constraint that the approximations used in writing down Eq. (24) should be valid. These constraints arise from the following considerations. In determining the OPE coefficients we used perturbative QCD. It is easy to see that $\alpha_s(Q^2)$, which depends logarithmically on Q^2, goes over[1] to $\alpha_s(M^2)$ under Borel transformation. Thus the neglect of α_s terms is justified for all M^2, for which $\alpha_s(M^2)\ll 1$. This implies choosing M^2 such that $M^2\gg\Lambda_{\mathrm{QCD}}^2$, where Λ_{QCD} is the QCD scale parameter. Furthermore, in writing down the sum rule (24), we neglected $O(1/Q^4)$ terms in C^m and C^G. It is easy to see that this is justified if we choose $M>m_1,m_2$. The neglect of the higher-dimensional condensates is justified since one expects the mass scales of the condensates to be related to the QCD scale parameter. The role of the Borel transformation is clear from Eq. (23). The dispersion integrand is now exponentially damped while the terms in the OPE are factorially suppressed.

The dispersion integral gets contributions from pseudoscalar hadronic states. Separating out the contribution from the pseudoscalar-meson poles, we have

$$\frac{1}{\pi}\int \mathrm{Im}\pi(s)e^{-s/M^2}ds = m_P{}^2f_P{}^2e^{-m_P{}^2/M^2} + \text{continuum} , \quad (27)$$

where m_p and f_p are the mass and the decay constant of the pseudoscalar meson. Our definition of f_p is so normalized that for the pion, $f_\pi\simeq 130$ MeV. The continuum contribution is given by

$$\text{continuum}=\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{Im}\pi(s)e^{-s/M^2}ds , \quad (28)$$

where s_0 is the lowest threshold for multihadronic contribution.

For the lightest quarks u or d, when the pseudoscalar meson is π, it is easy to see that the sum rule (24) collapses to the well-known current-algebra result in the approximate limit. To see this, note that the perturbative term C^I on the right-hand side of Eq. (24) is $O(m_{u,d}{}^2)$ whereas the quark condensate term is $O(m_{u,d})$. Thus, in the chiral limit ($m_{u,d}\to 0$, $m_\pi\to 0$), with pion-pole dominance, the sum rule (24) reproduces the current-algebra result[14]

$$m_\pi{}^2f_\pi{}^2=-(m_u+m_d)\langle 0|\bar{u}u+\bar{d}d|0\rangle . \quad (29)$$

For heavier pseudoscalar mesons ($K,D,F,B,\dots$), we can thus view the sum rule (24) as providing a generalization to the current-algebra result. A salient feature of this generalization is the perturbative term on the right-hand side of Eq. (24), which acquires greater importance for heavier quarks.

The current-algebra sum rule (29) can be used to determine the quark condensate if the current-algebra masses of u and d are known. Assuming that the quark condensate is SU(2) invariant, and $m_u+m_d\simeq 11$ MeV, one gets[1]

$$\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle\simeq-(250\ \text{MeV})^3 . \quad (30)$$

If the quark condensate is SU(3) invariant, the result (30) can also be extended to the strange quark. It is well known that nonvanishing values of quark condensates signal spontaneous breaking of chiral symmetry. Since spontaneously broken chiral symmetries involving the heavier quarks $c,b,\dots$, are not expected to be good symmetries, it would be dangerous to extend Eq. (30) to heavy-quark condensates. Shifman, Vainshtein, and Zakharov[1] have shown that for heavy quarks, the condensate in fact is suppressed by the heavy-quark mass

$$\langle 0|\bar{q}_hq_h|0\rangle=-\frac{1}{12m_h}\left\langle 0\left|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}\right|0\right\rangle + O\left[\frac{1}{m_h{}^2}\right]. \quad (31)$$

A simple proof of Eq. (31) is sketched in Appendix A. Using the value of the gluon condensate determined[1] from the J/ψ spectroscopy, it is easy to check that

$$\langle 0|\bar{c}c|0\rangle\sim\tfrac{1}{20}\langle 0|\bar{u}u|0\rangle$$

and

$$\langle 0 | \bar{b}b | 0 \rangle \simeq \tfrac{1}{66} \langle 0 | \bar{u}u | 0 \rangle .$$

Thus, the heavy-quark condensates may be neglected.

III. DECAY CONSTANTS OF PSEUDOSCALAR MESONS

We now apply the sum rule (24), to evaluate the decay constants f_p of the pseudoscalar mesons. Furthermore, this evaluation must be carried out consistently for all values of the parameter M^2 from $M^2 \sim$ a few GeV^2 to $M^2 = \infty$. As discussed in the last section we will take

$$M_{\min}{}^2 \leq M^2 \leq \infty \ . \tag{32}$$

For heavy pseudoscalar mesons $(D,B,\ldots)$ to be specific, we will choose $M_{\min}{}^2 = s_0$, where, as defined before, s_0 is the threshold for the hadronic continuum. For the K-meson case, $s_0 = (m_K + 2m_\pi)^2$ is too low a value for M^2, so we will choose $M_{\min}{}^2 \sim 1$ GeV^2 in this case.

The continuum contribution is difficult to calculate. However, some general constraints on this contribution can easily be obtained. First, since the spectral function $\text{Im}\pi(s)$ is positive definite, the continuum contribution must also be positive definite. Furthermore as $M^2 \to \infty$, the sum rule (24) develops a divergence, since the perturbative contribution on the right-hand side is proportional to M^2. In principle, this should be canceled from the continuum contribution. The simplest way to achieve this is to assume that the hadronic contribution for sufficiently large energies can effectively be represented by the quark-antiquark state. Thus, in Eq. (28), we take

$$\text{Im}\pi(s) = \text{Im}\pi^{q\bar{q}}(s), \quad s \geq \Lambda_0{}^2 \ . \tag{33}$$

The parameter Λ_0 represents the energy beyond which the replacement of the hadronic continuum by the quark-antiquark state is a good approximation. Clearly, we must have $\Lambda_0{}^2 > s_0$, the continuum threshold. We split the continuum contribution into two pieces

$$\text{continuum} = C_N + C_F \ , \tag{34}$$

where C_N and C_F are the near- and far-continuum contributions:

$$C_N = \frac{1}{\pi} \int_{s_0}^{\Lambda_0{}^2} \text{Im}\pi(s) e^{-s/M^2} ds \ , \tag{35}$$

$$C_F = \frac{1}{\pi} \int_{\Lambda_0{}^2}^{\infty} \text{Im}\pi^{q\bar{q}}(s) e^{-s/M^2} ds \ . \tag{36}$$

The far-continuum contribution C_F is very similar to the perturbative contribution C^I given by Eq. (25). In fact Eq. (25) can be written as

$$C^I = \frac{1}{\pi} \int_{s_0}^{\infty} \text{Im}\pi^{q\bar{q}}(s) e^{-s/M^2} ds \ . \tag{37}$$

$\text{Im}\pi^{q\bar{q}}(s)$ is the absorptive part of Fig. 1(a), and is given by

$$\text{Im}\pi^{q\bar{q}}(s) = \frac{3}{8\pi} \frac{(m_1+m_2)^2}{s^2} [s - (m_1 - m_2)^2]^{3/2} \times [s - (m_1+m_2)^2]^{1/2} \ . \tag{38}$$

It is easy to verify that Eqs. (37) and (38) lead to the result (25) if we make the substitution

$$s = \xi(x) \equiv \frac{m_1{}^2}{x} + \frac{m_2{}^2}{1-x} \ . \tag{39}$$

Using Eqs. (27) and (34) in the sum rule (24) and transferring C_F to the right-hand side, we may rewrite the Borel-transformed sum rule as

$$m_P{}^2 f_P{}^2 e^{-m_P{}^2/M^2} + C_N = \tilde{C}^I(M^2,\Lambda_0) - (m_1+m_2) \langle 0 | \bar{q}_1 q_1 + \bar{q}_2 q_2 | 0 \rangle + O\left[\frac{1}{M^2}\right] , \tag{40}$$

where

$$\tilde{C}^I(M^2,\Lambda_0) = C^I(M^2) - C_F(M^2,\Lambda_0) = \frac{1}{\pi} \int_{s_0}^{\Lambda_0{}^2} \text{Im}\pi^{q\bar{q}}(s) e^{-s/M^2} ds \ . \tag{41}$$

Using the result (38) and the substitution (39) in Eq. (41), we get

$$\tilde{C}^I(M^2,\Lambda_0) = \frac{3}{4\pi^2} M^2 (m_1+m_2) \int_{x_0^-}^{x_0^+} dx \, [m_1(1-x) + m_2 x] \times (e^{-\xi(x)/M^2} - e^{-\Lambda_0{}^2/M^2}) \ , \tag{42}$$

where $x_0^{\pm}$ are the roots of the quadratic equation $\xi(x) = \Lambda_0{}^2$:

$$x_0^{\pm} = \frac{1}{2}\left[1 - \frac{m_1{}^2 - m_2{}^2}{\Lambda_0{}^2}\right] \pm \frac{1}{2}\left[1 - \frac{(m_1-m_2)^2}{\Lambda_0{}^2}\right]^{1/2} \times \left[1 - \frac{(m_1+m_2)^2}{\Lambda_0{}^2}\right]^{1/2} . \tag{43}$$

Observe that as $M^2 \to \infty$, $\tilde{C}^I$ is finite, unlike C^I.

We now study the sum rule (40). For a given pseudoscalar meson, the problem is to determine f_P and Λ_0 that provide the best fit to the sum rule for the entire range of M values being considered. However in Eq. (40), the near-continuum contribution is unknown, except for the positivity constraint

$$C_N \geq 0 \ . \tag{44}$$

In practice, it should be noted that if one chooses Λ_0 incorrectly as too low, one could overestimate the far-continuum contribution on the basis of the approximation (33), which in turn could imply a negative C_N, in contradiction to the constraint (44). Thus the constraint (44), provides a lower bound on Λ_0, as we shall see below. It is also clear from Eq. (35) that C_N as a function of M would be smallest for the lowest allowed value of M. This is because the exponential damping in the integral would be most effective for this M value. In what follows we would consider C_N at $M^2 = M_{\min}{}^2$ to be negligible. Then for $M^2 > M_{\min}{}^2$ we expect C_N to be a small positive contribution, increasing with M^2, provided of course the far continuum has not been overestimated, i.e., provided the choice of Λ_0 is suitably made.

In practice, the determination of f_P and Λ_0 is achieved as follows. We plot the pole term and the right-hand side (RHS) of the sum rule (40), separately as a function of M for several different choices of the parameter Λ_0. The pole term contains the unknown parameter f_P. However, as discussed above, for each choice of Λ_0, we normalize the pole term to be equal to the RHS contribution at $M = M_{\min}$. We now check that for the range $M_{\min} < M \le \infty$, the pole term and the RHS of Eq. (38) satisfy the inequality

$$\text{pole term} \le \text{RHS} \tag{45}$$

which follows from Eq. (40) if we use the inequality (44). The choices of Λ_0 that violate the inequality (45) are automatically rejected. Finally, the determination of Λ_0 is achieved by the requirement that the two curves approximate each other as closely as possible, while still satisfying (45). In this case the effect of the near contribution would be small for the entire range of M values being considered. With Λ_0 determined, the numerical value of the RHS at $M = M_{\min}$, determines the normalization of the pole term, and hence the decay constant f_P.

We illustrate this procedure for the D meson in Fig. 2. As a function of M, the plot of the pole term is shown by $\times$ and of the RHS by $\circ$. We have displayed three graphs for different Λ_0 values. For $\Lambda_0 = 2.4$ GeV, the appropriately normalized pole curve lies above the curve for the RHS, thus violating the inequality (45). The case when $\Lambda_0 = 2.8$ GeV satisfies the requirement (45), but the gap between the two curves, which is a measure of C_N, is quite wide. The case when $\Lambda_0 = 2.6$ GeV, clearly gives the best fit according to the requirements listed above. Thus, for D meson, we determine $\Lambda_0 = 2.6$ GeV, and hence from the normalization of the pole term at $M^2 = s_0$, the decay constant $f_D = 0.19$ GeV.

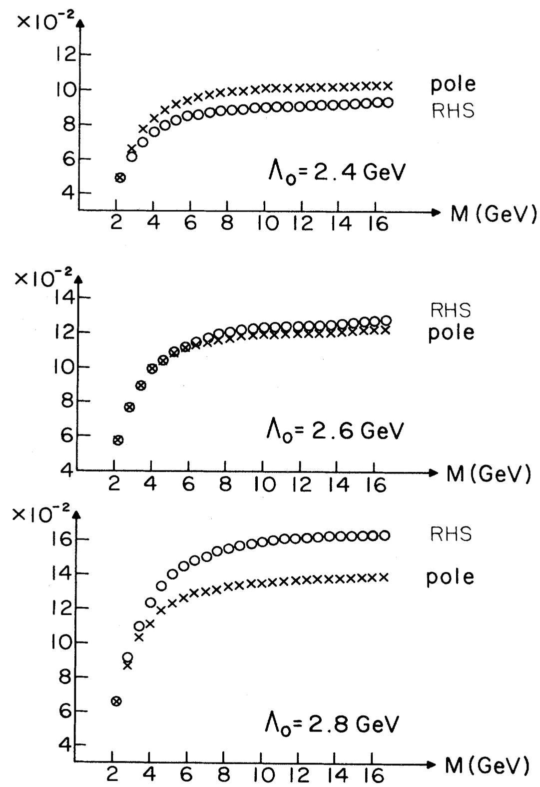

FIG. 2. The pole term ($\times$) and RHS ($\circ$) of the sum rule (40) for the D meson as functions of M with different values of Λ_0.

This procedure can easily be extended to other pseudoscalar mesons. Our results for the best fits to Λ_0 and f_P for various mesons are displayed in Table I. The mass of the pseudoscalar meson, m_P, and the current-algebra mass of the heavy quark, m_Q, used in the numerical work are also displayed in Table I. Our calculated average for f_K is in good agreement with the experimental result $f_K/f_\pi \simeq 1.2$. We similarly believe that our results for f_D and f_B are quite reliable, although it is difficult to assign a quantitative level of accuracy. The case of the T meson is clearly illustrative.

We close this section with a few comments.

(i) From Table I, we see that f_P increases very slowly as the mass of the pseudoscalar meson increases rapidly.

(ii) The quark condensate term in the sum rule (40) represents the nonperturbative contribution. The relative contribution of the nonperturbative term to the perturbative term $\tilde{C}^I$ in determining f_P is very important for low-mass mesons, but decreases rapidly as we go to heavier pseudoscalar mesons. This confirms the feature that bound states of heavy quarks can be regarded as perturbative systems in QCD.

IV. SU(2) MASS SPLITTINGS OF PSEUDOSCALAR MESONS

It is well known that isospin symmetry is broken by electromagnetism and by the u-d quark mass difference. The mass splitting between members of an SU(2) multiplet is, accordingly made up of electromagnetic and hadronic contributions, respectively. For the pseudoscalar mesons

$$P_d \equiv Q\bar{d}\ , \quad P_u \equiv Q\bar{u}\ , \tag{46}$$

the mass difference

$$\Delta m_P{}^2 \equiv m_{P_d}{}^2 - m_{P_u}{}^2 \tag{47}$$

can then be written as a sum of two pieces

TABLE I. Values of Λ_0 and f_P determined from the best fit to the sum rule.

Meson $P(Q\bar{q})$	m_P (GeV)	m_Q (GeV)	Λ_0 (GeV)	f_P (GeV)
K	0.494	0.112 (Ref. 15)	1.9	0.140
		0.150 (Ref. 16)		0.165
D	1.86	1.26 (Ref. 1)	2.6	0.19
B	5.2	4.24 (Ref. 3)	6.0	0.23
T	30	26	30–32	0.4–0.6

$$\Delta m_P{}^2=(\Delta m_P{}^2)_h+(\Delta m_P{}^2)_\gamma \; . \tag{48}$$

For pions, since the $\pi^{\pm}$-π^0 mass difference gets contributions from $I=2$ operators, and the u-d mass-difference term transforms as $I=1$, we have in the lowest order of the u-d mass difference

$$(\Delta m_\pi{}^2)_h=0 \; . \tag{49}$$

The only contribution to the $\pi^{\pm}$-π^0 mass difference thus comes from the electromagnetic interactions, which has been calculated[10] using standard current-algebra techniques. Note that for K, D, and B, the mass difference gets contributions from $I=1$ operators, so unlike pions, the hadronic component of the mass difference will not vanish.

The use of the current-algebra technique to calculate the hadronic or the electromagnetic mass differences is of limited applicability. It is a useful technique for pions, and to some extent for kaons. Certainly for D and B mesons, it is not applicable. Here[17] we propose to use the QCD sum rules to calculate the hadronic mass differences. The electromagnetic mass differences are much harder to evaluate, and we will estimate these using Dashen's theorem[18] and a suitable generalization of it.

A. Hadronic component of mass difference

We use the QCD sum rule (40) for P_d and P_u. Since the quark Q is much heavier than u or d, we can derive an approximate form for $\tilde{C}^I$. Setting $m_1=m_Q$ and $m_2=m$ (m_u or m_d) in Eq. (42), $\tilde{C}^I(M^2,\Lambda_0)$ can be written as

$$\begin{aligned}\tilde{C}^I(M^2,\Lambda_0)=\frac{3}{4\pi^2}M^2\{&m_Q{}^2[K_1(a,b)-\tfrac{1}{2}(1-b)^2e^{-a/b}]\\&+m_Qm\,[K_0(a,b)-(1-b)e^{-a/b}]\\&+O(m^2)\}\;,\end{aligned} \tag{50}$$

where

$$a=\frac{m_Q{}^2}{M^2},\quad b=\frac{m_Q{}^2}{\Lambda_0{}^2}\;. \tag{51}$$

and $K_{0,1}(a,b)$ are related to incomplete Γ functions:

$$\begin{aligned}K_0(a,b)&=a[\Gamma(-1,a)-\Gamma(-1,a/b)]\;,\\K_1(a,b)&=a[\Gamma(-1,a)-\Gamma(-1,a/b)]\\&\quad-a^2[\Gamma(-2,a)-\Gamma(-2,a/b)]\;.\end{aligned} \tag{52}$$

In the last section, we found that with the parameters f_p and Λ^0 chosen appropriately, the sum rule (40) is well satisfied for the entire range of M values, with a negligible near-continuum contribution. Accordingly, we will neglect C_N for all M. Subtracting the sum rules (40) for

$$\begin{aligned}&f_P{}^2[m_{P_d}{}^2\exp(-m_{P_d}{}^2/M^2)-m_{P_u}{}^2\exp(-m_{P_u}{}^2/M^2)]\\&\qquad=[\tilde{C}^I_d(M^2,\Lambda_0)-\tilde{C}^I_u(M^2,\Lambda_0)]-(m_d-m_u)\langle 0\,|\,\bar{q}q+\bar{Q}Q\,|\,0\rangle+O\left[\frac{1}{M^2}\right]\;,\end{aligned} \tag{53}$$

where we have used SU(2) invariance for the quark condensates, and used q to denote the light quark u or d. Furthermore, we have used the SU(2) result $f_{P_d}=f_{P_u}=f_P$ for the decay constants, which is expected to be good (see comment (i) at the end of Sec. III).[19] In order to obtain a simple formula for the hadronic mass difference from Eq. (53), we will, to start with, consider the case $M\rightarrow\infty$. From the asymptotic expansion of the incomplete Γ functions, Eq. (53) gives

$$(m_{P_d}{}^2-m_{P_u}{}^2)_h=\frac{(m_d-m_u)}{f_P{}^2}\left[\frac{3}{4\pi^2}m_Q\Lambda_0{}^2\left[1-\frac{m_Q{}^2}{\Lambda_0{}^2}+\frac{m_Q{}^2}{\Lambda_0{}^2}\ln\frac{m_Q{}^2}{\Lambda_0{}^2}\right]-\langle 0\,|\,\bar{q}q+\bar{Q}Q\,|\,0\rangle\right]\;. \tag{54}$$

Similarly, if we add the sum rules (40) for P_d and P_u and go through the same steps as indicated above, we get

$$2m_P{}^2=\frac{m_Q}{f_P{}^2}\left[\frac{3}{4\pi^2}m_Q\Lambda_0{}^2\left[1-\frac{m_Q{}^4}{\Lambda_0{}^4}+\frac{m_Q{}^2}{\Lambda_0{}^2}\ln\frac{m_Q{}^4}{\Lambda_0{}^4}\right]-2\langle 0\,|\,\bar{q}q+\bar{Q}Q\,|\,0\rangle\right]\;. \tag{55}$$

From Eqs. (54) and (55), we get

$$\left[\frac{m_{P_d}-m_{P_u}}{m_P}\right]_h=\frac{m_d-m_u}{m_Q}\gamma\;, \tag{56}$$

where

$$\gamma=\frac{1-m_Q{}^2/\Lambda_0{}^2+(m_Q{}^2/\Lambda_0{}^2)\ln m_Q{}^2/\Lambda_0{}^2-(4\pi^2/3)\langle 0\,|\,\bar{q}q+\bar{Q}Q\,|\,0\rangle/m_Q\Lambda_0{}^2}{1-m_Q{}^4/\Lambda_0{}^4+(m_Q{}^2/\Lambda_0{}^2)\ln m_Q{}^4/\Lambda_0{}^4-(8\pi^2/3)\langle 0\,|\,\bar{q}q+\bar{Q}Q\,|\,0\rangle/m_Q\Lambda_0{}^2}\;. \tag{57}$$

P_d and P_u, we get

If the continuum contribution in the sum rule (40) is completely neglected, i.e., we set $\Lambda_0 \to \infty$, Eq. (57) shows that $\gamma \to 1$, so that[20]

$$\frac{m_{P_d}-m_{P_u}}{m_P}=\frac{m_d-m_u}{m_Q} . \tag{58}$$

This looks like the naive result in constituent quark model, although the quark masses we are dealing with are not constituent quark masses but rather the current-algebra masses.

For a more realistic determination of γ, we adopt the Λ_0 values obtained in the previous section. This leads to

$$\gamma=\begin{cases} 0.67\ (0.70) & \text{for the } K \text{ meson ,} \\ 1.44 & \text{for the } D \text{ meson ,} \\ 2.60 & \text{for the } B \text{ meson ,} \end{cases} \tag{59}$$

where for the K meson, $\gamma=0.67$ for $m_s=112$ MeV and $\gamma=0.70$ for $m_s=150$ MeV.

B. Electromagnetic component of mass difference

The electromagnetic mass differences of kaons and pions are related by Dashen's theorem[18]

$$(m_{K^\pm}{}^2-m_{K^0}{}^2)_\gamma=(m_{\pi^\pm}{}^2-m_{\pi^0}{}^2)_\gamma . \tag{60}$$

The derivation of this result is based on U-spin invariance of electromagnetic interactions and the soft-pion technique of current algebra. In view of Eq. (49), the result (60) allows us to estimate $(m_{K^\pm}{}^2-m_{K^0}{}^2)_\gamma$ in terms of the experimentally known $\pi^\pm$-π^0 mass difference.

For the D and the B mesons, there is no clear way of determining the electromagnetic mass differences. If we assume that electromagnetic interactions are P-spin[21] $(c \leftrightarrow u)$ invariant, it is easy to see that in the soft-pion limit

$$(m_{D^\pm}{}^2-m_{D^0}{}^2)_\gamma=(m_{\pi^\pm}{}^2-m_{\pi^0}{}^2) . \tag{61}$$

For details, see Appendix B. In view of the rather different masses of the c and u quarks, the validity of P-spin invariance may be questionable. We will nevertheless use Eq. (61) to estimate the D^+-D^0 mass difference. Extending these considerations to $b \leftrightarrow s$ symmetry, we have

$$(m_{B^\pm}{}^2-m_{B^0}{}^2)_\gamma=(m_{D^\pm}{}^2-m_{D^0}{}^2)_\gamma =(m_{K^\pm}{}^2-m_{K^0}{}^2)_\gamma=m_{\pi^\pm}{}^2-m_{\pi^0}{}^2 . \tag{62}$$

From Eqs. (56) and (62), the total SU(2) mass splitting is given by

$$\frac{m_{P_d}-m_{P_u}}{m_P}=\frac{m_d-m_u}{m_Q}\gamma \pm \frac{m_{\pi^+}-m_{\pi^0}}{m_P}\frac{m_\pi}{m_P} , \tag{63}$$

where the $+$ $(-)$ sign arises if P_d is a charged (neutral) meson. In the literature, several estimates exist for the d-u mass difference with somewhat varying results, but it is generally of the order of a few MeV. Here we choose to evaluate it using the K mass difference as input. The experimental value of $m_{K^0}-m_{K^-}$ then gives

$$m_d-m_u=1.8\ (2.3)\ \text{MeV} \tag{64}$$

which is consistent with other determinations.[15,22] Using $m_d-m_u=2$ MeV, the D and B mass differences can be calculated from Eq. (63), and are displayed in Table II. Our result for the D mass difference is in good agreement with the experimental value. For the B mass difference, one has to wait for a more precise experimental determination. For the heavier mesons, the electromagnetic mass differences as determined from the extended Dashen theorem are small fractions of the total mass difference, so that the use of $c \leftrightarrow u$ or $b \leftrightarrow s$ symmetry in estimating them is perhaps not crucial to our final result.

V. M DEPENDENCE OF MASS-DIFFERENCE SUM RULE

So far we have analyzed the mass difference using the sum rule in the limit $M \to \infty$. This procedure is justified if the near-continuum contribution is negligible for all M in our chosen range of M values. It is desirable, however, to investigate the M dependence of the mass-difference sum rule (53) to check the stability of our result. For this purpose, we plot, as before, the RHS of Eq. (53) as well as the pole term on the LHS separately as a function of M in the range $M_{\min} \le M \le \infty$. We adopt the values of f_P and Λ_0 as obtained in Sec. IV. We also take $m_d-m_u=2$ MeV, and choose the hadronic mass difference between P_d and P_u as found in the $M \to \infty$ limit. Since we are now considering the difference sum rule, the near-continuum contribution to Eq. (53) need not be positive definite, so no inequality between the RHS and the pole term exists, as was the case in the original sum rule (40) discussed in Sec. III.

In Fig. 3, we display the results for the D meson. We see that the two curves match well over the range of M values considered. This implies that the sum rule (53) with the mass difference given by our calculations in the $M \to \infty$ limit, is indeed stable against variations in M.

TABLE II. SU(2) mass splittings.

Mass difference	Hadronic (MeV)	Electromagnetic (MeV)	Total (MeV)	Experimental result (MeV)
$m_{K^0}-m_{K^-}$	5.3 (input)	-1.3	4.0	4.01 ± 0.13 (Ref. 23)
$m_{D^+}-m_{D^0}$	4.3	0.35	4.7	4.7 ± 0.3 (Ref. 23)
$m_{B^0}-m_{B^-}$	5.9	-0.12	5.8	$3.4\pm3.0\pm2.0$ (Ref. 24)

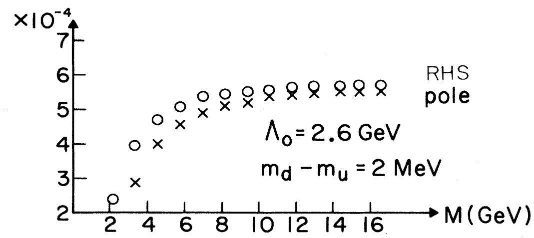

FIG. 3. Stability of the hadronic D mass splitting against variation in M.

Similar checks on the stability of the sum rule have been made in the case of other mass differences.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-76ER13065.

APPENDIX A: HEAVY-QUARK CONDENSATES

Consider the trace of the energy-momentum tensor[25]

$$\theta^{\mu}_{\mu}=\frac{\beta(g)}{2g}G^{a}_{\mu\nu}G^{a}_{\mu\nu}+\sum_i m_i\bar{q}_iq_i\ , \qquad \text{(A1)}$$

where the Gell-Mann—Low β function is given by[26]

$$\frac{\beta(g)}{2g}=-\frac{11-\frac{2}{3}n_f}{2(4\pi)^2}g^2+O(g^4)\ , \qquad \text{(A2)}$$

g is the QCD coupling constant, $g^2/4\pi=\alpha_s$, and n_f is the number of quark flavors. From Lorentz invariance, the vacuum energy density ϵ, is given by

$$\epsilon=\langle 0\,|\,\theta^0_0\,|\,0\rangle=\tfrac{1}{4}\langle 0\,|\,\theta^{\mu}_{\mu}\,|\,0\rangle\ . \qquad \text{(A3)}$$

Now, from the decoupling theorem of Appelquist and Carazzone,[27] we expect the contribution of a heavy quark q_h would be damped, and be of order $1/m_h$, where m_h is the mass of the heavy quark. For an extra flavor of heavy quark, we can readily find from Eqs. (A1)—(A3), the extra contribution to ϵ. Since this should be $O(1/m_h)$, we obtain

$$\frac{2}{3}\frac{g^2}{2(4\pi)^2}\langle 0\,|\,G^a_{\mu\nu}G^a_{\mu\nu}\,|\,0\rangle+m_h\langle 0\,|\,\bar{q}_hq_h\,|\,0\rangle=O\left[\frac{1}{m_h}\right]\ , \qquad \text{(A4)}$$

which yields the results (31) quoted in the text.

APPENDIX B: GENERALIZED DASHEN THEOREM

Assume that electromagnetic interactions are P-spin ($c\leftrightarrow u$) invariant. If we consider the P-spin doublet

$$\begin{bmatrix}D^+\\ \pi^+\end{bmatrix}=\begin{bmatrix}c\bar{d}\\ u\bar{d}\end{bmatrix}\ ,$$

clearly

$$m_\gamma{}^2(D^+)=m_\gamma{}^2(\pi^+)\ . \qquad \text{(B1)}$$

Now consider the P-spin triplet

$$\begin{bmatrix}D^0\\ \pi_P\\ \bar{D}^0\end{bmatrix}=\begin{bmatrix}c\bar{u}\\ \frac{1}{\sqrt{2}}(u\bar{u}-c\bar{c})\\ -u\bar{c}\end{bmatrix}\ . \qquad \text{(B2)}$$

In terms of the states

$$\pi^0=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})\ ,$$
$$\eta_8=\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})\ , \qquad \text{(B3)}$$
$$\eta_{15}=\frac{1}{\sqrt{12}}(u\bar{u}+d\bar{d}+s\bar{s}-3c\bar{c})\ ,$$

we can express π_P as

$$\pi_P=\frac{1}{2}\pi^0+\frac{1}{2\sqrt{3}}\eta_8+\left[\frac{2}{3}\right]^{1/2}\eta_{15}\ . \qquad \text{(B4)}$$

P-spin invariance of the electromagnetic interactions then implies for the triplet (B2)

$$\begin{aligned}m_\gamma{}^2(D^0)&=m_\gamma{}^2(\pi_P)\\ &=\tfrac{1}{4}m_\gamma{}^2(\pi^0)+\tfrac{1}{12}m_\gamma{}^2(\eta_8)+\tfrac{2}{3}m_\gamma{}^2(\eta_{15})\\ &\quad+\frac{1}{4\sqrt{3}}[m_\gamma{}^2(\eta_8\rightarrow\pi^0)+\text{c.c.}]\\ &\quad+\frac{1}{\sqrt{6}}[m_\gamma{}^2(\eta_{15}\rightarrow\pi^0)+\text{c.c.}]\\ &\quad+\frac{1}{3\sqrt{2}}[m_\gamma{}^2(\eta_{15}\rightarrow\eta_8)+\text{c.c.}]\ ,\end{aligned} \qquad \text{(B5)}$$

where

$$m_\gamma{}^2(a\rightarrow b)=\langle b\,|\,H^{\text{eff}}_{\text{EM}}\,|\,a\rangle\ . \qquad \text{(B6)}$$

Furthermore, if we consider the P-spin-singlet state

$$\eta_P=\frac{1}{\sqrt{6}}(u\bar{u}+c\bar{c}-2d\bar{d})=\frac{\sqrt{3}}{2}\pi^0-\frac{1}{6}\eta_8-\frac{\sqrt{2}}{3}\eta_{15}\ , \qquad \text{(B7)}$$

we have from P-spin invariance,

$$\langle\eta_P\,|\,H^{\text{eff}}_{\text{EM}}\,|\,\pi_P\rangle=0$$

which leads to

$$\begin{aligned}0&=\tfrac{3}{4}m_\gamma{}^2(\pi^0)-\tfrac{1}{12}m_\gamma{}^2(\eta_8)-\tfrac{2}{3}m_\gamma{}^2(\eta_{15})\\ &\quad+\frac{\sqrt{3}}{4}m_\gamma{}^2(\eta_8\rightarrow\pi^0)-\frac{\sqrt{3}}{12}m_\gamma{}^2(\pi^0\rightarrow\eta_8)\\ &\quad+\frac{\sqrt{3}}{2}m_\gamma{}^2(\eta_{15}\rightarrow\pi^0)-\frac{1}{\sqrt{6}}m_\gamma{}^2(\pi^0\rightarrow\eta_{15})\\ &\quad-\frac{1}{3\sqrt{2}}m_\gamma{}^2(\eta_{15}\rightarrow\eta_8)-\frac{1}{3\sqrt{2}}m_\gamma{}^2(\eta_8\rightarrow\eta_{15})\ .\end{aligned} \qquad \text{(B8)}$$

Adding (B5) and (B8), we get

$$m_\gamma{}^2(D^0)=m_\gamma{}^2(\pi^0)+\frac{1}{\sqrt{3}}\langle\pi^0\,|\,H^{\rm eff}_{\rm EM}\,|\,\eta_8\rangle+2\left[\frac{2}{3}\right]^{1/2}\langle\pi^0\,|\,H^{\rm eff}_{\rm EM}\,|\,\eta_{15}\rangle\ . \qquad (B9)$$

In the soft-pion limit, the last two terms vanish, and we get

$$m_\gamma{}^2(D^0)=m_\gamma{}^2(\pi^0)\ . \qquad (B10)$$

From Eqs. (B1) and (B10), we get the generalization of Dashen's theorem

$$[m^2(D^+)-m^2(D^0)]_\gamma=[m^2(\pi^+)-m^2(\pi^0)]_\gamma\ . \qquad (B11)$$

[1]M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

[2]V. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin, and V. Zakharov, Phys. Rep. 41, 1 (1978).

[3]L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B186, 109 (1981).

[4]V. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin, and V. Zakharov, Phys. Rev. Lett. 38, 626 (1977).

[5]V. S. Mathur and M. T. Yamawaki, Phys. Lett. 107B, 127 (1981).

[6]L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. 104B, 305 (1981).

[7]V. S. Mathur, in *Proceedings of 4th Annual Montreal-Rochester-Syracuse-Toronto High Energy Theory Conference, Montreal, 1982,* edited by B. Margolis and T. F. Morris (Department of Physics, McGill University, Montreal, 1982).

[8]V. S. Mathur and M. T. Yamawaki, in *MRST 83,* proceedings of 5th Annual Montreal-Rochester-Syracuse-Toronto Meeting on High Energy Theory, Toronto, 1983, edited by G. Kunstutter and P. J. O'Donnell (Physics Department, University of Toronto, Toronto, 1983).

[9]E. V. Shuryak, Nucl. Phys. B198, 83 (1982).

[10]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Lett. 18, 759 (1967).

[11]Shifman, Vainshtein, and Zakharov (Ref. 1) have shown that small-size instanton effects lead to a breakdown of the operator expansion, starting from some power $(Q^2)^{-d}$, where $d>5$. Since we consider only low-dimensional terms in the expansion, this effect may be ignored. The validity of the expansion (4) in nonperturbative vacuum is by no means obvious. In this connection, see S. Gupta and H. R. Quinn, Phys. Rev. D 26, 499 (1982); 27, 980 (1983); C. Taylor and B. McClain, *ibid.* 28, 1364 (1983).

[12]L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. 103B, 63 (1981); 97B, 257 (1980); 100B, 519(E) (1981).

[13]The gluon condensate contributes to the transverse function $\pi_t(Q^2)$, but not to the longitudinal function $\pi_l(Q^2)$. Since we are interested only in $\pi_l(Q^2)$ in this work, any uncertainties in the phenomenological determination of the gluon condensate term is irrelevant to us here. See, for instance, J. S. Bell and R. A. Bertleman, Nucl. Phys. B187, 285 (1981).

[14]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[15]H. Pagels and S. Stokar, Phys. Rev. D 22, 2876 (1980).

[16]S. Weinberg, in *Festschrift for I. I. Rabi,* edited by L. Motz (New York Academy of Sciences, New York, 1977).

[17]For completely different attempts at calculating the D^+-D^0 or B^0-B^- mass difference, see A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 398 (1976); K. Lane and S. Weinberg, *ibid.* 37, 717 (1976); R. Dutt and S. N. Sinha, Phys. Lett. 70B, 103 (1977); T. Fukuda, Prog. Theor. Phys. 59, 2179 (1978); A. Khare, Nucl. Phys. 163B, 269 (1980); E. Eichten, Phys. Rev. D 22, 1819 (1980); B. K. Barik and S. N. Jena, *ibid.* 24, 2905 (1981).

[18]R. Dashen, Phys. Rev. 183, 1245 (1969).

[19]See also E. Bagen *et al.*, CERN Report No. TH.3675, 1983 (unpublished).

[20]V. S. Mathur and M. T. Yamawaki, Phys. Rev. D 28, 669 (1983).

[21]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[22]See however J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982), who report a somewhat larger value of m_d-m_u.

[23]Particle Data Group, Phys. Lett. 111B, 1 (1982).

[24]S. Behrends *et al.*, Phys. Rev. Lett. 50, 881 (1983).

[25]R. Crewther, Phys. Rev. Lett. 28, 1421 (1972); M. Chanowitz and J. Ellis, Phys. Lett. 40B, 397 (1972); Phys. Rev. D 7, 2490 (1973).

[26]See D. J. Gross, in *Methods of Field Theory,* 1975 Les Houches Lectures, edited by R. Balian and J. Zinn-Justin (North Holland, Amsterdam, 1976).

[27]T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).