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Rotational bands in the baryon spectrum

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As suggested by the bag model, we assume that the ground states of the nucleon and the Δ are nearly spherical, but the excited even-parity states are quite deformed. This deformation is responsible for the observed low-lying excitations $N(1440)\frac{1}{2}^+$ and $\Delta(1600)\frac{3}{2}^+$, on which rotational bands are built.

I. INTRODUCTION

A striking feature in the excitation spectra of the nucleon and the Δ is the appearance of low-lying even-parity states. In the quark model, the energy spectrum of a baryon is generated by exciting one or more quarks to excited orbitals which appear in groups of alternating parity. This feature is true both in the bag model[1] and in the nonrelativistic constituent-quark model,[2] and this is the same as in the nuclear shell model.[3] Since the odd-parity states are seen at excitations of about 500 MeV, one should expect the even-parity single-particle states to appear at roughly twice this energy. Yet the $N(1440)\frac{1}{2}^+$ and $\Delta(1600)\frac{3}{2}^+$ are all found to be lower than the lowest odd-parity states. In the Isgur-Karl nonrelativistic model[2] (see also Forsyth and Cutkosky[4]), the even-parity states are lowered by splitting the $2\hbar\omega$ states using a local (spin-independent) two-body potential in a first-order perturbation calculation. This first-order calculation splits the states by the order of $\hbar\omega$, and the lowest even-parity excitation is shifted to about the energy of the odd-parity excitations. In this paper we are proposing a new model[5] for the even-parity excited baryonic states. We propose that the baryons are highly deformed in the excited even-parity states, and that these states may be simply described as members of rotational bands.[6]

Before proceeding with the description of the work, it may be helpful to give an analogy from nuclear physics. In ^{16}O, eight protons and eight neutrons form a closed-shell nucleus, with a gap of about 12 MeV between occupied $p_{3/2}$ orbitals and the unoccupied $d_{5/2}$ orbitals of the spherical mean field.[7] The first even-parity excited states in the single-particle picture should come around 20 MeV, yet experimentally a 0^+ state is found at 6.05 MeV. This state is a highly collective state, with two-particle—two-hole, and more importantly four-particle—four-hole excitations from the ground state, which strongly polarize and deform the core, thereby reducing the excitation energy.[8] The other low-lying even-parity states form a well defined rotational band with angular momentum 2^+, 4^+, 6^+, etc. This feature is quite common in nuclei where the ground state may be almost spherical, but the excited even-parity state is highly deformed, leading to the famous "coexistence" of collective and single-particle states.

We are proposing a similar model for baryons. In the bag model, it is expected that the nonlinear boundary conditions will cause the bag to deform in shape for the excited states. Calculations have been performed for the single-particle states of a bag as a function of the deformation parameter by various authors.[9] In a baryon, in addition to the three valence quarks, $q\bar{q}$ pairs and gluons may be generated with increasing excitation energy, causing further deformation of the bag. In this paper we want to examine the low-lying spectrum of the nucleon and Δ by assuming that in the excited states the valence quarks are moving in a deformed mean field. The intrinsic states are generated from a product of the single-particle deformed orbitals. Projection into states of good angular momentum then generates the characteristic $L=0^+,2^+,4^+,\ldots$ spectrum, which, combined with the total spin $S=\frac{1}{2}$ or $\frac{3}{2}$, would generate the spectrum with states of good J.

For the low-lying odd-parity $1\hbar\omega$ states belonging to the $\underline{70}$ 1^- representation, the mean field is probably much less deformed. In the bag model, the $1p_{1/2}$ orbital is the lowest with spherical symmetry, and the $1p_{3/2}$ state, which should be deformed, does not come down much in energy with deformation.[9] However, the states belonging to the $\underline{56}$ 1^- representation around 1900 MeV which have $3\hbar\omega$ orbitals should be very deformed in our picture, similar in situation to ^{16}O. This should result in the observed lowering in energy of these states, which in the spherical nonrelativistic quark model could only be lowered by introducing an extra term in the Hamiltonian.[4]

This paper will be of a qualitative nature where we shall concentrate on the even-parity excitations of the nucleon and the Δ. The lowering of the excitation energies in a deformed mean field will be illustrated in Sec. II by considering the nonrelativistic model of constituent quarks in a deformed oscillator potential. Similar considerations also hold in a deformed bag,[9] but some of the simplicity is lost. Section III will be devoted to a brief discussion of the decay properties of these states. In the final section, we summarize the main differences between the present model and the conventional quark model.

II. THE DEFORMED-OSCILLATOR MODEL

Consider a nonrelativistic constituent-quark model in which each valence quark is moving in an axially symmetric deformed oscillator:[7]

$$V=\tfrac{1}{2}m\omega_\perp^2(x^2+y^2)+\tfrac{1}{2}m\omega_3^2z^2 . \quad (1)$$

Such a potential has previously been considered[10] to describe the ground state of the nucleon. Define a deformation parameter δ and a mean frequency ω_0 through the equations

$$\delta=3\frac{(\omega_\perp-\omega_3)}{2\omega_\perp+\omega_3}\ , \quad \omega_0=\tfrac{1}{3}(2\omega_\perp+\omega_3) . \quad (2)$$

The single-particle eigenenergies $E(n_3,n_\perp)$ are then given by

$$E(n_3,n_\perp)=\hbar\omega_0(N+\tfrac{3}{2})-\tfrac{1}{3}\hbar\omega_0\delta(2n_3-n_\perp) , \quad (3)$$

where $N=(n_3+n_\perp)$, n_3 and $n_\perp$ being the number of excitation quanta along and perpendicular to the symmetry axis, respectively. This single-particle spectrum is shown in Fig. 1, where $E(n_3,n_\perp)/\hbar\omega_0$ is plotted against the deformation parameter δ. Note that with the onset of deformation, the orbital angular momentum l of the quark is no longer a good quantum number, but its projection Λ along the symmetry axis 3 is still conserved. The single-particle states with $n_\perp=0$, corresponding to $\Lambda=0$, come down in energy with increasing δ.

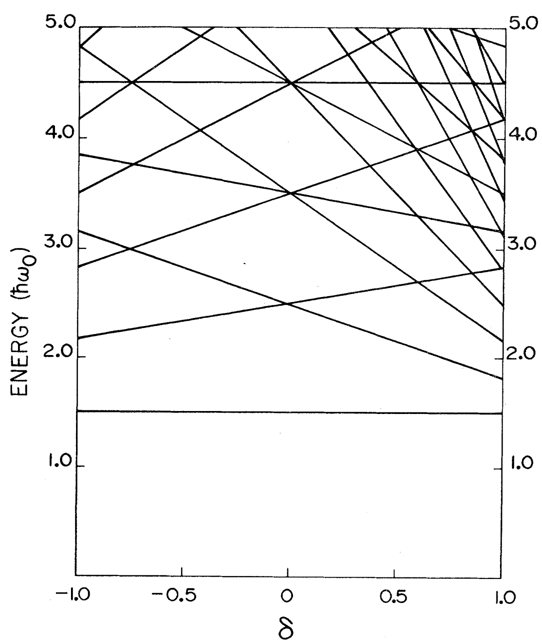

FIG. 1. The single-particle energy levels of an axially symmetric oscillator as a function of the deformation parameter δ. The parameters $\hbar\omega_0$ and δ are defined in Eq. (2). The shell gaps of $\hbar\omega_0$ occur for $\delta=0$, but the orbitals with $n_\perp=0$ come down most for prolate deformation (positive δ).

We shall assume that the ground state is spherical, and the deformation parameter δ is small for the $N=1$ excitations, but builds up to a bigger value for the $N=2$ states. This feature has been established in nuclei, where, as more and more particles occupy a given orbital, it gets more and more deformed. The lowest $N=2$ excitations, for a given δ, may obviously be obtained by promoting two quarks to $N=1$ orbitals with $\Lambda=0$, or by promoting a single quark to the $N=2$ orbital with $\Lambda=0$. In the absence of deformation the excitation energy of this state would be $2\hbar\omega_0$, but this will reduce to $\hbar\omega_0(2-\frac{4}{3}\delta)$ for nonzero δ. The projection of the $L=0$ state from this deformed intrinsic state would result in a further lowering in energy[11] by an amount $\langle L^2\rangle/2\mathscr{I}$, where the expectation value of L^2 is taken with respect to the intrinsic state, and $\mathscr{I}$ is the moment of inertia. We shall see that we expect $\hbar^2/2\mathscr{I}\sim 40$–$50$ MeV, so $\langle L^2\rangle/2\mathscr{I}$ could be about 300 MeV. If we take $h\omega_0\sim 500$ MeV, then the spherical $1\hbar\omega$ odd-parity excitation will appear at about the same energy as the $L=0^+$ excitation for $\delta\approx 0.5$.

The intrinsic state that we construct should have the proper permutation symmetry, so that by combining it with the appropriate SU(6) multiplet $\underline{56}$, $\underline{70}$, or $\underline{20}$, a totally symmetric wave function is obtained. As in the spherical model, we find one symmetric and two mixed-symmetry orbital states, but these cannot be labeled (in the nucleon) as ^2S_S, ^2S_M, and ^4D_M, because in the intrinsic state the l values are mixed. The $\underline{20}$ representation cannot be realized because the $L=1$ component is absent in the lowest-energy state. States of good J can be constructed by projecting out states of good L from the intrinsic state, and combining L with $S=\frac{1}{2}$ or $\frac{3}{2}$. We have one symmetric and two mixed-symmetry (MS) states from permutation. The symmetric states give, on projection, $L=0$, 2, 4, etc., states, and a combination of the two MS states also yields a $L=0,2,4$, band. In the nucleon, the symmetric band can only combine with $S=\frac{1}{2}$ to give rise to states of $J=\frac{1}{2}$, $(\frac{3}{2},\frac{5}{2})$, $(\frac{7}{2},\frac{9}{2})$, etc., while the MS band can combine with $S=\frac{1}{2}$ or $S=\frac{3}{2}$ to yield a set of states. These are shown schematically in Fig. 2(a) for the nucleon and Fig. 2(b) for the Δ. Up to now the classification scheme is the same as the O(3)×SU(6) of the standard quark model, except that the O(3) part of the spectrum has been generated by the motion of quarks in a deformed field. There are, however, some important differences in the mixing of the bands in the two models, with the consequence that experimental data may be able to distinguish between the two.

Up to now, we have ignored the effect of the residual quark-quark interaction on the rotational spectra. First, let us consider the spin-orbit potential that one expects to have from the one-gluon-exchange mechanism. In the presence of such a force, the three component of the orbital angular momentum Λ is no longer a good quantum number (J_3, of course, is). The spin-orbit potential mixes states of different Λ, but this mixing is inhibited due to the large splitting in energy between such states that is

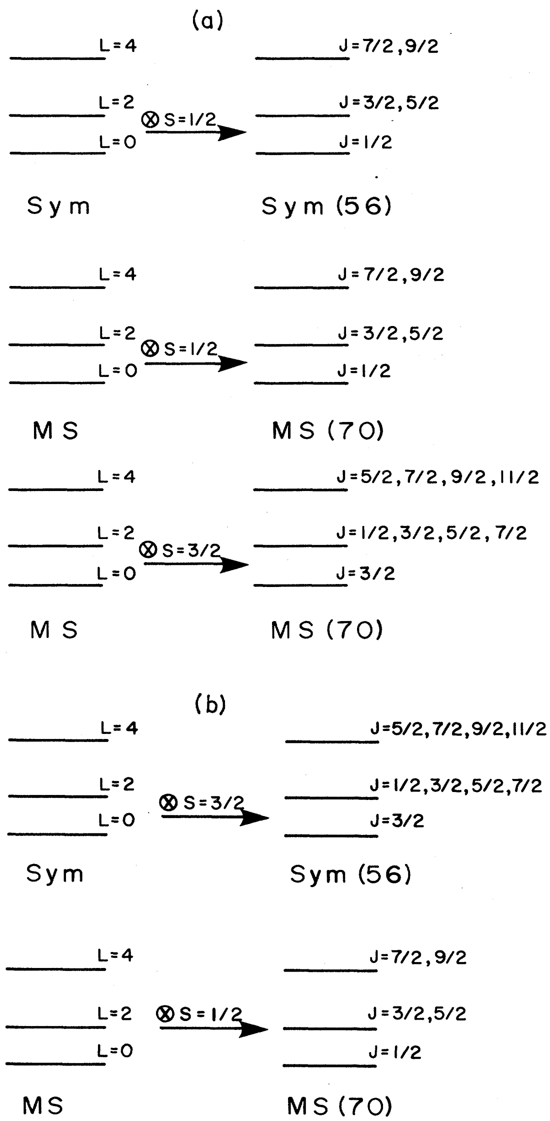

FIG. 2. (a) A schematic picture of the even-parity excited states of the nucleon. The states are grouped in rotational bands of orbital angular momentum (L), orbital permutation symmetry [symmetric (Sym) or mixed symmetry (MS)], spin (S), and the SU(6) representation (56 and 70). (b) A schematic description of the even-parity excited states of the Δ. The classification is done in the same way as in (a).

brought about by deformation. In the limit of large deformation, Λ becomes, again, a good quantum number. For the $\Lambda=0$ state that has come down most in energy, the spin-orbit contribution is zero. Thus in the rotational bands the spin-orbit force has little effect and the even-parity states are insensitive to the strength of this force. A similar argument does not hold for the tensor force, and, as in Ref. 4, we have to assume that it is small in order that the spins $S=\frac{1}{2}$ and $S=\frac{3}{2}$ couple weakly to the collective rotations, with little mixing between the two. We also assume that most of the central spin-independent long-range q-q interaction has already been absorbed in the mean field, unlike the conventional quark model where this played a crucial role. In our model, the most important residual force between the quarks is the very-short-range hyperfine interaction, which causes considerable mixing between the 56 and the 70 bands in the nucleon. In the conventional quark model, this mixing was largely inhibited by the splitting between these bands due to the central spin-independent force.

We now discuss the even-parity excitation spectrum in some detail. In Fig. 3, the experimentally established[12] states of the nucleon and Δ are shown by unbroken lines and the more uncertain data are given in dashed lines. We have grouped these in rotational bands according to our model, along with the expected, but as yet unseen members of the band by dot-dashed lines. We begin with Δ, which has the simpler spectrum. The lowest excited state listed[12] is the (two-star) 1550 $\frac{1}{2}^+$ state, and the next is the (three-star) 1600 $\frac{3}{2}^+$ state. The nominal energy of the latter state is 1600 MeV, although the baryon table quotes an uncertainty in its mass in the range 1500—1900 MeV. Recent phase-shift analyses[13,14] give its mass as 1600 ± 50 and 1522 MeV, respectively, so there is reason to believe that it is low lying. In our model we expect a 56 $J=\frac{3}{2}^+$ and a 70 $J=\frac{1}{2}^+$, which are the lowest-lying members (band heads) of two rotational bands. We expect these two states to be nearly degenerate in energy, and not split substantially by the central part of the qq interaction as in previous calculations.[4,15] The splitting is caused not by the hyperfine interaction, which cannot mix $S=\frac{1}{2}$ and $S=\frac{3}{2}$ states, but only by the (weak) tensor force. As in the previous calculations, we expect the 70 states to couple weakly to the elastic πN channel. The splitting between $N(1440)$ and $\Delta(1600)$ in previous models comes out a factor of 2 too large. This is because in the spherical model these two states have a structure similar to the respective ground states (being nodal excitations), and the hyperfine splitting is about the same as in the ground state. In the present model, both $N(1440)$ and $\Delta(1600)$ are deformed and have a different structure from the spherical ground state, and a very-short-range hyperfine interaction would yield less splitting. We expect near-degenerate states of Δ with spins $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{7}{2}^+$ coming from the 56 2^+ coupling to $S=\frac{3}{2}$, and these are seen between 1900 and 1950 MeV. We would also expect a degenerate pair in Δ with spins $\frac{3}{2}^+$ and $\frac{5}{2}^+$ coming from the 70 2^+ coupled to $S=\frac{1}{2}$. These should be weakly coupled in the πN channel and it is not surprising that they are not seen. Going higher up in the energy spectrum of Δ, we also expect the states $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{9}{2}^+$, and $\frac{11}{2}^+$ arising from the 56 4^+ states. The splitting between the $L=0^+$ and $L=2^+$ states in the rotational band of Δ is about 300 MeV. If we take a rotational spectrum of the form $(\hbar^2/2\mathscr{I})L(L+1)$, where $\mathscr{I}$ is the moment of inertia, then this gives $\hbar^2/2\mathscr{I}\approx50$ MeV. If the moment of inertia was constant, then the 4^+ states should be about 1000 MeV above the band head at 1600 MeV. An $\frac{11}{2}^+$ is seen at about 2420 MeV in Δ, a $\frac{9}{2}^+$ at 2300 MeV (two star), and a $\frac{7}{2}^+$ at 2400 MeV (one star, shown by a dashed line in Fig. 3). We believe these to be the 56 4^+ states.

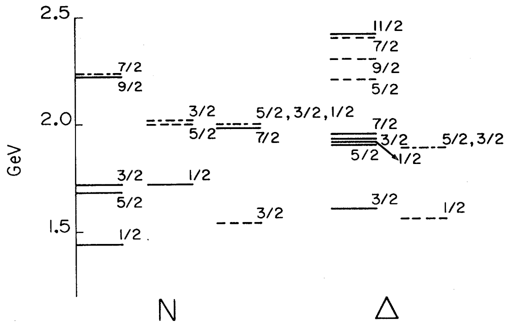

FIG. 3. The even-parity excited states in the nucleon and Δ arranged in rotational bands. The solid line corresponds to experimental states given a three- or four-star status by the Particle Data Group (Ref. 12). The dashed lines are one- and two-star states [with the exception of the Δ(2200) which has been seen in only one analysis and is not given a star status]. The dot-dashed lines correspond to states predicted by the model that have not been observed experimentally.

They are below the estimate of 2600 MeV because relativistic effects should cause the moment of inertia to increase for the more excited states.

The nucleon is slightly more complicated than the Δ. Here we expect three band heads with $L=0$: a $\underline{56}$ $J=\frac{1}{2}^+$, a $\underline{70}$ $J=\frac{1}{2}^+$, and a $\underline{70}$ $J=\frac{3}{2}^+$. We expect in the present model the $\underline{70}$ $J=\frac{3}{2}^+$ state to be nearly degenerate with the Δ(1600) and couple weakly to the πN channel. It is interesting that there is a weak (one-star) state, $N(1540)\frac{3}{2}^+$, which may be the $\underline{70}$ $J=\frac{3}{2}^+$ band head. With a weak tensor force, this should not mix with the $S=\frac{1}{2}$ states. The other two low-lying states, $\underline{56}$ $J=\frac{1}{2}^+$ and $\underline{70}$ $J=\frac{1}{2}^+$, can and do mix through the spin-dependent hyperfine interaction, giving much of the splitting between $N(1440)\frac{1}{2}^+$ and $N(1710)\frac{1}{2}^+$. From the decays we expect a 25% admixture of $\underline{70}$ $\frac{1}{2}^+$ in $N(1440)$, and correspondingly a 25% admixture of $\underline{56}$ $\frac{1}{2}^+$ in $N(1710)$ (see Sec. III).

We find the 2^+ state built on the $N(1440)$ at 1700 MeV $[N(1680)\frac{5}{2}^+$ and $N(1720)\frac{3}{2}^+]$, a splitting of 260 MeV from the 0^+. The moment of inertia is thus the same or slightly greater than the Δ. In the rotor model we would expect the 4^+ at 2300 MeV. We find the $\frac{9}{2}^+$ at 2200 MeV (2150–2300). Again a bit low, as expected, if the moment of inertia increases with excitation. The expected $\frac{7}{2}^+$ is not seen at this energy, but there is a (three-star) $\frac{7}{2}^+$ at 1990 MeV which couples rather weakly to the πN channel. Perhaps this state is the $L=2$ member of the $\underline{70}$ $J=\frac{3}{2}^+$ band, which should be in this energy range, although with very small coupling to the πN channel. If this is true, then the $\frac{7}{2}^+$ state in the $N(1440)$ band has not been seen yet possibly due to a large total width. Note also that we expect three other $L=2$ states in the $\underline{70}$ $\frac{3}{2}^+$ band as shown in Fig. 3, nearly degenerate with $N(1990)\frac{7}{2}^+$, which have not been seen either because of their small coupling to the πN channel. The 2^+ rotational state build on the $N(1710)$ is expected slightly above 2000 MeV. The $\frac{5}{2}^+$ state is seen at this energy, but the $\frac{3}{2}^+$ is unseen. There is a one-star $J=\frac{1}{2}^+ N(2100)$, which, if it really exists, may belong to the $\underline{20}$ representation, which would not move down in energy in our model since $\Lambda \neq 0$.

The Λ particle spectrum is quite similar to the nucleon and is generally consistent with the rotational model. Classification of the bands is very similar to the nucleon, and we shall not discuss it here. The Σ spectrum, on the other hand, has many more levels since both the $\underline{8}$ and $\underline{10}$ SU(3) representations are present. It is also complicated by poorer experimental data. There are many one- and two-star states.[12] Also, most of the predictions we would make for the Σ are just the SU(6) predictions and are not unique to our model. Thus at the present stage there is little to be learned about the validity of the rotational model from the Σ spectrum. However, we would like to make one prediction. The location of the state analogous to the Δ(1600) ($\underline{56}$ $S=\frac{3}{2}$) is not yet determined experimentally. However, there is evidence for the $L=2$ rotational states built with this state as the band head. The $L=2$ states occur between 2000 and 2100 MeV with the $\frac{7}{2}^+$ being particularly strong. Using the moment of inertia from the Δ, we predict the band head $(J=\frac{3}{2}^+)$ to be between 1650 and 1750 MeV and possibly identified with the bumps at 1690 MeV, whose spin and parity is as yet undetermined. This prediction is in disagreement with Isgur and Karl,[2] whose lowest $\frac{3}{2}^+\Sigma$ state is at 1865 MeV.

Before ending this section, we would like to comment on the moment-of-inertia parameter. From the fits to the experimental data, we have seen that

$$\frac{\hbar^2}{2\mathscr{I}} \approx 40\text{–}50 \text{ MeV} . \tag{4}$$

The rigid moment of inertia of a rotating sphere of mass M and radius R is $\frac{2}{5}MR^2$. Taking $M=1500$ MeV and $R=0.9$ fm yields $\hbar^2/2\mathscr{I}_{\text{rigid}}=40$ MeV. This very rough estimate is consistent with the value obtained from the fit.

III. DECAY PROPERTIES

Since we are using a basis of good SU(6), it makes sense to use the $SU(6)_W \times O(3)$ of Melosh[16] and Hey, Litchfield, and Cashmore[17] to describe the decays. Since this model is reasonably successful for the $\underline{56}$ 2^+ decays, we expect our rotational model to also be reasonable. The main difference from Hey, Litchfield, and Cashmore for the $\underline{56}$ 2^+ decays is that our states are not pure $\underline{56}$, but may contain appreciable mixtures of $\underline{70}$. This would tend to reduce the πN decay strengths somewhat for N^* decays compared to Δ^* decays. We would not expect the pure $\underline{70}$ states to couple to the πN channel, the same as in the nonrelativistic quark model.[4,14]

As pointed out earlier, two states where we differ from the usual nonrelativistic quark model are the $N(1440)$ and the $N(1710)$, both $J=\frac{1}{2}^+$ states. These we expect to be mixed perhaps as much as 75–25%. With or without mixing we cannot explain the large width of the $N(1440)$ found in Ref. 13. However, if we take the width of Ref. 14, the results are more reasonable. For the decay of the $N(1710)$ we would have zero for πN decay without mixing of the 70 and 56 representations. The partial decay width of 30 MeV is consistent with about 20% mixing. Stronger evidence for mixing comes from the photo-decay data.

The helicity amplitudes $A^p_{1/2}$ and $A^n_{1/2}$ for the $N(1440)\frac{1}{2}^+ \to N(940)\frac{1}{2}^+ + \gamma$ are known experimentally.[12] These are (in units of 10^{-3} GeV$^{1/2}$)

$$A^p_{1/2} = -70 \pm 9 \ , \quad A^n_{1/2} = 42 \pm 23 \ . \tag{5}$$

Let us briefly point out the difficulty of reproducing these data with spherical orbitals. For the $N(940)\frac{1}{2}^+$, let us take the state[2]

$$|N(940)\tfrac{1}{2}^+\rangle = 0.98\,|{}^2S_S\rangle - 0.199\,|{}^2S_M\rangle \tag{6}$$

with oscillator parameter $b = 0.7$ fm. For $N(1440)\frac{1}{2}^+$, the wave function taken by Koniuk and Isgur[15] is

$$|N(1440)\tfrac{1}{2}^+\rangle = 0.99\,|{}^2S'_S\rangle + 0.17\,|{}^2S_M\rangle + 0.01\,|{}^4D_M\rangle \tag{7}$$

with an oscillator parameter $b = 0.48$ fm. The calculated amplitudes are, in the same units as before,

$$A^p_{1/2} = -24 \ , \quad A^n_{1/2} = 16 \ .$$

To obtain the helicity amplitudes closer to experiment, we need a wave function[18]

$$|N(1440)\tfrac{1}{2}^+\rangle = 0.85\,|{}^2S'_S - 0.51\,|{}^2S_M\rangle - 0.11\,|{}^2S_S\rangle \ , \tag{8}$$

which yields

$$A^p_{1/2} = -64 \ , \quad A^n_{1/2} = 43 \ .$$

In other words, it is necessary to have about 25% or more of the mixed symmetric state combination in $N(1440)\frac{1}{2}^+$ to reproduce the data. This is not realized in the spherical model because of the large diagonal splitting of the $N(1440)$ and $N(1710)$ from the central part of the qq force. Now consider the state $N(1710)\frac{1}{2}^+$, which is the partner in mixing. Here the experimental data[12] is rather poor:

$$A^p_{1/2} = 3 \pm 15 \ , \quad A^n_{1/2} = 9 \pm 30 \ . \tag{9}$$

In the quark model with spherical orbitals, the wave function is now taken to be predominantly ${}^2S_M$,

$$|N(1710)\tfrac{1}{2}^+\rangle = 0.94\,|{}^2S_M\rangle - 0.15\,|{}^2S_S\rangle - 0.31\,|{}^4D_M\rangle - 0.07\,|{}^2P_A\rangle \ . \tag{10}$$

The oscillator parameter is again 0.48 fm. One then gets

$$A^p_{1/2} = -47 \ , \quad A^n_{1/2} = -21 \ .$$

In spite of the large error bars in the data, a fit is not possible unless the mixing between $|{}^2S_M\rangle$ and $|{}^2S_S\rangle$ is increased. In fact, if we take a wave function for $N(1710)\frac{1}{2}^+$, which is orthogonal to the state $N(1440)\frac{1}{2}^+$ given by Eq. (8), with the coefficients of $|{}^2S'_S\rangle$ and $|{}^2S_M\rangle$ interchanged in magnitude, the experimental values given by (9) are obtained.

IV. CONCLUSIONS

The main point of this paper is that although in the ground state the baryons are spherical, with excitation the valence quarks see a deformed mean field. A qualitative analysis has been made by assuming a deformed-harmonic-oscillator model, although it should be possible to do a similar analysis with deformed bag orbitals. In such a model, the $N = 2$ excitations quite naturally come lower in energy than the $N = 1$ odd-parity excitations if one assumes that the deformation tends to increase with the excitation quanta. This will also result in the $N = 3$ 56 1^- states coming lower in energy than the spherical quark model.

It has been shown that for those states that come down in energy most (with $L_3 = \Lambda = 0$), the spin-orbit qq potential is inoperative for large deformation. This is in contrast to the spherical quark model, where the spin-orbit force must be suppressed artificially to fit the data. The tensor force is assumed to be weak, otherwise there would be strong mixing between the $S = \frac{1}{2}$ and $S = \frac{3}{2}$ states, leading to the strong-coupling model.[5] Unlike the conventional models, we assume that the long-range central spin-independent qq force is mostly absorbed in the mean field, leaving only the short-range hyperfine interaction as the main residual interaction. This causes considerable mixing between the 56 $S = \frac{1}{2}$ and 70 $S = \frac{1}{2}$ states in the nucleon, giving the splitting between the $N(1440)\frac{1}{2}^+$ and $N(1710)\frac{1}{2}^+$, and improving the quality of fit in the photo-decay data. We also predict that the 70 0^+ states are considerably lower than in the conventional quark models. These states are hard to see experimentally because they couple weakly to the πN channel. It is important to find experimentally where these states are.

It is well known in nuclear physics that a rotor-type spectrum is generated by particles moving in a deformed mean field.[19] The single-particle strengths of the spherical orbitals are spread over these collective states. In our model, states in the representation 20 0^+ do not come down in energy because $\Lambda \neq 0$, and may be looked upon as single-particle states. For the rotational bands, we find that the moment of inertia parameter is close to the rigid value.

In the present paper, we have not given a prescription for finding the deformation parameter in a self-consistent manner for the different excited configurations of the baryon. For this one would need to start with a Hamiltonian which adequately describes the interaction of the valence quarks, and the interplay of the valence quarks with other degrees of freedom, such as the $q\bar{q}$ core.

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[1]T. A. DeGrand and R. L. Jaffe, Ann. Phys. (N.Y.) 100, 425 (1976); T. A. DeGrand, *ibid.* 101, 496 (1976).

[2]N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978); 19, 2653 (1979).

[3]M. G. Mayer, Phys. Rev. 75, 1969 (1949); O. Haxel, J. H. D. Jensen, and H. E. Suess, *ibid.* 75, 1766 (1949).

[4]C. P. Forsyth and R. E. Cutkosky, Z. Phys. C 18, 219 (1983).

[5]R. K. Bhaduri, B. K. Jennings, and J. C. Waddington, TRIUMF Report No. TRI-PP-83-50 (unpublished); in *MRST 83*, proceedings of the Fifth Annual Montreal-Rochester-Syracuse-Toronto Meeting, Toronto, 1983, edited by G. Kunstatter and P. J. O'Donnell, (Physics Department, University of Toronto, Toronto, 1983); H. Toki, J. Dey, and M. Dey, Phys. Lett. 133B, 20 (1983).

[6]A. Bohr, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 26 (No. 14) (1952).

[7]See, for example, M. A. Preston and R. K. Bhaduri, *Structure of the Nucleus* (Addison-Wesley, Reading, Mass., 1975), Chaps. 9 and 10.

[8]G. E. Brown and A. M. Green, Phys. Lett. 15, 320 (1966).

[9]K. Hahn, R. Goldflam, and L. Wilets, Phys. Rev. D 27, 635 (1983); R. D. Viollier, S. A. Chin, and A. K. Kerman, Nucl. Phys. A407, 269 (1983); R. Shanker, D. Vasak, C. S. Warke, W. Greiner, and B. Müller, Z. Phys. C 18, 327 (1983).

[10]B. Giraud and J. Le Tourneaux, Lett. Nuovo Cimento 27, 497 (1980).

[11]X. Campi, H. Flocard, and A. K. Kerman, Nucl. Phys. A251, 193 (1975).

[12]Particle Data Group, Phys. Lett. 111B, 16 (1982).

[13]R. E. Cutkosky, C. P. Forsyth, R. E. Hendrick, and R. L. Kelly, Phys. Rev. D 20, 2839 (1980); R. E. Cutkosky *et al.*, in *Baryon 1980*, proceedings of the IVth International Conference on Baryon Resonances, Toronto, edited by N. Isgur (University of Toronto, Toronto, 1980), p. 217.

[14]G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, *Handbook of Pion-Nucleon Scattering*, No. 12-1 of *Physics Data* (Fachinformationszentrum, Karlsruhe, 1979); R. Koch, in *Baryon 1980* (Ref. 13), p. 3.

[15]R. Koniuk and N. Isgur, Phys. Rev. Lett. 44, 845 (1980); Phys. Rev. D 21, 1868 (1980).

[16]H. J. Melosh, Phys. Rev. D 9, 1095 (1974).

[17]A. J. G. Hey, P. J. Litchfield, and R. J. Cashmore, Nucl. Phys. B95, 516 (1975).

[18]Mira Dey and M. V. N. Murthy (private communication).

[19]R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. London 70, 381 (1957).