Analytic expressions for the cascade decay $W \to$ heavy fermion $\to$ light fermion, and a signature study of the process $W \to$ heavy lepton $\to$ electron

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A covariant formalism for single-particle inclusive decay of the W and Z bosons is established. It is then applied to the cascade decay of the W to a light fermion via an intermediate fermion with an arbitrary mix of V and A coupling. Exact analytic expressions are presented for heavy-fermion production in W decay, for heavy-fermion decay, and for the complete decay chain. As a particular illustration of the cascade results, the electron signatures for W decay through the τ and through a new heavy lepton are studied in detail in the W rest frame and contrasted.

I. INTRODUCTION

With the recent discovery of the W and Z resonances[1] of the standard electroweak theory, it becomes reasonable to speculate on possible exotic decay modes of these resonances. In this paper we present a general formalism for the decay of the W or Z to a new heavy fermion, which subsequently decays to a final state with one particle singled out for observation. The W decay is of special interest since the nondiagonal nature of the W current allows production of a new fermion with mass approaching the W mass itself, if the new partner is light. Thus we focus our attention on the decay of the charged W to a new heavy fermion L accompanied by its weak-isospin partner, whose mass we assume to be negligible. As a particular application of the formalism we obtain an analytic expression for the cascade decay

$$W \to \nu_L L \to \nu_L \nu_L l \nu_l ,$$

where L and l independently represent quark, antiquark, lepton, or antilepton, and ν_L and ν_l denote their respective weak-isospin partners. Our analytic formulas neglect any final-state masses, but contain the exact dependence on the L mass, m. Particle spins and W-mass effects, including W-propagation effects in the L decay, are properly included. The $WL\nu_L$ coupling is allowed an arbitrary mixture of V and A.

The ease of electron and muon identification in experimental detectors suggests using a lepton from the heavy fermion's weak decay as the event signature. We illustrate the application of our formulas by showing results for heavy-lepton production and leptonic decay[2-4] in the W rest frame. Our interest in a new lepton is twofold: if discovered, its nature may offer insight into the mystery of family replication, and its existence could challenge the cosmological arguments which bound the number of light (<1 MeV) neutrino species.[5]

In Sec. II, a formalism is defined offering an invariant characterization of the single-particle spectrum from any W decay, direct or sequential. The W decay tensor is factorized into invariant functions containing the dynamics, and simple four-vectors. Analytic expressions for the invariant functions describing the cascade $W \to$ (heavy fermion)$\to$ (light fermion) are presented in Sec. III. These expressions are the heart of this work. The analytic expressions are applied to the particular decay of a W to an electron via a heavy-lepton intermediate in Sec. IV. The use of an electron signature in the W rest frame to isolate a new heavy lepton is discussed in detail. A brief summary constitutes Sec. V. Technical details, which should prove useful for any cascade-decay calculation, are collected in two appendices.

II. SINGLE-PARTICLE SPECTRUM FROM DECAY OF THE W AND Z: GENERAL FORMALISM

We seek the momentum spectrum of a single particle chosen from the W-decay products. Denote the W or Z four-momentum by Q^μ and that of the observed final-state particle, labeled f, by p^μ. Define the covariant tensor

$$W^{\mu\nu} = \frac{d^3p}{(2\pi)^3 2p_0} \int dP_{\rm LI} \langle 0 | J_W^{\nu\dagger}(Q) | f(p), F \rangle \times \langle f(p), F | J_W^\mu(Q) | 0 \rangle . \qquad (2.1)$$

J_W^μ is the current coupling to the W^- field in the weak Lagrangian, F labels the final-state particles excluding f, and

$$dP_{\rm LI} = (2\pi)^4 \delta^4 \left[Q^\mu - p^\mu - \sum_F p_F^\mu \right] \prod_F \frac{d^3p_F}{(2\pi)^3 2p_F^0} \qquad (2.2)$$

is the Lorentz-invariant phase space for the set F. If the polarization of f is not measured, then covariance implies the following tensor expansion:

$$W^{\mu\nu} = -g^{\mu\nu} W_1 + p^\mu p^\nu W_2 + i\epsilon^{\mu\nu\alpha\beta} p_\alpha Q_\beta W_3 + Q^\mu Q^\nu W_4 + p^{\{\mu} Q^{\nu\}} W_5 + i p^{[\mu} Q^{\nu]} W_6 . \qquad (2.3)$$

The notation $a^{[\mu}b^{\nu]} = a^\mu b^\nu - b^\mu a^\nu$ and $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + b^\mu a^\nu$ is employed. The W_i are Lorentz-invariant functions of the invariants Q^2, $p \cdot Q$, and p^2. They are real as a consequence of the manifest Hermiticity of $W^{\mu\nu}$ as defined in Eq. (2.1). The W quantum may be real, with

$Q^2 = M_W{}^2$, or virtual, with $Q^2 \neq M_W{}^2$. If the W is on its mass shell, or if the source of the W quantum is conserved, then W_4, W_5, and W_6 are unobservable. Ignoring phases in the mass-matrix and strong-interaction phases, T invariance of the weak interaction implies the vanishing of W_6. The tensor for W^+ decay to $\bar{f}$ is obtained from that for W^- decay to f by changing the signs of W_3 and W_6. The tensor $Z^{\mu\nu}$ describing single-particle inclusive Z decay is analogously defined. A simple example of the formalism is W^- decay to a fermion f_1 (observed) plus antifermion $\bar{f}_2$, via the standard $V-A$ coupling:

$$
\begin{aligned}
&W_1 = \tfrac{1}{2}(Q^2 - m_1{}^2 - m_2{}^2) \, , \\
&W_3 = -\tfrac{1}{2} W_2 = W_5 = 1 \, , \qquad (2.4) \\
&W_4 = W_6 = 0 \, ,
\end{aligned}
$$

all times the factor

$$\frac{G_F M_W{}^2 3^c}{\sqrt{2}\pi^2} \delta(Q^2 + m_1{}^2 - m_2{}^2 - 2Q\cdot p) \frac{d^3 p}{p_0} \, ,$$

with G_F the usual Fermi constant and $c = 0$ (1) if the fermion is a lepton (quark); 3^c is a color-averaging factor. For $\bar{f}_2$ observed, or for $W^+ \to \bar{f}_1$ plus f_2 (observed), or for a $V+A$ coupling, the sign of W_3 should be changed.

In general, the covariant W decay tensor $W^{\mu\nu}$ is to be contracted with a W production tensor $P_{\mu\nu}$ (which we take to inlcude the numerator factors from the W propagators) and multiplied by the square of the W propagator, whose functional dependence is often parametrized by a Breit-Wigner resonance function. $P_{\mu\nu}$ is determined by the particular process producing the W boson, independent of the subsequent W decay mode. On the other hand, the W_i are independent of the production process, but depend upon the particular decay mode of the W. The contraction between W production and W decay may be carried out immediately using our formulation. As a useful example, we present the tensor describing W production through unpolarized fermion-antifermion annihilation. From the usual $V-A$ current

$$\left[\frac{G_F M_W{}^2}{\sqrt{2}} \right]^{1/2} \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi \, ,$$

there results

$$P_{\mu\nu} = \frac{\sqrt{2}\, G_F M_W{}^2}{3^c} (q_\mu \bar{q}_\nu + q_\nu \bar{q}_\mu - g_{\mu\nu} q \cdot \bar{q} + i\epsilon_{\mu\nu\alpha\beta} \bar{q}^\alpha q^\beta) \, . \qquad (2.5)$$

q ($\bar{q}$) is the fermion (antifermion) momentum, and $c = 0$ (1) if the annihilating fermion is a lepton (quark). (We have neglected the fermion masses relative to M_W.) The cross section results from contracting $P_{\mu\nu}$ and $W^{\mu\nu}$, and multiplying by the Breit-Wigner function and standard flux factor. One easily finds

$$d\sigma = \frac{G_F M_W{}^2}{\sqrt{2}\, 3^c} \frac{1}{(Q^2 - M_W{}^2)^2 + (M_W \Gamma_W)^2} \times \left[W_1 + \frac{2p\cdot q \, p\cdot \bar{q}}{Q^2} W_2 + p\cdot(\bar{q} - q) W_3 \right] . \qquad (2.6)$$

We are interested in signatures for new matter in W decay. Thus we wish to calculate the W_i for sequential decay of the W through a heavy particle, which we label L. In particular, in the next section we calculate analytic expressions for the W_i describing the sequential decay chain: real or virtual $W \to \nu_L L \to \nu_L \nu_L l \nu_l$, with l observed. L and l may be quark, antiquark, lepton, or antilepton, independently of the other. ν_L and ν_l denote their respective weak-isospin partners. Our choice of particle notation anticipates the particular application to heavy-lepton production and subsequent decay to an electron or muon.

III. ANALYTIC FORMULAS FOR HEAVY-FERMION PRODUCTION AND DECAY

A virtue of looking for new matter in W decay is that the nondiagonal nature of the charged current allows production of new particles with mass m right up to the phase-space limit M_W. Near the W mass terms of all orders in (m/M_W) and the effects of the W propagator become important and must be included in rigorous calculations. We begin this section by generalizing well known formulas of $V-A$ theory to include these effects, and to include the possibility of an arbitrary V and A mixture.

The coupling of a new fermion to the W is defined by its current

$$J_W^\mu = \left[\frac{G_F M_W{}^2}{\sqrt{2}} \right]^{1/2} \bar{L} \gamma^\mu (v + a\gamma_5) \nu_L \, . \qquad (3.1)$$

The angular dependence for the decay of a polarized, on-shell W into the new fermion is then calculated to be

$$\frac{d\Gamma}{d\cos\theta}(W \to L \bar{\nu}_L) = \frac{G_F M_W{}^3 3^c}{16\pi\sqrt{2}} (1-\rho)^2 \left[\frac{v^2 + a^2}{2} \right] \times \left[1 + \frac{4va}{v^2 + a^2} \cos\theta + \cos^2\theta + \rho \sin^2\theta \right] . \qquad (3.2)$$

We have taken ν_L to be massless. The color weighting is $c = 1$ (0) if L is a quark (lepton). θ is the angle between the L momentum and the W spin, and

$$\rho = (m/M_W)^2 \, .$$

If L is an antifermion, we must reverse the sign of a. The $\rho \to 0$ distribution of $V-A$ theory, $(1 \mp \cos\theta)^2$, is recovered by setting $v = -a = 1$. But notice the significance of the "wrong-helicity" amplitude for L production, responsible for the $\sin^2\theta$ term.[6] For $V \mp A$ coupling the fraction of wrong-handed L's produced is $\rho/(2+\rho)$. As ρ approaches unity, one in three L's is wrong handed, and the angular distribution, shown in Fig. 1, shifts toward the linear dependence $1 \mp \cos\theta$. The fraction of wrong-helicity L's as a function of m is shown in Fig. 2. Also shown in Fig. 2 is the partial width, obtained by integrating Eq. (3.2):

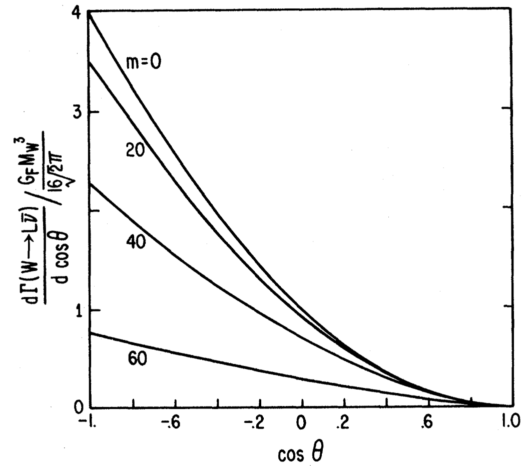

FIG. 1. Single-differential width for W^- decay to $V-A$ L^- as a function of the angle between the W spin and the lepton momentum, for lepton masses of 0, 20, 40, and 60 GeV. For a $V+A$ L^-, or for $W^+\to L^+$, we must reverse the sign of $\cos\theta$.

$$\Gamma(W\to L\nu)=\frac{G_F M_W{}^3 3^c}{6\sqrt{2}\pi}\left[\frac{v^2+a^2}{2}\right](1-\rho)^2(1+\tfrac{1}{2}\rho)\ . \tag{3.3}$$

The weak decay of a heavy fermion is more complicated. Defining $g_{L/R}=\frac{1}{2}(v\mp a)$, we find for arbitrary V and A coupling[7]

$$\frac{d\Gamma}{d\cos\theta\, dy}(L\to l\bar{\nu}_l\nu_L)$$
$$=\frac{3^c G_F{}^2 m^5}{2\times 192\pi^3}[g_L{}^2(\tilde{A}+\tilde{B}\cos\theta)+g_R{}^2\tilde{C}(1-\cos\theta)] \tag{3.4}$$

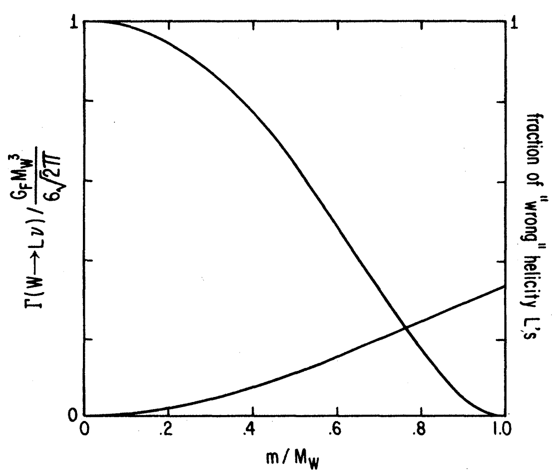

FIG. 2. Partial width for W decay into heavy lepton and neutrino vs lepton mass (as a fraction of W mass). Also, fraction of L's produced with right-handed helicity from a $V-A$ production vertex.

with

$$\tilde{A}(\rho,y)=\frac{12}{\rho^4}[y^2-y(2+\rho)+(2y-\rho-2)\ln(1-y)]\ ,$$
$$\tilde{B}(\rho,y)=\frac{12}{\rho^4}\left[y^2-y(2+\rho)+4\rho+\frac{1}{y}[2y^2-(2+3\rho)y+4\rho]\ln(1-y)\right]\ ,$$
$$\tilde{C}(\rho,y)=\frac{12}{\rho^4}(\rho-y)y^2/(1-y)\ .$$

Final-state masses are ignored, θ is the angle between the L spin and the l momentum, and

$$y\equiv 2mE_l/M_W{}^2\ .$$

This time the color weighting is $c=1$ (0) if l is a quark (lepton). For an incompletely polarized L, $\cos\theta$ is replaced by $\hat{p}\cdot\vec{w}$, where $\vec{w}$ is the polarization vector of the L.

Equation (3.4) describes the angle and energy distribution of a fermion resulting from weak decay of a fermion, e.g., $L^-\to\nu_L l^-\bar{\nu}_l$ with l^- observed. If an antifermion is observed, e.g., l^+ in $L^+\to\bar{\nu}_L l^+\nu_l$, we take $\cos\theta\to-\cos\theta$ as dictated by the CP invariance of the weak interaction. If the fermion number of L and the observed particle differ, as in $t\to bl^+\nu_l$ or $\bar{t}\to\bar{b}l^-\bar{\nu}_l$ with $l^\pm$ observed, we must interchange g_L and g_R as well.

$\tilde{A}$ and $\tilde{B}$ have the Taylor-series expansions

$$\tilde{A}=\frac{12}{\rho^4}\sum_{n=2}^{\infty}[\rho-2y/(n+1)]y^n/n\ ,$$
$$\tilde{B}=\frac{-12}{\rho^4}\sum_{n=2}^{\infty}[2y+(n-3)\rho]y^n/n(n+1)\ . \tag{3.5}$$

Integration over the electron energy gives

$$A\equiv\int_0^\rho dy\,\tilde{A}=\frac{12}{\rho^4}[\rho-\tfrac{1}{2}\rho^2-\tfrac{1}{6}\rho^3+(1-\rho)\ln(1-\rho)]\ ,$$
$$B\equiv\int_0^\rho dy\,\tilde{B}=\frac{12}{\rho^4}[\rho+\tfrac{11}{2}\rho^2-\tfrac{1}{6}\rho^3+(1+2\rho)(1-\rho)\ln(1-\rho)-4\rho\mathrm{Li}_2(\rho)]\ ,\tag{3.6}$$
$$C\equiv\int_0^\rho dy\,\tilde{C}=A\ .$$

The function

$$\mathrm{Li}_2(\rho)=-\int_0^\rho\frac{dx}{x}\ln(1-x)$$

is the dilogarithm, or Spence function. A and B have the Taylor-series expansions

$$A=12\sum_{m=0}^{\infty}\frac{\rho^m}{(m+3)(m+4)}\ ,$$
$$B=-12\sum_{m=0}^{\infty}\frac{\rho^m(m^2+5m+2)}{(m+4)(m+3)^2(m+2)}\ . \tag{3.7}$$

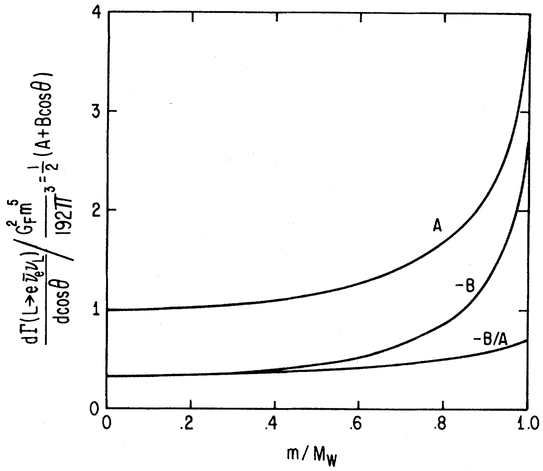

FIG. 3. Parameters A, B, and B/A vs heavy-lepton mass. These parameters determine the angular distribution of electrons produced from the decay of a polarized L.

As ρ approaches zero, one finds the well known low-mass limits

$$(A,B)\underset{\rho\to 0}{\longrightarrow}(1,-\tfrac{1}{3}) .$$

Interestingly, the $\rho\to 1$ limit is nonvanishing,

$$(A,B)\underset{\rho\to 1}{\longrightarrow}4(1,19-2\pi^2) .$$

The front-back asymmetry of a $V-A$ lepton, B/A, as well as A and B, are shown in Fig. 3. For a $V+A$ fermion, the asymmetry has its maximal value of -1 for all values of the fermion mass; angular momentum conservation forbids electron emission in the direction of the L spin. The partial width for $L\to l\bar{\nu}_l\nu_L$ is

$$\Gamma(L\to l\bar{\nu}_l\nu_L)=\frac{G_F{}^2m^5 3^c}{192\pi^3}(g_L{}^2+g_R{}^2)A . \tag{3.8}$$

Notice that a $V+A$ fermion has the same width as a $V-A$ fermion. Notice also the effect of including the W propagator in the heavy-fermion decay. As ρ approaches unity, the L width exceeds the naive m^5 extrapolation by a factor of 4 (Fig. 3).

The complete process, real or virtual $W\to\nu_L(L\to l\bar{\nu}_l\nu_L)$, is described by the six W_i. For the arbitrary V,A mixture defined in Eq. (3.1), we find, using the techniques in Appendices A and B,

$$W_i=\frac{(G_F M_W{}^2/\sqrt{2})^3 3^c}{M_W{}^4\pi^6 m\Gamma_L}\frac{d^3p}{p_0}\frac{1}{u}\int_{um^2/Q^2}^{\min\{u,m^2/2\}}dr\,\mathscr{W}_i ,$$

with the integrands given by

$$\begin{aligned}
\mathscr{W}_1&=g_L{}^4[\alpha\Sigma_1+(r-u)\Sigma_2]+g_R{}^4[2r\alpha+m^2(r-u)]\Sigma_3+g_L{}^2g_R{}^2[-(r-u)\Sigma_4+\alpha\Sigma_5] ,\\
\mathscr{W}_2&=\frac{2}{u^2}[g_L{}^4(\beta\Sigma_1+\gamma\Sigma_2)+g_R{}^4(2r\beta+m^2\gamma)\Sigma_3+g_L{}^2g_R{}^2(-\gamma\Sigma_4+\beta\Sigma_5)] ,\\
\mathscr{W}_3&=\frac{1}{u}\{g_L{}^4[\gamma/u\Sigma_1+(r-u)\Sigma_2]+g_R{}^4[-2r\gamma/u-m^2(r-u)]\Sigma_3\\
&\qquad+g_L{}^2g_R{}^2[-\gamma/u\Sigma_5+(r-u)(\Sigma_2-m^2\Sigma_3)]\} ,\\
\mathscr{W}_4&=\frac{2r(u-r)}{u^2}(g_L{}^4\Sigma_1+g_R{}^4 2r\Sigma_3+g_L{}^2g_R{}^2\Sigma_5) ,\\
\mathscr{W}_5&=\frac{1}{u}\{g_L{}^4[\delta\Sigma_1+(r-u)\Sigma_2]+g_R{}^4[2r\delta+m^2(r-u)]\Sigma_3+g_L{}^2g_R{}^2[(u-r)\Sigma_4+\delta\Sigma_5]\} ,\\
\mathscr{W}_6&=0 ,
\end{aligned} \tag{3.9}$$

where c is the number of quarks (0,1, or 2) in $\{L,l\}$, and Γ_L is the total width of L. New definitions are

$$\alpha\equiv t-2rt/u+Q^2r^2/u^2 ,$$

$$\beta\equiv -t^2+\frac{Q^2}{2}(6tr/u-3Q^2r^2/u^2-m^2)$$

$$\gamma\equiv tu-Q^2r ,$$

$$\delta\equiv t+m^2-(Q^2+4t)r/u+3Q^2r^2/u^2 ,$$

where

$$r=k\cdot p ,$$

$$t=Q\cdot k=\tfrac{1}{2}(m^2+Q^2) ,$$

$$u=Q\cdot p ,$$

$$y=2r/M_W{}^2 ,$$

and

$$\Sigma_1=\frac{\pi M_W{}^4}{4y}(\rho-y)[2-y+2(1/y-1)\ln(1-y)]\ ,$$

$$\Sigma_2=\frac{\pi M_W{}^4\rho}{4y^2}\left\{y^2-(2+\rho)y+4\rho+\frac{1}{y}[2y^2-(2+3\rho)y+4\rho]\ln(1-y)\right\}$$

$$\Sigma_3=\frac{\pi M_W{}^2(\rho-y)}{4(1-y)}\ ,$$

$$\Sigma_4=\Sigma_2+m^2\Sigma_3\ ,$$

$$\Sigma_5=\Sigma_1-\frac{y}{\rho}\Sigma_2\ .$$

It may be seen in this equation that a $V+A$ fermion is obtained from a $V-A$ fermion by the substitution

$$W_3\to-W_3,\ \ \Sigma_1\to 2r\Sigma_3,\ \ \Sigma_2\to m^2\Sigma_3\ .$$

These formulas, as written, describe the chain $W\to L\to l$ when L and l are fermions (quarks or leptons). If the observed particle l is an antifermion, Eq. (3.9) still holds when the sign of W_3 is reversed. If, in addition, L and l have opposite fermion number, as may happen if L or l or both are quarks, an interchange of g_R and g_L is required. Examples of this latter case include $W^+\to\bar{b}(t\to b\nu_e e^+)$ and $W^-\to b(\bar{t}\to\bar{b}\bar{\nu}_e e^-)$ with $e^\pm$ observed.

Finally, the r integration may be performed to give analytic results for the invariant $W_i(Q^2,u)$. These are presented in Appendix B for the $V-A$ heavy fermion.

The cumbersome factors in Eq. (3.9) are exact for all values of the intermediate mass m. For small mass, m^2 and r are small compared to t, u, and Q^2, so an expansion of Eq. (3.9) in powers of ρ and y is appropriate. Subsequent to the expansion, y may be simply integrated. The following small-mass formulas result for $V-A$ theory:

$$\begin{aligned}&W_1/W_0=Q^2/2\ ,\\&W_2/W_0=-2/x^2\ ,\\&W_3/W_0=W_5/W_0=1/x\ ,\\&W_4=W_6=0\end{aligned}\qquad(3.10)$$

with

$$W_0=\frac{G_F{}^3m^53^c}{144\sqrt{2}\pi^5\Gamma_L}\frac{M_W{}^2}{Q^2}\frac{(1-x^3)}{x}d^3p/p_0$$

and

$$x=2u/Q^2\ .$$

In the Q rest frame, $x=p_0/(p_0)_{\max}$. This approximate result can also be derived by convoluting the squared amplitude for W decay to L with the squared helicity amplitude for left-handed L decay. Equation (3.10) is valid to lowest order in $m^2/M_W{}^2$, and can be used to quantitatively describe the $W\to(\tau,c,b)\to l$ chains. Equation (3.10) characterizes the $W\to$fermion$\to$fermion sequence. If the intermediate particle is an antifermion, we reverse the sign of W_3. If the two particles in the decay chain have opposite fermion number, we replace the factor $(1-x^3)$ with $3(1-x)^3$. Approximate formulas for the τ chain have been presented previously in Refs. 8 and 9.

The analytic expressions presented in Eq. (3.9) for the W_i are the main contribution of this paper. These W_i are an essential input into any calculation of the l spectrum resulting from $W\to\bar{\nu}_L L\to\bar{\nu}_L\nu_L l\bar{\nu}_l$, regardless of the W's source. To illustrate the physics content of the invariant W_i we now proceed to analyze the momentum spectrum of l in the rest frame of a polarized W. For some processes, e.g., $e^+e^-\to W^+W^-$, the W rest frame is obtainable event by event, and the W polarization is completely calculable. For other processes, e.g., $p\bar{p}\to WX$, the W rest frame is in general not obtainable, as final-state momentum is lost down the beam axis, and the W polarization, determined in part by uncalculable initial-state strong-interaction corrections, must be modeled. The final-particle spectrum for $p\bar{p}\to W\to L\to l$ in the laboratory, showing the model-dependent smearing effects of an incomplete W polarization and integration over the W laboratory momentum, will be discussed in a separate paper.[10]

IV. THE SINGLE-ELECTRON SIGNATURE FOR A HEAVY-LEPTON CASCADE

This section presents the electron signature, in the W rest frame, for the decay chain $W\to L\to e$ having a heavy-lepton intermediate. The direct decay $W\to e\nu$, is not a background since the direct electron energy is kinematically fixed to be $x=1$, where $x\equiv 2p_0/\sqrt{Q^2}$ is the scaled electron energy. In contrast, the decay-chain electron is part of a four-body final state and has a scaled energy lying in the interval [0,1]. For the remainder of this section we make the simplifying assumption that the W is on-shell, $Q^2=M_W{}^2$.

If the spin correlations among the W, L, and e are neglected, a simple phase-space model for the cascade decay can be developed. The resulting electron-energy spectrum, derived in Appendix A, is

$$\frac{d\Gamma_{ps}}{dx}\propto B_L[x(1-\rho)/\rho\Theta(\rho-x)+(1-x)\Theta(x-\rho)]\ ,\qquad(4.1)$$

where B_L is the branching ratio for $L\to e\bar{\nu}_e\nu_L$. Since the branching ratio to a particular final state is less for the heavier of two given leptons, Eq. (4.1) embodies a *theorem:* neglecting spin correlations, the contribution to the electron energy spectrum from the production and decay of a lighter lepton will *exceed* the contribution from a heavier lepton *at all values of electron energy.* The relevant processes for this paper are the $W\to\tau\to e$ background to an electron signature originating from a heavier lepton L. The τ may decay to ν_τ plus $(e\nu_e)$, (μ,ν_μ), $(ud)\times 3$ colors, so one expects $B_\tau\simeq\frac{1}{5}$. A heavier lepton may also decay to (τ,ν_τ) and $(c,s)\times 3$ colors, yielding $B_L\simeq\frac{1}{9}$, or $B_L\simeq\frac{1}{12}$ if (tb) is kinematically allowed. Experimentally $B_\tau=17\%$, but for internal consistency we show all our results assuming $B=\frac{1}{5}$, $B_L=\frac{1}{9}$. A correct rescaling of the curves in Figs. 4–9 is obtained by multi-

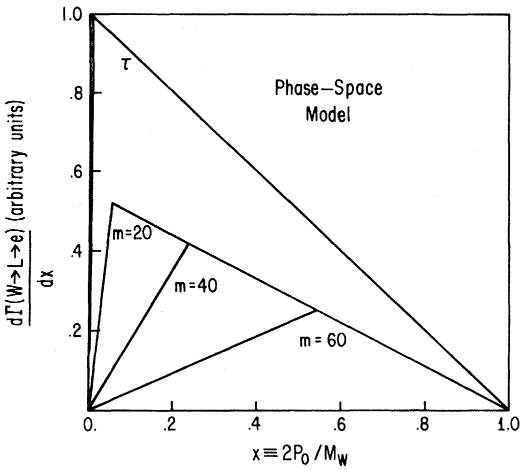

FIG. 4. Energy spectrum of electrons produced by the cascade decay of W through a heavy lepton (mass = 1.78, 20, 40, and 60 GeV) according to the phase-space model.

plying by $5B_\tau$ and $9B_L$ for the τ and heavy lepton, respectively. The electron spectrum from the τ chain is compared with the spectrum from the heavier-lepton cascade in Fig. 4. It is clear that ignoring spin effects, the electron signature for a heavy lepton is at best an enhancement factor of $B(L\rightarrow e\nu_e\nu_L)/B(\tau\rightarrow e\nu_e\nu_\tau)\sim 55\%$ in the τ chain spectrum at $x>\rho$.

One may hope to improve the electron signature by properly including the particle spins. For definiteness, we fix the W mass to be 80 GeV and let the W spin point along the z axis, i.e., $\epsilon^\mu=\epsilon_+^\mu=1/\sqrt{2}\,(0;1,i,0)$ is the polarization vector. Then, from Eq. (2.3),

$$d\Gamma(W^\mp\rightarrow L^\mp\rightarrow e^\mp)=\frac{1}{2M_W}\epsilon_+^\mu\epsilon_+^{\nu*}W_{\mu\nu}$$
$$=\frac{1}{2M_W}[W_1+\tfrac{1}{2}p_T{}^2W_2\mp M_Wp_zW_3] \tag{4.2}$$

in the W rest frame. The W_i are given in complete generality in Eq. (3.9), and in the small intermediate mass limit, applicable to the τ chain, in Eq. (3.11). Distributions in the single variables x and

$$x_T\equiv 2p_{Te}/M_W$$

are shown in Figs. 5 and 6 for $V\pm A$ lepton couplings. One sees that inclusion of spin correlations affects the spectra considerably. Yet still the τ chain dominates any heavier $V-A$ lepton at all values of x and x_T. However,

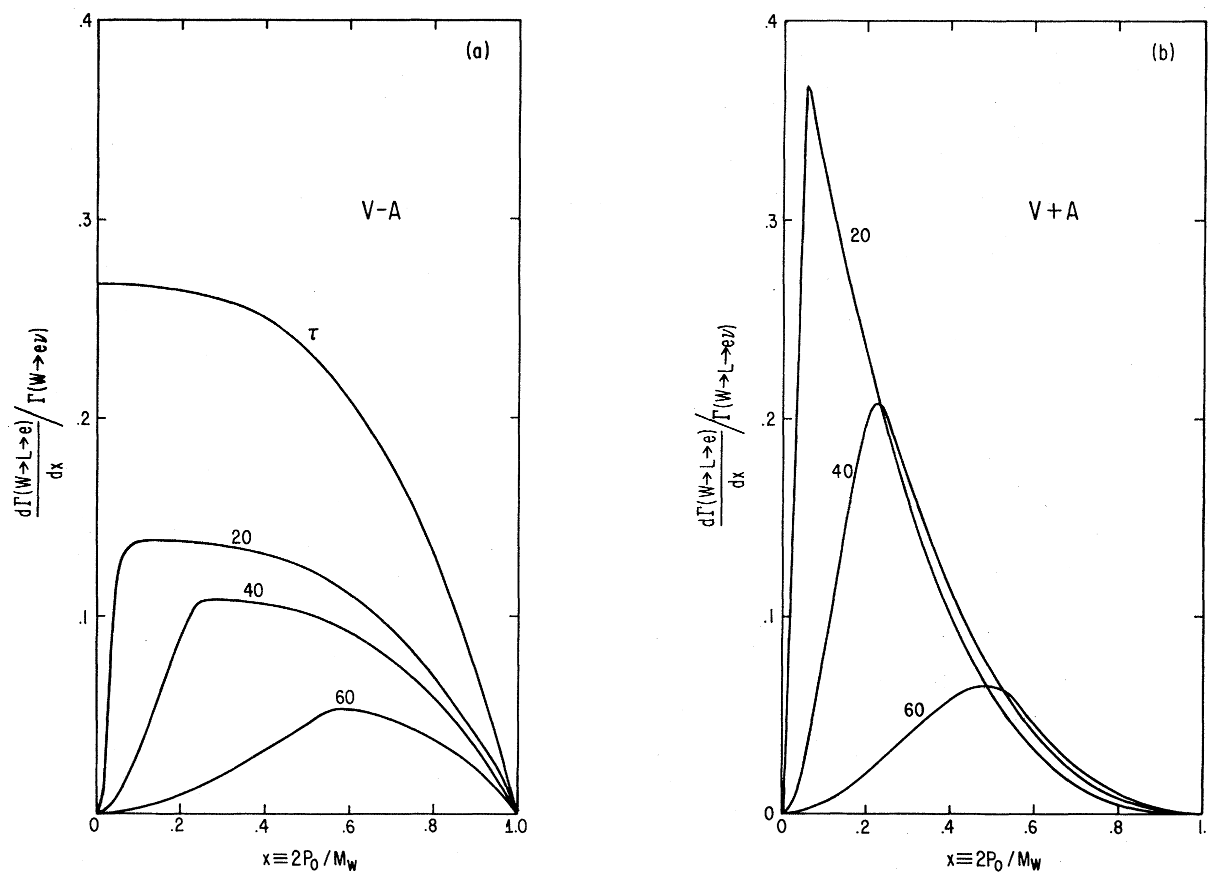

FIG. 5. (a) Energy spectrum of electrons produced by the decay of W through a $V-A$ heavy lepton for the same lepton masses as in Fig. 4. The spectrum is normalized relative to the direct $W\rightarrow e\nu$ decay width. (b) Same as (a) but for a $V+A$ heavy lepton.

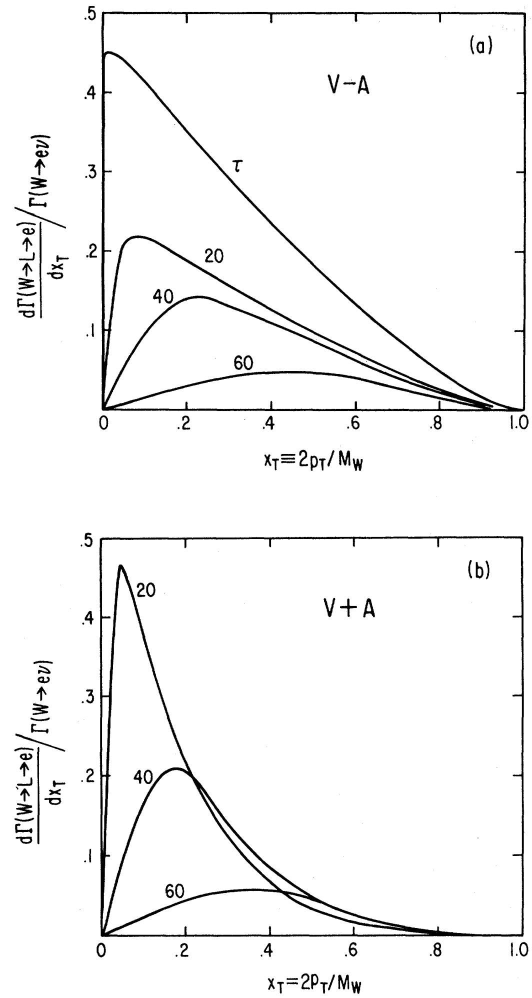

FIG. 6. (a) Electron distribution in x_T, the fraction of maximum momentum perpendicular to W spin, for $V-A$ L interaction. (b) Same as (a) but for $V+A$ L interaction.

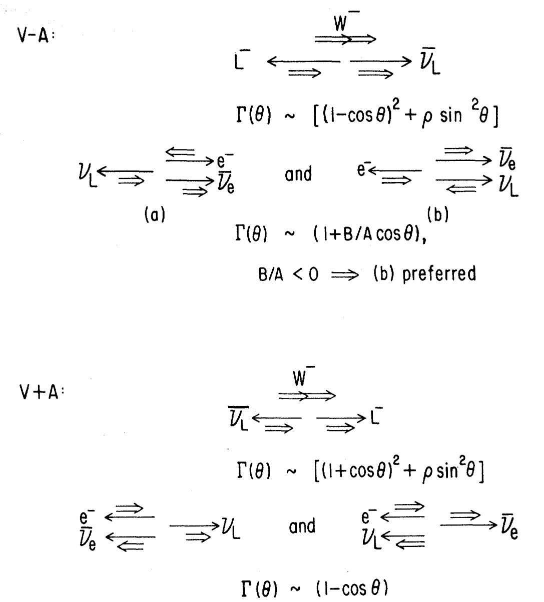

FIG. 7. Helicity diagram for W^- decay into L^- and $\bar{\nu}_L$ and subsequent three-body decay of L^-. θ denotes the angle between W spin and L momentum, and between L spin and electron momentum. The cases of $V-A$ and $V+A$ interactions for L^- are contrasted. For a W^+ decay chain, reverse the sign of $\cos\theta$.

a $V+A$ lepton of mass $\lesssim 40$ GeV contributes an amount comparable to the τ at low values ($\lesssim 0.2$) of x and x_T. The origin of the difference between the $V-A$ and $V+A$ spectra is qualitatively explained by the helicity diagrams of Fig. 7. In the heavy-lepton rest frame the favored direction of electron emission from a $V-A$ ($V+A$) L is in (against) the direction of the boost to the W rest frame. Moreover, for the decay of a $V-A$ L the emitted electron tends to emerge alone, balanced by $\bar{\nu}_e$ and ν_L. For the $V+A$ case, the electron and one of the neutrinos tend to balance the second neutrino. These two effects conspire to give a much harder electron spectrum from a $V-A$ intermediate as compared to a $V+A$ intermediate.

It is clear from Eq. (4.2) that the angular distribution is at most quadratic in $\cos\theta$. The $\cos\theta$ distribution is shown in Fig. 8. A marked difference between the τ^- chain and the heavier-lepton contributions is revealed. The τ-chain electron is very sharply peaked in the direction opposite the W^- spin. The angular distribution for electrons from heavier intermediates is considerably flatter, or in the case of a $\lesssim 30$ GeV $V+A$ intermediate, even peaked in the W spin direction. (For a positive W^+ decay chain, $\cos\theta \rightarrow -\cos\theta$ in accord with CP invariance, and the peaks reverse direction.) Referring again to Fig. 7, one easily understands this behavior. The electron's angular distribution results from a convolution of the angular factor from the $W^- \rightarrow L^- \nu_L$ decay with the angular factor from $L^- \rightarrow e^- \bar{\nu}_e \nu_L$ decay boosted to the W rest frame. The boost is parametrized by

$$\vec{\beta} = [(1-\rho)/(1+\rho)]\hat{k} \ .$$

For the $V-A$ case, $[(1-\cos\theta)^2 + \rho \sin^2\theta]$ is convoluted with a boosted $[1+(B/A)\cos\theta]$. B/A is negative, and furthermore the mean direction of the boost, $\langle \hat{k} \rangle$, is opposite to the W^- spin. For small m, all of these effects contribute to a distinct backward peaking. As m increases, the "wrong-helicity" $\sin^2\theta$ term becomes significant and the boost diminishes. Consequently, the backward peaking from a $V-A$ intermediate diminishes. As ρ approaches unity, the L rest frame approaches the W rest frame and the electron's angular distribution is given by

$$\lim_{\rho \rightarrow 1} \frac{d\Gamma}{d\cos\theta}(L^- \rightarrow e^- \bar{\nu}_e \nu_L) \ ,$$

i.e., $(1-0.74\cos\theta)$ (Ref. 11). For the $V+A$ case, $[(1+\cos\theta)^2 + \rho \sin^2\theta]$ is convoluted with a boosted $(1-\cos\theta)$. The two angular factors are in competition; which factor dominates depends on ρ through the boost. For $V+A$, the mean boost direction is along the W^-

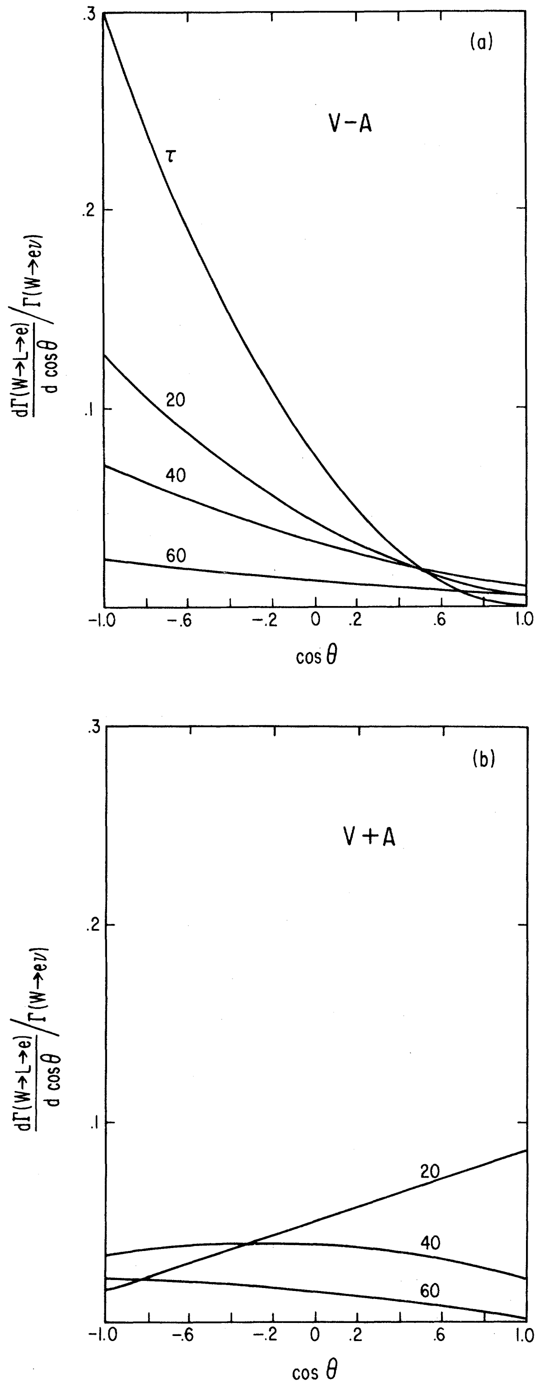

FIG. 8. (a) Angular distribution of electrons from cascade decay of W through τ and L for selected L masses. θ is the angle between the W spin and the electron momentum. (b) Same as (a) but for $V+A$ L interaction.

spin, thus enhancing a forward peaking. As ρ approaches unity, the boost vanishes and the angular distribution approaches $(1-\cos\theta)$.

The near vanishing of the τ-chain electron in the direction of the W^- spin suggests scrutiny of the forward hemisphere for a heavy-lepton signal. Figure 9 shows the electron energy spectrum for a fixed value of forward angle, $\theta=30°$. It is clear that a restriction to forward angles enhances the L-signal-to-τ-noise ratio. The price paid is a reduction in event rate. Figure 10 shows the ratio of electron events from the heavy-lepton chain to those from the τ chain, in the interval $0\leq\theta\leq 60°$, with p_0 cut away below 8 GeV to further reduce the τ contribution [cf. Fig. 5(a)]. The $V-A$ lepton arising from the decay of a completely polarized W contributes up to twice as much as the τ, while a 40-GeV $V+A$ lepton contributes five times as much as the τ. However, the extreme dominance of the τ chain in the backward direction, evident in Fig. 8(a), portends a loss of signal if the W polarization is less than complete. In Fig. 10 we also show the event ratio when the W vector polarization

$$\frac{N(\hat{s}=\hat{z})-N(\hat{s}=-\hat{z})}{N(\hat{s}=\hat{z})+N(\hat{s}=\hat{z})}$$

is 80%. The event ratio is considerably reduced. For a 50% polarization, the event ratio decreases monotonically as m increases, from 0.55 at $m=4$ GeV for $V-A$ and from 0.86 at $m=4$ GeV for $V+A$. It seems that unless the W is highly polarized, extraction of a heavy-lepton signal from the τ-chain background will be difficult. Our exact analytic formulas for the invariant W_i will help reduce uncertainty in the lepton search.

One may remove the τ background altogether by considering particular final states whose invariant mass exceeds the τ mass, e.g., $L\rightarrow\nu_L\bar{c}s$, or $L\rightarrow\nu_L\bar{t}b$ if $m_t+m_b<M_L$. Multiplying our results by 3 for color gives the strange-quark spectrum from $L\rightarrow\nu_L\bar{c}s$. To describe the b spectrum of the second reaction, our formulas need the modifications available in Appendix B, as the t mass may not be neglected. We do not consider these modes in this paper. The signature, jet plus missing energy, has its own background: any channel producing W's will also pair produce b quarks at a considerable rate. The semileptonic decay of the b then offers a significant background to this signature.[3]

We have also not considered the trickledown electrons from the chain $W\rightarrow L\rightarrow\tau\rightarrow e$. These increase the L contribution by a factor of $B(\tau\rightarrow e)$, and distort the spectrum we have calculated. We expect, however, that the contribution from this double cascade chain is mainly at soft electron energies which may be cut away.

V. SUMMARY

We have established a general covariant formalism for the single-particle inclusive decay of a spin-one boson. The formalism characterizes the decay with products of simple, channel-independent covariant tensors and channel-dependent invariant functions. We have applied the formalism to the cascade decay of a W boson through a new heavy fermion in order to provide a rigorous signature for the existence of any new fermion. An advantage of W-decay physics is that the nondiagonal nature of the charged current allows the possibility of W decay to a new fermion nearly as heavy as the W itself. We have chosen to concentrate on the single particle inclusive sig-

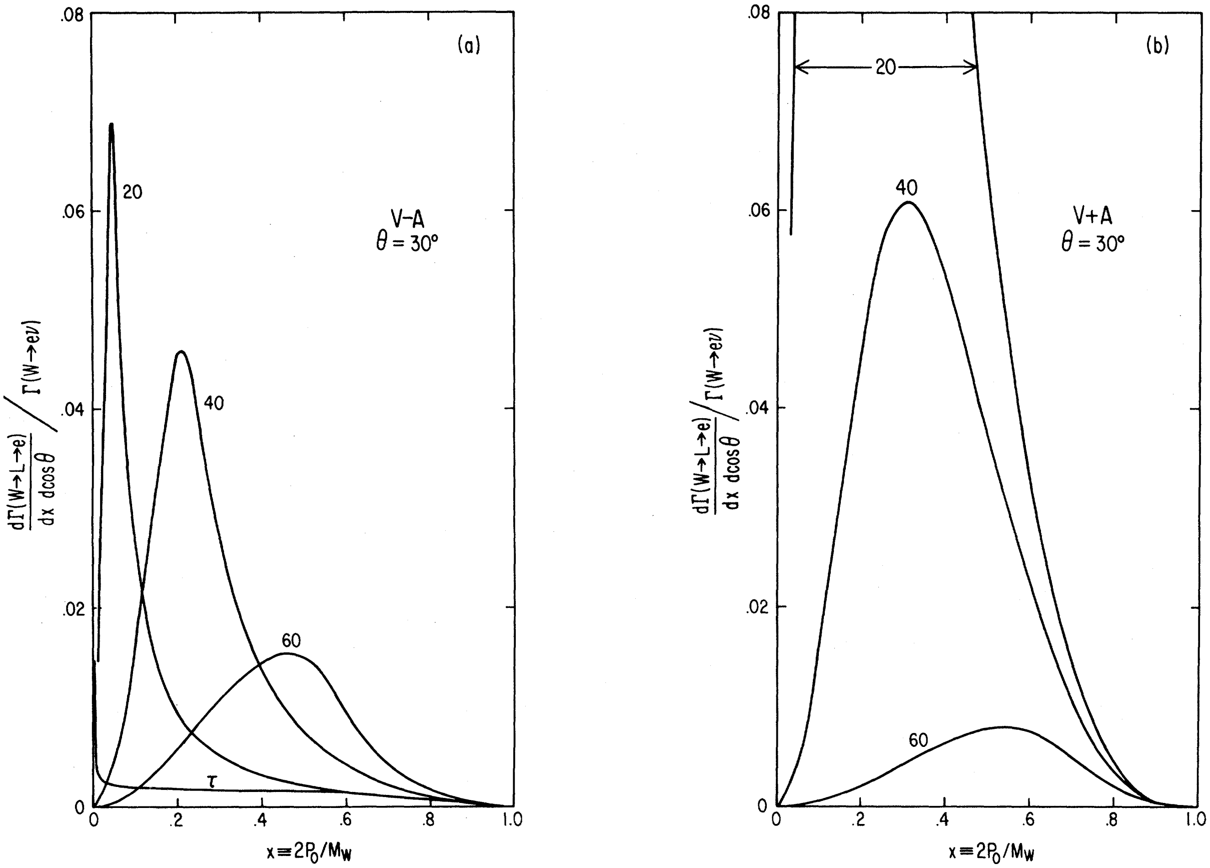

FIG. 9. (a) Fractional energy distribution of electrons at fixed angle of 30° with respect to W spin, for τ mass and selected L masses. (b) Same as (a) but for $V+A$ L interaction.

nature since electron plus missing energy is the signature already used for detection of the W. It will be a well studied channel.

Previously published formulas for weak production and decay of heavy fermions neglected the heavy-fermion polarization, or assumed a heavy-fermion mass small enough compared to the W mass that (i) its weak decay could be treated as a four-fermion interaction and (ii) its weak production was limited to a single helicity. This results in significant errors, especially when the particle mass is a significant fraction of the W mass, as will certainly be the case with a new quark or lepton. We present analytic formulas for W decay into a new heavy fermion, heavy-fermion decay into a light fermion, and, finally, light-fermion production from the complete cascade chain. Spin correlations, W-propagator effects, and the heavy-fermion mass are all properly included. Moreover, our calculations allow for an arbitrary mixture of V and A in the coupling of the W to the heavy-fermion current. The invariant functions we calculate are a necessary input into any quantitative evaluation of the process $W \to$heavy fermion$\to$light fermion, regardless of the W production mode. Our formulas are useful in the search for new quarks, new leptons, and the new fermions of supersymmetric models.

To display the physics of our formulas, we have analyzed the electron spectrum as a signature for polarized $W \to$new heavy lepton$\to$electron, in the W rest frame. From phase-space considerations alone it is shown that for all electron energies, the τ chain constitutes a formidable background. However, using our exact formulas we find that a $V+A$ lepton offers a clean signature, and that a $V-A$ lepton may still be found above the τ background if appropriate cuts are made. These results follow from the observation that, compared to the τ-chain electron, the electron from the new-lepton decay tends to be produced more in the direction of the W spin.

Of course, the present source of W bosons is the CERN $p\bar{p}$ collider. To analyze the $p\bar{p}$ reaction, one must deal with the model-dependent production mechanism of the bosons. In a subsequent paper we will do that.[10]

Note added. After completion of this paper we received a paper by V. Barger, H. Baer, A. D. Martin, E. W. N. Glover, and R. J. N. Phillips [Phys. Rev. D 29, 2020 (1984)] concerned with signals for heavy leptons in W decay. The emphasis of their work is on laboratory frame backgrounds for the $p\bar{p}$ reaction; our emphasis is on analytic formulas relevant to any reaction producing W's.

ACKNOWLEDGMENTS

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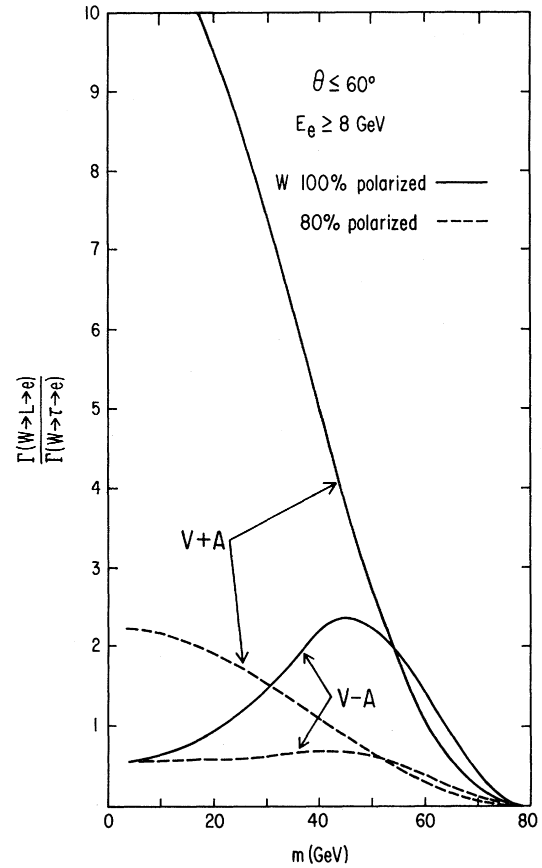

FIG. 10. Electrons from cascade through L vs cascade through τ as a function of L mass. Cuts are (i) electron momentum within 60° of W spin and (ii) electron energy above 8 GeV. Solid curves are for $V\pm A$ interaction with completely polarized W's. Dashed curves are for 80% W polarization.

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APPENDIX A: PHASE-SPACE MODEL OF THE DECAY CHAIN

The decay chain shown in Figs. 11 and 12 may be calculated without great difficulty if one takes advantage of covariant expansions. In this appendix we set up phase space in the most convenient way. The covariant expansions are presented in Appendix B.

The chain in Fig. 11 is a two-body decay, followed by a three-body decay. Although the intermediate-particle momentum k is determined to be $Q-l_3$, it is better to keep it as a variable. To this end we introduce

$$1=\int d^4k\,\delta^4(Q-k-l_3)$$

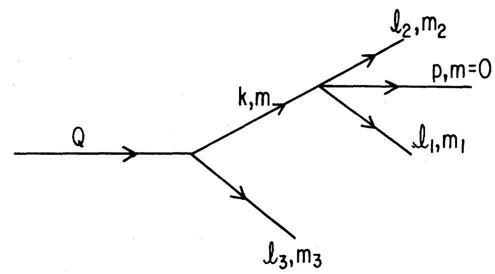

FIG. 11. Momenta of particles for the phase-space model of the cascade decay $W\to L+\nu_L\to l+\bar{\nu}_l+\bar{\nu}_L+\nu_L$.

into the usual Lorentz-invariant phase space:

$$dP_{\rm LI}=\frac{d^3l_1}{(2\pi)^3 2l_1^0}\frac{d^3l_2}{(2\pi)^3 2l_2^0}\frac{d^3l_3}{(2\pi)^3 2l_3^0}\frac{d^3p}{(2\pi)^3 2p^0}\times(2\pi)^4\delta^4(Q-l_1-l_2-l_3-p) \quad \text{(A1)}$$

and obtain a factorized phase space

$$dP_{\rm LI}=\left[\frac{d^3l_3}{(2\pi)^3 2l_3^0}\delta^4(Q-k-l_3)\right]\times\left[\frac{d^3l_1}{(2\pi)^3 2l_1^0}\frac{d^3l_2}{(2\pi)^3 2l_2^0}\delta^4(k-p-l_1-l_2)\right]\times(2\pi)^4 d^4k\frac{d^3p}{(2\pi)^3 2p^0}\ . \quad \text{(A2)}$$

The first set of square brackets is a factor in the two-body decay. The second is the part of the three-body decay corresponding to the unobserved particle momenta l_1,l_2 recoiling against momentum $k-p$.

Let us first consider the three-body decay. Since l_1 and l_2 are produced from the momentum $k-p$, $k-p$ must be timelike. The calculation is simplest in the l_1, l_2 rest frame. Letting $K=k-p$, the second set of square brackets becomes

$$\Theta(K_0-m_1-m_2)\Theta(K^2-(m_1+m_2)^2)\frac{1}{(2\pi)^6}\times\frac{\pi[\lambda(K^2,m_1{}^2,m_2{}^2]^{1/2}}{2K^2}\ , \quad \text{(A3)}$$

where $\lambda(a,b,c)=(a-b-c)^2-4bc$, is the triangle function, and m_1 and m_2 are the masses of the two unobserved particles.

Next the first set of square brackets is easily integrated, yielding $(2\pi)^{-3}\delta(Q^2+k^2-2Q\cdot k-m_3{}^2)$. So our phase space becomes

$$\frac{1}{(2\pi)^8}\frac{\pi}{2K^2}\sqrt{\lambda}\,d^4k\,\delta(Q^2+k^2-2Q\cdot k-m_3{}^2)\frac{d^3p}{2p_0}\Theta(k_0-p_0-m_1-m_2)\Theta(k^2+p^2-2k\cdot p-(m_1+m_2)^2)\ . \quad \text{(A4)}$$

The physical interpretation is clear. The first δ function puts the unobserved final-state particle from the initial two-body decay on shell. From this δ function one sees that at fixed k^2, the only integration is over the angles of k. The last Θ function restricts the energy of the observed particle, of momentum p. In the k rest frame, p is produced with two other particles, so it may come out at any angle, with any energy below its maximum,

$$[k^2-(m_1+m_2)^2]/(2\sqrt{k^2}) .$$

In our phase-space model, and our other calculations, we apply the narrow-width approximation (NWA) to the intermediate particle, replacing $g[(k^2-m^2)^2+(m\Gamma)^2]^{-1}$ with

$$\frac{\pi}{m\Gamma}\delta(k^2-m^2) ,$$

with Γ the total width of the intermediate particle. Once the intermediate particle is forced on shell, the magnitude of its momentum is fixed in the Q rest frame, and the final Θ function restricts its angular variables (if p is appropriately chosen so both Θ functions are nonvanishing). Since the final Θ function is written in a covariant way, it has a convenient interpretation in both the $\vec{k}$ rest frame and Q rest frame. It requires $\vec{k}$ to lie inside a paraboloid axisymmetric about $\vec{p}$,[12] given by the formula

$$k'_{\parallel}=\frac{p_0}{\sigma}k_{\perp}'^2+k_{\min} , \tag{A5}$$

with $\sigma=m^2-(m_1+m_2)^2$ and $k_{\min}=m^2p_0/\sigma-\sigma/4p_0$. The primes denote k coordinates with respect to the $\hat{p}$ axis.

The δ function constraint of Eq. (A4) is most simply realized in the Q rest frame, where it fixes $|\vec{k}|$ to be

$$|\vec{k}|=\frac{1}{2\sqrt{Q^2}}[\lambda(Q^2,m^2,m_3{}^2)]^{1/2} . \tag{A6}$$

Thus the integration region for d^3k/k^0 is that portion of the sphere of radius $|\vec{k}|$ contained within the paraboloid of Eq. (A5). For

$$p_0\le\frac{\sigma\sqrt{Q^2}}{Q^2+m^2-m_3{}^2+[\lambda(Q^2,m^2,m_3{}^2)]^{1/2}}$$

in the Q rest frame, the entire sphere is inside the paraboloid. It is convenient to change variables, viz.,

$$\frac{d^3k}{k_0}\delta(Q^2+m^2-m_3{}^2-2Q\cdot k)=\frac{\pi}{u}dr , \tag{A7}$$

where $r\equiv k\cdot p$, $u\equiv Q\cdot p$, and the azimuthal angle of $\vec{k}$ with respect to $\hat{p}$ has been trivially integrated out. The final Θ function of Eq. (A4) and the boundedness of $\cos\theta\in[-1,1]$ imply

$$r_-\le r\le\min\{r_+,\sigma/2\} ,$$
$$r_\pm=u(k^0\pm|\vec{k}|)/\sqrt{Q^2} \tag{A8}$$

with k^0, $|\vec{k}|$ fixed according to the invariant of Eq. (A6). Thus one has

$$(dP_{\rm LI})_{\rm NWA}=\frac{d^3p}{p_0}\frac{1}{2^{11}\pi^6m\Gamma}\frac{\pi}{u}\int dr\left[\frac{[\lambda(m^2-2r,m_1{}^2,m_2{}^2)]^{1/2}}{m^2-2r}\right] . \tag{A9}$$

In the limit $m_1=0=m_2=m_3$, the quantity in brackets becomes unity, r_+ (r_-) becomes u ($u\xi$), and one finds

$$(dP_{\rm LI})_{\rm NWA}=\frac{d^3p/p_0}{2^{11}\pi^5m\Gamma}[\min\{1,m^2/2u\}-\xi] , \tag{A10}$$

where $\xi\equiv m^2/Q^2$. Putting the intermediate particle on-shell reduces the range of u from

$$0\le u\le\tfrac{1}{2}[Q^2-(m_1+m_2+m_3)^2]$$

to

$$0\le u\le\{Q^2+m^2-m_3{}^2+[\lambda(Q^2,m^2,m_3{}^2)]^{1/2}\}\sigma/4m^2 . \tag{A11}$$

In the massless-final-state limit, a simple phase-space model of the differential cross section may now be constructed: the coupling of the W to the intermediate particle and its partner is pointlike, contributing a constant factor to the cross section. However, the decay of the intermediate particle contributes a squared amplitude equal to $2m\Gamma B$ divided by massless three-body phase space, where B is the branching ratio to the particular final state. Up to an overall constant, this factor is $B\Gamma/m$. Thus our phase-space-model cross section is

$$p_0\frac{d\sigma^{\rm PS}}{d^3p}=\text{constant}\times\frac{B}{\xi}[\min\{1,m^2/2u\}-\xi] . \tag{A12}$$

The electron-energy spectrum in the Q rest frame is obtained by integrating over the angle $\hat{p}\cdot\hat{Q}$. Defining the scaled energy $x\equiv 2p_0/\sqrt{Q^2}$, one finds

$$\frac{d\sigma^{\rm PS}}{dx}=\text{constant}\times B[x\frac{(1-\xi)}{\xi}\Theta(\xi-x)+(1-x)\Theta(x-\xi)] \tag{A13}$$

with $0\le x\le 1$. The energy spectrum is the union of two straight lines, with a peak at the intersection point $x=\xi$. From this formula it is clear that the single-particle inclusive cross section from production and decay of a lighter fermion will exceed the contribution of a heavier fermion *at all values of the single-particle energy,* unless the branching ratio of the heavier fermion or its coupling

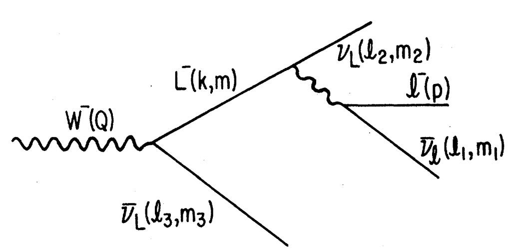

FIG. 12. Feynman diagram, with momentum and mass notation, for the process $W\rightarrow\bar{\nu}_L(L\rightarrow\nu_L l\bar{\nu}_l)$.

to the W exceeds that of the lighter.

The lighter decay chain will also dominate at all p_T values, since in this isotropic phase-space model the p_T distribution is just the projection of the energy distribution onto a plane. Defining $x_\perp \equiv 2p_T/\sqrt{Q^2}$, the explicit expression is

$$\frac{d\sigma^{\rm PS}}{dx_\perp{}^2}=\text{constant}\times B\left\{\Theta(\xi-x_\perp)\left[\frac{1}{\xi}\ln[\xi+(\xi^2-x_\perp{}^2)^{1/2}]-\ln[1+(1-x_\perp{}^2)^{1/2}]-\frac{(1-\xi)}{\xi}\ln x_\perp\right.\right.$$
$$\left.+\frac{1}{x_\perp}\arctan\left[\frac{x_\perp[(1-x_\perp{}^2)^{1/2}-(\xi^2-x_\perp^2)^{1/2}]}{x_\perp{}^2+[(1-x_\perp{}^2)(\xi^2-x_\perp{}^2)]^{1/2}}\right]\right]$$
$$\left.+\Theta(x_\perp-\xi)\left[\frac{1}{x_\perp}\arctan\left[\frac{(1-x_\perp{}^2)^{1/2}}{x_\perp}\right]-\ln\left[\frac{1+(1-x_\perp{}^2)^{1/2}}{x_\perp}\right]\right]\right\} \tag{A14}$$

with $0\le x_\perp \le 1$. This expression for the tranverse-momentum spectrum is valid in any frame connected to the Q rest frame by a boost along $\hat{z}$.

APPENDIX B: COVARIANT EXPANSIONS AND INVARIANT FUNCTIONS FOR THE DECAY OF THE INTERMEDIATE PARTICLE

To go beyond the phase-space model, one must calculate the amplitude corresponding to the Feynman diagram in Fig. 12. If one directly calculates the absolute squared amplitude, there will be two traces, one from the product of eight γ matrices, one from four. The usual expansion for the trace of 8 γ matrices has over 100 terms, so a direct attack is to be avoided. As in the phase-space model, consider first the three-body decay of the intermediate particle. It is convenient to Fierz transform the amplitude so that the final particles ν_L and $\bar{\nu}_l$ are part of the same spinor product. The relevant identity is

$$[\gamma^\sigma(v+a\gamma_5)]_{\alpha\beta}[\gamma_\sigma(1-\gamma_5)]_{\gamma\delta}=\tfrac{1}{2}(a-v)[\gamma^\sigma(1-\gamma_5)]_{\alpha\delta}[\gamma_\sigma(1-\gamma_5)]_{\gamma\beta}+(v+a)[1-\gamma_5]_{\alpha\delta}[1+\gamma_5]_{\gamma\beta}\,. \tag{B1}$$

Upon squaring the amplitude, the spinor product containing the ν_L and $\bar{\nu}_l$ spinors becomes a trace linear in l_1 and l_2. Accordingly, we define our first covariant expansion:[13]

$$I^{\alpha\beta}(k,p)=\int\frac{d^3l_1}{2l_1^0}\frac{d^3l_2}{2l_2^0}\frac{l_2^\alpha l_1^\beta}{[(l_1+p)^2-M_W{}^2]^2+(M_W\Gamma_W)^2}\delta^4(k-p-l_1-l_2)$$
$$\equiv C_1g^{\alpha\beta}+C_2p^\alpha p^\beta+C_3(p^\alpha k^\beta+p^\beta k^\alpha)+C_4(p^\alpha k^\beta-p^\beta k^\alpha)+C_5k^\alpha k^\beta\,. \tag{B2}$$

If final-state mass m_1 may not be neglected, the scalar mode of the W propagator does not decouple. We have not calculated the scalar-mode contribution to the cross section.

The invariant functions C_i are functions of k^2, p^2, and $p\cdot k$. They may be expanded in terms of the three invariant integrals

$$I_n=\int\frac{d^3l_1}{2l_1^0}\frac{d^3l_2}{2l_2^0}\frac{(l_1\cdot p)^n}{[(l_1+p)^2-M_W{}^2]^2+(M_W\Gamma_W)^2}\delta^4(k-p-l_1-l_2)\,,\quad n=0,1,2\,. \tag{B3}$$

W-propagator effects are included so as to correctly describe heavy-fermion decay. In the approximation $p^2=0$, setting $K=k-p$, we find

$$C_1=\tfrac{1}{2}\left[-m_1{}^2I_0+\frac{1}{r}(K^2+m_1{}^2-m_2{}^2)I_1-\frac{K^2}{r^2}I_2\right],$$
$$C_2=\frac{1}{r^2}\left\{-\tfrac{1}{4}[K^4+2K^2(2m_1{}^2-m_2{}^2)+2r(K^2+3m_1{}^2-m_2{}^2)+(m_1{}^2-m_2{}^2)^2]I_0\right.$$
$$\left.+\left[\frac{3K^2}{2r}(K^2+m_1{}^2-m_2{}^2)+3K^2+2(m_1{}^2-m_2{}^2)+r\right]I_1-\left[1+\frac{3K^2}{r}\left[1+\frac{K^2}{2r}\right]\right]I_2\right\},$$
$$C_3=\frac{1}{2r}\left[\tfrac{1}{2}(K^2+3m_1{}^2-m_2{}^2)I_0-\frac{1}{r}(3K^2+2m_1{}^2-2m_2{}^2+2r)I_1+\frac{1}{r^2}(3K^2+2r)I_2\right], \tag{B4}$$
$$C_4=\frac{1}{4r}\left[(m_2{}^2-m_1{}^2-K^2)I_0+\frac{2K^2}{r}I_1\right],$$
$$C_5=\frac{1}{r}\left[I_1-\frac{1}{r}I_2\right],$$

and

$$I_0=\frac{\pi}{4M_W{}^2r\epsilon}\arctan\left[\frac{\epsilon(u_+-u_-)}{u_+u_-+\epsilon^2}\right],$$

$$I_1=\tfrac{1}{2}(M_W{}^2-m_1{}^2)I_0-\frac{\pi}{16r}\ln\left[\frac{u_+{}^2+\epsilon^2}{u_-{}^2+\epsilon^2}\right],$$

$$I_2=\tfrac{1}{4}[(M_W{}^2-m_1{}^2)^2-M_W{}^4\epsilon^2]I_0-\frac{\pi}{16r}(M_W{}^2-m_1{}^2)\ln\left[\frac{u_+{}^2+\epsilon^2}{u_-{}^2+\epsilon^2}\right]+\pi\sqrt{\lambda}/8K^2\,,$$

where

$$\epsilon\equiv\Gamma_W/M_W\,,\quad u_\pm\equiv[K^2(M_W{}^2-m_1{}^2)-r(K^2+m_1{}^2-m_2{}^2\mp\sqrt{\lambda})]/K^2M_W{}^2\,,$$

$$\lambda\equiv\lambda(K^2,m_1{}^2,m_2{}^2)\,,\quad r=k\cdot p\,.$$

These exact formulas are applicable if heavy quark triggers are used as a signature for the intermediate particle's production and decay.

The massless-final-state approximation, used in the text, is given by the $m_1,m_2\to 0$ limit of Eq. (B4):

$$\begin{aligned}
C_1&=(1-m^2/2r)\left[-I_1+\frac{1}{r}I_2\right],\\
C_2&=\frac{m^2}{2r}(1-m^2/2r)I_0+\frac{1}{r}[1-(1-m^2/2r)3m^2/r]\left[I_1-\frac{1}{r}I_2\right],\\
C_3&=\frac{1}{4r}\left[(m^2-2r)I_0+\frac{2}{r^2}(4r-3m^2)(rI_1-I_2)\right],\\
C_4&=\frac{m^2-2r}{4r^2}(2I_1-rI_0)\,,\\
C_5&=\frac{1}{r^2}(rI_1-I_2)
\end{aligned}\tag{B5}$$

with

$$I_0=\frac{\pi}{2M_W{}^4y\epsilon}\arctan\left[\frac{\epsilon y}{1-y+\epsilon^2}\right]\underset{\epsilon\to 0}{\longrightarrow}\frac{\pi}{2M_W{}^4(1-y)}\,,$$

$$I_1=\frac{1}{2M_W{}^2}\left[I_0M_W{}^4+\frac{\pi}{4y}\ln\left[\frac{(1-y)^2+\epsilon^2}{1+\epsilon^2}\right]\right]\underset{\epsilon\to 0}{\longrightarrow}\frac{\pi}{4M_W{}^2(1-y)}\left[1+\left[\frac{1}{y}-1\right]\ln(1-y)\right],$$

$$I_2=\frac{\pi}{8}\left[1+\frac{1}{y}\ln\left[\frac{(1-y)^2+\epsilon^2}{1+\epsilon^2}\right]\right]+\frac{M_W{}^4}{4}(1-\epsilon^2)I_0\underset{\epsilon\to 0}{\longrightarrow}\frac{\pi}{4}\left[\frac{1-y/2}{1-y}+\frac{1}{y}\ln(1-y)\right],$$

where $y=2r/M_W{}^2$.

With l_1 and l_2 integrated out of the covariant expansion in Eq. (B2), the contraction of this expansion with the rest of the squared matrix element yields simple results. Use of $\not{p}\not{p}=0$ and $\not{k}\not{k}=m^2$ reduces the trace of eight γ matrices to traces of just four. After a trivial integration over the unobserved particle from the initial two-body decay of the W, we are left with an expression in terms of Q, p, and k. As above, we make use of covariant expansions and invariant integrals to proceed. The four-vector k^μ appears either linearly or quadratically in the integrand, multiplied by invariant functions. After integration over d^3k/k_0, k^μ must be replaced by the only available four-vectors, Q^μ and p^μ, times invariant functions. Similarly, the second-rank tensor $k^\mu k^\nu$ has a covariant expansion. Explicitly, for $V-A$ coupling, and massless final state,

$$W_{\mu\nu}=\frac{G_F{}^3M_W{}^2}{2\sqrt{2}\pi^7m\Gamma}\frac{d^3p}{2p_0}\int_{V_k}d^4k[\mathrm{Tr}(1+\gamma^5)\gamma^\mu(\not{Q}-\not{k})\gamma^\nu\not{k}\Sigma_1-\mathrm{Tr}(1+\gamma^5)\gamma^\mu(\not{Q}-\not{k})\gamma^\nu\not{p}\Sigma_2]\tag{B6}$$

with the region of support V_k given by

$$\Theta(Q_0-k_0)\delta(k^2-m^2)\Theta(m^2-2p\cdot k)\Theta(k_0-p_0)\delta(Q^2-2Q\cdot k+m^2)\,.$$

The Σ_i are combinations of invariants, and are given explicitly in the text, Eq. (3.9). Define

$$\Gamma^i=\int_{V_k} d^4k\, \Sigma_1(p\cdot k)^i \tag{B7}$$

for $i=0,1,2$. We find the covariant replacements

$$\int_{V_k} d^4k\, \Sigma_1 k_\mu \to p_\mu \left[\frac{t}{u}\Gamma^0-\frac{Q^2}{u^2}\Gamma^1\right]+Q_\mu\frac{\Gamma^1}{u} ,$$

$$\int_{V_k} d^4k\, \Sigma_1 k_\mu k_\nu \to g_{\mu\nu}\left[\frac{m^2}{2}\Gamma^0-\frac{t}{u}\Gamma^1+\frac{1}{2}\frac{Q^2}{u^2}\Gamma^2\right]+Q_\mu Q_\nu\frac{\Gamma^2}{u^2}+p_\mu p_\nu\left[\frac{t^2+\frac{1}{2}m^2Q^2}{u^2}\Gamma^0-\frac{3Q^2t}{u^3}\Gamma^1+\frac{3}{2}\frac{Q^4}{u^4}\Gamma^2\right]$$
$$+(p_\mu Q_\nu+p_\nu Q_\mu)\left[\frac{-m^2}{2u}\Gamma^0+\frac{2t}{u^2}\Gamma^1-\frac{3}{2}\frac{Q^2}{u^3}\Gamma^2\right] \tag{B8}$$

with $u\equiv Q\cdot p$, $t\equiv Q\cdot k$, as in the text. Similarly, defining Λ^i with Σ_2 replacing Σ_1 in Eq. (B7), we arrive at the expansion for W_i

$$W_i=\frac{G_F{}^3M_W{}^2\sqrt{2}}{\pi^7 m\Gamma}\frac{d^3p}{2p_0}\widetilde{W}_i ,$$

with

$$\begin{aligned}
\widetilde{W}_1&=-u\Lambda^0+\Lambda^1+t\Gamma^0-\frac{2t}{u}\Gamma^1+\frac{Q^2}{u^2}\Gamma^2 ,\\
\widetilde{W}_2&=\frac{2t}{u}\Lambda^0-\frac{2Q^2}{u^2}\Lambda^1-\frac{Q^2m^2+2t^2}{u^2}\Gamma^0+\frac{6Q^2t}{u^3}\Gamma^1-\frac{3Q^4}{u^4}\Gamma^2 ,\\
\widetilde{W}_3&=-\Lambda^0+\frac{1}{u}\Lambda^1+\frac{t}{u}\Gamma^0-\frac{Q^2}{u^2}\Gamma^1 ,\\
\widetilde{W}_4&=\frac{2}{u}\Gamma^1-\frac{2}{u^2}\Gamma^2 ,\\
\widetilde{W}_5&=-\Lambda^0+\frac{1}{u}\Lambda^1+\frac{m^2+t}{u}\Gamma^0-\frac{4t+Q^2}{u^2}\Gamma^1+\frac{3Q^2}{u^3}\Gamma^2 ,\\
\widetilde{W}_6&=0 .
\end{aligned} \tag{B9}$$

Λ^i, Γ^i are defined as four-dimensional integrals over k. The narrow-width approximation applied to the intermediate fermion and the mass-shell condition for the first emitted neutrino fix the magnitude of $\vec{k}$. The function $\Theta(m^2-2p\cdot k)$ restricts the angle between $\vec{k}$ and $\vec{p}$, while the azimuthal integration is trivial. Changing variables from $\hat{k}\cdot\hat{p}$ to $y=2k\cdot p/M_W{}^2$,

$$\Gamma^i=\frac{\pi}{4}\frac{M_W{}^2}{u}\left[\frac{M_W{}^2}{2}\right]^i\int_a^b dy\, \Sigma_1(y)y^i \tag{B10}$$

with $a=2u\rho/Q^2$, $b=2u/M_W{}^2$ for $0\le u\le\frac{1}{2}m^2$, and $b=\rho$ for $\frac{1}{2}m^2\le u\le\frac{1}{2}Q^2$. A similar expression holds for Λ^i. The indefinite integrals are

$$\Gamma^0=\frac{\pi^2M_W{}^6}{16u}[-(4+\rho)y+y^2/2-2(1-y)(1+\rho/y)\ln(1-y)+2(1+\rho)\mathrm{Li}_2(y)]\,\Big|_a^b ,$$

$$\Gamma^1=\frac{\pi^2M_W{}^8}{32u}\left[(4\rho+1)y-\frac{3+\rho}{2}y^2+y^3/3+(1-y)(1-y+2\rho)\ln(1-y)-2\rho\mathrm{Li}_2(y)\right]\Bigg|_a^b ,$$

$$\Gamma^2=\frac{\pi^2M_W{}^{10}}{64u}\{(\tfrac{1}{3}-\rho)y+\tfrac{1}{6}(1+9\rho)y^2-(\tfrac{8}{9}+\rho/3)y^3+y^4/4+(1-y)^2[\tfrac{1}{3}(1+2y)-\rho]\ln(1-y)\}\,\Big|_a^b , \tag{B11}$$

$$\Lambda^0=\frac{\pi^2 M_W{}^6\rho}{16u}\left[\frac{-2\rho}{y}+y+\frac{(1-y)}{y}(2+\rho-2\rho/y)\ln(1-y)-2\mathrm{Li}_2(y)\right]\Bigg|_a^b ,$$

$$\Lambda^1=\frac{\pi^2 M_W{}^8\rho}{32u}[-(4+\rho)y+y^2/2-2(1-y)(1+2\rho/y)\ln(1-y)+(2+3\rho)\mathrm{Li}_2(y)]\,|_a^b .$$

Equations (B9) and (B11) comprise the exact analytic formulas for the invariant functions describing the chain $W\rightarrow V-A$ fermion$\rightarrow$ massless final state (see Fig. 12). Analytic results for an arbitrary V and A mixture can be similarly generated.

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[11]Figure 3(a) of Ref. 2 shows the muon spectrum from a 40-GeV $V-A$ intermediate lepton peaking in a direction opposite to the muons from the τ intermediate or from the direct $W\rightarrow\mu\nu_\mu$ decay. We hope we have convinced the reader that this cannot be. We believe the background histogram should be reflected about $\cos\theta=0$ for W^- decay, or that the heavy lepton curve should be reflected for W^+ decay. Then we are in qualitative agreement with the Ref. 2 result.

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