Chiral Lagrangian for proton decay

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We extend the recent chiral model of Claudson, Wise, and Hall to include vector and axial-vector mesons as gauge bosons of an $SU(3)_L \times SU(3)_R$ chiral symmetry. The resulting baryon-number-violating interaction Lagrangian contains an additional free parameter and modifies significantly the two-body branching ratios of protons. Without some experimental input, it is not possible to predict definite branching ratios even in the minimal SU(5) model of grand unification. Depending on the values of the effective parameters of the chiral Lagrangian, several interesting possibilities for the branching ratios arise, and they are discussed in some detail. We provide, within the framework of the model, general formulas for two-body decays of the proton for an arbitrary grand unified theory including both Higgs- and gauge-boson exchanges. We also discuss how the effective parameters in our chiral model can be related to quark and bag models, and thereby show how various models can be tested for their consistency with the chiral aspects of strong interactions.

I. INTRODUCTION

One of the most striking consequences of unifying strong and electroweak interactions is the instability of the proton[1] through baryon- and lepton-number-violating interactions. Over the past few years, a great deal of theoretical and experimental effort has been devoted to the study of this phenomenon. Recently a lower bound of 2×10^{21} yr for the proton lifetime into $\pi^0 e^+$ channel has been quoted by the Irvine-Michigan-Brookhaven (IMB) collaboration experiment.[2] This bound seems to be in conflict with the theoretical expectations based on the minimal SU(5) theory.[3] In this paper, we discuss some of the basic assumptions underlying the theoretical estimates of the proton lifetime and its branching ratios into various two-body decay modes, in order to have a better understanding of the experimental situation.

At the present time, there is no unique grand unification scheme. There are several possible schemes. The properties of baryon-number- and lepton-number-violating interactions, such as their strengths and selection rules, obviously depend upon the choice of the model. In a given model, the extrapolation of these interactions from the grand unification mass of $10^{14}-10^{15}$ GeV down to 1 GeV depends crucially upon the use of renormalization-group equations[4] and the symmetry-breaking patterns. There are all the nagging questions concerning the choice of Higgs representations, the form of the Higgs potential, the relative dominance of Higgs-boson exchanges over the relevant gauge-boson exchanges, and so on. The conclusion in a nutshell is that there are too many ambiguities unless one drastically restricts the models. This is the main reason why a great deal of attention has been focused upon the minimal SU(5) scheme which has a natural one-stage symmetry breaking from $SU(5)\rightarrow SU(3)\times SU(2)\times U(1)$ and consequently one high mass scale M_X. Even so the determination of M_X is by no means unambiguous. At present, its determination is dependent upon the QCD parameter Λ in the renormalization-group equations. The current world-average $\Lambda_{\overline{MS}}=0.16^{+0.10}_{-0.08}$ GeV (Ref. 5) ($\overline{MS}$ denotes the modified minimal-subtraction scheme) leads to a value of $M_X=(1-3)\times10^{14}$ GeV and an uncertainty in the proton lifetime by a factor of 10^2 can easily arise. In the future, this situation may be improved by the precise determintaion of the mass M_W of the recently discovered W boson. The latter can then be used in the renormalization-group equations instead of Λ_{QCD}.[6]

Our paper is concerned mainly with another important source of ambiguity, namely, the evaluation of the low-energy hadronic matrix elements. For this purpose, several approximation schemes are available and they have all been utilized. Thus the nonrelativistic-quark-model[7] and bag-model[8] wave functions have been employed to calculate the overlap of quarks inside the nucleon. The matrix elements can also be estimated using current algebra and PCAC (partial conservation of axial-vector current).[9] The various approaches have between them a variation of approximately a factor 30 in the proton lifetime.

In view of this situation, it is both important and desirable to continue to study the various methods and their refinements to estimate the hadronic matrix elements and see how they affect the quantities of interest. In this paper, we use the method of phenomenological Lagrangians based on the chiral $SU(3)_L\times SU(3)_R$ symmetry, which has been used quite successfully in the past to describe low-energy hadronic physics. Recently, Claudson, Wise, and Hall[10] have constructed an effective Lagrangian for proton decay and used it to estimate the various two-body branching ratios into pseudoscalar mesonic channels. Their approach includes automatically the three-quark-fusion diagrams in the form of the baryon-pole diagrams.[11] Further, Isgur and Wise[12] have shown that the inclusion of such diagrams in quark-model calculations removes the discrepancy that existed between quark-model and chiral-model calculations. We generalize the chiral Lagrangian of Ref. 10 to include the vector and axial-vector mesons. In the past such generalizations have provided more accurate predictions of low-energy hadronic scattering parameters by providing specific extrapolations of current-algebra results to nonzero momenta.[13] In the context of proton decay, as there are some sizable vector-meson decay modes in models that use SU(6) or bag-model wave functions, we deem it important to consider such a generalization. As we shall show, the generalization has indeed important effects on the branching ratios into both the pseudoscalar mesons and vector mesons.

The organization of this paper is as follows. In Sec. II we give an outline of the chiral Lagrangian for the strong interactions of baryons and pseudoscalar, vector, and axial-vector mesons. Various aspects of the chiral-symmetry breaking will be displayed in some detail. In Sec. III, we present the baryon-number-changing effective Lagrangian as well as its selection rules.

In Sec. IV, we compute the two-body decay rates for the case of minimal SU(5). A comparison with other approaches to the problem of proton decay will be presented in Sec. V. Section VI will summarize our conclusions. In an appendix, we present the general expressions for the two-body decay rates which are applicable to any grand unified theory for which the $\Delta B=1$ interactions are described by an effective four-fermion Lagrangian.

II. CHIRAL LAGRANGIAN FOR STRONG INTERACTIONS

Phenomenological Lagrangians based on chiral symmetry have been used extensively in the past to represent the current-algebra results in the zero-momentum limit and they have provided an extrapolation of these results for nonzero momenta. The interested reader can find this subject discussed in a review by Gasiorowicz and Geffen.[14] In what follows, we present a brief description of the chiral Lagrangian we will be using for the strong vertices of our problem. The Lagrangian utilizes a nonlinear realization of the chiral group $SU(3)_L \times SU(3)_R$ within the general framework formulated by Coleman, Wess, and Zumino.[15] Claudson, Wise, and Hall[10] have used such a Lagrangian for the strong interactions of pseudoscalar-meson and baryon octets. We extend their model by including vector and axial-vector gauge fields of a local $SU(3)_L \times SU(3)_R$. For convenience, we have chosen our notation to coincide with theirs in the pseudoscalar meson, baryon sector.

The pseudoscalar-meson octet is introduced as the parametrization of the coset space

$$\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$$

and, as such, the pseudoscalar mesons are the pseudo-Goldstone bosons of the chiral symmetry. Consider the special unitary matrix $\underline{\xi}$,

$$\underline{\xi}=\exp(i\underline{M}/f_\pi)\ , \tag{2.1}$$

where f_π is the pion decay constant ($f_\pi=128$ MeV), and $\underline{M}$ is the familiar meson octet

$$\underline{M}=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} . \tag{2.2}$$

Given an element (L,R) of $SU(3)_L \times SU(3)_R$, $\underline{\xi}$ transforms as

$$\underline{\xi}\to\underline{\xi}'=\underline{L}\,\underline{\xi}\,\underline{U}^\dagger=\underline{U}\,\underline{\xi}\,\underline{R}^\dagger\ . \tag{2.3}$$

In the above transformation law, $\underline{U}$ is a unitary matrix defined by (2.3) and it depends nonlinearly on (L,R). However, the transformation becomes linear for the diagonal subgroup $SU(3)_V$ with $\underline{L}=\underline{R}=\underline{U}$. The transformation law for the baryon octet $\underline{B}$ is

$$\underline{B}\to\underline{B}'=\underline{U}\,\underline{B}\,\underline{U}^\dagger\ . \tag{2.4}$$

For the local group $SU(3)_L \times SU(3)_R$, we can introduce the vector and axial-vector gauge fields $\underline{v}_\mu$ and $\underline{a}_\mu$ with the usual transformation properties, namely,

$$\begin{aligned} \underline{v}_\mu+\underline{a}_\mu &\to(\underline{v}_\mu+\underline{a}_\mu)'=\underline{R}\left[\underline{v}_\mu+\underline{a}_\mu+\frac{i}{g}\partial_\mu\right]\underline{R}^\dagger\ , \\ \underline{v}_\mu-\underline{a}_\mu &\to(\underline{v}_\mu-\underline{a}_\mu)'=\underline{L}\left[\underline{v}_\mu-\underline{a}_\mu+\frac{i}{g}\partial_\mu\right]\underline{L}^\dagger\ , \end{aligned} \tag{2.5}$$

where $\sqrt{2g}$ is the $\rho\pi\pi$ coupling constant with

$$\frac{2g^2}{4\pi}=2.84\pm0.5\ . \tag{2.6}$$

In order to couple the vector-meson octets to baryons and pseudoscalar mesons, it is convenient to introduce the fields $\underline{P}_\mu$ and $\underline{V}_\mu$ which have simple transformation properties under the nonlinear realization of the group. They are given by

$$\begin{aligned} \underline{P}_\mu= {} & \tfrac{1}{2}\underline{\xi}[i\partial_\mu+g(\underline{v}_\mu+\underline{a}_\mu)]\underline{\xi}^\dagger \\ & -\tfrac{1}{2}\underline{\xi}^\dagger[i\partial_\mu+g(\underline{v}_\mu-\underline{a}_\mu)]\underline{\xi}\ , \end{aligned} \tag{2.7a}$$

$$\begin{aligned} \underline{V}_\mu= {} & \tfrac{1}{2}\underline{\xi}[i\partial_\mu+g(\underline{v}_\mu+\underline{a}_\mu)]\underline{\xi}^\dagger \\ & +\tfrac{1}{2}\underline{\xi}^\dagger[i\partial_\mu+g(\underline{v}_\mu-\underline{a}_\mu)]\underline{\xi}\ . \end{aligned} \tag{2.7b}$$

Using (2.3) and (2.5), we can easily prove

$$\underline{P}_\mu\to\underline{P}'_\mu=\underline{U}\,\underline{P}_\mu\underline{U}^\dagger\ , \tag{2.8a}$$

$$\underline{V}_\mu\to\underline{V}'_\mu=\underline{U}[\underline{V}_\mu+i\partial_\mu]\underline{U}^\dagger\ , \tag{2.8b}$$

and in view of the last equation, a chiral covariant derivative $D_\mu\underline{B}$ for the baryon octet $\underline{B}$ can be defined,

$$D_\mu\underline{B}=\partial_\mu\underline{B}-i[\underline{V}_\mu,\underline{B}]\ , \tag{2.9}$$

with the transformation property,

$$D_\mu\underline{B}\to(D_\mu\underline{B})'=\underline{U}(D_\mu\underline{B})\underline{U}^\dagger\ . \tag{2.10}$$

The field-strength tensors $\underline{F}^{(\pm)}_{\mu\nu}$ are defined in the usual way, namely,

$$\underline{F}^{(\pm)}_{\mu\nu}=\partial_\mu(\underline{v}_\nu+\underline{a}_\nu)-\partial_\nu(\underline{v}_\mu\pm\underline{a}_\mu)-ig[\underline{v}_\mu\pm\underline{a}_\mu,\underline{v}_\nu\pm\underline{a}_\nu]\ . \tag{2.11}$$

A minimal Lagrangian $\mathscr{L}_0$ involving baryons, pseudoscalar mesons, and vector and axial-vector gauge fields can now be written as

$$\mathscr{L}_0=\tfrac{1}{2}f_0{}^2\mathrm{Tr}(\underline{P}_\mu\underline{P}^\mu)+\mathrm{Tr}[\underline{\bar{B}}(i\gamma_\mu D^\mu-M_B)\underline{B}]+D_0\mathrm{Tr}(\underline{\bar{B}}\gamma^\mu\gamma_5\{P_\mu,\underline{B}\}_+)+F_0\mathrm{Tr}(\underline{\bar{B}}\gamma^\mu\gamma_5[\underline{P}_\mu,\underline{B}]) -\tfrac{1}{8}\mathrm{Tr}(\underline{F}^{(+)}_{\mu\nu}\underline{F}^{(+)\mu\nu}+F^{(-)}_{\mu\nu}\underline{F}^{(-)\mu\nu})+\tfrac{1}{2}m_0{}^2\mathrm{Tr}(\underline{v}_\mu\underline{v}^\mu+\underline{a}_\mu\underline{a}^\mu)\ . \tag{2.12}$$

The above Lagrangian is invariant under the local $SU(3)_L\times SU(3)_R$, except for the last term which is invariant only under the global group. We include this term, because otherwise the pseudoscalar mesons can be gauged away from $\mathscr{L}_0$, leaving massive axial-vector mesons through the familiar Higgs mechanism.

The quadratic part of $\mathscr{L}_0$ is diagonalized by the substitution

$$\underline{a}_\mu=\underline{A}_\mu-\frac{gf_\pi}{m_0{}^2}\partial_\mu\underline{M} \tag{2.13}$$

and $\underline{A}_\mu$ describes the physical axial-vector mesons. The standard normalization for the kinetic-energy term of the pseudoscalar mesons is ensured by

$$Zf_0{}^2=f_\pi{}^2,\ \ Z=1-\left[\frac{gf_\pi}{m_0}\right]^2 . \tag{2.14}$$

After these substitutions, the masses of $\underline{v}_\mu$ and $\underline{A}_\mu$ can be read off to be

$$m_v{}^2\equiv m_\rho{}^2=m_0{}^2,\ \ m_A{}^2=\frac{m_0{}^2}{Z}\ . \tag{2.15}$$

From the value of the ρ-meson mass and (2.14), we find

$$Z\cong\tfrac{1}{2}\ \text{and}\ m_A{}^2\cong 2m_\rho{}^2\ , \tag{2.16}$$

which is a well-known relation due to Weinberg.[16] The composite field $\underline{P}_\mu$ can now be expanded as

$$\underline{P}_\mu=-\frac{Z}{f_\pi}\partial_\mu\underline{M}-g\underline{A}_\mu+\cdots\ , \tag{2.17}$$

which implies that the coefficients D_0 and F_0 will be renormalized to D and F given by

$$D=ZD_0,\ \ F=ZF_0\ . \tag{2.18}$$

From the axial-vector-current matrix elements, one obtains[17]

$$D=0.813,\ \ F=0.4395\ . \tag{2.19}$$

We now turn our attention to the $SU(3)_V$ breaking terms to be added to $\mathscr{L}_0$. Since the quark mass terms transform according to $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ under $SU(3)_L\times SU(3)_R$, one can write down an $SU(3)_V$-breaking Lagrangian with the same transformation properties. With the quark mass matrix,

$$\underline{\mathscr{M}}=\begin{bmatrix} m_u & & \\ & m_d & \\ & & m_s\end{bmatrix}, \tag{2.20}$$

and

$$\underline{\Sigma}=\underline{\xi}^2 \tag{2.21}$$

an $SU(3)_V$-breaking Lagrangian is

$$\mathscr{L}_1=v^3\mathrm{Tr}(\underline{\mathscr{M}}\underline{\Sigma}+\underline{\Sigma}^\dagger\underline{\mathscr{M}})+a_1\mathrm{Tr}[\underline{\bar{B}}(\underline{\xi}^\dagger\underline{\mathscr{M}}\,\underline{\xi}^\dagger+\underline{\xi}\underline{\mathscr{M}}\underline{\xi})\underline{B}]+a_2\mathrm{Tr}[\underline{\bar{B}}\,\underline{B}(\underline{\xi}^\dagger\underline{\mathscr{M}}\underline{\xi}^\dagger+\underline{\xi}\,\underline{\mathscr{M}}\,\underline{\xi})] +b_1\mathrm{Tr}[\underline{\bar{B}}\gamma_5(\underline{\xi}^\dagger\underline{\mathscr{M}}\,\underline{\xi}^\dagger-\underline{\xi}\,\underline{\mathscr{M}}\,\underline{\xi})\underline{B}]+b_2\mathrm{Tr}[\underline{\bar{B}}\gamma_5\underline{B}(\underline{\xi}^\dagger\,\underline{\mathscr{M}}\,\underline{\xi}^\dagger-\underline{\xi}\,\underline{\mathscr{M}}\,\underline{\xi})] -\frac{\kappa_1}{16}\mathrm{Tr}[\underline{F}^{(-)}_{\mu\nu}\underline{F}^{(-)\mu\nu}(\underline{\mathscr{M}}\,\underline{\Sigma}^\dagger+\underline{\Sigma}\,\underline{\mathscr{M}})+\underline{F}^{(+)}_{\mu\nu}\underline{F}^{(+)\mu\nu}(\underline{\mathscr{M}}\,\underline{\Sigma}+\underline{\Sigma}^\dagger\,\underline{\mathscr{M}})]-\frac{\kappa_2}{8}\mathrm{Tr}(\underline{F}^{(-)}_{\mu\nu}\underline{\Sigma}\,\underline{F}^{(+)\mu\nu}\underline{\mathscr{M}}+\underline{F}^{(+)}_{\mu\nu}\underline{\Sigma}^\dagger\,\underline{F}^{(-)\mu\nu}\underline{\mathscr{M}})\ . \tag{2.22}$$

Considering the isospin-invariance limit $m_u=m_d=\overline{m}$, the parameters a_1, a_2, and $m_s/\overline{m}$ can be obtained from the masses of pseudoscalar mesons and baryons. We find

$$a_1\approx-0.45,\ \ a_2\approx 0.88,\ \ \frac{m_s}{\overline{m}}\approx 24\ . \tag{2.23}$$

With $m_s=150$ MeV, one has $\overline{m}=6.2$ MeV and $v=186$ MeV. The parameters b_1 and b_2 do not contribute to the baryon masses and they can presumably be obtained from the meson-nucleon scattering lengths.

As for the vector masses, we have chosen to induce symmetry breaking in the kinetic terms rather than the mass term. There are good arguments in favor of this procedure.[18] This leads to renormalization of the vector-meson fields,

$$\rho_R=Z_\rho{}^{1/2}\rho\ ,$$
$$K^*_R=Z_{K^*}{}^{1/2}K^*\ , \tag{2.24}$$
$$\phi_{8R}=Z_8{}^{1/2}\phi_8\ ,$$

and similarly for the axial-vector-meson fields. Here, we have,

$$Z_\rho = 1+\bar{m}(\kappa_1+\kappa_2)\ ,$$
$$Z_{K^*} = 1+\left[\frac{\bar{m}+m_s}{2}\right](\kappa_1+\kappa_2)\ , \tag{2.25}$$
$$Z_8 = 1+\left[\frac{\bar{m}+2m_s}{3}\right](\kappa_1+\kappa_2)\ .$$

From the masses of ρ and K^*,

$$m_\rho{}^2 = \frac{m_0{}^2}{Z_\rho},\quad m_{K^*}{}^2 = \frac{m_0{}^2}{Z_{K^*}}\ , \tag{2.26}$$

one finds that, $\bar{m}(\kappa_1+\kappa_2) = -0.021$, and

$$Z_\rho \cong 1,\quad Z_{K^*} \simeq \tfrac{3}{4},\quad Z_8 \cong \tfrac{2}{3}\ . \tag{2.27}$$

These values will be used in the vector-meson vertices of the proton decay matrix elements.

Finally, we treat the ω-ϕ mixing[19] by simply expressing

$$\phi_{8R} = \sin\theta_V\omega+\cos\theta_V\phi,\quad \theta_V = 40.3^\circ\ . \tag{2.28}$$

III. CHIRAL LAGRANGIAN FOR $\Delta B = 1$ INTERACTIONS

For a grand unified theory with a unification mass as high as 10^{14} GeV and with a low-energy spectrum consisting only of ordinary quarks and leptons, the leading effective operators contributing to the baryon-number-changing processes are the four-fermion operators constructed out of the quark and lepton fields. There are only four families of such operators consistent with the low-energy $SU(3)_C\times SU(2)_L\times U(1)$ symmetry.[20] In a two-component notation for the spinor fields, these are

$$\begin{aligned} O^{(1)}_{abcd} &= \epsilon_{\alpha\beta\gamma}\epsilon_{ij}(d^\alpha_{aR}u^\beta_{bR})(q^{\gamma i}_{cL}l^j_{dL})\ ,\\ O^{(2)}_{abcd} &= \epsilon_{\alpha\beta\gamma}\epsilon_{ij}(q^{\alpha i}_{aL}q^{\beta j}_{bL})(u^\gamma_{cR}l_{dR})\ ,\\ O^{(3)}_{abcd} &= \epsilon_{\alpha\beta\gamma}\epsilon_{im}\epsilon_{jk}(q^{\alpha i}_{aL}q^{\beta j}_{bL})(q^{\gamma k}_{cL}l^m_{dL})\ ,\\ O^{(4)}_{abcd} &= \epsilon_{\alpha\beta\gamma}(d^\alpha_{aR}u^\beta_{bR})(u^\gamma_{cR}l_{dR})\ , \end{aligned} \tag{3.1}$$

where α,β,γ are the color indices, i,j,k,m the $SU(2)_L$ indices, while a,b,c,d denote the quark-lepton generations. The fields appearing in these expressions are in the gauge-group representation basis. We will, however, neglect the quark-mixing angles and take them as mass eigenstates. In this case, the only operators relevant for a proton or bound neutron decay, are those containing at most one strange quark. For a given lepton generation d, the expressions in (3.1) contain ten such operators, four of which represent $\Delta S = 0$, and the remaining six, $\Delta S = 1$ transitions. Thus,

$$\begin{aligned} \Delta S = 0:\quad Q^{(1)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_R u^\beta_R)(u^\gamma_L e_{dL}-d^\gamma_L\nu_{dL})\ ,\\ Q^{(2)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_L u^\beta_L)(u^\gamma_R e_{dR})\ ,\\ Q^{(3)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_L u^\beta_L)(u^\gamma_L e_{dL}-d^\gamma_L\nu_{dL})\ ,\\ Q^{(4)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_R u^\beta_R)(u^\gamma_R e_{dR})\ . \end{aligned} \tag{3.2}$$

$$\begin{aligned} \Delta S = 1:\quad \tilde{Q}^{(1)}_d &= \epsilon_{\alpha\beta\gamma}(s^\alpha_R u_R)(u^\gamma_L e_{dL}-d^\gamma_L\nu_{dL})\ ,\\ \tilde{Q}^{(2)}_d &= \epsilon_{\alpha\beta\gamma}(s^\alpha_L u^\beta_L)(u^\gamma_R e_{dR})\ ,\\ \tilde{Q}^{(3)}_d &= \epsilon_{\alpha\beta\gamma}(s^\alpha_L u^\beta_L)(u^\gamma_L e_{dL}-d^\gamma_L\mu_{dL})\ ,\\ \tilde{Q}^{(4)}_d &= \epsilon_{\alpha\beta\gamma}(s^\alpha_R u^\beta_R)(u^\gamma_R e_{dR})\ ,\\ \tilde{Q}^{(5)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_R u^\beta_R)(s^\gamma_L\nu_{dL})\ ,\\ \tilde{Q}^{(6)}_d &= \epsilon_{\alpha\beta\gamma}(d^\alpha_L u^\beta_L)(s^\gamma_L\nu_{dL})\ . \end{aligned} \tag{3.3}$$

The effective four-fermion Lagrangian for $\Delta B = 1$ decays is then

$$\mathscr{L}^{\Delta B=1} = \sum_{i=1}^{4}\sum_d C^{(i)}_d Q^{(i)}_d + \sum_{i=1}^{6}\sum_d \tilde{C}^{(i)}_d\tilde{Q}^{(i)}_d + \text{H.c.} \tag{3.4}$$

The coefficients $C^{(i)}_d$ and $\tilde{C}^{(i)}_d$ in (3.4) are determined from the corresponding coefficients at the unification mass through a renormalization-group analysis[21] down to ordinary energies.

Chiral Lagrangians for $\Delta B = 1$ processes can now be constructed in terms of hadron fields, by analyzing parity and $SU(3)_L\times SU(3)_R$ transformation properties of the operators in (3.2) and (3.3). Such an analysis has been carried out in Ref. 10 and for the case involving no vector-meson fields, Claudson, Wise, and Hall obtained the following chiral Lagrangian, which we express in four-component notation:

$$\begin{aligned} \mathscr{L}^{\Delta B=1}_{(1)} = \alpha\sum_d[\overline{e^+_{dR}}\,\mathrm{Tr}(\underline{C}^{(1)}_d\underline{\xi}\,\underline{B}\,\underline{\xi}) - \overline{e^+_{dL}}\,\mathrm{Tr}(\underline{C}^{(2)}_d\underline{\xi}^\dagger\underline{B}\,\underline{\xi}^\dagger) - \overline{\nu^c_{dR}}\,\mathrm{Tr}(\underline{F}^{(1)}_d\underline{\xi}\,\underline{B}\,\underline{\xi})] + \text{H.c.}\\ +\beta\sum_d[\overline{e^+_{dR}}\,\mathrm{Tr}(\underline{C}^{(3)}_d\underline{\xi}\,\underline{B}\,\underline{\xi}^\dagger) - \overline{e^+_{dL}}\,\mathrm{Tr}(\underline{C}^{(4)}_d\underline{\xi}^\dagger\underline{B}\,\underline{\xi}) - \overline{\nu^c_{dR}}\,\mathrm{Tr}(\underline{F}^{(3)}_d\underline{\xi}\,\underline{B}\,\underline{\xi}^\dagger)] + \text{H.c.} \end{aligned} \tag{3.5}$$

The matrices

$$\underline{C}^{(i)}_d = \begin{bmatrix} 0 & 0 & 0\\ -\tilde{C}^{(i)}_d & 0 & 0\\ C^{(i)}_d & 0 & 0 \end{bmatrix},\quad i=1,2,3,4\ ,\quad \underline{F}^{(1)}_d = \begin{bmatrix} 0 & 0 & 0\\ 0 & -\tilde{C}^{(1)}_d & 0\\ 0 & C^{(1)}_d & -\tilde{C}^{(5)}_d \end{bmatrix},\quad \underline{F}^{(3)}_d = \begin{bmatrix} 0 & 0 & 0\\ 0 & -\tilde{C}^{(3)}_d & 0\\ 0 & C^{(3)}_d & -\tilde{C}^{(6)}_d \end{bmatrix} \tag{3.6}$$

summarize the projections corresponding to the operators in (3.2) and (3.3) and the coefficients α and β play the role of reduced matrix elements of a Wigner-Eckart-type analysis.

It is now straightforward to write a chiral Lagrangian involving vector and axial-vector mesons by simply replacing the baryon octet $\underline{B}$ in (3.5) by $i\gamma_\mu D^\mu\underline{B}$ where $D^\mu\underline{B}$ is the chiral covariant derivative of the baryon octet introduced in

(2.9). We recall from (2.4) and (2.10) that $\underline{B}$ and $D^{\mu}\underline{B}$ transform in the same way under $SU(3)_L \times SU(3)_R$, and therefore the Lagrangian

$$\begin{aligned}\mathscr{L}^{\Delta B=1}_{(2)} = \frac{\gamma}{M_p}\sum_{d}\{&\overline{e^{+}_{dR}}i\gamma_{\mu}\mathrm{Tr}[\underline{C}_d^{(1)}\underline{\xi}(D^{\mu}\underline{B})\underline{\xi}]-\overline{e^{+}_{dL}}i\gamma_{\mu}\mathrm{Tr}[\underline{C}_d^{(2)}\underline{\xi}^{\dagger}(D^{\mu}\underline{B})\underline{\xi}^{\dagger}]-\overline{\nu^{c}_{dR}}i\gamma_{\mu}\mathrm{Tr}[\underline{F}_d^{(1)}\underline{\xi}(D^{\mu}\underline{B})\underline{\xi}]\}+\mathrm{H.c.}\\ +\frac{\delta}{M_p}\sum_{p}\{&\overline{e^{+}_{dR}}i\gamma_{\mu}\mathrm{Tr}[\underline{C}_d^{(3)}\underline{\xi}(D^{\mu}\underline{B})\underline{\xi}^{\dagger}]\overline{e^{+}_{dL}}i\gamma_{\mu}\mathrm{Tr}[\underline{C}_d^{(4)}\underline{\xi}^{\dagger}(D^{\mu}\underline{B})\underline{\xi}]-\overline{\nu^{c}_{dR}}i\gamma_{\mu}\mathrm{Tr}[\underline{F}_d^{(3)}\underline{\xi}(D^{\mu}\underline{B})\underline{\xi}^{\dagger}]\}+\mathrm{H.c.}\end{aligned} \qquad (3.7)$$

shares with $\mathscr{L}^{\Delta B=1}_{(1)}$ the same transformation properties under the chiral group. We thus take

$$\mathscr{L}^{\Delta B=1}=\mathscr{L}^{\Delta B=1}_{(1)}+\mathscr{L}^{\Delta B=1}_{(2)} \qquad (3.8)$$

as the chiral Lagrangian for $\Delta B=1$ decays.[22]

Before presenting our results in the next section, it is useful to recall some of the selection rules[23] governing the nucleon decay.

(1) For $\Delta S=0$ transitions, the four-fermion operators in (3.2) or the hadronic operators in (3.5) and (3.7) all satisfy $\Delta I=\frac{1}{2}$ rule. This leads immediately to the relations

$$\Gamma(n\to X^{-}e^{+})=2\Gamma(p\to X^{0}e^{+}) , \qquad (3.9)$$

$$\Gamma(p\to X^{+}\overline{\nu}_e)=2\Gamma(n\to X^{0}\overline{\nu}_e) , \quad X=\pi \text{ or } \rho . \qquad (3.10)$$

(2) Furthermore, the hadronic operators multiplying the antineutrino and the right-hand positron fields in $\mathscr{L}^{\Delta B=1}$ together form the components of an irreducible tensor with $I=\frac{1}{2}$, which leads to

$$\Gamma(p\to Xe^{+}_R)=\Gamma(n\to X\overline{\nu}_e) , \quad X=\pi^0,\rho^0,\eta,\omega . \qquad (3.11)$$

(3) If the baryon decay is due to the exchange of a gauge boson, then the effective four-fermion Lagrangian contains only the operators of the type $O^{(1)}_{abcd}$ and $O^{(2)}_{abcd}$ in (3.1). In this case, the coefficients $C^{(3)}_d$, $C^{(4)}_d$, $\widetilde{C}^{(3)}_d$, $\widetilde{C}^{(4)}_d$, and $\widetilde{C}^{(6)}_d$ all vanish. For $\Delta S=0$ transitions, one then obtains

$$\Gamma(N\to Xe^{+}_L)=\left|\frac{C^{(2)}_1}{C^{(1)}_1}\right|^2\Gamma(N\to Xe^{+}_R) , \qquad (3.12)$$

where X is any nonstrange hadron. Neglecting the lepton mass, (3.11) and (3.12) give in this case

$$\Gamma(p\to Xe^{+})=\left[1+\left|\frac{C^{(2)}_1}{C^{(1)}_1}\right|^2\right]\Gamma(n\to X\overline{\nu}_e) , \quad X=\pi^0,\rho^0,\eta,\omega . \qquad (3.13)$$

By combining (3.13) and (3.10), one also gets

$$\Gamma(p\to X^{0}e^{+})=\tfrac{1}{2}\left[1+\left|\frac{C^{(2)}_1}{C^{(1)}_1}\right|^2\right]\Gamma(p\to X^{+}\overline{\nu}_e) , \quad X=\pi \text{ or } \rho . \qquad (3.14)$$

(4) Finally, we obtain

$$\begin{aligned}\Gamma(N\to\omega\overline{l})&=\frac{3\sin^2\theta_V}{Z_8}\Gamma(N\to\rho^0\overline{l})\\ &=1.9\Gamma(N\to\rho^0\overline{l}) ,\end{aligned} \qquad (3.15)$$

where the factor 3 is the SU(3) result, Z_8 is the renormalization factor for ϕ_8 (2.27), and $\sin^2\theta_V$ comes from the ω-ϕ mixing (2.28).

IV. TWO-BODY DECAY RATES

The evaluation of the two-body decay rates is based on tree diagrams of the Lagrangian,

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_1+\mathscr{L}^{\Delta B=1} , \qquad (4.1)$$

where $\mathscr{L}_0$, $\mathscr{L}_1$, and $\mathscr{L}^{\Delta B=1}$ are given by (2.12), (2.22), and (3.8), respectively. There are two types of diagrams that contribute to two-body decay amplitudes. The first type is the direct conversion diagram (Fig. 1) which is given by the three-body vertices of $\mathscr{L}^{\Delta B=1}$. The second type is the baryon pole diagram (Fig. 2), where the nucleon-meson-baryon vertex is taken from the strong interaction Lagrangian $\mathscr{L}_0+\mathscr{L}_1$ and the conversion of the intermediate virtual baryon into the final antilepton is given by the two-body vertices of $\mathscr{L}^{\Delta B=1}$. These pole diagrams correspond to three-quark fusion diagrams of quark model analysis.[11] The relevant vertices in each case can be read off from the Lagrangian by expanding $\underline{\xi}$ and $D^{\mu}\underline{B}$,

$$\underline{\xi}=\exp\left[\frac{i}{f_\pi}\underline{M}\right]=1+\frac{i}{f_\pi}\underline{M}+\cdots , \qquad (4.2)$$

$$\begin{aligned}D^{\mu}\underline{B}&=\partial^{\mu}\underline{B}-[\underline{V}^{\mu},\underline{B}]\\ &=\partial^{\mu}\underline{B}-ig[\underline{v}^{\mu},\underline{B}]+\cdots ,\end{aligned} \qquad (4.3)$$

where in the last equation, the expression (2.7b) for $\underline{V}_{\mu}$ has been expanded. For the vector-meson octet $\underline{v}^{\mu}$, one then carries out the renormalization prescription (2.24).

The Lagrangian $\mathscr{L}^{\Delta B=1}_{(2)}$ (3.7), contains baryon-vector meson—antilepton vertices. We note however that, as a consequence of the inclusion of the vector mesons into the

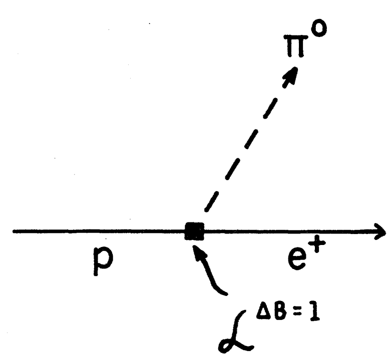

FIG. 1. Direct-conversion diagram for $p\to\pi^0e^{+}$ amplitude.

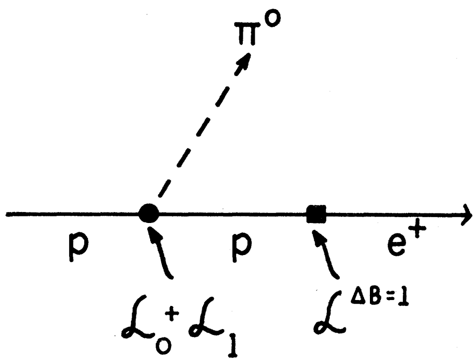

FIG. 2. Pole diagram for $p\to\pi^0 e^+$ amplitude.

chiral Lagrangian in a chirally covariant way, this Lagrangian, through the terms involving $\partial_\mu \underline{B}$, also contributes to the baryon—pseudoscalar-meson—antilepton diagrams as well as the pole diagrams. These amplitudes should be added to the amplitudes coming from $\mathcal{L}_{(1)}^{\Delta B=1}$ (3.5) for the evaluation of decay rates into the pseudoscalar channels. As we will see in Sec. V, the amplitudes

$$\langle \text{pseudoscalar meson}; \bar{l}\,|\,\mathcal{L}_{(1)}^{\Delta B=1}\,|\,\text{nucleon}\rangle$$

and

$$\langle \text{pseudoscalar meson}; \bar{l}\,|\,\mathcal{L}_{(2)}^{\Delta B=1}\,|\,\text{nucleon}\rangle$$

become equal to each other in the limit of flavor-SU(3) symmetry and it does not matter whether one uses $\mathcal{L}_{(1)}^{\Delta B=1}$ or $\mathcal{L}_{(2)}^{\Delta B=1}$ for the pseudoscalar channels in that limit. However, without flavor-SU(3) symmetry, these amplitudes will have different contributions to the decay rates. In fact, in Sec. V, we will exploit the breaking of flavor-SU(3) symmetry to determine the relative contributions of $\mathcal{L}_{(1)}^{\Delta B=1}$ and $\mathcal{L}_{(2)}^{\Delta B=1}$ to the pseudoscalar channel decay rates.

We find it useful to express the two-body decay rates in terms of dimensionless constants. To this end, we define

$$C_d^{(i)}=\frac{g_{\rm GUT}^2}{M_X^2}c_d^{(i)},\quad i=1,2,3,4\ ,$$
$$\tilde{C}_d^{(i)}=\frac{g_{\rm GUT}^2}{M_X^2}\tilde{c}_d^{(i)},\quad i=1,\ldots,6\ , \tag{4.4}$$

where $g_{\rm GUT}$ is the coupling constant of the grand unified theory at the unification mass M_X. The constants $\alpha,\beta,\gamma,\delta$ appearing in $\mathcal{L}_{(1)}^{\Delta B=1}$ and $\mathcal{L}_{(2)}^{\Delta B=1}$ all have the dimension of $(\text{mass})^3$. It is convenient to express them in units of 1 GeV and define the dimensionless constants $\hat\alpha,\hat\beta,\hat\gamma,\hat\delta$ as

$$\begin{bmatrix}\alpha\\ \beta\\ \gamma\\ \delta\end{bmatrix}=(1\ \text{GeV})^3\begin{bmatrix}\hat\alpha\\ \hat\beta\\ \hat\gamma\\ \hat\delta\end{bmatrix}. \tag{4.5}$$

The decay rates then contain an overall multiplicative factor Γ_0,

$$\Gamma_0=\pi(\alpha_{\rm GUT})^2\frac{M_p}{M_X{}^4 f_\pi{}^2}\ \text{GeV}^6\ , \tag{4.6}$$

where $4\pi\alpha_{\rm GUT}=g_{\rm GUT}{}^2$. Using the tree diagrams of the Lagrangian (4.1), we have computed the two-body decay rates in an arbitrary grand unified theory. The results are presented in the Appendix. In this section, we will specialize to the case of minimal SU(5), in which case, the only nonvanishing coefficients are

$$c_1^{(1)}=\lambda,\quad c_1^{(2)}=r\lambda,\quad \tilde{c}_2^{(1)}=\lambda,\quad \tilde{c}_2^{(2)}=\frac{r}{2}\lambda\ , \tag{4.7}$$

where λ is the renormalization-group enhancement factor,[24]

$$\lambda=\left[\frac{\alpha_s(M_p)}{\alpha_5}\right]^{6/(33-2N_f)}\left[\frac{\alpha_2(M_W)}{\alpha_5}\right]^{27/(86-8N_f)}\left[\frac{5\alpha_1(M_W)}{3\alpha_5}\right]^{-33/(40N_f+6)} \tag{4.8}$$

and

$$r=2\left[\frac{5\alpha_1(M_W)}{3\alpha_5}\right]^{-18/(20N_f+3)}. \tag{4.9}$$

N_f is the number of flavors; α_1, α_2, and α_3 are the U(1), SU(2), and SU(3)$_c$ running coupling strengths at the indicated mass scales. $\alpha_5=\alpha_{\rm GUT}$. Using the formulas in the Appendix and evaluating the kinematic factors, we find the following expressions for the various two-body decay rates as functions of Γ_0,λ,r and $\hat\alpha,\hat\gamma$:

$$\Gamma(p\to\pi^0 e^+)=\Gamma_0\lambda^2(1+r^2)(1.22)(\hat\alpha+\hat\gamma)^2\ , \tag{4.10}$$

$$\Gamma(p\to\eta e^+)=\Gamma_0\lambda^2(1+r^2)(0.0088)(\hat\alpha+\hat\gamma)^2(1+2.6b_2)^2\ , \tag{4.11}$$

$$\Gamma(p\to K^0\mu^+)=\Gamma_0\lambda^2\left[(0.166)\left[1+\frac{r}{2}\right]^2[1.24\hat\alpha(1+0.19b_2)+1.3\hat\gamma(1+0.23b_2)]^2\right.$$
$$\left.+(0.088)\left[1-\frac{r}{2}\right]^2[1.36\hat\alpha(1+0.21b_2)+1.46\hat\gamma(1+0.25b_2)]^2\right]\ , \tag{4.12}$$

$$\Gamma(p\to K^+\bar\nu_\mu)=\Gamma_0\lambda^2(0.065)\{0.89\hat\alpha(1+0.21b_1+0.19b_2)+1.084\hat\gamma[1+0.2(b_1+b_2)]\}^2\ , \tag{4.13}$$

$$\Gamma(n\to K^0\bar{\nu}_\mu)=\Gamma_0\lambda^2(0.26)\{0.85\hat{\alpha}[1+0.21b_2-0.11b_1]+0.83\hat{\gamma}[1+0.13(2b_2-b_1)]\}^2 , \tag{4.14}$$

$$\Gamma(p\to\rho^0e^+)=\Gamma_0\lambda^2(1+r^2)(0.0288)(\hat{\alpha}+2\hat{\gamma})^2 , \tag{4.15}$$

$$\Gamma(p\to K^{*+}\bar{\nu}_\mu)=\Gamma_0\lambda^2(4.7\times10^{-6})\hat{\alpha}^2 , \tag{4.16}$$

$$\Gamma(n\to K^{*0}\bar{\nu}_\mu)=\Gamma_0\lambda^2(0.0016)(1.63\hat{\alpha}+4\hat{\gamma})^2 . \tag{4.17}$$

The decay rates for the remaining channels can be obtained from these by the use of the symmetry relations (3.9)–(3.15). We note that the widths for the η channel and some of the $\Delta S=1$ channels depend on the SU(3)-symmetry-breaking parameters b_1 and b_2. These strong interaction parameters can, in principle, be related to and hence estimated from an analysis of baryon-meson scattering lengths. We have not carried out such an analysis at the present time. We shall assume that their effects are small and set $b_1=b_2=0$. This will not introduce any series uncertainty in decay rates except for the ηe^+ channel.[25]

From the expressions (4.10), (4.11), and (4.15), we have the following useful results:

$$\frac{\Gamma(p\to\eta e^+)}{\Gamma(p\to\pi^0e^+)}=0.0072 , \tag{4.18}$$

$$\frac{\Gamma(p\to\rho^0e^+)}{\Gamma(p\to\pi^0e^+)}=0.0236\left[\frac{\hat{\alpha}+2\hat{\gamma}}{\hat{\alpha}+\hat{\gamma}}\right]^2 . \tag{4.19}$$

These results are in fact valid for any grand unified theory, for which the leading $\Delta B=1$ interactions are mediated by a gauge-boson exchange. Equation (4.18) was already obtained in Ref. 10 based on $\mathscr{L}_{(1)}^{\Delta B=1}$ (3.5) and it remains valid in the more general case (3.8) that we are considering. This ratio is sensitive to the SU(3)-symmetry-breaking parameter b_2 which we set equal to zero and, as already noted, a further analysis is needed to estimate the value of b_2 which will give a more reliable result for the ratio in (4.18). As for (4.19), we observe that the relative importance of π^0e^+ and ρ^0e^+ modes is solely determined by the value of $\hat{\gamma}/\hat{\alpha}$. While this ratio can be estimated by considering a constituent picture of hadrons, as will be done in the next section, at the level of effective Lagrangian (3.8), it is completely arbitrary. Thus, $\hat{\gamma}/\hat{\alpha}$ parametrizes a genuine ambiguity in the problem of proton decay, which is well reflected in the literature since various approaches lead to totally different results for the relative importance of π^0e^+ and ρ^0e^+ modes.

To proceed further, we shall use, for the various coupling constants appearing in (4.6), (4.8), and (4.9), typical values from Marciano,[6] namely,

$$\begin{aligned} &M_W=83.0\text{ GeV},\quad \Lambda_{\overline{\text{MS}}}=0.16\text{ GeV} ,\\ &\alpha_1(M_W)=0.0166,\quad \alpha_2(M_W)=0.0365 ,\\ &\alpha_3(M_N)=\frac{4\pi}{7\ln(M_N{}^2)/\Lambda_{\overline{\text{MS}}}{}^2)}=0.507 ,\\ &\alpha_5=\alpha_{\text{GUT}}=0.0242 . \end{aligned} \tag{4.20}$$

Then,

$$\lambda=3.36,\quad r=2.11,\quad M_X=2.1\times10^{14}\text{ GeV} . \tag{4.21}$$

As noted earlier, the results for the total decay rate depend sensitively on M_X which in turn depends crucially on the QCD parameter $\Lambda_{\overline{\text{MS}}}$. To display this sensitivity explicitly so that our results can be reevaluated easily if warranted in the future, we shall write the multiplicative factor Γ_0 (4.6) in the form

$$\Gamma_0=(3.85\times10^{26}\text{ yr})^{-1}\left[\frac{2.1\times10^{14}\text{ GeV}}{M_X}\right]^4 . \tag{4.22}$$

With (4.21) and (4.22), the two-body decay modes are functions of two arbitrary parameters $\hat{\alpha}$ and $\hat{\gamma}$. Within the strict framework of chiral symmetry and the phenomenological Lagrangian, $\hat{\alpha}$ and $\hat{\gamma}$ are arbitrary parameters. Some experimental information is necessary to determine them. While in the next section, we shall derive expressions for them in terms of the quark- and bag-model parameters, we shall close this section by giving results in Tables I and II, for the various branching ratios in three different cases: (i) $\hat{\gamma}=0$. This case corresponds to the one treated in Ref. 10, but including the vector mesons in the strong-interaction Lagrangian. (ii) $\hat{\alpha}=0$. This is an extreme case which includes both pseudoscalar and vector

TABLE I. Proton decay rate in minimal SU(5).

Decay rates (in units of $\Gamma_0\lambda^2$)	Two-body branching ratios $\hat{\gamma}=0$	$\hat{\alpha}=0$	$\hat{\alpha}+\hat{\gamma}=0$
$\Gamma(p\to\pi^0e^+)=(6.62)(\alpha+\gamma)^2$	61.7	53.3	0
$\Gamma(p\to\pi^+\bar{\nu}_e)=(0.367)\Gamma(p\to\pi^0e^+)$	22.6	19.6	0
$\Gamma(p\to\eta e^+)=(0.0072)\Gamma(p\to\pi^0e^+)$	0.4	0.4	0
$\Gamma(p\to K^0\mu^+)=(0.7)(1.24\hat{\alpha}+1.3\hat{\gamma})^2$	10	9.5	0.5
$\Gamma(p\to K^+\bar{\nu}_\mu)=(0.065)(0.89\hat{\alpha}+1.08\hat{\gamma})^2$	0.5	0.6	0.5
$\Gamma(p\to\rho^0e^+)=(0.157)(\hat{\alpha}+2\hat{\gamma})^2$	1.5	5.1	30.3
$\Gamma(p\to\rho^+\bar{\nu}_e)=(0.367)(p\to\rho^0e^+)$	0.5	1.9	11.1
$\Gamma(p\to\omega e^+)=(1.9)\Gamma(p\to\rho^0e^+)$	2.8	9.6	57.6
$\Gamma(p\to K^{*+}\bar{\nu}_\mu)=4.7\times10^{-6}\hat{\alpha}^2$	~0	0	~0

TABLE II. Neutron decay rates in minimal SU(5).

Decay rates (in units of $\Gamma_0\lambda^2$)	Two-body branching ratios $\hat{\gamma}=0$	$\hat{\alpha}=0$	$\hat{\alpha}+\hat{\gamma}=0$
$\Gamma(n\to\pi^- e^+)=2\Gamma(p\to\pi^- e^+)$	87.8	81.4	0
$\Gamma(n\to\pi^0\bar{\nu}_e)=(0.183)\Gamma(p\to\pi^0 e^+)$	8.1	7.5	0
$\Gamma(n\to\eta\bar{\nu}_e)=(0.183)\Gamma(p\to\eta e^+)$	0.1	0.1	0
$\Gamma(n\to K^0\bar{\nu}_\mu)=0.26(0.85\hat{\alpha}+0.83\hat{\gamma})^2$	1.3	1.1	~ 0
$\Gamma(n\to\rho^- e^+)=2\Gamma(p\to\rho^0 e^+)$	2.1	7.7	77.2
$\Gamma(n\to\rho^0\bar{\nu}_e)=(0.183)\Gamma(p\to\rho^0 e^+)$	0.2	0.7	7.1
$\Gamma(n\to\omega\bar{\nu}_e)=(0.349)\Gamma(p\to\rho^0 e^+)$	0.4	1.3	13.5
$\Gamma(n\to K^{+0}\bar{\nu}_\mu)=(0.0016)(1.63\hat{\alpha}+4\hat{\gamma})^2$	~ 0	0.2	2.2

mesons in baryon-number-violating interactions in the chiral framework. (iii) $\hat{\alpha}+\hat{\gamma}=0$. The examination of this case is prompted by the recent results of the IMB experiment which does not see the $p\to\pi^0 e^+$ mode.

V. COMPARISON WITH OTHER APPROACHES

In this section, we have obtained the nucleon two-body decay rates as a function of the parameters α and γ of the chiral Lagrangian (3.8). In the spirit of an effective Lagrangian approach, such parameters are to be determined from experimental data. In the absence of such a determination, we shall make use of the chiral Lagrangian to correlate various other approaches. To this end, we follow a method suggested by Donoghue and Golowich[26] and consider, for the case of minimal SU(5), the following quantities:

$$\langle e^+(p)\,|\,H_{\rm GUT}(0)\,|\,\text{proton}(p)\rangle = \frac{G_u}{\sqrt{2}}B_p\bar{U}_e(p)(3\gamma_5+1)U_p(p)\ , \quad (5.1)$$

$$\langle \mu^+(p)\,|\,H_{\rm GUT}(0)\,|\,\Sigma^+(p)\rangle = -\frac{G_u}{\sqrt{2}}2B_\Sigma\bar{U}_\mu(p)\gamma_5U_\Sigma(p)\ , \quad (5.2)$$

$$\langle \bar{\nu}_\mu(p)\,|\,H_{\rm GUT}(0)\,|\,\Lambda^0(p)\rangle = -\frac{G_u}{\sqrt{2}}\frac{B_\Lambda}{\sqrt{6}}\bar{U}_{\nu_\mu}(p)(1-\gamma_5)U_\Lambda(p)\ , \quad (5.3)$$

where

$$\frac{G_u}{\sqrt{2}}=\frac{g_{\rm GUT}{}^2}{8M_X{}^2}\ . \quad (5.4)$$

The form factors B_p, B_Σ, and B_Λ thus introduced, can be calculated from a constituent-quark picture for baryons, when $H_{\rm GUT}$ is expressed in terms of quark and lepton fields. Donoghue and Golowich suggest that various approaches to the problem of proton decay may be compared with each other at the level of these form factors. Since the above amplitudes do not involve any final-state mesons, such a comparison has the advantage of being free from the ambiguities associated with introducing explicit quark wave functions for mesons.

We have calculated these form factors using the chiral Lagrangian (3.8) and our results are

$$B_p=4(\alpha+\gamma)\ , \quad (5.5)$$

$$B_\Sigma=4\left[\alpha+\frac{M_\Sigma}{M_N}\gamma\right]\ , \quad (5.6)$$

$$B_\Lambda=4\left[\alpha+\frac{M_\Lambda}{M_N}\gamma\right]\ . \quad (5.7)$$

We note that in the limit of flavor-SU(3) symmetry, B_Σ and B_Λ equal B_p. Thus the introduction of the Lagrangian $\mathscr{L}_{(2)}^{\Delta B=1}$ (3.7), which was needed to account for the three-body vector-meson—baryon—lepton vertices, has also provided a splitting of these form factors from one another in terms proportional to γ, due to SU(3)-symmetry breaking.

Given a constituent model computation of B_p, B_Σ, and B_Λ, the strategy would then be to solve Eqs. (5.5)—(5.7) for α and γ. Since we have three equations for α and γ, the form factors B_p, B_Σ, and B_Λ ought to satisfy the following consistency condition:

$$\frac{B_\Sigma-B_p}{B_\Lambda-B_p}=\frac{M_\Sigma-M_N}{M_\Lambda-M_N}\ . \quad (5.8)$$

We expect that, an evaluation of B_p, B_Σ, and B_Λ which takes flavor-SU(3) breaking into account will lead to the values of these form factors which will be consistent with (5.8).

From the bag-model analysis of Donoghue and Golowich,[26] we have

$$B_p=0.0127\ \text{GeV}^3,\ B_\Sigma=0.014\ \text{GeV}^3\ ,\qquad B_\Lambda=0.0137\ \text{GeV}^3\ . \quad (5.9)$$

These values are consistent with (5.8) and from Eqs. (5.5) and (5.6) we obtain

$$\alpha=0.0019\ \text{GeV}^3, \gamma=0.001\,27\ \text{GeV}^3\ . \quad (5.10)$$

From the ratio

$$\frac{\gamma}{\alpha}=0.67\ , \quad (5.11)$$

we have evaluated the two-body branching ratios of the nucleon and our results are presented in Table III. Thus, with the above determination of γ/α, the dominant decay mode of the proton is the $\pi^0 e^+$ channel. Among the various estimations that appeared in the literature for the two-body branching ratios of the nucleon, our results are closer to those of Tomozawa.[27] We feel that it will be in-

TABLE III. Branching ratios for proton and neutron corresponding to $\gamma/\alpha=0.67$.

Proton		Neutron	
Channel	Branching ratio	Channel	Branching ratio
$p\to\pi^0 e^+$	58.7	$n\to\pi^- e^+$	85.6
$p\to\pi^+\bar{\nu}_e$	21.5	$n\to\pi^0\bar{\nu}_e$	7.9
$p\to\eta e^+$	0.4	$n\to\eta\bar{\nu}_e$	0.1
$p\to K^0\mu^+$	10	$n\to K^0\bar{\nu}_\mu$	1.2
$p\to K^+\bar{\nu}_\mu$	0.5	$n\to\rho^- e^+$	4
$p\to\rho^0 e^+$	2.7	$n\to\rho^0\bar{\nu}_e$	0.4
$p\to\rho^+\bar{\nu}_e$	1	$n\to\omega\bar{\nu}_e$	0.7
$p\to\omega e^+$	5.2	$n\to K^{*0}\bar{\nu}_\mu$	0.1
$p\to K^{*+}\bar{\nu}_\mu$	~ 0		

structive to carry out in the future the analysis that has been described in this section, for the case of the nonrelativistic quark model and to obtain an independent determination of the ratio in (5.11). Since the branching ratios depend only on γ/α, this will constitute a check on our results in Table III.

For the remainder of this section, we shall discuss the decay width for the $\pi^0 e^+$ channel in various models. From (5.5) and substituting minimal SU(5) values (4.21) for λ and r into (4.10), we have

$$\tau(p\to\pi^0 e^+)=\frac{0.82\times 10^{26}\ \text{yr}}{B_p{}^2}\left[\frac{M_X}{2.1\times 10^{14}\ \text{GeV}}\right]^4, \quad (5.12)$$

where B_p is in GeV^3. For the bag-model calculation of Donoghue and Golowich, B_p is given by (5.9). For the nonrelativistic quark model with harmonic-oscillator potential,[28] we have calculated B_p, taking the static limit[29] of Dirac spinors throughout the calculation (a computation that is implicit in the work of Isgur and Wise[12]), and we find

$$B_p=\left[\frac{2}{\pi}\right]^{3/2} 3^{1/4}\alpha_B{}^3\ , \quad (5.13)$$

where α_B is the Gaussian parameter for the baryons. For $\alpha_B=0.32$ GeV, which is the value used by Isgur and Wise,[12] $B_p=21.9\times 10^{-3}$ GeV^3. As emphasized in Ref. 16, a more appropriate value α_B may be the one that correctly reproduces the charge radius of the proton. In our normalization, this corresponds to $\alpha_B=0.2$ GeV and $B_p=5.4\times 10^{-3}$ (GeV)^3. Table IV summarizes the corresponding values of $\tau(p\to\pi^0 e^+)$ for these various choices of B_p and for three different values of M_X. The values for M_x, 1.3×10^{14}, 2.1×10^{14}, and 3.5×10^{14} GeV correspond to $\Lambda_{\overline{\text{MS}}}=0.10$, 0.16, and 0.26 GeV, respectively.[6] It is also worth noting again that we have calculated the decay amplitudes by using the tree diagrams of the effective Lagrangian, and in so doing, we have implicitly neglected the possible momentum dependence of the effective vertices. This procedure is justified if the momentum carried by the relevant meson is small, which is typically the case for the problem of proton decay except for the pionic modes. These off-shell effects could be estimated within the present framework using chiral perturbation theory which we have not done so far. Isgur and Wise,[12] however, have evaluated such form factors in nonrelativistic quark models, and from their work we can infer that the off-shell effects will increase the lifetimes into pionic modes by a factor of about 1.5.

As expected, the major uncertainty comes from the value of M_X. If M_X can be as high as 3.5×10^{14} GeV, corresponding to $\Lambda_{\overline{\text{MS}}}=0.26$ GeV, then, at least for the nonrelativistic quark model with $\alpha_B=0.2$ GeV, the proton lifetime into $\pi^0 e^+$ channel becomes compatible with the experimental lower bound of 2×10^{31} yr.

VI. CONCLUDING REMARKS

Baryon-number- and lepton-number-violating processes in the low-energy region are perhaps the most dramatic consequences of grand unified theories. Their terrestrial observation would certainly confirm the correctness of the basic ideas of grand unification. Hence, prompted by the desire to have predictions amenable to immediate experimental scrutiny, one has proceeded to calculate by making perhaps too many simplifying assumptions. In this paper we have attempted to show that this is the situation in the case of proton decay.

There are at the onset the well-known difficulties inherent in the grand unification schemes: The choice of the grand unification group G, the symmetry-breaking pattern, the appearance of arbitrary parameters if the symmetry breaking involves more than one stage, the choice of Higgs representation, and Higgs potential, and so on. These difficulties aside, given the extrapolated, renormalized $\Delta B\neq 0$ interaction Lagrangian in terms of quark and lepton fields, the evaluation of the low-energy

TABLE IV. Estimates of $\tau(p\to\pi^0 e^+)$ for various values of B_p and M_x. The off-shell effects in vertices mentioned in the text increase the liftetimes in the last three columns approximately by 1.5.

Constituent model used for B_p	B_p (GeV^3)	$\tau(p\to\pi^0 e^+)$ (yr) $\Lambda_{\overline{\text{MS}}}=0.1$ GeV $M_X=1.3\times 10^{14}$ GeV	$\Lambda_{\overline{\text{MS}}}=0.16$ GeV $M_X=2.1\times 10^{14}$ GeV	$\Lambda_{\overline{\text{MS}}}=0.26$ GeV $M_X=3.5\times 10^{14}$ GeV
Nonrelativistic quark model ($\alpha_B=0.32$ GeV)	0.0219	2.5×10^{28}	1.7×10^{29}	1.3×10^{30}
Bag model (Donoghue and Golowich)	0.0127	7.5×10^{28}	5.1×10^{29}	3.9×10^{30}
Nonrelativistic quark model ($\alpha_B=0.2$ GeV)	0.0054	4.1×10^{29}	2.8×10^{30}	2.2×10^{31}

matrix elements of this interaction Lagrangian responsible for proton decay is not unique. Although the overall total lifetime is mainly governed by the grand unification mass scale M_X, the various observable branching ratios are sensitively dependent upon the method of evaluation of the low-energy matrix elements.

In this paper, we have chosen to emphasize a phenomenological approach based on broken chiral symmetry. Such approaches in the past have provided reasonably successful descriptions of several aspects of low-energy strong-interaction physics and currently phenomenological or effective Lagrangians incorporating appropriate symmetries are proving to be valuable in bridging the gap between fundamental Lagrangian in terms of confined quark fields (such as QCD Lagrangian) and a calculable Lagrangian in terms of observable hadronic fields. Until we have better, quantitatively rigorous methods to deal with confinement (or for that matter bound states with ordinary or nonconfined constituents), effective Lagrangians with a few phenomenological parameters are best suited to correlate experimental data. In the case of proton decay, alternate methods based on quark and bag models are too sensitive to the details that go into the calculations. We do not intend here to venture into a detailed discussion of the implicit assumptions, the uncertanties, and to the extent to which they have provided quantitative descriptions of the phenomena. The fact that these models applied to proton decay give results differing by orders of magnitude should be sufficient to conclude that they are at best qualitatively successful and not quantitatively.

On the other hand, phenomenological approaches suffer from the appearance of arbitrary parameters the determination of which requires experimental information. Our extension of the work in Ref. 10 to include the vector and axial-vector mesons introduces an additional parameter, so that even in minimal SU(5) with all the customary simplifying assumptions, one is not able to make definite predictions concerning the branching ratios of the various two-body decay modes. It is somewhat disappointing that unless some proton decay mode and its branching ratio is observed with reasonable accuracy, one really cannot arrive at any definite conclusion.

Nonetheless it is useful to have general results with a few parameters as in our case. The results provided in the Appendix are applicable to any grand unified theory including both Higgs and gauge boson exchanges. They also contain SU(3)-symmetry-breaking effects in terms of the parameters b_1 and b_2 which, in principle, can be calculated by appealing to information on low-energy scattering lengths in meson-baryon scattering. Furthermore, the effective Lagrangian contains information about uncorrelated multileptonic decays ($p \to \pi^0\pi^0 e^+$, $\pi^+\pi^- e^+$, for instance). These and other refinements need further investigation.

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APPENDIX: GENERAL EXPRESSIONS FOR TWO-BODY DECAY RATES

In this appendix, we present the two-body decay rates for an arbitrary grand unified theory for which the leading baryon-number-violating interactions are described by an effective four-fermion Lagrangian as in (3.4). We denote the lepton mass by μ and the (pseudoscalar or vector) meson by m. The index d denotes the lepton generation. In the evaluation of the pole diagrams, we encounter the following expressions:

$$S_1(\mu)=\frac{M_N-\mu}{M_N+\mu}(D+F)+4b_1\frac{\bar{m}}{M_N+\mu}\,, \tag{A1}$$

$$S_2(\mu)=\frac{M_N-\mu}{M_N+\mu}(3F-D)+4\frac{b_1\bar{m}-2b_2m_s}{M_N+\mu}\,, \tag{A2}$$

$$S_3(\mu)=\frac{M_N-\mu}{M_\Sigma+\mu}(D-F)+2b_2\frac{\bar{m}+m_s}{M_\Sigma+\mu}\,, \tag{A3}$$

$$S_4=\frac{M_N}{M_\Lambda}(D+3F)+2(2b_1-b_2)\frac{\bar{m}+m_s}{M_\Lambda}\,, \tag{A4}$$

where $\bar{m}$ and m_s denote the quark masses, and the strong-interaction constants D, F, b_1, and b_2 are defined in Sec. II. The expressions for ω modes as well as $\Delta S=0$ decay modes of the neutron are not explicitly given due to symmetry relations (3.9), (3.10), and (3.15).

The following expressions are useful for the kinematics of the problem:

$$\lambda_1(\mu,m)=\left[1+\frac{\mu}{M_N}\right]^2-\left[\frac{m}{M_N}\right]^2, \tag{A5}$$

$$\lambda_2(\mu,m)=\lambda_1(\mu,m)+\frac{4}{3}\frac{|\vec{q}\,|^2}{m^2}\,. \tag{A6}$$

The center-of-mass momentum of the decay particles, $|\vec{q}\,|$, is given by

$$|\vec{q}\,|=\frac{1}{2}\frac{W(M_N{}^2,\mu^2,m^2)}{M_N}\,, \tag{A7}$$

where

$$W(a,b,c)=[a^2+b^2+c^2-2(ab+ac+bc)]^{1/2}\,. \tag{A8}$$

1. Nucleon $\to$ pseudoscalar meson $+\, e_d^+$

The decay rates are given by

$$\Gamma=2\Gamma_0\frac{|\vec{q}\,|}{M_N}[\lambda_1(\mu,m)\,|A\,|^2+\lambda_1(-\mu,m)\,|B\,|^2]\,, \tag{A9}$$

$$B(\mu_d,c_d^{(1)},c_d^{(3)},\tilde{c}_d^{(1)},\tilde{c}_d^{(3)})=A(-\mu_d,-c_d^{(1)},-c_d^{(3)},-\tilde{c}_d^{(1)},-\tilde{c}_d^{(3)})\,. \tag{A10}$$

(a) $p \to \pi^0 e_d^+$:

$$A = -\frac{1}{2\sqrt{2}}[(\hat{\alpha}+\hat{\gamma})(c_d^{(1)}+c_d^{(2)})+(\hat{\beta}+\hat{\delta})(c_d^{(3)}+c_d^{(4)})][1+S_1(\mu_d)] . \tag{A11}$$

(b) $p \to \eta e_d^+$:

$$A = \frac{1}{2\sqrt{6}}\{(\hat{\alpha}+\hat{\gamma})(c_d^{(1)}+c_d^{(2)})[1-S_2(\mu_d)]-(\hat{\beta}+\hat{\delta})(c_d^{(3)}+c_d^{(4)})[3+S_2(\mu_d)]\} . \tag{A12}$$

(c) $p \to K^0 e_d^+$:

$$A = \tfrac{1}{2}[(\hat{\alpha}+\hat{\gamma})(\tilde{c}_d^{(1)}+\tilde{c}_d^{(2)})-(\hat{\beta}+\hat{\delta})(\tilde{c}_d^{(3)}+\tilde{c}_d^{(4)})]+\tfrac{1}{2}\left[\left[\hat{\alpha}+\frac{M_\Sigma}{M_N}\hat{\gamma}\right](\tilde{c}_d^{(1)}+\tilde{c}_d^{(2)})+\left[\hat{\beta}+\frac{M_\Sigma}{M_B}\hat{\delta}\right](\tilde{c}_d^{(3)}+\tilde{c}_d^{(4)})\right]S_3(\mu_d) . \tag{A13}$$

2. Nucleon→pseudoscalar meson $+\bar{\nu}_d$

The decay rates are given by

$$\Gamma = 2\Gamma_0 \frac{|\vec{q}|^2}{M_N{}^2}|A|^2 . \tag{A14}$$

(a) $p \to \pi^+ \bar{\nu}_d$:

$$A = [(\hat{\alpha}+\hat{\gamma})c_d^{(1)}+(\hat{\beta}+\hat{\delta})c_d^{(3)}][1+S_1(0)] . \tag{A15}$$

(b) $p \to K^+ \bar{\nu}_d$:

$$A = -[(\hat{\alpha}+\hat{\gamma})\tilde{c}_d^{(5)}+(\hat{\beta}+\hat{\delta})\tilde{c}_d^{(6)}]+\frac{1}{2}\left[\left[\hat{\alpha}+\frac{M_\Sigma}{M_N}\hat{\gamma}\right]\tilde{c}_d^{(1)}+\left[\hat{\beta}+\frac{M_\Sigma}{M_N}\hat{\delta}\right]\tilde{c}_d^{(3)}\right]S_3(0)$$
$$+\frac{1}{6}\left[\left[\hat{\alpha}+\frac{M_\Lambda}{M_N}\hat{\gamma}\right](\tilde{c}_d^{(1)}-2\tilde{c}_d^{(5)})+\left[\hat{\beta}+\frac{M_\Lambda}{M_N}\hat{\delta}\right](\tilde{c}_d^{(3)}-2\tilde{c}_d^{(6)})\right]S_4 . \tag{A16}$$

(c) $n \to \eta \bar{\nu}_d$:

$$A = \frac{1}{\sqrt{6}}[(\hat{\alpha}+\hat{\gamma})c_d^{(1)}(S_2(0)-1)+(\hat{\beta}+\hat{\delta})c_d^{(3)}(S_2(0)+3)] . \tag{A17}$$

(d) $n \to K^0 \bar{\nu}_d$:

$$A = -[(\hat{\alpha}+\hat{\gamma})(\tilde{c}_d^{(1)}+\tilde{c}_d^{(5)})+(\hat{\beta}+\hat{\delta})(\tilde{c}_d^{(6)}-\tilde{c}_d^{(3)})]-\frac{1}{2}\left[\left[\hat{\alpha}+\frac{M_\Sigma}{M_N}\hat{\gamma}\right]\tilde{c}_d^{(1)}+\left[\hat{\beta}+\frac{M_\Sigma}{M_N}\hat{\delta}\right]\tilde{c}_d^{(3)}\right]S_3(0)$$
$$+\frac{1}{6}\left[\left[\hat{\alpha}+\frac{M_\Lambda}{M_N}\hat{\gamma}\right](\tilde{c}_d^{(1)}-2\tilde{c}_d^{(5)})+\left[\hat{\beta}+\frac{M_\Lambda}{M_N}\hat{\delta}\right](\tilde{c}_d^{(3)}-2\tilde{c}_d^{(6)})\right]S_4 . \tag{A18}$$

3. Nucleon→vector meson $+e_d^+$

The decay rates are given by

$$\Gamma = 6\Gamma_0 \frac{|\vec{q}|}{M_N}\frac{1}{Z}\left[\frac{gf_\pi}{M_N}\right]^2[\lambda_2(\mu,m)|A|^2+\lambda_2(-\mu,m)|B|^2] , \tag{A19}$$

$$B(\mu_d,c_d^{(1)},c_d^{(3)},\tilde{c}_d^{(1)},\tilde{c}_d^{(3)}) = A(-\mu_d,-c_d^{(1)},-c_d^{(3)},-\tilde{c}_d^{(1)},-\tilde{c}_d^{(3)}) . \tag{A20}$$

In the expression for Γ, $\sqrt{2}g$ is the ρ-π-π coupling constant (2.6) and Z is the relevant renormalization constant (2.27).

(a) $p \to \rho^0 e_d^+$:

$$A = \frac{1}{2\sqrt{2}}\left\{\hat{\gamma}(c_d^{(1)}+c_d^{(2)})+\hat{\delta}(c_d^{(3)}+c_d^{(4)})+[(\hat{\alpha}+\hat{\gamma})(c_d^{(1)}+c_d^{(2)})+(\hat{\beta}+\hat{\delta})(c_d^{(3)}+c_d^{(4)})]\frac{M_N}{M_N+\mu_d}\right\} . \tag{A21}$$

(b) $p \to K^{*0} e_d^+$: From phase space, the final lepton can only be an electron and $d=1$. We have

$$A = \frac{1}{2}\left\{\hat{\gamma}(\tilde{c}_d^{(1)}+\tilde{c}_d^{(2)})+\hat{\delta}(\tilde{c}_d^{(3)}+\tilde{c}_d^{(4)})+\left[\left[\hat{\alpha}+\frac{M_\Sigma}{M_N}\hat{\gamma}\right](\tilde{c}_d^{(1)}+\tilde{c}_d^{(2)})+\left[\hat{\beta}+\frac{M_\Sigma}{M_N}\hat{\delta}\right](\tilde{c}_d^{(3)}+\tilde{c}_d^{(4)})\right]\frac{M_N}{M_\Sigma+\mu_e}\right\} . \tag{A22}$$

4. Nucleon → vector meson $+\bar{\nu}_d$

The decay rates are given by

$$\Gamma=4\Gamma_0\frac{|\vec{q}\,|^2}{M_N{}^2}\left[1+\frac{1}{2}\frac{M_N{}^2}{m^2}\right]\frac{1}{Z}\left[\frac{gf_\pi}{M_N}\right]^2|A|^2 . \tag{A23}$$

(a) $p\to\rho^+\bar{\nu}_d$:

$$A=(\hat{\alpha}+2\hat{\gamma})c_d^{(1)}+(\hat{\beta}+2\hat{\delta})c_d^{(3)} . \tag{A24}$$

(b) $p\to K^{*+}\bar{\nu}_d$:

$$A=-[\hat{\gamma}\tilde{c}_d^{(5)}+\hat{\delta}\tilde{c}_d^{(6)}]+\frac{1}{2}\left[\left[\frac{M_N}{M_\Lambda}\hat{\alpha}+\hat{\gamma}\right](\tilde{c}_d^{(1)}-2\tilde{c}_d^{(5)})+\left[\frac{M_N}{M_\Lambda}\hat{\beta}+\hat{\delta}\right](\tilde{c}_d^{(3)}-2\tilde{c}_d^{(6)})\right]$$
$$-\frac{1}{2}\left[\left[\frac{M_N}{M_\Sigma}\hat{\alpha}+\hat{\gamma}\right]\tilde{c}_d^{(1)}+\left[\frac{M_N}{M_\Sigma}\hat{\beta}+\hat{\delta}\right]\tilde{c}_d^{(3)}\right] . \tag{A25}$$

(c) $n\to K^{*0}\bar{\nu}_d$:

$$A=[\hat{\gamma}(\tilde{c}_d^{(1)}-\tilde{c}_d^{(5)}+\hat{\delta}(\tilde{c}_d^{(3)}-\tilde{c}_d^{(6)})]+\frac{1}{2}\left[\left[\frac{M_N}{M_\Lambda}\hat{\alpha}+\hat{\gamma}\right](\tilde{c}_d^{(1)}-2\tilde{c}_d^{(5)})+\left[\frac{M_N}{M_\Lambda}\hat{\beta}+\hat{\delta}\right](\tilde{c}_d^{(3)}-2\tilde{c}_d^{(6)}\right]$$
$$+\frac{1}{2}\left[\left[\frac{M_N}{M_\Sigma}\hat{\alpha}+\hat{\gamma}\right]\tilde{c}_d^{(1)}+\left[\frac{M_N}{M_\Sigma}\hat{\beta}+\hat{\delta}\right]\tilde{c}_d^{(3)}\right] . \tag{A26}$$

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tribute to two-body nucleon decays. If we then use the Dirac equation $\gamma_\mu \partial^\mu \underline{B} = M_p \underline{B}$, the structure of $\mathscr{L}_{(2)}^{\Delta B=1}$ becomes identical to $\mathscr{L}_{(1)}^{\Delta B=1}$. By redefining $\alpha+\gamma=\alpha'$ and $\beta+\delta=\beta'$, we thus get the $\Delta B=1$ Lagrangian of Ref. 10. We thank the referee for emphasizing that we point out this consistency with the work of Claudson, Wise, and Hall in the limit where the vector-meson fields are set to zero.

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[24]In Ref. 6, Marciano tabulates the values of M_X corresponding to the various values of $\Lambda_{\overline{\mathrm{MS}}}$.

[25]The dependence of $\Gamma(p \to \eta e^+)/\Gamma(p \to \pi^0 e^+)$ on the symmetry-breaking parameter b_2 was analyzed in Ref. 10, and for $|b_2| < 1$ it was shown that this ratio was less than 10%. This corresponds to the case where $\hat{\gamma}=0$.

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