# Study of $\pi^0 \to \mu^\pm e^\mp$ decays

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We deduce an experimental upper bound $\frac{1}{2}[\Gamma(\pi^0 \to \mu^+ e^-) + \Gamma(\pi^0 \to \mu^- e^+)]/\Gamma(\pi^0 \to \text{all}) < 7 \times 10^{-8}$ (90% C.L.) on the branching ratios of $\pi^0 \to \mu^\pm e^\mp$ decays from available data. We consider the existing constraints on the underlying muon-number-violating interaction and find that the present experimental information on $\mu^- \to e^-$ conversion in sulfur implies an upper limit on the $\pi^0 \to \mu^\pm e^\mp$ branching ratios which is not likely to be larger than $10^{-15}$.

## I. INTRODUCTION

In the minimal standard $SU(2)_L \times U(1)$ gauge theory of the electroweak interactions[1] (the standard model with a single Higgs doublet) muon-number violation can arise only if the neutrinos have mass. In this theory, if there are no further lepton families beyond the three we know, the largest possible branching ratios for muon-number-violating processes (such as $\mu \to e\gamma$, $\mu^- N \to e^- N$, $K_L \to \mu e$) are orders of magnitude below the present experimental limits.[2,3] This is due to the suppression factors $m_\nu^2/M_W^2$ contained in the amplitudes, and the experimental limits on the neutrino masses.[4] However, the branching ratios of muon-number-violating processes could be larger in many theoretical schemes that go beyond the minimal standard model. The possibilities include: existence of flavor-changing neutral gauge bosons (for example, the gauge bosons associated with horizontal gauge interactions,[5] or the gauge bosons present in extended hypercolor theories[6]); existence of flavor-changing neutral Higgs bosons (in the standard model this would require the presence of more than one Higgs doublets[7]); composite models;[8] muon-number violation mediated by light leptoquarks (present in some grand unified theories[9] and in extended hypercolor theories[6]); muon-number violation mediated by supersymmetric partners of the usual $SU(2)_L \times U(1)$ gauge bosons;[10] existence of new electroweak interactions.[11] Muon-number violation may be stronger also in the minimal standard model if further lepton families, involving heavier neutrinos, exist.

The relative size of the rates of various muon-number-violating reactions depends on the mechanism of muon-number violation, and also on the model and its parameters. It is therefore important to search for all possible muon-number-violating processes.

The purpose of this paper is to investigate the decays $\pi^0 \to \mu^\pm e^\mp$, potentially interesting for facilities with high-intensity pion beams.[12] A preliminary report on this study was given in Ref. 13. Subsequently, the decays $\pi^0 \to \mu^\pm e^\mp$ have been investigated in Ref. 14.

In Sec. II we deduce an experimental upper limit on the branching ratios of $\pi^0 \to \mu^\pm e^\mp$ from existing data. In Sec. III, the $\pi^0 \to \mu^\pm e^\mp$ amplitudes are given for a general local effective interaction. In Sec. IV we explore what constraints are imposed on the coupling constants of the effective interaction (and thus on the $\pi^0 \to \mu^\pm e^\mp$ branching ratios) by the present experimental information on $\mu^- \to e^-$ conversion in nuclei. We consider here also the effective interaction in the light of possible sources of muon-number violation suggested by current theories. The last section contains our conclusions.

## II. EXPERIMENTAL LIMIT ON THE $\pi^0 \to \mu^\pm e^\mp$ BRANCHING RATIOS

To date, no experiment has been carried out to search for the decays $\pi^0 \to \mu^\pm e^\mp$.[15] To determine the best existing limit for the branching ratios

$$B(\pi^0 \to \mu^\pm e^\mp) \equiv \Gamma(\pi^0 \to \mu^\pm e^\mp)/\Gamma(\pi^0 \to \text{all})$$

we have examined all past experiments that might have been sensitive to $\pi^0 \to \mu^\pm e^\mp$. An ideal experiment would have a copious source of $\pi^0$'s and some means of identifying the particles in the final state. The best limit we can find comes from a high-statistics experiment[16,17] designed to study $K^\pm \to \pi^+ \pi^- e^\pm \nu_e$ decays. This experiment, which was sensitive to about $10^9$ charged-kaon decays, also searched for various rare decay modes of the charged kaon, including $K^\pm \to \pi^\pm \mu^\mp e^\pm$. The $K^\pm \to \pi^\pm \pi^0$ decay mode, with a branching ratio of 21%, provided a copious, tagged source of neutral pions. The decays $\pi^0 \to \mu^\pm e^\mp$ would appear in charged-kaon decays through the cascade $K^{(\pm)} \to \pi^{(\pm)} \pi^0 \to \pi^{(\pm)} \mu^\pm e^\mp$. The experiment used two segmented threshold Cherenkov counters to identify $e^\pm$ with the same sign of charge as the decaying kaon. The excellent electron identification enabled them to search for $K^+ \to \pi^+ e^+ \mu^-$ and $K^- \to \pi^- e^- \mu^+$ without requiring a cut on the $\mu e$ effective mass. Their limit on $\Gamma(K^\pm \to \pi^\pm e^\pm \mu^\mp)$ and the fact that 56% (44%) of their data had incident positive (negative) kaons[17] yields

$$0.56\Gamma(K^+ \to \pi^+ e^+ \mu^-)/\Gamma(K^+ \to \text{all}) + 0.44\Gamma(K^- \to \pi^- \mu^+ e^-)/\Gamma(K^- \to \text{all}) < 1.4 \times 10^{-8} \text{ (90\% C.L.)} . \quad (1)$$

As

 

$$\Gamma(K^+\to\pi^+e^+\mu^-)/\Gamma(K^+\to\text{all})\ge B(\pi^0\to\mu^-e^+)\Gamma(K^+\to\pi^+\pi^0)/\Gamma(K^+\to\text{all})\ ,$$

$$\Gamma(K^-\to\pi^-e^-\mu^+)/\Gamma(K^-\to\text{all})\ge B(\pi^0\to\mu^+e^-)\Gamma(K^-\to\pi^-\pi^0)/\Gamma(K^-\to\text{all})\ ,$$

and since

$$\Gamma(K^+\to\pi^+\pi^0)/\Gamma(K^+\to\text{all})\simeq\Gamma(K^-\to\pi^-\pi^0)/\Gamma(K^-\to\text{all})$$

(Ref. 18), the bound (1) implies[13]

$$\tfrac{1}{2}[B(\pi^0\to\mu^+e^-)+B(\pi^0\to\mu^-e^+)]_{\text{expt}}<7\times10^{-8}\ (90\%\ \text{C.L.})\ . \quad (2)$$

An experiment specifically designed to search for $\pi^0\to\mu^\pm e^\mp$ could improve the sensitivity to this process. For example, an experiment designed to detect $K^+\to\pi^+\mu^\pm e^\mp$ (Ref. 19) could search for $\pi^0\to\mu^\pm e^\mp$ through the cascade $K^+\to\pi^+\pi^0\to\pi^+\mu^\pm e^\mp$. The detection of the $\pi^+$ with the right kinematics to indicate a missing mass of 135 MeV/$c^2$ tags the $\pi^0$. The experiment of Ref. 19 hopes to be sensitive to $\sim10^{11}$ $K^+$ decays. Since 21% of all $K^+$ decays are into the $\pi^+\pi^0$ channel, it may be possible to search for $\pi^0\to\mu^\pm e^\mp$ with a branching ratio as low as $10^{-10}$. A more sensitive search for $\pi^0\to\mu^\pm e^\mp$ would require a higher-intensity kaon beam than is presently available.

## III. THE $\pi^0\to\mu^\pm e^\mp$ AMPLITUDES

Diagrams describing some possible mechanisms for the decays $\pi^0\to\mu^\pm e^\mp$ are shown on Figs. 1 and 2. Regardless of the underlying decay mechanism, the amplitudes are of the general form[20]

$$\mathcal{M}(\pi^0\to\mu^+e^-)=C\bar{u}(p_-)i\gamma_5 v(q_+)+D\bar{u}(p_-)v(q_+)\ , \quad (3)$$

$$\mathcal{M}(\pi^0\to\mu^-e^+)=\tilde{C}\bar{u}(q_-)i\gamma_5 v(p_+)+\tilde{D}\bar{u}(q_-)v(p_+)\ , \quad (4)$$

where $p_-,p_+,q_-,q_+$ are the momenta of $e^-,e^+,\mu^-,\mu^+$ and $C,D,\tilde{C},\tilde{D}$ are constants. The decay rate for $\pi^0\to\mu^+e^-$ is given by

$$\Gamma(\pi^0\to\mu^+e^-)=\frac{m_\pi}{8\pi}[(1-r_e{}^2)+r_\mu{}^4-2r_\mu{}^2(1+r_e{}^2)]^{1/2}(1-r_e{}^2-r_\mu{}^2)$$
$$\times\left[|C|^2\left[1+2\frac{r_er_\mu}{1-r_\mu{}^2-r_e{}^2}\right]+|D|^2\left[1-2\frac{r_er_\mu}{1-r_\mu{}^2-r_e{}^2}\right]\right]$$
$$\simeq\frac{m_\pi}{8\pi}(1-r_\mu{}^2)^2(|C|^2+|D|^2)\ , \quad (5)$$

where $r_e=m_e/m_\pi$ and $r_\mu=m_\mu/m_\pi$ ($m_e,m_\mu$, and $m_\pi$ are the masses of the electron, muon, and pion). The same expression holds for $\Gamma(\pi^0\to\mu^-e^+)$, but with $C$ and $D$ replaced by $\tilde{C}$ and $\tilde{D}$, respectively.[21]

In the following we shall assume that the muon-number-violating quark-lepton interaction responsible for $\pi^0\to\mu^\pm e^\mp$ can be represented by a local Hermitian[21] effective Hamiltonian of the form

$$H_{\text{eff}}=\frac{G}{\sqrt{2}}[(f_{VA}\bar{e}\gamma^\lambda\mu+f_{AA}\bar{e}\gamma^\lambda\gamma_5\mu)J_\lambda^A$$
$$+(f_{SP}\bar{e}\mu+f_{PP}\bar{e}i\gamma_5\mu)J^P]+\text{H.c.}\ , \quad (6)$$

where $f_{VA}$, $f_{AA}$, $f_{SP}$, and $f_{PP}$ are constants, $G\equiv$ Fermi constant ($\simeq10^{-5}m_p{}^{-2}$), and $J_\lambda^A$, $J^P$ are isovector operators given by

$$J_\lambda^A=\tfrac{1}{2}(\bar{u}\gamma_\lambda\gamma_5u-\bar{d}_{\gamma_\lambda\gamma_5}d)\ , \quad (7)$$

$$J^P=\tfrac{1}{2}(\bar{u}i\gamma_5u-\bar{d}i\gamma_5d)\ . \quad (8)$$

The interaction (6) is the most general local nonderivative four-fermion coupling that can contribute to $\pi^0\to\mu^\pm e^\mp$. It accounts for the contribution of the diagrams of Fig. 1 (cf. Sec. IV), and also for the contribution of the diagrams in Fig. 2 unless the square of the mass of the mediating boson is comparable to or smaller than the square of the four-momentum it can carry. We shall comment on the

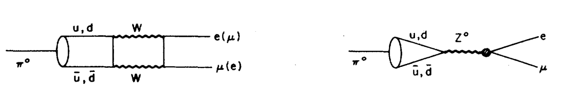

FIG. 1. Electroweak contributions to $\pi^0\to\mu^\pm e^\mp$. (a) Box diagrams; (b) induced $Z^0$ contribution.

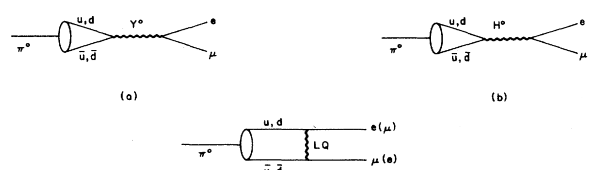

FIG. 2. Nonelectroweak contributions to $\pi^0\to\mu^\pm e^\mp$. (a) Contribution of a neutral flavor-changing gauge boson; (b) neutral flavor-changing Higgs-boson contribution; (c) leptoquark contribution.

latter possibility in Sec. V.

The Hamiltonian (6) gives rise to the following $\pi^0 \to \mu^\pm e^\mp$ amplitudes:

$$C = \frac{G}{\sqrt{2}}[f_{AA}(m_e+m_\mu)m_\pi \rho_A + f_{PP} m_\pi{}^2 \rho_P] , \quad (9)$$

$$D = \frac{G}{\sqrt{2}}[i f_{VA}(m_e-m_\mu)m_\pi \rho_A + f_{SP} m_\pi{}^2 \rho_P] , \quad (10)$$

$$\tilde{C} = \frac{G}{\sqrt{2}}[f^*_{AA}(m_e+m_\mu)m_\pi \rho_A + f^*_{PP} m_\pi{}^2 \rho_P] , \quad (11)$$

$$\tilde{D} = \frac{G}{\sqrt{2}}[i f^*_{VA}(m_\mu-m_e)m_\pi \rho_A + f^*_{SP} m_\pi{}^2 \rho_P] , \quad (12)$$

where $\rho_A$ and $\rho_P$ are defined by

$$\langle 0 | J_\lambda^A | \pi^0(p) \rangle = i m_\pi \rho_A p_\lambda , \quad (13)$$

$$\langle 0 | J^P | \pi^0(p) \rangle = m_\pi{}^2 \rho_P . \quad (14)$$

The constant $\rho_A$ is related to the charged-pion decay constant $f_\pi$ [defined by

$$\langle 0 | \bar{d}\gamma_\lambda \gamma_5 u \mid \pi^+(p) = i f_\pi p_\lambda ,$$

$f_\pi \simeq 131.9$ MeV (Ref. 22)] as

$$\rho_A = -f_\pi / m_\pi \sqrt{2} \simeq \pm 0.7 .$$

$\rho_P$ can be related to $\rho_A$ using the equation

$$\partial^\lambda J_\lambda^A = m_u \bar{u} i \gamma_5 u - m_d \bar{d} i \gamma_5 d , \quad (15)$$

where $m_u$ and $m_d$ are the (current) masses of $u$ and $d$. One finds[23]

$$\rho_P = \rho_A m_\pi / (m_u + m_d) . \quad (16)$$

Hence we obtain (neglecting the mass of the electron)

$$|C| = |\tilde{C}| \simeq |(8\times 10^{-8}) f_{AA} + 10^{-7} \omega f_{PP}| , \quad (17)$$

$$|D| = |\tilde{D}| \simeq |(8\times 10^{-8}) i f_{VA} + 10^{-7} \omega f_{SP}| , \quad (18)$$

where we have denoted $\omega = m_\pi/(m_u+m_d)$. The corresponding branching ratios are [using $\Gamma(\pi^0 \to \text{all}) \simeq 7.9$ eV (Ref. 24)]

$$B(\pi^0 \to \mu^+ e^-) = B(\pi^0 \to \mu^- e^+) \simeq 6.7\times 10^{-10}(|f_{AA} + 1.3\omega f_{PP}|^2 + |i f_{VA} + 1.3\omega f_{SP}|^2) . \quad (19)$$

It follows that the experimental limit (2) constrains the coupling constants of (6) as

$$(|f_{AA} + 1.3\omega f_{PP}|^2 + |i f_{VA} + 1.3\omega f_{SP}|^2)^{1/2} \lesssim 10 . \quad (20)$$

## IV. CONSTRAINTS ON THE MUON-NUMBER-VIOLATING COUPLINGS FROM $\mu^- \to e^-$ CONVERSION

Could $\pi^0 \to \mu^\pm e^\mp$ occur with a branching ratio near the experimental limit (2)? To give an answer, we have to investigate what constraints are imposed on the muon-number-violating couplings by other data. To date, the only further direct information on strangeness-conserving muon-number-violating couplings between quarks and leptons comes from experiments searching for conversion of negative muons into electrons in nuclei.[25–27] Among these we find the most constraints for $\pi^0 \to \mu^\pm e^\mp$ to be the results of a recent search for $\mu^- \to e^-$ conversion in sulfur.[26]

In the experiment of Ref. 26 no electrons were seen in the momentum range 96–112 MeV/$c$. This implies a limit[26]

$$B_{\mu e}^{\text{coh}}({}^{32}\text{S})_{\text{expt}} \equiv [\Gamma_{\mu e}^{\text{coh}}({}^{32}\text{S}) / \Gamma_{\mu\nu}^{\text{tot}}({}^{32}\text{S})]_{\text{expt}} < 7\times 10^{-11} \text{ (90\% C.L.)} \quad (21)$$

on the branching ratio of the coherent conversion process [the process in which the nucleus remains in the ground state, and consequently the electrons are monoenergetic, with the maximum possible energy ($E_e^{\max} = 104.7$ MeV for a ${}^{32}$S target)]. In Eq. (21), $\Gamma_{\mu\nu}^{\text{tot}}({}^{32}\text{S})$ is the total $\mu^- + {}^{32}\text{S} \to \nu_\mu + {}^{32}\text{P}^*$ capture rate.

Coherent $\mu^- \to e^-$ conversion would take place if the muon-number-violating interaction contained terms involving scalar or vector quark densities. Such terms do not contribute to $\pi^0 \to \mu^\pm e^\mp$ decays. To contribute to coherent conversion on ${}^{32}$S the couplings must also be isoscalar, the ground state of ${}^{32}$S being an $I=0$ state. For couplings of the forms

$$\mathscr{L}_V^{I=0} = \frac{G}{\sqrt{2}}(f_{VV}^{(0)} \bar{e}\gamma^\lambda \mu + f_{AV}^{(0)} \bar{e}\gamma^\lambda \gamma_5 \mu)\tfrac{1}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d) \quad (22)$$

and

$$\mathscr{L}_S^{I=0} = \frac{G}{\sqrt{2}}(f_{SS}^{(0)} \bar{e}\mu + f_{PS}^{(0)} \bar{e} i\gamma_5 \mu)\tfrac{1}{2}(\bar{u}u + \bar{d}d) , \quad (23)$$

the branching ratios of coherent $\mu^- + {}^{32}\text{S} \to e^- + {}^{32}\text{S}$ conversion are[28]

$$B_{\mu e}^{\text{coh}}({}^{32}\text{S}) = 76.3(|f_{VV}^{(0)}|^2 + |f_{AV}^{(0)}|^2) , \quad (24)$$

and

$$B_{\mu e}^{\text{coh}}({}^{32}\text{S}) = 69.1(|f_{SS}^{(0)}|^2 + |f_{PS}^{(0)}|^2) ; \quad (25)$$

consequently, the limit (21) implies

$$(|f_{VV}^{(0)}|^2 + |f_{AV}^{(0)}|^2)^{1/2} \lesssim 9.6\times 10^{-7} \quad (26)$$

and

$$(|f_{SS}^{(0)}|^2 + |f_{PS}^{(0)}|^2)^{1/2} \lesssim 10^{-6} . \quad (27)$$

The experiment of Ref. 26 constrains also the coupling constants of the interaction (6). For couplings of this type the conversion strength is not concentrated at the end of the electron momentum spectrum but instead the average electron energy is expected to be around 80 MeV, as the average neutrino energy in ordinary muon capture.[29,30] The electron spectrum in Ref. 26 was measured for electron momenta above 80 MeV/$c$. Bounds on the muon-number-violating coupling constants could be obtained by calculating capture rates to particular excited states of ${}^{32}$S and comparing the results with experimental upper limits for these rates, deduced from the measured electron yields and the theoretical electron momentum spectrum due to known processes. For our purposes it will be sufficient to concentrate on the quantity

$$B_{\mu e}(^{32}\text{S};80\text{ MeV}/c)=\Gamma_{\mu e}(^{32}\text{S};80\text{ MeV}/c)/\Gamma_{\mu\nu}^{\text{tot}}(^{32}\text{S})\equiv\sum_{p_-=80\text{ MeV}/c}^{p_-=p_-^{\max}}\Gamma_{\mu e}(^{32}\text{S};p_-)/\Gamma_{\mu\nu}^{\text{tot}}(^{32}\text{S})\,, \quad (28)$$

i.e., on the branching ratio corresponding to the sum of branching ratios of $\mu^-\to e^-$ transitions over the full momentum range in which the electrons were searched for. The experimental electron momentum spectrum agrees with the expected spectrum above 80 MeV/$c$ from $\mu^-$ decay in orbit and from radiative $\mu$ capture in both shape and rate to within $\sim 20\%$. Taking this as an estimate for the largest $\mu^-\to e^-$ signal which could be present in the data, we find

$$B_{\mu e}(^{32}\text{S};80\text{ MeV}/c)_{\text{expt}}\lesssim 4\times 10^{-9}\,. \quad (29)$$

Since for a given nucleus a substantial fraction of all $\mu^-\to e^-$ transitions is expected to result in electrons with momenta above 80 MeV/$c$, the experimental limit (29) enables us to set a rather secure upper bound on the total $\mu^-\to e^-$ conversion rate $\Gamma_{\mu e}^{\text{tot}}(^{32}\text{S})$, a quantity which can be estimated without the knowledge of the nuclear wave functions. Assuming that $\Gamma_{\mu e}(^{32}\text{S};\,80\text{ MeV}/c)$ accounts for a half or more of all $\mu^-\to e^-$ transitions, (29) implies

$$B_{\mu e}^{\text{tot}}(^{32}\text{S})_{\text{expt}}\equiv[\Gamma_{\mu e}^{\text{tot}}(^{32}\text{S})/\Gamma_{\mu\nu}^{\text{tot}}(^{32}\text{S})/]_{\text{expt}}\lesssim 8\times 10^{-9}\,. \quad (30)$$

The contribution of (6) to $\Gamma_{\mu e}^{\text{tot}}(^{32}\text{S})$ can be estimated using the closure approximation,[29] which gives a good description of total rates for ordinary muon capture for a wide range of elements.[31] For the total $\mu^-\to e^-$ conversion rate on a nucleus of charge $Z$ and atomic number $A$ we find, following the procedure of Ref. 29,

$$\Gamma_{\mu e}^{\text{tot}}(Z,A)=\frac{G^2}{16\pi^2}\frac{m_\mu{}^3\bar{E}^2}{(137)^3}Z^3A\,|\varphi_\mu|^2{}_{\text{av}}\lambda\left[1-\frac{Z^2+(A-Z)^2}{2A^2}\delta^{(n)}\right], \quad (31)$$

where $\bar{E}$ is the average electron energy, $|\varphi_\mu|^2{}_{\text{av}}$ represents an average of $|\varphi_\mu|^2$ [$\varphi_\mu\equiv$ muon wave function normalized as $\varphi_\mu(0)=1$] over the nuclear volume, and $\delta^{(n)}$ is a nucleon-nucleon correlation parameter,[32] analogous to the quantity $\delta$ appearing in the closure formula for the total $\mu^-+Z\to\nu_\mu+(Z-1)$ capture rate.[29] The quantity $\lambda$ is given by

$$\lambda=\left[3g_A{}^2+g_A(g_A+m_\mu g_P)\frac{\bar{E}}{M}+(g_A+m_\mu g_P)^2(\bar{E}/2M)^2\right](|f_{VA}|^2+|f_{AA}|^2)+h_P{}^2(\bar{E}/2M)^2(|f_{SP}|^2+|f_{PP}|^2)$$
$$+h_P(\bar{E}/M)\left[g_A+(g_A+m_\mu g_P)\frac{\bar{E}}{2M}\right]\text{Re}(if_{VA}f_{SP}^*-f_{AA}f_{PP}^*)\,. \quad (32)$$

In Eq. (32) $M$ is the nucleon mass and $g_A,g_P,h_P$ are the form factors in single-nucleon matrix elements of $J_\lambda^A$ and $J^P$

$$\langle N(p')|J_\lambda^A|N(p)\rangle=\bar{N}(p')[g_A(q^2)\gamma_\lambda\gamma_5+g_P(q^2)\gamma_5 q_\lambda]\frac{\tau_3}{2}N(p)\,,$$
$$\langle N(p')|J^P|N(p)\rangle=\bar{N}(p')\left[h_P(q^2)i\gamma_5\frac{\tau_3}{2}\right]N(p)\,, \quad (33)$$

evaluated at $q^2=m_\mu{}^2-2m_\mu\bar{E}\,(q=p_--q_-)$.

To estimate $\Gamma_{\mu e}^{\text{tot}}(Z,A)$ we shall take $\bar{E}=80$ MeV, suggested by the magnitude of the average neutrino energy in ordinary muon capture. It follows that $q^2\simeq -0.5m_\mu{}^2$. Thus

$$g_A\equiv g_A(q^2=-0.5m_\mu{}^2)=(1+0.5m_\mu{}^2/m_A{}^2)^{-2}g_A(0)\simeq 1.25\,, \quad (34)$$

where we have used $m_A=1.07$ GeV (Ref. 33) and $g_A(0)=1.26$ (Ref. 34). For $g_P$ we shall use the PCAC (partial conservation of axial-vector current) prediction[35]

$$m_\mu g_P(q^2)\simeq -\sqrt{2}f_\pi g_{\pi NN}\frac{m_\mu}{m_\pi{}^2-q^2}+(4Mm_\mu/m_A{}^2)g_A(0)\,, \quad (35)$$

where $g_{\pi NN}$ ($\simeq 13.4$) (Ref. 36) and $f_\pi$ ($\simeq 131.9$ MeV) (Ref. 22) are the pion-nucleon coupling constant and the pion decay constant, respectively. Consequently,

$$m_\mu g_P(q^2=-0.5m_\mu{}^2)\simeq -10.7\,. \quad (36)$$

$h_P$ can be estimated using the relation (15). Neglecting the isoscalar contribution (proportional to $m_u-m_d$), we find

$$h_P(q^2)=(\omega/m_\pi)[2Mg_A(q^2)-q^2g_P(q^2)]\,, \quad (37)$$

and consequently

$$h_P=h_P(q^2=-0.5m_\mu{}^2)\simeq 13.2\omega\,, \quad (38)$$

where, as before, $\omega=m_\pi/(m_u+m_d)$. Using (34), (36), (38), $|\varphi|^2{}_{\text{av}}\simeq 0.53$ (Ref. 37), $\delta^{(n)}\simeq 3$ (Ref. 32), and $\Gamma_{\mu\nu}^{\text{tot}}(^{32}\text{S})_{\text{expt}}=(1.352\pm 0.003)\times 10^6\text{ sec}^{-1}$ (Ref. 31), we obtain

$$B_{\mu e}^{\text{tot}}(^{32}\text{S})\simeq(1.9\times 10^{-1})(|f_{VA}|^2+|f_{AA}|^2)+(1.6\times 10^{-2})(|f_{SP}|^2+|f_{PP}|^2)\omega^2+(4.7\times 10^{-2})\omega\,\text{Re}(if_{VA}f_{SP}^*-f_{AA}f_{PP}^*)\,. \quad (39)$$

Hence the experimental bound (30) implies

$$[(|f_{VA}|^2+|f_{AA}|^2)+(8.2\times10^{-2})\omega^2(|f_{SP}|^2+|f_{PP}|^2)+(2.5\times10^{-1})\omega\,\mathrm{Re}(if_{VA}f_{SP}^*-f_{AA}f_{PP}^*)]^{1/2}\lesssim 2\times10^{-4} \quad (40)$$

for the coupling constants of (6).[38]

Let us consider now the consequences of the bound (40) on the $\pi^0\to\mu^\pm e^\mp$ branching ratios.

For a pure axial-vector-type coupling (40) yields

$$(|f_{VA}|^2+|f_{AA}|^2)^{1/2}\lesssim 2\times10^{-4} . \quad (41)$$

As a result, we obtain the bound [cf. Eq. (19)]

$$B(\pi^0\to\mu^\pm e^\mp)\lesssim 3\times10^{-17} . \quad (42)$$

For the pseudoscalar-type couplings the constraint is

$$(|f_{SP}|^2+|f_{PP}|^2)^{1/2}\lesssim(7\times10^{-4})/\omega , \quad (43)$$

implying

$$B(\pi^0\to\mu^\pm e^\mp)\lesssim 6\times10^{-16} , \quad (44)$$

independent of the quark masses.[39]

To conclude this section, we shall consider yet briefly some possible sources of the effective interaction (6).

### A. The minimal standard model

The lowest-order diagrams describing the transitions $u\bar{u}\to\mu^\pm e^\mp, d\bar{d}\to\mu^\pm e^\mp$ relevant to $\pi^0\to\mu^\pm e^\mp$ are shown in Fig. 1. Their contribution to the effective coupling constants of the Hamiltonian (6) is given by[40]

$$f_{VA}=-f_{AA}$$
$$=\sum_{j=1}^{n}\frac{3Gm_j^2}{4\pi^2\sqrt{2}}U_{\mu j}^*U_{ej}\left[\left[1+\frac{m_j^2}{m_W^2}\right]\ln\frac{M_W^2}{m_j^2}+\frac{1}{3}\right] , \quad (45)$$
$$f_{SP}=f_{PP}=0 ,$$

where $U_{ej}, U_{\mu j}$ are elements of the neutrino mixing matrix, $m_j$ are the neutrino masses, and $n$ is the number of fermion generations.

With only the three known lepton families, the largest possible neutrino mass is given by the upper limit of 250 MeV for $m_3$ (Ref. 4). Using the constraint $|U_{e3}U_{\mu 3}^*|\lesssim10^{-2}$ valid for 35 MeV$\lesssim m_3\lesssim$300 MeV (Ref. 41), we obtain

$$|f_{VA}|=|f_{AA}|\lesssim10^{-8} , \quad (46)$$

and consequently $B(\pi^0\to\mu^\pm e^\mp)\lesssim10^{-25}$. $|f_{VA}|, |f_{AA}|$ could be larger than the upper bound (46) if further lepton families exist. It should be noted however that for a wide range of values of the heavy-neutrino mass $B(\mu\to e\gamma)$ and $B_{\mu e}^{\mathrm{coh}}(^{32}\mathrm{S})$ are more constraining for $f_{VA}, f_{AA}$ than $B_{\mu e}^{\mathrm{tot}}(^{32}\mathrm{S})$.

### B. Extended electroweak models

An attractive extension of the standard model is a gauge theory of the electroweak interactions based on $SU(2)_L\times SU(2)_R\times U(1)$. In some versions of these models muon-number-violating processes may have large rates even in the three-generation case, due to the presence of heavy right-handed neutrinos.[11] The dominant diagrams for $u\bar{u}\to\mu^\pm e^\mp, d\bar{d}\to\mu^\pm e^\mp$ are expected to be the same as those in Fig. 1, but with the left-handed gauge bosons replaced by their right-handed counterparts. The corresponding coupling constants $f_{VA}$ and $f_{AA}$ are proportional to

$$[(M_1^2-M_2^2)/m_{W_R}^2](m_{W_L}^2/m_{W_R}^2)$$

($M_1, M_2$ are heavy-neutrino masses),[11] and could be larger than the bound (46). To our knowledge, sufficiently detailed calculations are not available, but also in these models $B(\mu\to e\gamma)$ and $B_{\mu e}^{\mathrm{coh}}(^{32}\mathrm{S})$ are expected to constrain $f_{VA}, f_{AA}$ more than $\Gamma_{\mu e}^{\mathrm{tot}}(^{32}\mathrm{S})$ for a wide range of values of the heavy neutrino masses.

### C. Flavor-changing neutral-gauge-boson exchange [diagram (a) in Fig. 2]

Examples are the gauge bosons associated with possible horizontal gauge interactions.[5] The simplest possibility is horizontal interactions governed by a U(1) gauge group.[42] The associated gauge boson $Y$ is Hermitian and couples to the fermion mass eigenstates as[43]

$$\mathscr{L}_h=g_h(\eta_V\bar{u}\gamma_\lambda u+\eta_A\bar{u}\gamma_\lambda\gamma_5 u+\xi_V\bar{d}\gamma_\lambda d$$
$$+\xi_A\bar{d}\gamma_\lambda\gamma_5 d+\beta_V\bar{s}\gamma_\lambda d+\cdots$$
$$+\sigma_V\bar{\mu}\gamma_\lambda e+\sigma_A\bar{\mu}\gamma_\lambda\gamma_5 e$$
$$+\rho_V\bar{e}\gamma_\lambda e+\cdots)Y^\lambda+\mathrm{H.c.} , \quad (47)$$

where $g_h$ is the horizontal gauge coupling constant, and the quantities $\eta_V, \eta_A, \ldots$, depend on the U(1) quantum-number assignment and on the various mixing angles and phase parameters. In the absence of generation mixing the interaction (47) is flavor conserving. The Lagrangian (47) generates an effective four-fermion interaction (assuming $m_Y^2\gg p_Y^2$; $p_Y$= four-momentum of $Y$) of the form (6), with

$$f_{KA}=\sqrt{2}g_h^2\sigma_K^*(\eta_A-\xi_A)/Gm_Y^2 \quad (k=V,A) . \quad (48)$$

Values of $f_{VA}, f_{AA}$ as large as the bound (41) cannot be ruled out.[44] The bound (41) implies $m_Y\gtrsim 25g_h|\eta_A-\xi_A|^{1/2}(|\sigma_V|^2+|\sigma_A|^2)^{1/2}$ TeV.

Flavor-changing neutral color-singlet gauge bosons are present also in extended hypercolor models.[6] Since they generate the masses of the ordinary fermions, their masses (unlike those of the horizontal bosons discussed above) cannot be arbitrarily large. $\mu^-\to e^-$ conversion in the observable range is expected,[6] unless the pertinent mixing angles are too small.

### D. Flavor-changing neutral-Higgs-boson exchange [diagram (b) in Fig. 2]

In the standard model of the electroweak interactions with more than one Higgs doublets the interactions of the neutral Higgs bosons with the fermions contain in general flavor-changing terms.[7] The coupling of a Hermitian

Higgs field $\phi_H$ to the leptons and the quarks has the general form

$$\mathscr{L}_H = (g_S^{(u)}\bar{u}u + g_P^{(u)}\bar{u}i\gamma_5 u + g_S^{(d)}\bar{d}d + g_P^{(d)}\bar{d}i\gamma_5 d + g_S^{(sd)}\bar{s}d + \cdots + g_S\bar{\mu}e + g_S^*\bar{e}\mu + g_P\bar{\mu}i\gamma_5 e + g_P^*\bar{e}i\gamma_5\mu + \cdots)\phi_H \ . \quad (49)$$

The corresponding effective constants $f_{SP}, f_{PP}$ are (assuming $m_H{}^2 \gg P_H{}^2$)

$$f_{kP} = \sqrt{2}\, g_k^*(g_P^{(u)} - g_P^{(d)})/Gm_H{}^2 \quad (k = S,P) \ . \quad (50)$$

Values of $f_{SP}$ and $f_{PP}$ as large as the bound (43) are not ruled out.[44] Unlike in the model with one Higgs doublet, the Higgs-field–fermion couplings are undetermined. In one special case they are proportional to the heaviest fermion mass in the given charge sector.[45] Assuming $g_S = g_P = 2^{1/4}\sqrt{G}\, m_\tau$, $g_P^{(u)} = 2^{1/4}\sqrt{G}\, m_t$, $g_P^{(d)} = 2^{1/4}\sqrt{G}\, m_b$ (ignoring mixing angles), the bound (43) would imply [with $m_u = 4.2$ MeV, $m_d = 7.5$ MeV (Ref. 23), and assuming $m_t = 30$ GeV] $m_H \gtrsim 1.4$ TeV.

Contributions to the effective interaction (6) may also come from Higgs bosons associated with extensions of the standard model. In extended hypercolor theories the states corresponding to Higgs bosons are the color-singlet pseudo-Goldstone bosons.[6]

### E. Leptoquark exchange [diagram (c) in Fig. 2]

Leptoquarks are present in grand unified theories and also in extended hypercolor theories. In extended hypercolor theories[6] and also in some classes of grand unified theories[9] they are sufficiently light to mediate some rare processes with rates in the observable range.[46] In lowest order leptoquark exchange generates only semileptonic interactions. Both spin-one and spin-zero leptoquarks occur.

The most general four-fermion interaction for $u\bar{u} \to \mu^\pm e^\mp$ and $d\bar{d} \to \mu^\pm e^\mp$ transitions arising from the exchange of a spin-one leptoquark is of the form (assuming $m_{\rm LQ}{}^2 \gg p_{\rm LQ}{}^2$)

$$H_{\rm eff} = \frac{G}{\sqrt{2}}[(n_{VV}^{(u)}\bar{u}\gamma_\lambda\mu\bar{e}\gamma^\lambda u + n_{AA}^{(u)}\bar{u}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda\gamma_5 u + n_{VA}^{(u)}\bar{u}\gamma_\lambda\mu\bar{e}\gamma^\lambda\gamma_5 u + n_{AV}^{(u)}\bar{u}\gamma_\lambda\gamma_5\mu\bar{e}\gamma^\lambda u) + (u \to d)] + \text{H.c.} \quad (51)$$

With the fields rearranged in the $\bar{e}\Gamma_i\mu\bar{q}\Gamma_j q$ order, the Hamiltonian (51) is a sum of $V$-, $A$-, $S$-, and $P$-type couplings, with $f_{AA}^{(q)} = f_{VV}^{(q)}$, $f_{VA}^{(q)} = f_{AV}^{(q)}$, $f_{PP}^{(q)} = f_{SS}^{(q)}$, $f_{SP}^{(q)} = -f_{PS}^{(q)}$ $(q = u,d)$. As a consequence $f_{AA} = f_{VV}$, $f_{VA} = f_{AV}$, $f_{PP} = f_{SS}$, and $f_{SP} = -f_{PS}$ where the constants $f_{VV}$, $f_{AV}$, $f_{SS}$, and $f_{PS}$ are defined by

$$H = \frac{G}{\sqrt{2}}[(f_{VV}\bar{e}\gamma^\lambda\mu + f_{AV}\bar{e}\gamma^\lambda\gamma_5\mu)\tfrac{1}{2}(\bar{u}\gamma_\lambda u - \bar{d}\gamma_\lambda d) + (f_{SS}\bar{e}\mu + f_{PS}\bar{e}i\gamma_5\mu)\tfrac{1}{2}(\bar{u}u - \bar{d}d)] + \text{H.c.} \quad (52)$$

Thus limits on $f_{iA}$ $(i = V,A)$ and $f_{iP}$ $(i = S,P)$ can be obtained in this case also from coherent $\mu^- \to e^-$ transitions sensitive to isovector couplings. The best existing limits (barring cancellations among the contributing terms in the $\mu^- \to e^-$ amplitude) come from the experimental results of Ref. 27, and are comparable to the bounds (41) and (43).[47]

The effective interaction due to spin-zero leptoquark exchange is of the same form as (51) but involving scalar and pseudoscalar, rather than vector and axial-vector densities. In the Fierz-transformed form $f_{AA}^{(q)} = -f_{VV}^{(q)}$, $f_{VA}^{(q)} = -f_{AV}^{(q)}$, $f_{SP}^{(q)} = f_{PS}^{(q)}$, $f_{PP}^{(q)} = f_{SS}^{(q)}$ $(q = u,d)$, so that limits on $f_{VV}, f_{AV}$ $(f_{SS}, f_{PS})$ apply to $f_{AA}, f_{VA}$ $(f_{PP}, f_{SP})$ also in this case. The novel feature is the presence of tensor couplings

$$H_T = \frac{G}{\sqrt{2}}\{[(g_{TT}^{(u)}\bar{e}\sigma_{\lambda\nu}\mu + g_{TT}'^{(u)}\bar{e}\sigma_{\lambda\nu}i\gamma_5\mu)\bar{u}\sigma^{\lambda\nu}u] + [u \to d]\} + \text{H.c.} \quad (53)$$

(which contribute to incoherent $\mu^- \to e^-$ conversion, but not to $\pi^0 \to \mu^\pm e^\mp$) with $g_{TT}^{(q)} = g_{SS}^{(q)}$ and $g_{TT}'^{(q)} = ig_{PS}^{(q)}$ $(q = u,d)$.

## V. CONCLUSIONS

Our main results are the experimental limit (2), and the (considerably lower) phenomenological upper bounds (42) and (44) on the $\pi^0 \to \mu^\pm e^\mp$ branching ratios.

The bounds (42) and (44) would be weaker if the average electron energy and/or the ratio $\Gamma_{\mu e}(^{32}\text{S}$; 80 MeV$/c)/\Gamma_{\mu e}^{\rm tot}(^{32}\text{S})$ were smaller. However, the values of these quantities are not likely to differ appreciably from those we took. Effective coupling constants exceeding the bounds (41) and (43) are not completely ruled out also because there could be cancellations among the various possible contributions to $\mu^- \to e^-$ conversion.

The bounds (42) or (44), which correspond to the local interaction (6), could be violated considerably if—a remote possibility—the $\bar{q}q \to \mu e$ $(q = u,d)$ transition is mediated by a light boson of mass $(m_B)$ near $m_\pi$, exchanged in the $s$ channel. The right-hand sides of (42) or (44) would have to be multiplied then by the factor $[(m_B{}^2 + 0.5m_\mu{}^2)/(m_B{}^2 - m_\pi{}^2)]^2$, arising from the momentum-transfer dependence of the boson propagator.[48] Other nonlocal terms can be present in the effective muon-number-violating interaction from nontree contributions, but is unlikely that their effect would be appreciable.

We conclude that the present experimental information on $\mu^- \to e^-$ conversion[49] implies an upper bound on the $\pi^0 \to \mu^\pm e^\mp$ branching ratios which is not likely to be larger than about $10^{-15}$. Such branching ratios cannot be ruled out in some extensions of the three-generation minimal standard model. They are however too small to be experimentally accessible in the foreseeable future.

*Note added in proof.* It has recently been brought to our attention that the experimental limit on $\pi^0 \to \mu e$ obtained by Bryman (Ref. 14) is, in fact,

$$B(\pi^0 \to \mu^+ e^-) + B(\pi^0 \to \mu^- e^+) < 7 \times 10^{-8} \ (90\% \text{ C.L.}) .$$

We have verified this by examining the experiment from which Bryman obtained his limit. This limit is a factor of 2 more sensitive than our limit [Eq. (2)]. We thank Professor T. G. Trippe for pointing out this to us.


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$$B(\pi^0 \to \mu^+ e^-) + B(\pi^0 \to \mu^- e^+) < 7 \times 10^{-8}$$

given here should have read

$$\tfrac{1}{2}[B(\pi^0 \to \mu^+ e^-) + B(\pi^0 \to \mu^- e^+)] < 7 \times 10^{-8} .$$

The phenomenological upper bound of $10^{-12}$ for $B(\pi^0 \to \mu^\pm e^\mp)$ we gave here is less stringent than the one we have set now (cf. Sec. IV), in part because the experimental information on $\mu^- \to e^-$ conversion was less constraining then, and also because we took $h_P/\rho_P$ [defined in Eqs. (14) and (33)] to be of the order of one, which turns out to be an underestimate for this quantity [cf. Eqs. (16) and (38)].

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$$B_{\mu e}^{\rm tot}({\rm Cu}) \simeq 0.3(|f_{VA}|^2 + |f_{AA}|^2) + 0.02(|f_{SP}|^2 + |f_{PP}|^2)\omega^2 + 0.07\omega\,{\rm Re}(if_{VA}f_{SP}^* - f_{AA}f_{PP}^*)\;,$$

and the experiment of Conforto *et al.* (Ref. 25). In the latter five events were observed with $E_e > 75$ MeV. This number of events is consistent with the expected background from ordinary muon-decay events with pile-up energy in the NaI detector. From this we find

$$B_{\mu e}({\rm Cu}; 75\ {\rm MeV}/c)_{\rm expt} < 3 \times 10^{-7}\ (90\%\ {\rm C.L.})\;.$$

Although the experiment of Bryman *et al.* (Ref. 25) set a more stringent limit for coherent $\mu^- \to e^-$ conversion, no limit for the incoherent conversion can be obtained because the measured electron spectrum is unavailable. The experiment of Ref. 27 has set an upper limit of $2 \times 10^{-9}$ (90% C.L.) on coherent $\mu^- \to e^-$ conversion in Mn, but, again, the measured electron spectrum is unavailable.

[39]Note that the ratio $\Gamma(\pi^0 \to \mu e)/\Gamma_{\mu e}^{\rm tot}(^{32}{\rm S})$ is $\sim 30$ and $\sim 600$ for axial-vector- and pseudoscalar-type couplings, respectively. $B(\pi^0 \to \mu e)/B_{\mu e}^{\rm tot}(^{32}{\rm S})$ is suppressed however by the small ratio

$$\Gamma_{\mu\nu}^{\rm tot}(^{32}{\rm S})/\Gamma(\pi^0 \to {\rm all})\ (\simeq 10^{-10})\;.$$

[40]Altarelli, Baulieu, Cabibbo, Maiani, and Petronzio (Ref. 2).

[41]From a measurement of the positron spectrum in $\pi^+ \to e^+\nu_e$ one has $|U_{e3}|^2 \lesssim 10^{-4}$ for 35 MeV $\lesssim m_3 \lesssim 120$ MeV [D. A. Bryman *et al.*, Phys. Rev. Lett. 50, 1546 (1983)], and a measurement of the momentum spectrum in $K^+ \to \pi^+\nu_\mu$ yields $|U_{\mu 3}|^2 \lesssim 10^{-4}$ in the range 70 MeV $\lesssim m_3 \lesssim 300$ MeV [R. S. Hayano *et al.*, Phys. Rev. Lett. 49, 1305 (1982)]. Thus $|U_{e3}U_{\mu e}^*| \lesssim 10^{-2}$ for 35 MeV $\lesssim m_3 \lesssim 300$ MeV. For $\omega_3 = 250$ one has $|f_{VA}| = |f_{AA}| \lesssim 4.7 \times 10^{-9}$. If $m_3 < 35$ MeV, then $|f_{VA}| = |f_{AA}| \lesssim 10^{-8}$ even for $|U_{e3}U_{\mu e}^*| \lesssim 1$.

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[47]Using the calculations of O. Shanker [Phys. Rev. D 20, 1608 (1979)] and the experimental bound of Ref. 27 on coherent $\mu^- \to e^-$ conversion in Mn we obtain

$$(|f_{VV}|^2 + |f_{AV}|^2)^{1/2} \lesssim 1.3 \times 10^{-4}$$

and

$$(|f_{SS}|^2 + |f_{PS}|^2)^{1/2} \lesssim 1.4 \times 10^{-4}\;.$$

[48]Note that $[(m_B{}^2 + 0.5m_\mu{}^2)/(m_B{}^2 - m_\pi{}^2)]^2 > 1$ for $m_B{}^2 > \frac{1}{2}(m_\pi{}^2 - 0.5m_\mu{}^2)$. We have neglected the width of $B$.

[49]It should be noted that a detailed calculation of partial $\mu^- + {}^{32}{\rm S} \to e^- + {}^{32}{\rm S}^*$ conversion rates in the region $p_- \gtrsim 80$ MeV/$c$ might yield a bound on the coupling constants which is more stringent than (41) and (43) if an important $\mu^- \to e^-$ transition is found in an electron momentum band where the background is small.