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Polarization of recoil-electron beam in high-energy Møller and Bhabha scattering

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Longitudinal and transverse polarizations of the final electron beam in Møller and Bhabha scattering are analyzed in electroweak gauge models involving more than one neutral vector boson. In pure QED, for given initial polarizations of incoming particles, the polarization of the outgoing electron beam is the same for Møller and Bhabha scattering and is energy independent. However, in various electroweak gauge models, we find that the energy dependence and final-state polarizations are in general different for the two scattering processes; thus, observing polarization effects in such scattering processes may be useful to test various gauge models at high energy.

I. INTRODUCTION

The discovery of the weak intermediate vector bosons $W^{\pm}$ (Refs. 1 and 2) and Z^0 (Refs. 3 and 4) at the CERN $p\bar{p}$ collider supports strongly the standard model[5] of electroweak theory and sets limits to viable alternative models. The left-right-symmetric model of Rizzo and Senjanović[6] predicts too light a mass for the intermediate vector boson Z^0 and a bit too large a value for $\sin^2\theta_W$. Another model based on the gauge group $SU(2)_L \times T_{3R} \times U(1)_{B\text{-}L}$, which was considered by Deshpande and Johnson,[7] predicts also some deviation from the result of the standard model. However, despite much information from recent experiments, there exists considerable freedom with left-right models. In a left-right model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B\text{-}L}$, as considered by Barger, Ma, and Whisnant,[8,9] the first neutral vector boson is indistinguishable in mass and fermion couplings from Z^0 of the standard model while the second neutral vector boson can have a mass as low as 200 GeV. The possible existence of a second neutral vector boson with a smaller effective coupling strength is not ruled out yet.

Møller (e^-e^-) and Bhabha (e^-e^+) scattering can be useful methods to distinguish various gauge models, because the effect of Z-boson exchange becomes more important at high energy. These processes have relatively large cross sections compared to other electroweak processes, and high-energy experimental data for them may become available in the near future from the Stanford Linear Collider (SLC) and CERN LEP.

The general covariant formula for the spin dependence of the cross sections for Møller and Bhabha scattering, within the one-photon-exchange approximation (i.e., in pure QED), was given by Bincer[10] and by Ford and Mullin.[11] They discussed longitudinally polarized incident particles. Raczka and Raczka[12] considered transversely polarized incident particles in Møller scattering. The polarization of the outgoing electron beam was considered by Kresnin and Rozentsveig.[13]

In the framework of the electroweak models, Møller scattering has been considered by Llewellyn Smith and Nanopoulos,[14] Gastmans and Van Ham,[15] DeRaad,[16] Budny,[17] and more recently by Puhala, Rizzo, and Young,[18] Olsen and Osland,[19] and Anders *et al.*[20] On the other hand, Bhabha scattering has been considered by Dicus,[21] Llewellyn Smith and Nanopoulos,[14] Budny and McDonald,[22] Budny,[23] and more recently by Hollik and Zepeda,[24] Olsen and Osland,[19] and Anders *et al.*[20] The complete expressions for the Møller- and Bhabha-scattering cross sections at high energy, for arbitrary initial polarization, have been obtained by Olsen and Osland.[19] Puhala, Rizzo, and Young[18] have calculated the differential cross section for Møller scattering when all particles are longitudinally polarized.

As is well known, the initially unpolarized electron and positron beams in the colliding machine become transversely polarized naturally due to the synchrotron radiation in the magnetic field. Therefore, it is worthwhile to study arbitrary polarizations of high-energy electrons and positrons in Møller and Bhabha scattering. Cooper *et al.*[25] have shown that the polarization of high-energy electrons can be measured by observing Møller scattering. Also the transverse polarization can be measured with almost the same degree of difficulty.

The purpose of this paper is to give the explicit form for the longitudinal and transverse polarizations of the recoil-electron beam in Møller and Bhabha scattering when incident beams are polarized arbitrarily. Our result is model independent but we have examined the predictions for QED, the standard model,[5] and the left-right model of Barger, Ma, and Whisnant[8,9] at the energy values of $\sqrt{S}=40$, 100 and 200 GeV which correspond to the energies of the present SLAC PEP and DESY PETRA, SLC, and LEP, respectively. The model dependence of the polarization of the recoil-electron beam can be seen at high energy.

II. MØLLER SCATTERING OF A POLARIZED BEAM

In Møller scattering, let (p_1^μ, s_1^μ) and (p_2^μ, s_2^μ) denote the momenta and polarizations of incoming electrons and $(p_1'^\mu, s_1^{f\mu})$ and $(p_2'^\mu, s_2^{f\mu})$ the corresponding quantities for outgoing electrons. Given the initial state

$$U^i_{\alpha\beta} = u_\alpha(p_1 s_1) u_\beta(p_2 s_2)\ , \qquad (1)$$

the final state $U^f_{\alpha\beta}$ can be obtained in the form $(MU^i)_{\alpha\beta}$. A general electroweak model with more than one neutral Z boson, together with one-photon exchange, can be described by the Lagrangian

$$L=\sum_f\left[eQ_f\bar{\psi}_f\gamma_\mu\psi_f A^\mu+\sum_i g_{Z_i}\bar{\psi}_f\gamma_\mu[\epsilon^f_{L_i}(1-\gamma_5)+\epsilon^f_{R_i}(1+\gamma_5)]\psi_f Z_i^\mu\right], \tag{2}$$

where f indicates specific fermions, and g_{Z_i}, $\epsilon^f_{L_i}$, and $\epsilon^f_{R_i}$ are model-dependent parameters. The final state can then be written as

$$\begin{aligned}U^f_{\alpha\beta}\equiv(MU^i)_{\alpha\beta}=&-\frac{e^2}{t}\left[\left[\frac{m+p\!\!/'_1}{2m}\right]\gamma_\mu u(p_1s_1)\right]_\alpha\left[\left[\frac{m+p\!\!/'_2}{2m}\right]\gamma^\mu u(p_2s_2)\right]_\beta\\&+\frac{e^2}{u}\left[\left[\frac{m+p\!\!/'_1}{2m}\right]\gamma_\mu u(p_2s_2)\right]_\alpha\left[\left[\frac{m+p\!\!/'_2}{2m}\right]\gamma^\mu u(p_1s_1)\right]_\beta\\&-\sum_i\frac{g_{Z_i}{}^2}{t-M_{Z_i}{}^2}\left[\left[\frac{m+p\!\!/'_1}{2m}\right]\gamma_\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_1s_1)\right]_\alpha\\&\times\left[\left[\frac{m+p\!\!/'_2}{2m}\right]\gamma^\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_2s_2)\right]_\beta\\&+\sum_i\frac{g_{Z_i}{}^2}{u-M_{Z_i}{}^2}\left[\left[\frac{m+p\!\!/'_1}{2m}\right]\gamma_\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_2s_2)\right]_\alpha\\&\times\left[\left[\frac{m+p\!\!/'_2}{2m}\right]\gamma^\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_1s_1)\right]_\beta,\end{aligned} \tag{3}$$

where t and u are equal to $(p_1-p'_1)^2$ and $(p_1-p'_2)^2$, respectively.

The density matrix for the final state is obtained from $U^f\bar{U}^f$. In particular, if one looks at only one of the outgoing electrons in the process, the density matrix for its polarization is given by

$$\rho^f_{\alpha\alpha'}=U^f_{\alpha\beta}\bar{U}^f_{\alpha'\beta}/\mathrm{Tr}(U^f\bar{U}^f). \tag{4}$$

The differential cross section with polarized incident electron, when summed over the final-state polarization, will be

$$d\sigma\propto\mathrm{Tr}(U^f\bar{U}^f). \tag{5}$$

Also note that the polarization four-vector of the outgoing electron beam can be expressed as

$$s^{f\mu}=\mathrm{Tr}(\gamma^\mu\gamma_5\rho^f). \tag{6}$$

Since the polarization four-vector s^μ is defined to be $(0,\underline{\vec{s}})$ in the particle's rest frame ($\underline{\vec{s}}$: general spin vector), it becomes at high energy

$$\vec{s}=\underline{\vec{s}}+\frac{\vec{p}(\vec{p}\cdot\underline{\vec{s}})}{m(E+m)}\cong\hat{p}\cdot\underline{\vec{s}}\frac{\vec{p}}{m}+\vec{s}_T, \tag{7a}$$

$$s^0=\frac{\vec{p}\cdot\underline{\vec{s}}}{m}\cong\hat{p}\cdot\underline{\vec{s}}\frac{E}{m}. \tag{7b}$$

Therefore, one may approximate

$$s^\mu=\frac{p^\mu}{m}\hat{p}\cdot\underline{\vec{s}}+s^\mu_T, \tag{7c}$$

where $\vec{s}_T\equiv\underline{\vec{s}}-\hat{p}\hat{p}\cdot\underline{\vec{s}}$ is the transverse component of $\underline{\vec{s}}$ (or of $\vec{s}$), and $\hat{p}$ is a unit vector in the direction of $\vec{p}$.

From Eqs. (3), (4), and (6), the final polarization of the outgoing electron beam can be obtained. For the calculation, we have found[26] that the following relations are especially useful:

$$\frac{1}{4}\left[1+\frac{p\!\!/}{m}\right](1+\gamma_5 s\!\!/)\cong\frac{p\!\!/}{4m}(1-\hat{p}\cdot\underline{\vec{s}}\gamma_5-s\!\!/_T\gamma_5), \tag{8a}$$

$$\left[1+\frac{p\!\!/}{m}\right]\gamma^\mu\gamma_5\left[1+\frac{p\!\!/}{m}\right]\cong2\frac{p^\mu}{m^2}\gamma_5 p\!\!/-\frac{2}{m}\gamma_5\gamma^\mu p\!\!/. \tag{8b}$$

With $s_1^{f\mu}$ divided into two parts as shown in Eq. (7c), the longitudinal polarization can be obtained from the coefficient of $p_1'^\mu/m$ while the transverse polarization $\vec{s}^{\,f}_{1T}$ is available from the explicit value of $\vec{s}^{\,f}_1$ multiplied by $(1-\hat{p}'_1\hat{p}'_1)$.

We now give the results of our calculations. In the center-of-mass frame, the longitudinal polarization of an outgoing electron beam with $(p'_1s^f_1)$ is given by

$$\hat{p}'_1\cdot\vec{s}^f_1=\frac{1}{D_M}\left\{(1+\hat{p}_1\cdot\vec{s}_1)(1+\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{R_i}{}^2\right]\right]^2\right.$$

$$-(1-\hat{p}_1\cdot\vec{s}_1)(1-\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}{}^2\right]\right]^2$$

$$\left.+2(\hat{p}_1\cdot\vec{s}_1-\hat{p}_2\cdot\vec{s}_2)\left[\cot^4(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right]^2-\tan^4(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}\epsilon_{R_i}\right]^2\right]\right\}, \quad (9)$$

and the transverse polarization by

$$\vec{s}^f_{1T}=-\frac{1}{D_M}\left[\hat{p}'_1\times[(\hat{p}'_1+\hat{p}_1)\times\vec{s}_{1T}]\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right]\right.$$

$$\times\left\{(1-\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}{}^2\right]\right]\right.$$

$$\left.+(1+\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{R_i}{}^2\right]\right]\right\}$$

$$+\hat{p}'_1\times[(\hat{p}'_1-\hat{p}_1)\times\vec{s}_{2T}]\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}\epsilon_{R_i}\right]$$

$$\times\left\{(1-\hat{p}_1\cdot\vec{s}_1)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}{}^2\right]\right]\right.$$

$$\left.\left.+(1+\hat{p}_1\cdot\vec{s}_1)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{R_i}{}^2\right]\right]\right\}\right]. \quad (10)$$

In the expressions (9) and (10), θ denotes the angle between $\vec{p}_1$ and $\vec{p}'_1$, and $h_i(t)$, $h_i(u)$ are defined as

$$h_i(t)=4g_{Z_i}{}^2t/[e^2(t-M_{Z_i}{}^2)], \quad (11a)$$

$$h_i(u)=4g_{Z_i}{}^2u/[e^2(u-M_{Z_i}{}^2)], \quad (11b)$$

and D_M by

$$D_M=(1-\hat{p}_1\cdot\vec{s}_1)(1-\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}{}^2\right]\right]^2$$

$$+(1+\hat{p}_1\cdot\vec{s}_1)(1+\hat{p}_2\cdot\vec{s}_2)\left[\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]+\cos^{-2}(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{R_i}{}^2\right]\right]^2$$

$$+2(1-\hat{p}_1\cdot\vec{s}_1\hat{p}_2\cdot\vec{s}_2)\left[\cot^4(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right]^2+\tan^4(\theta/2)\left[1+\sum_i h_i(u)\epsilon_{L_i}\epsilon_{R_i}\right]^2\right]$$

$$+4(\vec{s}_{1T}\cdot\vec{s}_{2T}-2\sin^{-2}\theta\,\hat{p}'_1\cdot\vec{s}_{1T}\hat{p}'_1\cdot\vec{s}_{2T})\left[1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right]\left[1+\sum_j h_j(u)\epsilon_{L_j}\epsilon_{R_j}\right]. \quad (12)$$

If we here choose the vectors $\hat{p}_1$, $\hat{p}'_1$, $\vec{s}_{1T}$, and $\vec{s}_{2T}$ according to

$$\hat{p}_1 = -\hat{p}_2 = (0,0,1)\ , \tag{13a}$$

$$\hat{p}'_1 = -\hat{p}'_2 = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\ , \tag{13b}$$

$$\vec{s}_{1T} = |\vec{s}_{1T}|(\cos\phi_1, \sin\phi_1, 0)\ , \tag{13c}$$

$$\vec{s}_{2T} = |\vec{s}_{2T}|(\cos\phi_2, \sin\phi_2, 0)\ , \tag{13d}$$

as in Ref. 19, the following relation can be used in the last term of Eq. (12),

$$\vec{s}_{1T}\cdot\vec{s}_{2T} - 2\sin^{-2}\theta\,\hat{p}'_1\cdot\vec{s}_{1T}\hat{p}'_1\cdot\vec{s}_{2T} = -|\vec{s}_{1T}|\,|\vec{s}_{2T}|\cos(2\phi-\phi_1-\phi_2)\ , \tag{14}$$

and the differential cross section obtained by Olsen and Osland[19] can be recovered:

$$\frac{d\sigma^M}{d\Omega} = \frac{\alpha^2}{4s} D_M \quad [s = (p_1+p_2)^2]\ . \tag{15}$$

III. BHABHA SCATTERING OF POLARIZED ELECTRON-POSITRON BEAMS

In Bhabha scattering, for the initial state of electron and positron as given by

$$U^i_{\alpha\beta} = u_\alpha(p_1 s_1)\bar{v}_\beta(p_2 s_2)\ , \tag{16}$$

the corresponding final state can be described by

$$\begin{aligned}
U^f_{\alpha\beta} = &-\frac{e^2}{t}\left[\left[\frac{m+\not{p}'_1}{2m}\right]\gamma_\mu u(p_1 s_1)\right]_\alpha \left[\bar{v}(p_2 s_2)\gamma^\mu\left[\frac{m-\not{p}'_2}{2m}\right]\right]_\beta \\
&+\frac{e^2}{s}\bar{v}(p_2 s_2)\gamma_\mu u(p_1 s_1)\left[\left[\frac{m+\not{p}'_1}{2m}\right]\gamma^\mu\left[\frac{m-\not{p}'_2}{2m}\right]\right]_{\alpha\beta} \\
&-\sum_i \frac{g_{Z_i}{}^2}{(t-M_{Z_i}{}^2)}\left[\left[\frac{m+\not{p}'_1}{2m}\right]\gamma_\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_1 s_1)\right]_\alpha \\
&\times\left[\bar{v}(p_2 s_2)\gamma^\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]\left[\frac{m-\not{p}'_2}{2m}\right]\right]_\beta \\
&+\sum_i \frac{g_{Z_i}{}^2}{s-M_{Z_i}{}^2+iM_{Z_i}\Gamma_i}\bar{v}(p_2 s_2)\gamma_\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]u(p_1 s_1) \\
&\times\left[\left[\frac{m+\not{p}'_1}{2m}\right]\gamma^\mu[\epsilon_{L_i}(1-\gamma_5)+\epsilon_{R_i}(1+\gamma_5)]\left[\frac{m-\not{p}'_2}{2m}\right]\right]_{\alpha\beta}\ .
\end{aligned} \tag{17}$$

In Bhabha scattering, note that timelike vector-boson propagators become important near the resonance energies, and Γ_i in Eq. (17) denote the resonance widths.

Using the same method as in Sec. II, we have obtained the longitudinal polarization of the outgoing electron beam for Bhabha scattering in the center-of-mass frame:

$$\begin{aligned}
\hat{p}'_1\cdot\vec{s}^f_1 = \frac{1}{D_B}\Bigg[&\cos^4(\theta/2)\Bigg[(1+\hat{p}_1\cdot\vec{s}_1)(1-\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{R_i}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]\right|^2 \\
&-(1-\hat{p}_1\cdot\vec{s}_1)(1+\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{L_i}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]\right|^2\Bigg] \\
&-2\sin^4(\theta/2)(\hat{p}_1\cdot\vec{s}_1-\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{L_i}\epsilon_{R_i}\right|^2+2\sin^{-4}(\theta/2)(\hat{p}_1\cdot\vec{s}_1+\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right|^2 \\
&-2\sin^2(\theta/2)\cos^2(\theta/2)|\vec{s}_{1T}|\,|\vec{s}_{2T}| \\
&\times\Bigg\{\cos(2\phi-\phi_1-\phi_2)\mathrm{Re}\left[\left[1+\sum_i h^*_i(s)\epsilon_{L_i}\epsilon_{R_i}\right]\sum_j(\epsilon_{L_j}{}^2-\epsilon_{R_j}{}^2)[h_j(s)-\sin^{-2}(\theta/2)h_j(t)]\right] \\
&+\sin(2\phi-\phi_1-\phi_2)\mathrm{Im}\Bigg[\sum_i h_i(s)(\epsilon_{L_i}-\epsilon_{R_i})^2 \\
&+\sin^{-2}(\theta/2)\sum_i h_i(s)\epsilon_{L_i}\epsilon_{R_i}\left[2+\sum_j h_j(t)(\epsilon_{L_j}{}^2+\epsilon_{R_j}{}^2)\right]\Bigg]\Bigg\}\Bigg]\ .
\end{aligned} \tag{18}$$

In Eq. (18), $h_i(s)$ is a complex function defined as

$$h_j(s)=4g_{Z_j}{}^2 s/[e^2(s-M_{Z_j}{}^2+iM_{Z_j}\Gamma_j)] \quad (19a)$$

$$=\mathrm{Re}h_j(s)+i\,\mathrm{Im}h_j(s) \quad (19b)$$

and the quantity D_B is given by

$$\begin{aligned} D_B=\cos^4(\theta/2)\Bigg[&(1+\hat{p}_1\cdot\vec{s}_1)(1-\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{R_i}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{R_i}{}^2\right]\right|^2 \\ &+(1-\hat{p}_1\cdot\vec{s}_1)(1+\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{L_i}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}{}^2\right]\right|^2\Bigg] \\ &+2\sin^4(\theta/2)(1-\hat{p}_1\cdot\vec{s}_1\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(s)\epsilon_{L_i}\epsilon_{R_i}\right|^2+2\sin^{-4}(\theta/2)(1+\hat{p}_1\cdot\vec{s}_1\hat{p}_2\cdot\vec{s}_2)\left|1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right|^2 \\ &+2\sin^2(\theta/2)\cos^2(\theta/2)\,|\vec{s}_{1T}|\,|\vec{s}_{2T}| \\ &\times\Bigg\{\cos(2\phi-\phi_1-\phi_2)\mathrm{Re}\Bigg[\left[1+\sum_i h_i^*(s)\epsilon_{L_i}\epsilon_{R_i}\right]\Bigg[\left[2+\sum_j h_j(s)(\epsilon_{L_j}{}^2+\epsilon_{R_j}{}^2)\right] \\ &\qquad-\sin^{-2}(\theta/2)\left[2+\sum_j h_j(t)(\epsilon_{L_j}{}^2+\epsilon_{R_j}{}^2)\right]\Bigg]\Bigg] \\ &+\sin(2\phi-\phi_1-\phi_2)\sum_i \mathrm{Im}h_i(s)(\epsilon_{L_i}{}^2-\epsilon_{R_i}{}^2)\left[1+\sin^{-2}(\theta/2)\sum_j h_j(t)\epsilon_{L_j}\epsilon_{R_j}\right]\Bigg\}\,. \end{aligned} \quad (20)$$

The corresponding final transverse polarization is

$$\begin{aligned} \vec{s}^{\,f}_{1T}=\frac{1}{D_B}&\sin^{-2}(\theta/2)\left[1+\sum_i h_i(t)\epsilon_{L_i}\epsilon_{R_i}\right] \\ &\times\Bigg[\hat{p}'_1\times[(\hat{p}'_1+\hat{p}_1)\times\vec{s}_{1T}] \\ &\times\Bigg\{(1+\hat{p}_2\cdot\vec{s}_2)\left[1+\sum_j \mathrm{Re}h_j(s)\epsilon_{L_j}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_j h_j(t)\epsilon_{L_j}{}^2\right]\right] \\ &\qquad+(1-\hat{p}_2\cdot\vec{s}_2)\left[1+\sum_j \mathrm{Re}h_j(s)\epsilon_{R_j}{}^2-\sin^{-2}(\theta/2)\left[1+\sum_j h_j(t)\epsilon_{R_j}{}^2\right]\right]\Bigg\} \\ &-2\hat{p}'_1\times[(\hat{p}'_1-\hat{p}_1)\times\vec{s}_{2T}]\left[1+\sum_i \mathrm{Re}h_i(s)\epsilon_{L_i}\epsilon_{R_i}\right] \\ &+\hat{p}'_1\times\{\hat{p}'_1\times[(\hat{p}'_1+\hat{p}_1)\times\vec{s}_{1T}]\}\left[(1+\hat{p}_2\cdot\vec{s}_2)\sum_i \mathrm{Im}h_i(s)\epsilon_{L_i}{}^2-(1-\hat{p}_2\cdot\vec{s}_2)\sum_i \mathrm{Im}h_i(s)\epsilon_{R_i}{}^2\right] \\ &-2\hat{p}'_1\times\{\hat{p}'_1\times[(\hat{p}'_1-\hat{p}_1)\times\vec{s}_{2T}]\}\hat{p}\cdot\vec{s}_1\sum_i \mathrm{Im}h_i(s)\epsilon_{L_i}\epsilon_{R_i}\Bigg]\,. \end{aligned} \quad (21)$$

The differential cross section for Bhabha scattering with polarized incident electron and positron beams, when summed over the final-state polarization, is equal to

$$\frac{d\sigma^B}{d\Omega}=\frac{\alpha^2}{4s}D_B\,. \quad (22)$$

The result (22) has been previously obtained by Olsen and Osland.[19]

IV. DISCUSSIONS

In pure QED, the polarization of the outgoing electron beam can be obtained from Eqs. (9), (10), and (12) for

Møller scattering and from Eqs. (18), (20), and (21) for Bhabha scattering, if we neglect ϵ_{L_i} and ϵ_{R_i} terms entirely. Explicitly, for Møller scattering, they are

$$\hat{p}'_1\cdot\vec{s}^{\,f}_1=\frac{1}{2A}[2(\hat{p}_1\cdot\vec{s}_1+\hat{p}_2\cdot\vec{s}_2)+(\hat{p}_1\cdot\vec{s}_1-\hat{p}_2\cdot\vec{s}_2)\cos\theta(1+\cos^2\theta)]\ , \tag{23}$$

$$\vec{s}^{\,f}_{1T}=-\frac{1}{A}\{\cos^2(\theta/2)\hat{p}'_1\times[(\hat{p}'_1+\hat{p}_1)\times\vec{s}_{1T}]+\sin^2(\theta/2)\hat{p}'_1\times[(\hat{p}'_1-\hat{p}_1)\times\vec{s}_{2T}]\}\ , \tag{24}$$

where A is defined as

$$A=\{(1+\hat{p}_1\cdot\vec{s}_1\hat{p}_2\cdot\vec{s}_2)+(1-\hat{p}_1\cdot\vec{s}_1\hat{p}_2\cdot\vec{s}_2)[\sin^8(\theta/2)+\cos^8(\theta/2)]-2\sin^4(\theta/2)\cos^4(\theta/2)\,|\vec{s}_{1T}||\vec{s}_{2T}|\cos(2\phi-\phi_1-\phi_2)\}\ . \tag{25}$$

For Bhabha scattering, they become

$$\hat{p}'_1\cdot\vec{s}^{\,f}_1|_{\text{Bhabha}}=\hat{p}'_1\cdot\vec{s}^{\,f}_1|_{\text{Møller}}\ , \tag{26}$$

$$\vec{s}^{\,f}_{1T}|_{\text{Bhabha}}=\vec{s}^{\,f}_{1T}|_{\text{Møller}}\ , \tag{27}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Bhabha}}=\cos^4(\theta/2)\left.\frac{d\sigma}{d\Omega}\right|_{\text{Møller}}\ , \tag{28}$$

if we use (p_2,s_2) to denote the momenta and polarizations of the incoming positron for Bhabha scattering and the corresponding electron for Møller scattering. Therefore, if incident particles are unpolarized in Møller and Bhabha scattering, the final electron beam will not be polarized in QED if only one outgoing electron beam is observed. This can be immediately seen from Eqs. (23) and (24). But, it can be longitudinally polarized in general electroweak theories. Also the polarizations of the outgoing electron beam in Møller and Bhabha scattering are identical and energy independent in QED; but this will not be true in the general Z-exchange gauge models.

To obtain the polarization of the outgoing electron beam in the framework of the standard model, we may set

$$g_{Z_1}=\frac{e}{2\sin\theta_W\cos\theta_W}\ , \tag{29a}$$

$$\epsilon_{L_1}=I_3-Q_f\sin^2\theta_W=-\tfrac{1}{2}+\sin^2\theta_W\ , \tag{29b}$$

$$\epsilon_{R_1}=-Q_f\sin^2\theta_W=\sin^2\theta_W\ , \tag{29c}$$

and

$$g_{Z_2}=\epsilon_{L_2}=\epsilon_{R_2}=0\ , \tag{29d}$$

in Eqs. (9), (10), and (12) for Møller scattering and in Eqs. (18), (20), and (21) for Bhabha scattering.

Barger, Ma, and Whisnant[9] (BMW) have considered the case of the left-right model with the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$, where the first neutral vector boson is indistinguishable in mass and fermion couplings from the standard model while for the second neutral vector bosons g_{Z_2}, ϵ_{L_2}, and ϵ_{R_2} are given by

$$g_{Z_2}=g_{Z_1}=\frac{e}{2\sin\theta_W\cos\theta_W}\ , \tag{30a}$$

$$\epsilon_{L_2}=\frac{\sin\theta_W}{[x_R(1-x_R-\sin^2\theta_W)]^{1/2}}x_R(I_3-Q_f)\ , \tag{30b}$$

$$\epsilon_{R_2}=\frac{\sin\theta_W}{[x_R(1-x_R-\sin^2\theta_W)]^{1/2}}[I_3(1-\sin^2\theta_W)+x_R]\ . \tag{30c}$$

In these equations, I_3 and Q_f are, respectively, $-\frac{1}{2}$ and -1 for electrons. On the other hand, x_R and M_{Z_2} are not independent parameters and the allowed region for them has been discussed in Ref. 9.

To get some feelings for the numerical values of the outgoing-electron polarization in the standard model and in the BMW model, we have chosen the following values:

$$M_{Z_1}=93\text{ GeV}\ , \tag{31a}$$

$$\sin^2\theta_W=0.23\ , \tag{31b}$$

$$\Gamma_{Z_1}=3\text{ GeV}\ , \tag{31c}$$

and

$$M_{Z_2}=210\text{ GeV}\ , \tag{32a}$$

$$\Gamma_{Z_2}=4.2\text{ GeV}\ , \tag{32b}$$

$$x_R=0.5\ . \tag{32c}$$

The values for M_{Z_1}, $\sin^2\theta_W$, and Γ_{Z_1} are consistent with experiments, and x_R and M_{Z_2} are also allowed values as discussed in Ref. 9. On the other hand, Γ_{Z_2} is chosen roughly on the basis that, if Z_1 and Z_2 have the same type of decay modes into lepton and quark pairs, one can get the ratio of the total widths of Z_1 and Z_2 by using the coupling-constant values given in Eqs. (29) and (30), i.e.,

$$\frac{\Gamma_{Z_2}}{\Gamma_{Z_1}}=\frac{M_{Z_2}}{M_{Z_1}}\frac{\sin^2\theta_W}{x_R(1-x_R-\sin^2\theta_W)}\times\frac{3(1-x_R-\sin^2\theta_W)+2x_R{}^2}{(3-6\sin^2\theta_W+8\sin^4\theta_W)}\cong 1.4. \tag{33}$$

It is not certain what the suitable value for Γ_{Z_2} is for Bhabha scattering. The $\text{Im}h_i(s)$ terms are usually negligible except at the resonance energy since Γ_i is known to be

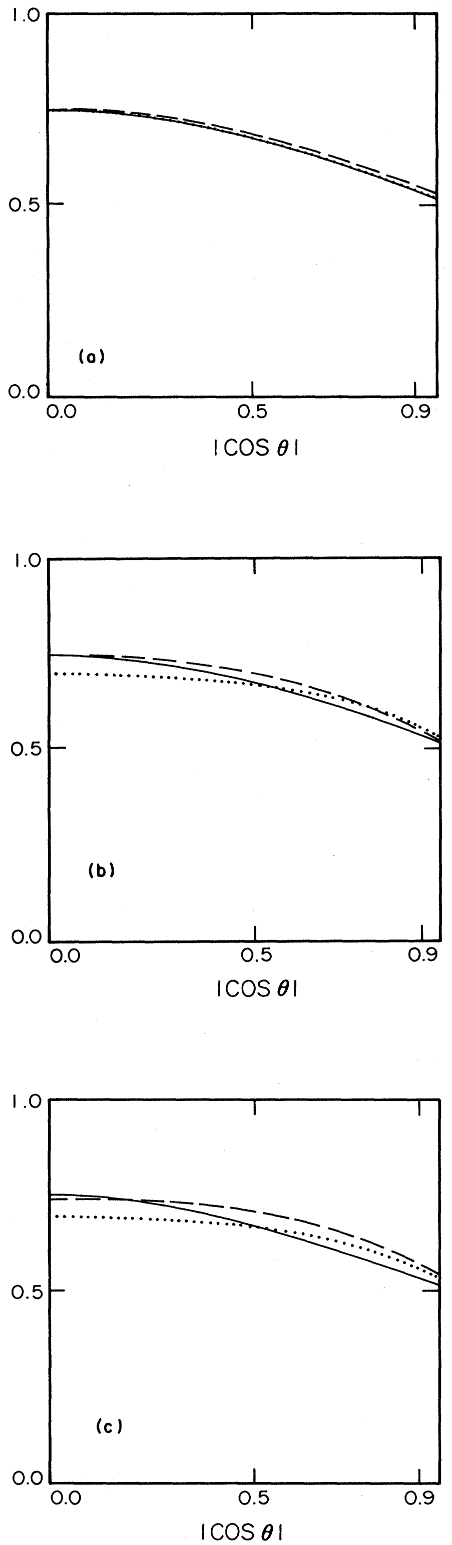

FIG. 1. (a) Longitudinal polarization of final electron beam in Møller scattering as a function of $\cos\theta$ for $\sqrt{S}=40$ GeV, $\hat{p}_1\cdot\vec{s}_1=\hat{p}_2\cdot\vec{s}_2=0.5$, $|\vec{s}_{1T}|=|\vec{s}_{2T}|=0$. The solid curve is for QED, the dashed curve for the standard model, and the dotted curve for BMW model. (b) Same as in (a) but for $\sqrt{S}=100$ GeV. (c) Same as in (a) but for $\sqrt{S}=200$ GeV.

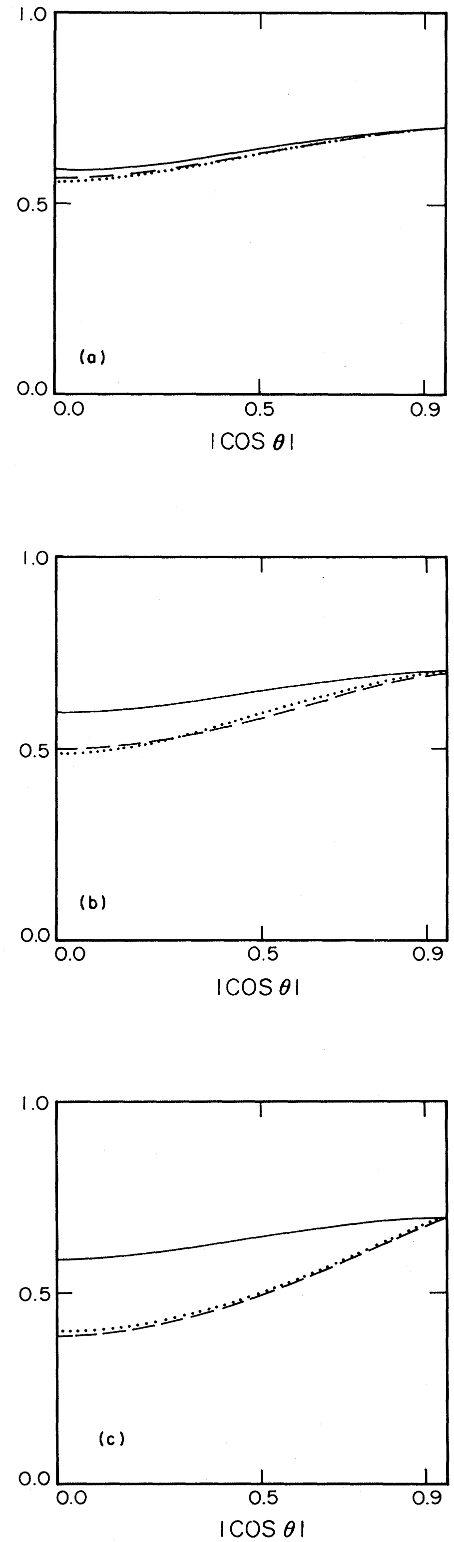

FIG. 2. (a) Transverse polarization of final electron beam $[\vec{s}\,'_{1T}\cdot(\vec{p}_1\times\vec{p}\,'_1)/|\vec{p}_1\times\vec{p}\,'_1|]$ in Møller scattering as a function of $\cos\theta$ for $\sqrt{S}=40$ GeV, $\hat{p}_1\cdot\vec{s}_1=\hat{p}_2\cdot\vec{s}_2=0$, $|\vec{s}_{1T}|=|\vec{s}_{2T}|=0.7$. $\phi=0$, $\phi_1=\phi_2=\pi/2$. (b) Same as in (a) but for $\sqrt{S}=100$ GeV. (c) Same as in (a) but for $\sqrt{S}=200$ GeV.

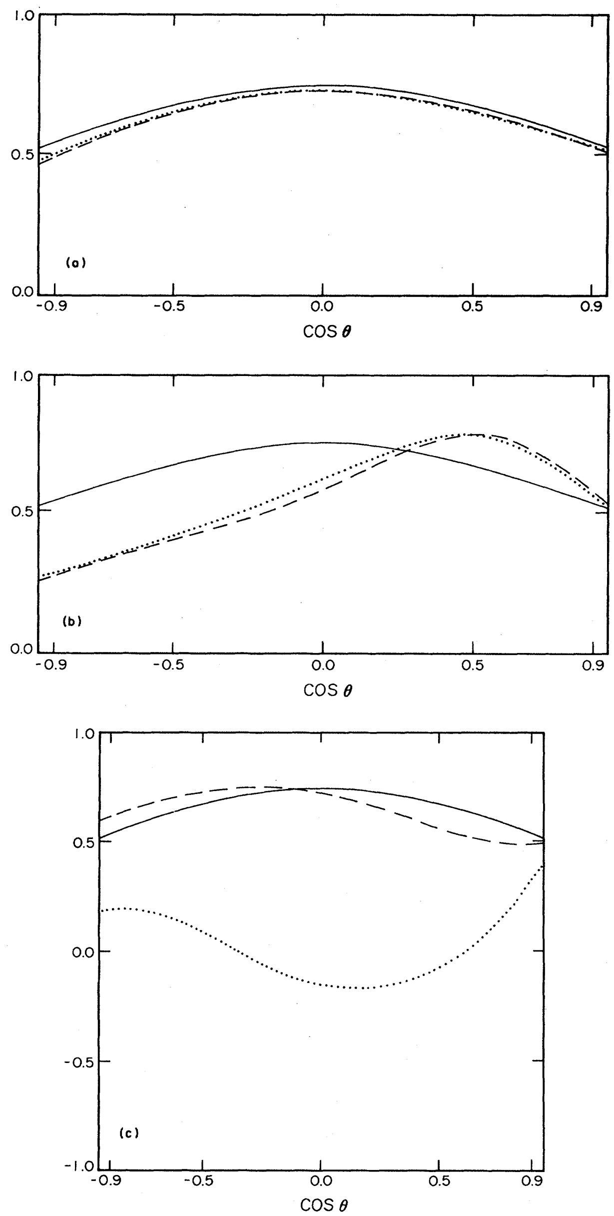

FIG. 3. (a) Longitudinal polarization of final electron beam in Bhabha scattering as a function of $\cos\theta$ for $\sqrt{S}=40$ GeV, $\hat{p}_1\cdot\vec{s}_1=\hat{p}_2\cdot\vec{s}_2=0.5$, $|\vec{s}_{1T}|=|\vec{s}_{2T}|=0$, and $\phi=0$, $\phi_1=-\phi_2=\pi/2$. (b) Same as in (a) but for $\sqrt{S}=100$ GeV. (c) Same as in (a) but for $\sqrt{S}=200$ GeV.

a few GeV while M_{Z_i} are larger than 90 GeV. When the total energy of the incident electron and positron beam reaches the resonance energy, the $\mathrm{Im}h_i(s)$ terms become important and one can test various electroweak theories around the resonance energies. However, since we do not know the exact resonance-energy value for Z_2, we have considered the (slightly) off-resonance regions for both Z_1 and Z_2 to get the numerical values for the polarization.

The experimental value of the transverse polarization can increase up to 0.92 ideally.[27] But, because of the depolarization effect,[28] it will be usually less than 0.92.

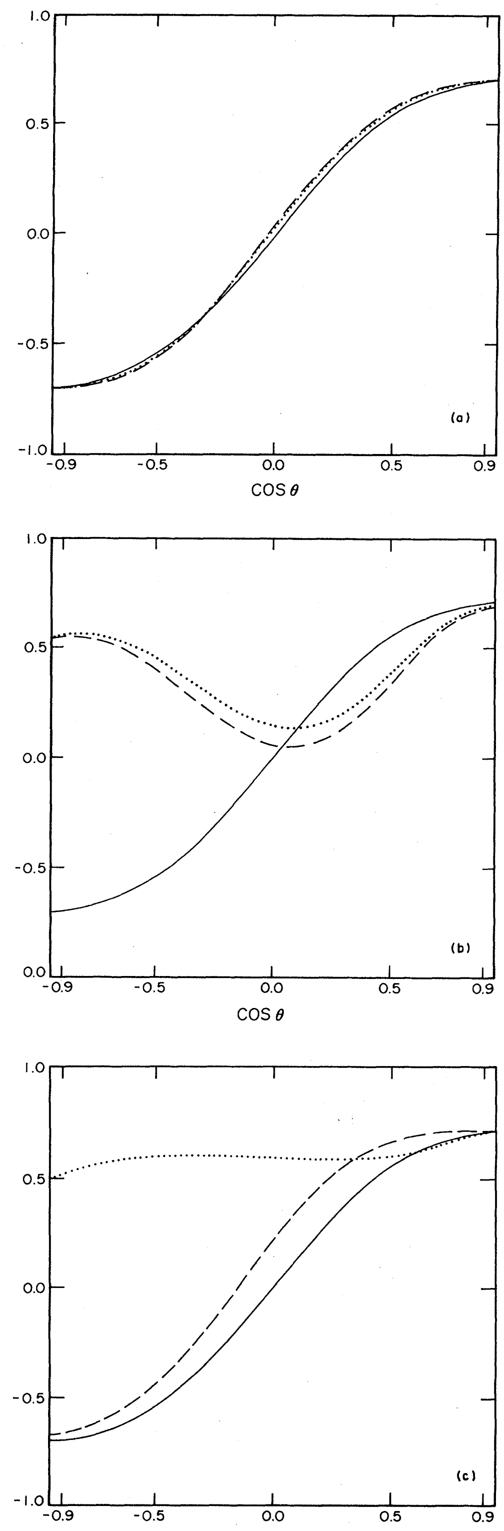

FIG. 4. (a) Transverse polarization of final electron beam $[\vec{s}\,^f_{1T}\cdot(\vec{p}_1\times\vec{p}\,'_1)/|\vec{p}_1\times\vec{p}\,'_1|]$ in Bhabha scattering as a function of $\cos\theta$ for $\sqrt{S}=40$ GeV, $\hat{p}_1\cdot\vec{s}_1=\hat{p}_2\cdot\vec{s}_2=0$, $|\vec{s}_{1T}|=|\vec{s}_{2T}|=0.7$, and $\phi=0$, $\phi_1=-\phi_2=\pi/2$. (b) Same as in (a) but for $\sqrt{S}=100$ GeV. (c) Same as in (a) but for $\sqrt{S}=200$ GeV.

We have chosen a few values for the initial polarization to see how these values effect the polarization of the final electron beam.

From Figs. 1(a), 2(a), 3(a), and 4(a), one can see that numerical values of the final-electron-beam polarizations predicted by QED, the standard model, and the BMW model are almost the same at $\sqrt{S}=40$ GeV. The polarization is symmetric at about $\cos\theta=0$ in Møller scattering as in Figs. 1 and 2.

Longitudinal polarization of the final electron beam in Møller scattering provides a simple method to distinguish between the standard model and the left-right-symmetric model of Rizzo and Senjanović[6] as shown in Ref. 18. But it does not apply to the models we considered here. The model dependence can be seen clearly by observing the final transverse polarization in Møller scattering and the final polarization in Bhabha scattering at high energy.

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