Strong magnetic field in spontaneously broken gauge theories

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We make a few observations about the calculations of effective potentials of spontaneously broken gauge theories, in the presence of a strong magnetic field, by summation over vacuum energies.

INTRODUCTION

A few recent papers have used the method of summation of vacuum energies for the purpose of deriving the form of the one-loop effective potential in spontaneously broken gauge theories in presence of an external magnetic field.[1] These calculations have been motivated by the possibility of observing a phase transition in these theories,[2] similar to the transition from the superconducting to the normal phase in solids. The purpose of this paper is to point out that (1) the summation of vacuum energies provides a rather inconclusive answer to the above problem, and (2) the alternative approach of using the background-field method for the calculation of the effective potential does not resolve the issue.

SUMMATION OF VACUUM ENERGIES

The earlier work in this area used the method of summation of vacuum energies to calculate the effective potential. In the original work[1] the authors obtained their results for low values of the external magnetic field. Since a low value of magnetic field is unlikely to effect a phase transition—one naively needs a magnetic field strength of the order of typical vacuum expectation values of the scalar fields in the theory—an extension of the work to the case of strong magnetic fields was made.[2] Apart from the inconvenience of having to deal with the generalized Riemann zeta functions, the two most serious ambiguities are the following:

(1) The effective potential turns out to have a part that is imaginary.[3] This is because the vacuum energy spectrum of a non-Abelian gauge boson in a magnetic field is given by

$$E_n{}^2 = k_H{}^2 + M^2 + (2n+1-2qS_H)\,eH \ , \tag{1}$$

where k_H and S_H denote the momenta and spin along the direction of the magnetic field, respectively. For massive non-Abelian gauge bosons, S_H takes the values 0, ± 1. For a strong magnetic field, the $S_H=1$, $n=0$ state has imaginary energy. Thus, when a summation is carried out over vacuum energies, it leads to a part that is imaginary.

Early work on the subject tended to ignore this imaginary part of the effective potential. Such a neglect was justified on the basis that perhaps the imaginary-energy state is unphysical.

Importance of the imaginary-energy state has been brought out in the work of several authors.[4] They have argued that it is precisely this imaginary energy that leads to confinement in the non-Abelian gauge theories. Thus, as the magnetic field is increased, the onset of the imaginary part in the effective potential may signify that a transition takes place to a confinement phase. This is true irrespective of what happens in the fermion or the scalar contribution to the effective potential. We believe that conceptually this is the most important hurdle that one has to overcome in order to arrive at a definite conclusion on the possibility of a phase transition in this method of summation of vacuum energies.

(2) There are some technical difficulties that also need to be recalled. In the presence of a strong magnetic field, if a phase transition does occur (maybe to a confinement phase as indicated above), then the degrees of freedom of the non-Abelian gauge bosons are limited to $S_H = \pm 1$. Thus the summation has to be carried out over $S_H = \pm 1$. This provides a source of ambiguity in the result.

One further remark in this context is that the effective potential is an expansion in terms of Planck's constant h. However, gauge interactions mix the orders of $\hbar$. It is necessary to be careful about this point.[3]

BACKGROUND-FIELD METHOD

It is because of the above-mentioned reasons that the background-field method of calculation of the effective potential seems less ambiguous.[5] There are no imaginary parts to the effective potential.

However, since the scalars, the fermions, and the gauge bosons contribute differently to the effective potential, the possibility of a phase transition depends on the specific model that one considers. In particular, it is the "relative abundance" of the spin-0, spin-$\frac{1}{2}$, and spin-1 particles that determines whether a transition occurs. Details of the calculations using the background-field method have been presented to us.[3]

In conclusion, we would like to remark that the problem of phase transition in the presence of an external magnetic field is both fascinating and challenging. Since the results are model dependent, it is difficult to come to very general conclusions. The fact that the vacuum energy picks up an imaginary part may indicate that the transition takes place to a completely confining or a partially confining phase. The details are not yet known.

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[1]A. Salam and J. Strathdee, Nucl. Phys. B90, 203 (1975).
[2]J. Chakrabarti, Phys. Rev. D 24, 2232 (1981); see also P. Roy, preceding paper, *ibid.* 29, 1856 (1984).
[3]J. Chakrabarti, Phys. Rev. D 28, 2657 (1983).
[4]N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978); see also the references quoted in this paper.
[5]B. S. DeWitt, Phys. Rev. 162, 1195 (1967); M. R. Brown and M. J. Duff, Phys. Rev. D 8, 2124 (1975); S. Midorikawa, *ibid.* 22, 2045 (1980); G. Shore, Ann. Phys. (N.Y.) 137, 262 (1981); see also J. Chakrabarti in Ref. 3.