PHYSICAL REVIEW D VOLUME 29, NUMBER 8 15 APRIL 1984

CP(N-1) model with holomorphic constraints in $D=2$ and $D=4$ dimensions

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(Received 15 February 1983; revised manuscript received 22 September 1983)

We introduce additional holomorphic constraints in the CP(N-1) model and derive, in the $1/N$ expansion, the effective action for neutral and charged composite fields. Then we discuss in four dimensions the generalized CP(N-1) model with four-linear Lagrangian and additional constraints. We also present a supersymmetric generalization of the model.

I. INTRODUCTION

The nonlinear two-dimensional σ models with composite gauge fields have recently received much attention because of their interesting features in common with four-dimensional QCD such as the presence of instantons, asymptotic freedom, and confinement. The first σ model was introduced as the O(4) model of pion dynamics and afterwards as a family of the O(N) models.[1] Later, the quantum properties of σ models were also investigated.[2] Many authors formulated and discussed their generalization to other manifolds[3] such as, for example, the Stiefel or Grassmann manifolds.

In particular, the models with fields taking values in the complex Grassmann manifolds have many interesting properties. The Grassmann manifold $CG(N,n)=U(N)/U(N-n)\times U(n)$ has complex structure, and it is known to admit parametrization by means of $N\times n$ complex rectangular matrices, satisfying the condition

$$z^{\dagger}z=\mathbb{1}_n,\quad z=\{z^{Jk}\}^{J=1,\ldots,N;k=1,\ldots,n} \tag{1.1}$$

provided that we identify the $U(n)$ gauge-equivalent quantities

$$z\sim z'=zG,\quad G\in U(n)\ . \tag{1.2}$$

In particular, if $n=1$ we obtain $U(N)/U(N-1)\times U(1)$—the $(N-1)$ dimensional complex projective space $CP(N-1)$. The nonlinear model with fields taking values in $U(N)/U(N-1)\times U(1)$ [so-called $CP(N-1)$ model] was discussed in great detail by D'Adda, Lüscher, and DiVecchia.[4] According to (1.1) and (1.2) one constructs the $CP(N-1)$ model by introducing N complex scalar fields $z_i(x)$, $i=1,\ldots,N$, which are constrained by

$$\sum_{i=1}^{N}\bar{z}_i(x)z_i(x)=1\ . \tag{1.3}$$

The fields related by local U(1) gauge transformation

$$z_i(x)\sim z_i'(x)=e^{i\Lambda(x)}z_i(x) \tag{1.4}$$

are to be identified.

The action

$$S=\frac{N}{2f}\int d^2x\,\overline{\nabla_\mu z}\cdot\nabla_\mu z,\quad \nabla_\mu=\partial_\mu+iA_\mu \tag{1.5}$$

possesses the global O(N) symmetry as well as the local U(1) gauge symmetry. The composite gauge fields

$$A_\mu=\tfrac{1}{2}i\bar{z}\cdot\overleftrightarrow{\partial}_\mu z \tag{1.6}$$

transform like the Abelian U(1) gauge fields

$$A_\mu\to A_\mu'=A_\mu-\partial_\mu\Lambda\ . \tag{1.7}$$

In two dimensions this model has several interesting properties. It is conformally invariant and has classical instanton solutions for all N. Its quantum version is asympotically free. Using the $1/N$-expansion method one can show that particles $z,\bar{z}$ (with a mass m generated dynamically by a Higgs-type effect) interacting with U(1) gauge composite fields λ_μ and the neutral scalar composite field α are confined. The confinement in the $CP(N-1)$ model is also "generated dynamically" and is identical to that which is already well known in other two-dimensional models.

One can investigate also other manifolds defined by imposing holomorphic constraints on $CP(N-1)$. Such manifolds are called algebraic. All σ models on the algebraic manifolds describe the dynamics of interacting composite fields.

In this paper we extend the $CP(N-1)$ dynamics by introducing additional constraints $z\cdot z=\bar{z}\cdot z=0$. As a result we obtain interacting charged composite fields. We investigate their dynamics, using the $1/N$ expansion to obtain the effective action and find it to be the same as the dynamics of the α field. The saddle-point condition in our model coincides with the one obtained in the $CP(N-1)$ model. We show that our model is also asymptotically free and the particles z, $\bar{z}$ are confined.

In Sec. II we discuss the equations of motion and the instanton solutions of the model. The quantization via the $1/N$ expansion, using the method presented in Ref. 4, is performed in Sec. III. In Sec. IV, following Ref. 5, we investigate the four-linear $CP(N-1)$ model in $D=4$ dimensions with the holomorphic constraints $z^2=\bar{z}^2=0$. Using the dimensional-regularization technique we show the renormalizability of the effective action. In Sec. V we describe a supersymmetric generalization of the model.

II. RG(N,2) MODEL—THE CLASSICAL EQUATIONS OF MOTION AND INSTANTONS

In this section we will discuss the classical σ model in two-dimensional Euclidean space, with fields taking values on the Grassmann manifold GR(N,2)=O(N)/O($N-2$)×O(2). This manifold has a standard real parametrization since it is a symmetric space. However, it is also a Kahler manifold and has complex structure. There is a homeomorphism between RG(N,2) and the complex hyperquadric in projective space CP($N-1$).[6,14]

This observation lets us introduce the GR(N,2) σ model as a field theory with N scalar fields, where

$$\begin{aligned} &\sum_i \bar{z}_i(x) z_i(x)=1 , \\ &\sum_i z_i(x) z_i(x)=\sum_i \bar{z}_i(x)\bar{z}_i(x)=0 , \\ &z_i(x)\sim z_i'(x)=e^{i\Lambda(x)} z_i(x) . \end{aligned} \tag{2.1}$$

The action formula for $z_i(x)$ is

$$S=\int d^2x\, \overline{\nabla_\mu z}\cdot \nabla_\mu z , \tag{2.2}$$

where

$$\nabla_\mu=\partial_\mu+iA_\mu \tag{2.3}$$

and it has the same form as in the two-dimensional CP($N-1$) model.[4] Solving the algebraic equation of motion for the gauge fields A_μ we find

$$A_\mu(z,\bar{z})=i\bar{z}\cdot\partial_\mu z . \tag{2.4}$$

The action (2.2) is conformally invariant. It is also invariant under global O(N) transformations and local U(1) gauge transformations. So, the local symmetry group in our model is exactly the same as in the CP($N-1$) model.

Using the Lagrangian multiplier fields $\alpha(x)$, $\mu(x)$, $\bar{\mu}(x)$ one can rewrite the Lagrangian as

$$\mathscr{L}=\overline{\nabla_\mu z}\cdot\nabla_\mu z+\alpha(\bar{z}\cdot z-1)+\mu z^2+\bar{\mu}\,\bar{z}^2 . \tag{2.5}$$

Equations of motion are

$$\begin{aligned} &A_\mu=i\bar{z}\cdot\partial_\mu z,\quad \bar{z}\cdot z=1 ,\quad z\cdot z=\bar{z}\cdot\bar{z}=0 , \\ &\nabla_\mu\nabla_\mu z+\alpha z+2\bar{\mu}\,\bar{z}=0 , \\ &\overline{\nabla_\mu\nabla_\mu z}+\alpha\bar{z}+2\mu z=0 . \end{aligned} \tag{2.6}$$

In this way we incorporate the constraints in the Lagrangian and may treat the fields $z_i(x)$ as unconstrained variables.

Eliminating the auxiliary fields $\alpha,\mu,\bar{\mu}$ by means of the algebraic equations of motion

$$\begin{aligned} &\alpha(z,\bar{z})=\overline{\nabla_\mu z}\cdot\nabla_\mu z , \\ &\mu(z,\bar{z})=\tfrac{1}{2}\overline{\nabla_\mu z}\cdot\overline{\nabla_\mu z} , \\ &\bar{\mu}(z,\bar{z})=\tfrac{1}{2}\nabla_\mu z\cdot\nabla_\mu z \end{aligned} \tag{2.7}$$

and inserting (2.7) into (2.6) we finally obtain the classical equations of motion for scalar fields in the form

$$\begin{aligned} &\nabla_\mu\nabla_\mu z+(\overline{\nabla_\mu z}\cdot\nabla_\mu z)z+(\nabla_\mu z\cdot\nabla_\mu z)\bar{z}=0 , \\ &\bar{z}\cdot z-1=z\cdot z=\bar{z}\cdot\bar{z}=0 . \end{aligned} \tag{2.8}$$

Under the gauge transformation

$$z_i(x)\to z_i'(x)=e^{i\Lambda(x)} z_i(x) \tag{2.9}$$

the composite fields transform as

$$\begin{aligned} &\alpha(x)\to\alpha'(x)=\alpha(x) , \\ &A_\mu(x)\to A_\mu'(x)=A_\mu(x)-\partial_\mu\Lambda(x) , \\ &\mu(x)\to\mu'(x)=e^{-2i\Lambda(x)}\mu(x) , \\ &\bar{\mu}(x)\to\bar{\mu}'(x)=e^{+2i\Lambda(x)}\bar{\mu}(x) . \end{aligned} \tag{2.10}$$

The RG(N,2) model is topologically nontrivial and in the two-dimensional space-time it has instanton solutions for all N. Following Ref. 4 one can define the topological density $q(x)$ and the topological charge Q:

$$\begin{aligned} &q(x)=\frac{1}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}=\frac{1}{2\pi}\epsilon_{\mu\nu}\partial_\mu A_\nu , \\ &Q=\int d^2x\, q(x),\quad \epsilon_{12}=-\epsilon_{21}=1 . \end{aligned} \tag{2.11}$$

One can show[7] that the topological charge Q is an integer. Q labels the homotopy classes of fields $z(x)$, i.e., any two fields with equal charge can be continuously deformed one into another and, furthermore, for any integer p there exists fields such that $Q=p$.

To obtain the instanton equations we rewrite the topological density as

$$q=\frac{i}{2\pi}\epsilon_{\mu\nu}\overline{\nabla_\mu z}\cdot\nabla_\nu z . \tag{2.12}$$

Let us consider the unequality

$$|\nabla_\mu z\pm i\epsilon_{\mu\nu}\nabla_\nu z|^2\geq 0 , \tag{2.13}$$

which, after some transformations, takes the form

$$\mathscr{L}=\overline{\nabla_\mu z}\cdot\nabla_\mu z\geq\pm i\epsilon_{\mu\nu}\overline{\nabla_\mu z}\cdot\nabla_\nu z=\pm 2\pi q . \tag{2.14}$$

Integrating over x we get

$$S\geq 12\pi|Q| . \tag{2.15}$$

The equality is saturated if and only if

$$\nabla_\mu z=\pm i\epsilon_{\mu\nu}\nabla_\nu z . \tag{2.16}$$

The finite-action solutions of these self-duality equations are called instantons (anti-instantons) and, because of (2.15), they are also solutions of the equations (2.8).

In the CP($N-1$) model the situation is the same. One can solve Eq. (2.16) by introducing unconstrained coordinates $W_i(x)$ Ref. 4,

$$z_i(x)=e^{i\Lambda(x)}W_i(x)/|W(x)| , \tag{2.17}$$

which satisfy the Cauchy-Riemann equations

$$\partial_\mu W(x)=\pm i\epsilon_{\mu\nu}\partial_\nu W(x) . \tag{2.18}$$

But now, in our model, we have the additional condition $z^2=\bar{z}^2=0$ and therefore fields $W_i(x)$ are only "partially unconstrained", i.e., we must impose the condition to be fulfilled by $W_i(x)$:

$$\sum_i W_i(x)W_i(x)=0 . \tag{2.19}$$

Finally we can say that the most general solution of Eq. (2.16) is given by (2.17) where $\Lambda(x)$ is a real function, $W_i(x)$ are meromorphic functions satisfying (2.19), and $W_j(x)=1$ for some $j=\alpha$.

III. 1/N EXPANSION OF THE QUANTUM RG(N,2) MODEL IN TWO DIMENSIONS

The method of the $1/N$ expansion has been applied to a number of theories, in particular, to the Gross-Neveu model,[8] the O(N) σ models,[9] ϕ^4 theory,[10] and the CP($N-1$) model.[4,7]

In this paper, in order to quantize the RG(N,2) model, we apply the method used by D'Adda, Lüscher, and Di Vecchia in two-dimensional CP($N-1$).[4] Let us start from the classical action

$$S=\frac{N}{2f}\int d^2x\overline{\nabla_\mu z}\cdot\nabla_\mu z,\quad \bar z\cdot z-1=z\cdot z=\bar z\cdot\bar z=0$$

where N is the number of components of the field $z(x)$ and f is a dimensionless coupling constant.

The vacuum generating functional for Euclidean Green's functions of the quantum RG(N,2) model is

$$Z=\int \mathscr{D}z\,\mathscr{D}\bar z\,\mathscr{D}\alpha\,\mathscr{D}\mu\,\mathscr{D}\bar\mu\,\exp\{-S+\int d^2x[i\alpha(\bar z\cdot z-1)+i\mu z^2+i\bar\mu\bar z^2]\}\ . \tag{3.1}$$

For convenience we calculate the functional integral in Euclidean space. If necessary the final result can be rotated back into Minkowski space. We have introduced the Lagrange multiplier fields $\bar\mu(x)$, $\mu(x)$, $\alpha(x)$. After performing the following rescaling of the fields

$$\begin{aligned}&z_i(x)\to(N/2f)^{1/2}z_i(x)\ ,\quad \alpha(x)\to(2f/\sqrt{N})\alpha(x)\ ,\\&\mu(x)\to(2f/\sqrt{N})\mu(x),\quad A_\mu(x)\to(2f/N)A_\mu(x)=\frac{f}{N}i\bar z\cdot\overleftrightarrow{\partial}_\mu z\end{aligned} \tag{3.2}$$

we get

$$Z=\int \mathscr{D}z\,\mathscr{D}\bar z\,\mathscr{D}\mu\,\mathscr{D}\bar\mu\,\mathscr{D}\alpha\,\mathscr{D}\lambda_\mu\exp\left\{-\int d^2x\left[\bar z\cdot\left[-\Box-\frac{i}{\sqrt{N}}\alpha+\frac{1}{N}\lambda^2-\frac{2i}{\sqrt{N}}\lambda_\mu\partial_\mu\right]z+\frac{i\sqrt{N}}{2f}\alpha-\frac{i}{\sqrt{N}}(\mu z^2+\bar\mu\bar z^2)\right]\right\}.$$

We have also introduced a new integration over the λ_μ field, in order to get rid of four-linear terms.

We would like to point out that we do not keep track of any constant (infinite) factors in front of z. The Gaussian path integration over z, $\bar z$ may now be performed and we get

$$\begin{aligned}Z&=\int \mathscr{D}\bar z\,\mathscr{D}z\,\mathscr{D}\bar\mu\,\mathscr{D}\mu\,\mathscr{D}\alpha\,\mathscr{D}\lambda_\mu\exp\left\{-\int d^2x\left[\tfrac{1}{2}(\bar z,z)\begin{bmatrix}\Delta&(2i/\sqrt{N})\bar\mu\\(2i/\sqrt{N})\mu&\Delta^T\end{bmatrix}\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}\bar z\\z\end{bmatrix}+\frac{i\sqrt{N}}{2f}\alpha(x)\right]\right\}\\&=\int\mathscr{D}\mu\,\mathscr{D}\bar\mu\,\mathscr{D}\alpha\,\mathscr{D}\lambda_\mu\exp(-S_{\rm eff})\ ,\end{aligned} \tag{3.3}$$

where

$$D_\mu=\partial_\mu+\frac{i}{\sqrt{N}}\lambda_\mu\ ,$$

$$\Delta=-D_\mu D_\mu-\frac{i}{\sqrt{N}}\alpha\ ,$$

$$S_{\rm eff}=\tfrac{1}{2}N\,{\rm Tr}\ln\begin{bmatrix}\Delta&\frac{2i}{\sqrt{N}}\bar\mu\\\frac{2i}{\sqrt{N}}\mu&\Delta^T\end{bmatrix}+\frac{i\sqrt{N}}{2f}\int d^2x\,\alpha(x)\ .$$

The tracing operation is to be understood in the matrix as well as in the functional sense. The integrations over the auxiliary fields $\alpha,\mu,\bar\mu$ cannot be performed exactly, therefore a stationary-phase approximation is used.

Variation of the effective action with respect to $\mu,\bar\mu$ gives the equations of motion

$$\frac{\delta S_{\rm eff}}{\delta\mu}=\frac{\delta S_{\rm eff}}{\delta\bar\mu}=0\ , \tag{3.4}$$

which are solved by the zero values of the fields. In effect we obtain the saddle-point condition as the classical equation of motion for α ($\lambda_\mu^c=\mu^c=\bar\mu^c=0$):

$$\left.\frac{\delta S_{\rm eff}}{\delta\alpha}\right|_{\alpha=\alpha^c\neq0}=\frac{\delta}{\delta\alpha}\left[N\,{\rm Tr}\ln\Delta+\int d^2x\frac{i\sqrt{N}}{2f}\alpha(x)\right]_{\alpha=\alpha^c}=0\ .$$

If we introduce a new parameter $m^2>0$ such that

$$\alpha^c=i\sqrt{N}m^2 \tag{3.5}$$

we get

$$i\sqrt{N}\left[\frac{1}{2f}-\int\frac{d^2p}{(2\pi)^2}\frac{1}{p^2+m^2}\right]=0\ . \tag{3.6}$$

As we can see the saddle-point condition in our model coincides with the one obtained in the CP($N-1$) model.

Renormalizing the bare coupling constant f as

$$\frac{2\pi}{f}=\ln\frac{\Lambda^2}{\mu^2}+\frac{2\pi}{f_R(\mu)}\ , \tag{3.7}$$

one gets the equation for the arbitrary parameter $m^2>0$:

$$\frac{1}{4\pi}\ln\frac{m^2}{\mu^2}-\frac{1}{2f_R(\mu)}=0\ . \tag{3.8}$$

The composite scalar field α has a nonzero vacuum expectation value m^2, so the symmetry is broken. As a result of this mechanism we get a mass for the scalar fields $z,\bar{z}$, even though $z,\bar{z}$ were first introduced as massless fields. We perform a shift in the effective action

$$\alpha\to\alpha+\alpha^c=\alpha+i\sqrt{N}m^2\ . \tag{3.9}$$

Now, one can expand the effective action in a power series of $1/N$ around the classical minimum $\lambda_\mu^c=\mu^c=\bar{\mu}^c=0$, $\alpha^c\neq 0$. We write

$$S_{\text{eff}}=\sum_{\nu=1}^{\infty}\left[\frac{1}{N}\right]^{\nu/2-1}S^{(\nu)}+\text{const}\ . \tag{3.10}$$

The term proportional to $\sqrt{N}$ in this expansion vanishes due to the saddle-point condition. The quadratic term of S_{eff} may be written in the form

$$S^{(2)}=\tfrac{1}{2}\int\int d^2x\,d^2y[\alpha(x)\Gamma^\alpha(x-y)\alpha(y)+2\bar{\mu}(x)\Gamma^\mu(x-y)\mu(y)+\lambda_\mu(x)\Gamma^\lambda_{\mu\nu}(x-y)\lambda_\nu(y)]\ , \tag{3.11}$$

where the Fourier transforms of the nonlocal vertices are

$$\tilde{\Gamma}^\alpha(p)=\int\frac{d^2q}{(2\pi)^2}\{(q^2+m^2)[(q+p)^2+m^2]\}^{-1}=\tilde{\Gamma}^\mu(p)\ , \tag{3.12}$$

$$\tilde{\Gamma}^\lambda_{\mu\nu}(p)=2\delta_{\mu\nu}\int\frac{d^2q}{(2\pi)^2}(q^2+m^2)^{-1}-\int\frac{d^2q}{(2\pi)^2}\frac{(2q+p)_\mu(2q+p)_\nu}{(q^2+m^2)[(q+p)^2+m^2)]}\ . \tag{3.13}$$

We see that the charged composite particles $\mu,\bar{\mu}$ have the same propagator as the neutral particle α. The integral (3.12) is easily evaluated:

$$\tilde{\Gamma}^\alpha(p)=\tilde{\Gamma}^\mu(p)=\frac{1}{2\pi}[p^2(p^2+4m^2)]^{-1/2}\ln\frac{(p^2+4m^2)^{1/2}+(p^2)^{1/2}}{(p^2+4m^2)^{1/2}-(p^2)^{1/2}}\equiv A(p)\ . \tag{3.14}$$

The one-loop integrals (3.13) are both divergent. However, in two dimensions, regularizing by a cutoff Λ, the divergent parts cancel[4] and we get

$$\tilde{\Gamma}^\lambda_{\mu\nu}(p)=\left[\delta_{\mu\nu}-\frac{1}{p^2}p_\mu p_\nu\right]\left[(p^2+4m^2)A(p)-\frac{1}{\pi}\right]\ . \tag{3.15}$$

All the other terms in the effective action are convergent, which ends the proof of its finiteness.

The calculated terms may be represented in the form of Feynman diagrams. We introduce the following graphical representation for propagators and vertices (see Figs. 1 and 2). Internal propagators for $z,\bar{z}$ fields are integrated with measure $d^2q/(2\pi)^2$. The terms $S^{(1)}$, $S^{(2)}$ considered above may be now represented in the form shown in Fig. 3. This is the graphical technique for calculating the effective action. Following Refs. 11 and 4 one can establish a modified Feynman technique with inverted bilinear parts of the effective action. But to invert the expression $\tilde{\Gamma}^\lambda_{\mu\nu}(p)$ it is necessary to fix the gauge. The tadpole and self-energy diagrams shown in Fig. 3, which were already taken into account in α, λ, and μ propagators, are now forbidden.

Let us point out that the presence of new particles $\mu,\bar{\mu}$ will modify the result of the CP($N-1$) model in higher orders of the $1/N$ expansion, e.g., the self-energy diagram for λ_μ fields in our model in order $1/N$ can be written as in Fig. 4. There are additional diagrams in our model for nonlocal vertices as well.[11]

The Feynman rules for the $1/N$ expansion, derived above, show that the quantum RG(N,2) model has similar properties as the CP($N-1$) model. The RG(N,2) model describes the O(N) vector of charged interacting particles

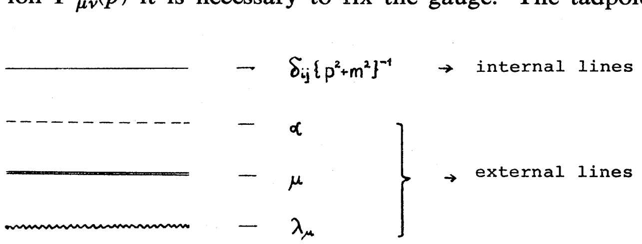

FIG. 1. The propagators in the $1/N$ expansion.

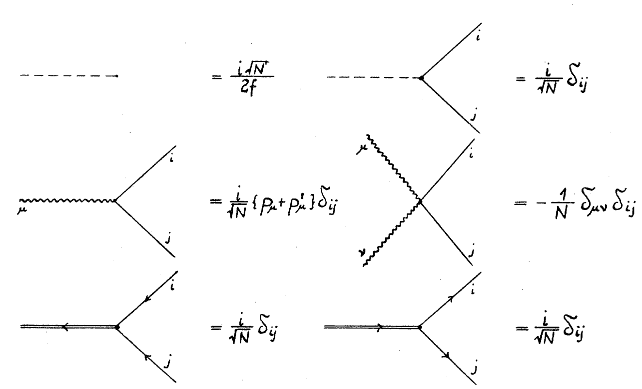

FIG. 2. The vertices in the $1/N$ expansion.

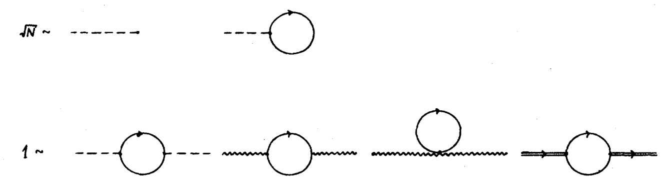

FIG. 3. The tadpole and self-energy diagrams.

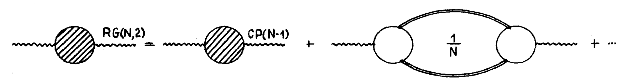

FIG. 4. The $1/N$ correction to the full gauge field propagation.

with a mass m. The mass for $z,\bar{z}$ is generated dynamically and m^2 is the positive vacuum expectation value of the composite neutral field α. Particles $z,\bar{z}$ interact by exchanging α,λ_μ and $\mu,\bar{\mu}$ quanta. The α exchange [as in CP$(N-1)$ (Ref. 4)] leads to weak short-range interaction among z and $\bar{z}$, so the α particle does not propagate in the effective theory. The exchange of the $\mu,\bar{\mu}$ charged composite particles leads to the same effect because of the relation (3.12).

In two dimensions the λ_μ interaction has the same effect as a linear Coulomb potential so that the particles $z,\bar{z}$ are confined.[7] The one-loop corrections generate a kinetic energy term for the composite gauge field λ_μ. Our model is also asymptotically free as can be seen from Eq. (3.7).

IV. FOUR-LINEAR RG$(N,2)$ MODEL IN FOUR DIMENSIONS

It is interesting to perform the renormalization procedure of our model for $D=4$. We obtain the result that the effective action is not renormalizable unless we add some additional terms to the action. In this section we will discuss this problem in great detail.

We start from the four-linear Lagrangian in $D=4$ Euclidean space first introduced in Ref. 5,

$$\mathscr{L}=\frac{N}{f}\left[\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}+\frac{\kappa^2}{2}\overline{\nabla_\mu z}\cdot\nabla_\mu z\right], \tag{4.1}$$

where f is the coupling constant, κ^2 the masslike parameter, and

$$\begin{aligned}\nabla_\mu&=\partial_\mu+iA_\mu=\partial_\mu-\bar{z}\cdot\partial_\mu z\ ,\\ F_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu=i\partial_{[\mu}\bar{z}\cdot\partial_{\nu]}z\ .\end{aligned} \tag{4.2}$$

The additional four-linear term $\frac{1}{4}F_{\mu\nu}{}^2$ describes the kinetic energy for the composite gauge potential A_μ. The constraints of the model are the same as in the $D=2$ case.

One can write the generating functional

$$Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\delta(\bar{z}\cdot z-1)\delta(z^2)\delta(\bar{z}^2)\exp\left[-\int d^4x\frac{N}{f}\left(\tfrac{1}{4}F_{\mu\nu}{}^2+\tfrac{1}{2}\kappa^2\overline{\nabla_\mu z}\cdot\nabla_\mu z\right)\right]. \tag{4.3}$$

As usual we introduce the Lagrange multiplier fields, $\alpha,\mu,\bar{\mu}$, and Z takes the form

$$Z=\int\mathscr{D}z\,\mathscr{D}\bar{z}\,\mathscr{D}\mu\,\mathscr{D}\bar{\mu}\,\mathscr{D}\alpha\exp\left\{-\int d^4x\left[\frac{N}{f}\tfrac{1}{4}F_{\mu\nu}{}^2+\frac{N}{2f}\kappa^2\overline{\nabla_\mu z}\cdot\nabla_\mu z-i\kappa^2\alpha(\bar{z}\cdot i\kappa^2(\mu z^2+\bar{\mu}\bar{z}^2)\right]\right]\right\}. \tag{4.4}$$

Rescaling the fields as in Sec. III, we introduce a new integration over the field $\lambda_\mu(x)$ and perform some standard calculations[5,11] and obtain

$$\begin{aligned}Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\mathscr{D}\bar{\mu}\,\mathscr{D}\mu\,\mathscr{D}\alpha\,\mathscr{D}\lambda_\mu\exp\Bigg\{&\int\int d^4x\,d^4y\frac{1}{2f}\lambda_\mu(x)G_{\mu\nu}(x-y)\lambda_\nu(y)\\ &-\int d^4x\left[\kappa^2\bar{z}\cdot\Delta z-\frac{i\kappa^2}{\sqrt{N}}(\mu z^2+\bar{\mu}\,\bar{z}^2)+\frac{i\kappa^2\sqrt{N}}{2f}\alpha\right]\Bigg\}\ ,\end{aligned} \tag{4.5}$$

where

$$\begin{aligned}G_{\mu\nu}(x-y)&=(\Box\delta_{\mu\nu}-\partial_\mu\partial_\nu)\left[1+\frac{\Box}{\kappa^2}\right]^{-1}(x-y)\ ,\\ \Delta&=-D_\mu D_\mu-\frac{i}{\sqrt{N}}\alpha\ ,\\ D_\mu&=\partial_\mu+\frac{i}{\sqrt{N}}\lambda_\mu\ .\end{aligned} \tag{4.6}$$

The Gaussian path integrals over $z,\bar{z}$ are the same as in the two-dimensional model discussed in Sec. III. However, now the expressions for the propagators are the four-dimensional divergent integrals. Therefore, the effective action obtained from Lagrangian (4.1) after performing integrations over $z,\bar{z}$ is not renormalizable. To solve this problem, we must add a counterterm to our Lagrangian:

$$\frac{1}{2\lambda}\alpha^2+\frac{1}{\xi}\bar{\mu}\mu\ . \tag{4.7}$$

The expression (4.7) destroys the constraint conditions in the model because our modified Lagrangian is classically equivalent to

$$\mathscr{L}'=\overline{\nabla_\mu z}\cdot\nabla_\mu z+\frac{3\kappa^4}{N}\xi\bar{z}^2z^2-\frac{3\lambda\kappa^4}{2N}\left[\bar{z}\cdot z-\frac{N}{2f}\right]^2 .$$

However, after quantization, if the bare quantities κ,λ,ξ satisfy suitable conditions, the constraints of the model are not changed, because the δ functional may be approximated as

$$\delta\left[\bar{z}\cdot z-\frac{N}{2f}\right]\to\exp\left[\frac{\lambda\kappa^4}{2N}\left[\bar{z}\cdot z-\frac{N}{2f}\right]^2\right],\quad \delta(\bar{z}^2)\delta(z^2)\to\exp\left[\frac{\xi\kappa^4}{2N}\bar{z}^2z^2\right], \tag{4.8}$$

and

$$\lambda\kappa^4\to\infty,\quad \xi\kappa^4\to\infty . \tag{4.9}$$

Using a suitable renormalization procedure we can fulfill the conditions (4.9). With this additional term effective action of the model is

$$S_{\rm eff}=\tfrac{1}{2}N\,{\rm Tr}\ln\begin{bmatrix}\Delta & \dfrac{2i}{\sqrt{N}}\bar{\mu}\\ \dfrac{2i}{\sqrt{N}}\mu & \Delta^T\end{bmatrix}+\int d^4x\left[\frac{i\kappa^2\sqrt{N}}{2f}\alpha+\frac{1}{2\lambda}\alpha^2+\frac{1}{\xi}\bar{\mu}\mu\right]+\int\int d^4x\,d^4y\frac{1}{2f}\lambda_\mu(x)G_{\mu\nu}(x-y)\lambda_\nu(y) . \tag{4.10}$$

Since the integration over α,μ,λ_μ cannot be performed exactly, the expansion around the saddle point is used. Variation of the effective action with respect to $\mu,\bar{\mu}$ gives the equations of motion which are solved by the zero values of the fields, and therefore we get, with $\mu^c=\bar{\mu}^c=\lambda_\mu^c=0$, $\alpha^c\neq 0$,

$$\left.\frac{\delta S_{\rm eff}}{\delta\alpha}\right|_{\alpha=\alpha^c}=-\frac{i}{\sqrt{N}}N\,{\rm Tr}\ln\left[-\Box-\frac{i\alpha^c}{\sqrt{N}}\right]^{-1}+\frac{i\kappa^2}{2f}\sqrt{N}+\frac{1}{\lambda}\alpha^c=0 . \tag{4.11}$$

If, similarly as in Sec. III, we define

$$\alpha^c=i\sqrt{N}m^2 , \tag{4.12}$$

then Eq. (4.11) takes the form

$$\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2}-\frac{\kappa^2}{2f}-\frac{m^2}{\lambda}=0 . \tag{4.13}$$

The one-loop integral in (4.13) is divergent. However, we can extract the regular part

$$\left[\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2}\right]_{\rm reg}=\int\frac{d^4p}{(2\pi)^4}\left[\frac{1}{p^2+m^2}-\frac{1}{p^2}+\frac{m^2}{(p^2+M^2)^2}\right] \tag{4.14}$$

(M^2 is an arbitrary mass parameter), and renormalizing the bare parameters properly,

$$\frac{\kappa^2}{2f}=\frac{\kappa_{\rm ren}{}^2}{2f_{\rm ren}}+\int_0^{\Lambda^2}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}\underset{\Lambda\to\infty}{\sim}\Lambda^2 ,$$
$$\frac{1}{\lambda}=\frac{1}{\lambda_{\rm ren}}-\int_0^{\Lambda^2}\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+M^2)^2}\underset{\Lambda\to\infty}{\sim}\ln\Lambda^2 , \tag{4.15}$$

we obtain the following equation for the spontaneously generated mass m:

$$\left[\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2}\right]_{\rm reg}=\frac{\kappa_{\rm ren}{}^2}{2f_{\rm ren}}+\frac{m^2}{\lambda_{\rm ren}}=\frac{1}{16\pi^2}m^2\ln\frac{m^2}{eM^2} . \tag{4.16}$$

Now, one can expand the effective action around the saddlepoint $\lambda_\mu^c=\mu^c=\bar{\mu}^c=0$, $\alpha^c\neq 0$:

$$S_{\rm eff}=\sum_{\nu=1}^{\infty}\left[\frac{1}{N}\right]^{\nu/2-1}S^{(\nu)}+{\rm const} . \tag{4.17}$$

The term which is proportional to $\sqrt{N}$ vanishes due to the saddle-point condition (4.13). The quadratic part of the effective action leads to the following propagators:

$$\tilde{\Gamma}^\alpha(p)=\frac{1}{\lambda}+\int\frac{d^4q}{(2\pi)^4}\{(q^2+m^2)[(q+p)^2+m^2]\}^{-1} , \tag{4.18}$$

$$\tilde{\Gamma}^\mu(p)=\frac{1}{\xi}+\int\frac{d^4q}{(2\pi)^4}\{(q^2+m^2)[(q+p)^2+m^2]\}^{-1} , \tag{4.19}$$

$$\tilde{\Gamma}^\lambda_{\mu\nu}(p)=\frac{1}{f}\left[1-\frac{p^2}{\kappa^2}\right]^{-1}(p^2\delta_{\mu\nu}-p_\mu p_\nu)+\int\frac{d^4q}{(2\pi)^4}\left[2\delta_{\mu\nu}\frac{2}{q^2+m^2}-\frac{(p+2q)_\mu(p+2q)_\nu}{(q^2+m^2)[(p+q)^2+m^2]}\right] . \tag{4.20}$$

The one-loop integral for α and μ propagators has only logarithmic divergences. On the other hand, the integrals in (4.20) have quadratic divergences as well.

Let us consider the α propagator. It may be made finite by the subtraction

$$\widetilde{\Gamma}^{\alpha}(p)=\widetilde{\Gamma}^{\alpha}_{\rm ren}(p)+1/\lambda_{\rm ren} ,$$

where

$$\widetilde{\Gamma}^{\alpha}_{\rm ren}(p)=\frac{1}{16\pi^2}\left[\ln\frac{m^2}{M^2}-2+\left[\frac{4m^2+p^2}{p^2}\right]^{1/2}\ln\frac{(p^2+4m^2)^{1/2}+(p^2)^{1/2}}{(p^2+4m^2)^{1/2}-(p^2)^{1/2}}\right] . \tag{4.21}$$

The dynamics of the field α depend on the choice of the parameter $\lambda_{\rm ren}$. In particular, when we take

$$1/\lambda_{\rm ren}=-\frac{1}{16\pi^2}\ln\frac{m^2}{M^2} , \tag{4.22}$$

we get

$$\widetilde{\Gamma}^{\alpha}(p)\sim p^2 ,$$

so the composite scalar field α is massless. Choosing $M^2=m^2$ we obtain $1/\lambda_{\rm ren}=0$.

The renormalization procedure for the μ propagator is similar. Renormalizing the constant ξ as

$$\frac{1}{\xi}=\frac{1}{\xi_{\rm ren}}-\int_0^{\Lambda^2}\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+M^2)^2} , \tag{4.23}$$

one obtains

$$\widetilde{\Gamma}^{\mu}(p)=\frac{1}{16\pi^2}\left[\ln\frac{m^2}{M_1^2}-2+(4m^2+p^2)A(p)\right]+\frac{1}{\xi_{\rm ren}} . \tag{4.24}$$

In particular, choosing

$$\frac{1}{\xi_{\rm ren}}=-\frac{1}{16\pi^2}\ln\frac{m^2}{M_1^2} \tag{4.25}$$

we get

$$\widetilde{\Gamma}^{\mu}(p)=\frac{1}{16\pi^2}[(4m^2+p^2)A(p)-2]\sim p^2 , \tag{4.26}$$

so μ is a charged massless field. If we take $m^2=M^2=M_1^2$ then $\widetilde{\Gamma}^{\alpha}(p)=\widetilde{\Gamma}^{\mu}(p)$ and $1/\xi_{\rm ren}=1/\lambda_{\rm ren}=0$. However, we would like to point out that in this case the propagators for α and $\mu,\bar{\mu}$ fields are generally not equal. They can be made equal by the suitable choice of $\lambda_{\rm ren}$ and $\xi_{\rm ren}$, namely,

$$\frac{1}{16\pi^2}\ln\frac{M_1^2}{M^2}=\frac{1}{\xi_{\rm ren}}-\frac{1}{\lambda_{\rm ren}} . \tag{4.27}$$

Now let us consider the λ_μ propagator. Here one can use the cutoff method, and renormalizing the coupling constant f obtain cancellation of the divergences. Such a method was presented in Ref. 5. However, the calculations are rather complicated. In fact, the difficulties mentioned above are connected with the gauge invariance. It is well known that the regularization by means of a cutoff Λ destroys this invariance. To avoid this problem we will use the dimensional-regularization technique.

Using the Feynman-Schwinger integral formula and introducing a complex continuous dimension 2ω we write $\widetilde{\Gamma}^{\lambda}_{\mu\nu}(q)$ as follows:

$$\widetilde{\Gamma}^{\lambda}_{\mu\nu}(q,2\omega)=\frac{1}{f}\left[1-\frac{q^2}{\kappa^2}\right]^{-1}(q^2\delta_{\mu\nu}-q_\mu q_\nu)+\int_0^1 d\alpha\int\frac{d^{2\omega}p}{(2\pi)^{2\omega}}\frac{2\delta_{\mu\nu}(p^2+\alpha^2q^2+m^2)-4p_\mu p_\nu-(2\alpha-1)^2q_\mu q_\nu}{[p^2+m^2+\alpha(1-\alpha)q^2]^2} . \tag{4.28}$$

This integral may be evaluated (see Refs. 11 and 12), and finally we obtain

$$\widetilde{\Gamma}^{\lambda}_{\mu\nu}(q,2\omega)=(q^2\delta_{\mu\nu}-q_\mu q_\nu)\left[\frac{1}{f}\left[1-\frac{q^2}{\kappa^2}\right]^{-1}+\Pi^{\rm div}(0,2\omega)-\Pi^{\rm reg}(q^2,2\omega)\right] , \tag{4.29}$$

where

$$\Pi^{\rm div}(0,2\omega)=\frac{1}{3}\frac{\pi^\omega}{(2\pi)^{2\omega}}m^{2\omega-4}\Gamma(2-\omega) , \tag{4.30}$$

$$\Pi^{\rm reg}(q^2,2\omega)=\frac{\pi^\omega}{(2\pi)^{2\omega}}m^{2\omega-4}\Gamma(3-\omega)\int_0^1 d\alpha(2\alpha-1)^2\ln\left[1+\alpha(1-\alpha)\frac{q^2}{m^2}\right] . \tag{4.31}$$

The expression (4.31) is finite when $\omega\to 2$,

$$\Pi^{\rm reg}(q^2,4)=\frac{1}{16\pi^2}\int_0^1 d\alpha(2\alpha-1)^2\ln\left[1+\alpha(1-\alpha)\frac{q^2}{m^2}\right] , \tag{4.32}$$

whereas $\Pi^{\rm div}(0,2\omega)$ is divergent when $\omega\to 2$. However, this infinity may be canceled by the renormalization of the coupling constant f. Namely,

$$\frac{1}{f}+\Pi^{\mathrm{div}}(0,2\omega)=\frac{1}{f_{\mathrm{ren}}},\quad \omega\to 2\ . \tag{4.33}$$

As a result we get the finite propagator for the gauge field:

$$\tilde{\Gamma}^{\lambda}_{\mu\nu}(q,4)=(q^2\delta_{\mu\nu}-q_\mu q_\nu)\left\{\frac{1}{f_{\mathrm{ren}}}-\frac{1}{16\pi^2}\int_0^1 d\alpha(2\alpha-1)^2\ln\left[1+\alpha(1-\alpha)\frac{q^2}{m^2}\right]\right\}. \tag{4.34}$$

In this case, similarly as for α,μ particles, the dynamics of the composite gauge fields depends on the choice of the renormalized coupling constant f_{ren}. In particular, if we choose f_{ren} such that

$$\frac{1}{f_{\mathrm{ren}}}-\Pi^{\mathrm{reg}}(q^2,4)\sim q^2\ , \tag{4.35}$$

then we obtain the λ_μ propagator with massless double pole.

Now we should investigate a connection between the parameter of the cutoff Λ and the parameter of the dimensional regularization $2-\omega$. Discussing the saddle-point condition we assumed that $\kappa^2/2f\sim\Lambda^2\to\infty$. On the other hand, from Eq. (4.33) we get $1/f\sim 1/(2-\omega)\to\infty$. So, if we assume that

$$(2-\omega)^{-1}\sim\ln\Lambda^2,\quad \frac{1}{f}\sim\frac{1}{2-\omega}\sim\ln\Lambda^2\to\infty\ , \tag{4.36}$$

we get the asymptotic behavior of the masslike parameter

$$\kappa^2\sim\Lambda^2/\ln\Lambda^2\to\infty\ .$$

One can easily check that asymptotic conditions (4.11) are also fulfilled.

There are some additional divergent terms in higher orders of the $1/N$ expansion. However, as was pointed out in Ref. 5, these divergences cancel pairwise due to the Abelian Ward identities. All other diagrams are convergent, which ends the proof of the renormalizability of the effective action.

V. THE SUPERSYMMETRIC RG(N,2) MODEL IN TWO DIMENSIONS

The supersymmetric CP($N-1$) model in four dimensions and its coupling to supergravity has been introduced first by Cremmer and Scherk.[13] In two dimensions it was also discussed by D'Adda, Lüscher, and Di Vecchia.[14] Following these references, in this section we will construct a supersymmetric generalization of our model.

To get the supersymmetric invariant action one starts with a superfield $\phi_i(x,\theta)$, $i=1,\ldots,N$,

$$\phi_i(x,\theta)=z_i(x)+i\theta\chi_i(x)+\tfrac{1}{2}i\theta\gamma_5\theta F_i(x)\ , \tag{5.1}$$

where the components $z_i(x)$, $F_i(x)$ are complex scalar fields, $\chi_i(x)$ is a spinor field, and θ_α is a real two-component spinor coordinate. All components transform according to the fundamental representation of O(N).

We introduce also the supersymmetric gauge transformation

$$\phi'(x,\theta)\to\phi(x,\theta)e^{i\Lambda(x,\theta)}\ , \tag{5.2}$$

where $\Lambda(x,\theta)$ is a real scalar superfield. One can construct a supersymmetric, gauge, and O(N)-invariant action using the supercovariant derivative

$$S=\frac{N}{8f}\int d^2x\, d\theta\,\gamma_5 d\theta\,\overline{\nabla\phi}\cdot\gamma_5\nabla\phi\ , \tag{5.3}$$

where

$$\nabla_\alpha=D_\alpha-A_\alpha\ , \tag{5.4}$$

$$D_\alpha=\partial_\alpha+i\theta_\alpha\partial\ . \tag{5.5}$$

The bar denotes here complex conjunction, and we use the following representation for the Euclidean γ matrices:

$$\gamma_0=\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$

$$\gamma_1=\begin{bmatrix}0&1\\1&0\end{bmatrix},$$

$$\gamma_5=\gamma_0\gamma_1=\begin{bmatrix}0&1\\-1&0\end{bmatrix}.$$

The constraints are

$$\begin{gathered}\bar\phi\cdot\phi=1\ ,\\ \phi\cdot\phi=\bar\phi\cdot\bar\phi=0\ .\end{gathered} \tag{5.6}$$

In terms of the component fields it reads

$$\begin{gathered}\bar z\cdot z=1,\quad z^2=\bar z^2=0\ ,\\ \bar z\cdot\chi-\bar\chi\cdot z=0\ ,\quad z\cdot\chi=\bar z\cdot\bar\chi=0\ ,\\ \bar z\cdot F+\bar F\cdot z=i\bar\chi\cdot\gamma_5\chi,\quad z\cdot F=\tfrac{1}{2}i\chi\cdot\gamma_5\chi\ .\end{gathered} \tag{5.7}$$

The action (5.3) with constraints (5.6) gives a supersymmetric extension of the RG(N,2) model. Classically, in terms of the superfields we can write the topological charge, the equations of motion, and the supersymmetric version of the self-duality equations presented in Sec. II.

The fermionic superfluid $A_\alpha(x,\theta)$ may be eliminated by using its equation of motion:

$$A_\alpha=\bar\phi\cdot D_\alpha\phi\ . \tag{5.8}$$

Now we rewrite the action (5.3) in terms of components. Performing the integration over θ_1,θ_2, using constraints and equations of motion for auxiliary fields, we finally obtain[14]

$$S=\frac{N}{2f}\int d^2x\{\overline{D_\mu z}\cdot D_\mu z-i\bar\psi\cdot\not{D}\psi+\tfrac{1}{4}[(\bar\psi\cdot\psi)^2+(\bar\psi\cdot\gamma_5\psi)^2-(\bar\psi\cdot\gamma_\mu\psi)^2]\}\ , \tag{5.9}$$

where $D_\mu=\partial_\mu-\bar{z}\cdot\partial_\mu z$ and a new fermionic field ψ is given by

$$\psi_i=\chi_i-z_i(\bar{z}\cdot\chi)\ . \tag{5.10}$$

The constraints now supplementing the action are

$$\bar{z}\cdot z=1,\ \ z\cdot z=\bar{z}\cdot\bar{z}=0\ ,\ \ \bar{z}\cdot\psi=\bar{\psi}\cdot z=z\cdot\psi=\bar{\psi}\cdot\bar{z}=0\ , \tag{5.11}$$

and the gauge transformations

$$z'(x)=e^{i\Lambda(x)}z(x),\ \ \psi'(x)=e^{i\Lambda(x)}\psi(x)\ . \tag{5.12}$$

The action is also invariant under the supersymmetry transformations[14]

$$\delta z_i=i\epsilon\psi_i,\ \ \delta\psi_i=-\tfrac{1}{2}i\epsilon z_i(\bar{\psi}\cdot\psi)+\tfrac{1}{2}i\gamma_5\epsilon z_i(\bar{\psi}\cdot\gamma_5\psi)+\gamma_\mu\epsilon[D_\mu z_i-\tfrac{1}{2}iz_i(\bar{\psi}\cdot\gamma_\mu\psi)]\ . \tag{5.13}$$

One can write the generating functional for Euclidean Green's functions of our model:

$$Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi\,\delta(\bar{z}\cdot z-1)\delta(\bar{\psi}\cdot z)\delta(\psi\cdot\bar{z})\delta(z^2)\delta(\bar{z}^2)\delta(\psi\cdot z)\delta(\bar{\psi}\cdot\bar{z})$$
$$\times\exp\left[-\frac{N}{2f}\int d^2x\{\overline{D_\mu z}\cdot D_\mu z-i\bar{\psi}\cdot\not{D}\psi+\tfrac{1}{4}[(\bar{\psi}\cdot\psi)^2+(\bar{\psi}\cdot\gamma_5\psi)^2-(\bar{\psi}\cdot\gamma_\mu\psi)^2]\}\right]\ . \tag{5.14}$$

The exponentiation of the constraints leads to the appearance of the Lagrange multiplier fields $\alpha(x)$, $\mu(x)$, $\bar{\mu}(x)$, $c(x)$, $\bar{c}(x)$, $\sigma(x)$, $\bar{\sigma}(x)$ and (5.14) reads

$$Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi\,\mathscr{D}\alpha\,\mathscr{D}\bar{c}\,\mathscr{D}c\,\mathscr{D}\bar{\mu}\,\mathscr{D}\mu\,\mathscr{D}\bar{\sigma}\,\mathscr{D}\sigma\exp[-S+i(z\cdot z-1)\alpha+i\mu z^2+i\bar{\mu}\bar{z}^2+i\bar{c}\bar{z}\cdot\psi$$
$$+i\bar{\psi}\cdot zc+i\bar{\sigma}z\cdot\psi+i\bar{\psi}\cdot\bar{z}\sigma]\ . \tag{5.15}$$

Rescaling the fields

$$z\to\left[\frac{N}{2f}\right]^{1/2}z,\ \ \bar{z}\to\left[\frac{N}{2f}\right]^{1/2}\bar{z},\ \ \psi\to\left[\frac{N}{2f}\right]^{1/2}\psi,\ \ \bar{\psi}\to i\left[\frac{N}{2f}\right]^{1/2}\bar{\psi}\ ,$$
$$\alpha\to\frac{2f}{\sqrt{N}}\alpha,\ \ (\mu,\bar{\mu})\to\frac{2f}{\sqrt{N}}(\mu,\bar{\mu})\ ,\ \ (\bar{c},c)\to\frac{2f}{\sqrt{N}}(\bar{c},c),\ \ (\bar{\sigma},\sigma)\to\frac{2f}{\sqrt{N}}(\bar{\sigma},\sigma) \tag{5.16}$$

and introducing the integral over the auxiliary fields we obtain

$$Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi\,\mathscr{D}\alpha\,\mathscr{D}\bar{c}\,\mathscr{D}c\,\mathscr{D}\bar{\mu}\,\mathscr{D}\mu\,\mathscr{D}\bar{\sigma}\,\mathscr{D}\sigma\,\mathscr{D}\lambda_\mu\mathscr{D}\phi\,\mathscr{D}\phi_5$$
$$\times\exp\left[\int d^2x\left[\bar{z}\cdot\Box z+\lambda_\mu[(\bar{z}\cdot\overleftrightarrow{\partial}_\mu z)-\bar{\psi}\cdot\gamma_\mu\psi]\frac{i}{\sqrt{N}}+\frac{i\alpha}{\sqrt{N}}\bar{z}\cdot z-\frac{i\sqrt{N}}{2f}\alpha-\bar{\psi}\cdot\partial\psi+\frac{i}{\sqrt{N}}(\mu z^2+\bar{\mu}\bar{z}^2)\right.\right.$$
$$+\frac{i}{\sqrt{N}}(\bar{c}\bar{z}\cdot\psi+\bar{\psi}\cdot zc)+\frac{i}{\sqrt{N}}(\bar{\sigma}\psi\cdot z+\bar{\psi}\cdot\bar{z}\sigma)+\frac{1}{\sqrt{N}}(\phi\bar{\psi}\cdot\psi+\phi_5\bar{\psi}\cdot\gamma_5\psi)$$
$$\left.\left.-\frac{1}{2f}(\phi^2+\phi_5{}^2)-\frac{1}{N}\lambda^2\bar{z}\cdot z\right]\right]\ . \tag{5.17}$$

After performing some simple transformations we get

$$Z=\int\mathscr{D}\bar{z}\,\mathscr{D}z\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi\,\mathscr{D}\alpha\,\mathscr{D}\bar{c}\,\mathscr{D}c\,\mathscr{D}\bar{\mu}\,\mathscr{D}\mu\,\mathscr{D}\bar{\sigma}\,\mathscr{D}\sigma\,\mathscr{D}\lambda_\mu\mathscr{D}\phi\,\mathscr{D}\phi_5$$
$$\times\left[\int d^2x\left[\bar{z}\cdot\Delta_B z+\bar{\psi}\cdot\Delta_F\psi+\frac{i\sqrt{N}}{2f}\alpha+\tfrac{1}{2}(\phi^2+\phi_5{}^2)-\frac{i}{\sqrt{N}}[\mu z^2+\bar{\mu}\bar{z}^2+(\bar{c}\bar{z}+\bar{\sigma}z)\cdot\psi+\bar{\psi}\cdot(zc+\bar{z}\sigma)]\right]\right]\ , \tag{5.18}$$

where we use the notation

$$\Delta_B=-D_\mu D_\mu-\frac{i}{\sqrt{N}}\alpha,\ \ D_\mu=\partial_\mu+\frac{i}{\sqrt{N}}\lambda_\mu,\ \ \Delta_F=\not{D}-\frac{1}{\sqrt{N}}(\phi+\phi_5\gamma_5)\ . \tag{5.19}$$

We can perform the Gaussian path integration over fermionic fields $\bar{\psi}$, ψ and over $z,\bar{z}$ getting the effective action of the model

$$S_{\rm eff}=\tfrac{1}{2}N\,{\rm Tr}\ln\begin{bmatrix}\Delta_B+\frac{2}{N}\bar{c}\Delta_F{}^{-1}c & \frac{2i}{\sqrt{N}}\bar{\mu}+\frac{2}{N}\bar{c}\Delta_F{}^{-1}\sigma\\ \frac{2i}{\sqrt{N}}\mu+\frac{2}{N}\bar{\sigma}\Delta_F{}^{-1}c & \Delta_B^T+\frac{2}{N}\bar{\sigma}\Delta_F{}^{-1}\sigma\end{bmatrix}-N\,{\rm Tr}\ln\Delta_F+\int d^2x\left[\frac{i\sqrt{N}}{2f}\alpha+\frac{1}{2f}\phi^2+\frac{1}{2f}\phi_5{}^2\right]. \quad (5.20)$$

As before, we wish to now expand the effective action in a power series of $1/\sqrt{N}$ around a classical minimum. Such a minimum occurs at nonzero constant values of the α,ϕ fields.

We write the saddle-point condition as

$$\begin{aligned}&c^c=\bar{c}^c=\sigma^c=\bar{\sigma}^c=\mu^c=\bar{\mu}^c=\lambda_\mu^c=\phi_5^c=0\ ,\\&\alpha^c\neq 0\neq\phi^c\ ,\\&\left.\frac{\delta S_{\rm eff}}{\delta\alpha}\right|_{\alpha=\alpha^c}=\left.\frac{\delta S_{\rm eff}}{\delta\phi}\right|_{\phi=\phi^c}=0\ .\end{aligned}\quad (5.21)$$

If we define $\alpha^c=i\sqrt{N}m^2$, $\phi^c=\sqrt{N}m$, Eqs. (5.21) read

$$\frac{1}{2f}-\int\frac{d^2p}{(2\pi)^2}\frac{1}{p^2+m^2}=0 \quad (5.22)$$

and the saddle-point condition coincides with the one obtained in the supersymmetric CP($N-1$) model.[14] The divergent integral in (5.22) may be regularized with a cutoff Λ and as a result we obtain the finite equation for an arbitrary parameter m^2.

Performing suitable shifts in $S_{\rm eff}$,

$$\begin{aligned}&\alpha\to\alpha+i\sqrt{N}m^2\ ,\\&\phi\to\phi+\sqrt{N}m\ ,\end{aligned}\quad (5.23)$$

we get the spontaneously generated mass for fermions and scalars. Expanding the effective action around the minimum we find that the term proportional to $\sqrt{N}$ vanishes due to the saddle-point condition. The quadratic part of $S_{\rm eff}$ is given by

$$\begin{aligned}S^{(2)}=&\tfrac{1}{2}\alpha\Gamma^\alpha\alpha+\tfrac{1}{2}\lambda_\mu\Gamma^\nu_{\mu\nu}\lambda_\nu+\bar{\mu}\Gamma^\mu\mu+\tfrac{1}{2}\phi\Gamma^\phi\phi\\&+\tfrac{1}{2}\phi_5\Gamma^{\phi_5}\phi_5+\lambda_\mu\Gamma^{\lambda\phi}_\mu\phi_5+\bar{c}\Gamma^{\bar{c}c}c+\bar{\sigma}\Gamma^{\bar{\sigma}\sigma}\sigma\ ,\end{aligned}$$

where the Fourier transforms of propagators are[14]

$$\tilde{\Gamma}^{\alpha}(p)=A(p)\ , \quad (5.24)$$

$$\tilde{\Gamma}^{\mu}(p)=A(p)\ , \quad (5.25)$$

$$\tilde{\Gamma}^{\lambda}_{\mu\nu}(p)=(p^2\delta_{\mu\nu}-p_\mu p_\nu)A(p)\ , \quad (5.26)$$

$$\tilde{\Gamma}^{\phi}(p)=(p^2+4m)A(p)\ , \quad (5.27)$$

$$\tilde{\Gamma}^{\phi_5}(p)=p^2A(p)\ , \quad (5.28)$$

$$\tilde{\Gamma}^{\lambda\phi}(p)=-\epsilon_{\mu\nu}p_\nu 2mA(p)\ , \quad (5.29)$$

$$\tilde{\Gamma}^{c}(p)=(\tfrac{1}{2}i\not{p}-m)A(p)\ , \quad (5.30)$$

$$\tilde{\Gamma}^{\sigma}(p)=(\tfrac{1}{2}i\not{p}-m)A(p)\ . \quad (5.31)$$

The propagators for the auxiliary fields calculated in the $1/N$ expansion are the same as in the supersymmetric CP($N-1$) model. However, in our case we have new particles connected with additional constraints: the charged composite scalar particles $\mu,\bar{\mu}$ of the dynamics, which is the same as the dynamics of α, and the spinor particles $\sigma,\bar{\sigma}$ of the dynamics, which is the same as the dynamics of $c,\bar{c}$.

The masses for scalar and spinor fields are thus generated. However, $m_z=m_\psi$ and the supersymmetry is not broken. Following Ref. 14, one can establish the Feynman rules and show that the higher-order diagrams in the $1/N$ expansion are finite and need not be renormalized.

ACKNOWLEDGMENTS

I would like to thank Professor J. Lukierski for suggesting the subject of this paper, discussions, and much useful advice. I would also like to thank Dr. B. Milewski for many useful discussions.

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