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Light-cone gauge in Yang-Mills theory

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A prescription for massless Feynman integrals in the light-cone gauge $n_\mu A_\mu^a=0$, $n^2=0$, is suggested which leads to well-defined and exact, but Lorentz-noninvariant, integrals. As a result the Yang-Mills self-energy is likewise Lorentz noninvariant, but remains transverse in agreement with the Ward and Becchi-Rouet-Stora (BRS) identities. It is also shown that the assumption of Lorentz invariance of the integrals, coupled with the validity of the Ward (BRS) identity, leads to a nonlocal Yang-Mills self-energy.

I. INTRODUCTION

The light-cone gauge is one of the latest of physical but noncovariant gauges to find favor among theoreticians. It belongs, like the axial and planar gauges, to the class of general axial gauges characterized by an arbitrary but constant vector n_μ. For the axial and planar gauges, n_μ need only satisfy $n^2\neq 0$—a relatively harmless condition compared with $n^2=0$ in the light-cone gauge. The constraint $n^2=0$ hides, as we shall see, new technical problems that endow the light-cone formalism with some unexpected properties.

In Yang-Mills theory, with the Lagrangian density

$$L_{\rm YM}=-\tfrac{1}{4}(F_{\mu\nu}^a)^2-\frac{1}{2\alpha}(n\cdot A^a)^2\ , \quad (1)$$

the light-cone gauge is specified by

$$n\cdot A^a=0,\ \ n^2=0\ , \quad (2)$$

where A_μ^a is a massless gauge field and α the gauge parameter. The corresponding propagator (we use a $+---$ metric and employ dimensional regularization with a space-time of 2ω dimensions)

$$G_{\mu\nu}^{ab}(q)=\frac{-i\delta^{ab}}{(2\pi)^{2\omega}(q^2+i\epsilon)}\times\left[\delta_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{n\cdot q}-\frac{\alpha q^2 q_\mu q_\nu}{(n\cdot q)^2}\right] \quad (3)$$

leads to integrals of the form

$$\int\frac{d^{2\omega}q\, f(q^2,q_\mu,n_\mu,q\cdot p)}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}\ , \quad (4)$$

p_μ being an external momentum. Our aim is to prescribe a definition of these integrals, and then to evaluate them consistently and unambiguously.

II. PROBLEMS WITH PREVIOUS LIGHT-CONE GAUGE PRESCRIPTIONS

It is not difficult to see that the usual principal-value prescription[1]

$$\frac{1}{q\cdot n}\Longrightarrow {\rm P.V.}\frac{1}{q\cdot n}\equiv\tfrac{1}{2}\lim_{\mu\to 0}\left[\frac{1}{q\cdot n+i\mu}+\frac{1}{q\cdot n-i\mu}\right] \quad (5)$$

leads to problems when $n^2=0$, both in Minkowski and Euclidean spaces. In Minkowski space, both poles of $(q\cdot n\pm i\mu)^{-1}$, namely, $q_0=(\vec{q}\cdot\vec{n}\pm i\mu)/n_0$, lie on a line parallel to the ${\rm Im}q_0$ axis. Their location prevents a Wick rotation to Euclidean momenta without encircling one of the singularities so that the conventional prescription (5) is, clearly, of no help unless the use of Euclidean space could be avoided. In Appendix B we perform a sample calculation of several integrals in Minkowski space using the conventional P.V. prescription (5). We find that this method leads to integrals which are inconsistent and which give a Yang-Mills self-energy which contradicts the Ward identity and Becchi-Rouet-Stora (BRS) identity.

A similar conclusion holds if one assumes, at the very outset, that $q\cdot n=n_4q_4+\vec{n}\cdot\vec{q}$ is defined over Euclidean space, and then tries to apply (5). The reason is that $n^2=0$ implies $n_4=\pm i\,|\,\vec{n}\,|$, so that $(q\cdot n)^{-1}$ is already complex and nothing is being gained by adding $\pm i\mu$, as in (5): the denominator of $(q\cdot n\pm i\mu)^{-1}$ remains ill defined.

These comments may help explain why the conventional light-cone prescription(s) available in the literature[1] lead to poorly defined integrals such as[2]

$$\int_0^1 dx\, x^{\omega-2}(1-x)^{-\omega}=\Gamma(\omega-1)\Gamma(1-\omega)/\Gamma(0)\ .$$

III. DIFFERENT APPROACH

We should now like to describe the following different prescription for $(q\cdot n)^{-1}$ in the light-cone gauge. Remembering that $n_4=\pm i\,|\,\vec{n}\,|$ in Euclidean space, we first write $(q\cdot n)^{-1}=(-\vec{q}\cdot\vec{n}+i\,|\,\vec{n}\,|\,q_4)^{-1}$, where the minus sign has been chosen. The second step is to rationalize the denominator,

$$\frac{1}{q\cdot n}=\frac{-(\vec{q}\cdot\vec{n}+i\,|\,\vec{n}\,|\,q_4)}{(\vec{q}\cdot\vec{n})^2+\vec{n}^{\,2}q_4^{\,2}}\ , \quad (6)$$

and *then* add a small, but real number μ^2, $\mu>0$, to ensure positive definiteness of the denominator. Our prescription in Euclidean space amounts, therefore, to replacing $(q\cdot n)^{-1}$ by

$$\frac{1}{q\cdot n}=-\lim_{\mu\to 0}\frac{\vec{q}\cdot\vec{n}+i\,|\,\vec{n}\,|\,q_4}{(\vec{q}\cdot\vec{n})^2+\vec{n}^{\,2}q_4{}^2+\mu^2} \tag{7}$$

in the integrand of (4).

In Minkowski space, we first write

$$(q\cdot n)^{-1}=(n_0q_0-\vec{n}\cdot\vec{q})^{-1}$$
$$=\frac{n_0q_0+\vec{n}\cdot\vec{q}}{n_0{}^2q_0{}^2-(\vec{n}\cdot\vec{q})^2}\,, \tag{8a}$$

and then add $+i\epsilon$, $\epsilon>0$, which places the poles in the second and fourth quadrants of the complex q_0 plane. Our prescription in Minkowski space amounts, therefore, to replacing $(q\cdot n)^{-1}$ by

$$\frac{1}{q\cdot n}=\lim_{\epsilon\to 0}\left[\frac{n_0q_0+\vec{n}\cdot\vec{q}}{n_0{}^2q_0{}^2-(\vec{n}\cdot\vec{q})^2+i\epsilon}\right]. \tag{8b}$$

It is "safe" now, in the case of a typical integral, to combine the various propagators according to Feynman, provided the same $+i\epsilon$ is used in each propagator. If so desired, one could then evaluate the integral entirely in Minkowski space by first finding the residue(s) in the complex q_0 plane and then using dimensional regularization to integrate over the remaining $(2\omega-1)$ components. This procedure is discussed in detail in Appendix A.

On the other hand, a Wick rotation in (8a) with $q_0=iq_4$ leads to

$$(q\cdot n)^{-1}=\frac{-(in_0q_4+\vec{n}\cdot\vec{q})}{n_0{}^2q_4{}^2+(\vec{n}\cdot\vec{q})^2}\,.$$

For a consistent evaluation of the integrals it is important to add a small number λ^2, $\lambda>0$, just as in the Euclidean case discussed above:

$$\frac{1}{q\cdot n}=-\lim_{\lambda\to 0}\frac{\vec{q}\cdot\vec{n}+in_0q_4}{(\vec{n}\cdot\vec{q})^2+n_0{}^2q_4{}^2+\lambda^2}\,.$$

Note that the n_4 vector remains fixed here, i.e., $n_0=n_4$. A somewhat different prescription, but also not a principal value, has been used by Mandelstam.[3]

IV. SAMPLE CALCULATION IN EUCLIDEAN SPACE

As a sample calculation let us evaluate one of the basic integrals,

$$\int d^{2\omega}q\{[(q-p)^2+i\epsilon]q\cdot n\}^{-1}\equiv I\,,$$

in the light-cone gauge. Using dimensional regularization and omitting henceforth the $i\epsilon$ in $[(q-p)^2+i\epsilon]$, we get in Euclidean space

$$I=i\lim_{\mu\to 0}\int\frac{d^{2\omega}q\,(\vec{q}\cdot\vec{n}+in_0q_4)}{(q-p)^2[n_0{}^2q_4{}^2+(\vec{n}\cdot\vec{q})^2+\mu^2]}\,. \tag{9}$$

Since the integrand is *not* Lorentz invariant, it is necessary to divide the region of integration according to

$$d^{2\omega}q=d^{2\omega-1}\vec{q}\,dq_4$$

and to employ exponential parametrization for the two propagators:

$$I=i\lim_{\mu\to 0}\int_0^\infty d\alpha\,d\beta\int d^{2\omega-1}\vec{q}\exp(-E_1)\int_{-\infty}^{+\infty}dq_4[\exp(-E_2)](\vec{q}\cdot\vec{n}+in_0q_4) \tag{10}$$

with

$$E_1=\alpha\mu^2+\beta p^2+\beta\vec{q}^{\,2}-2\beta\vec{q}\cdot\vec{p}+\alpha(\vec{n}\cdot\vec{q})^2\,,$$
$$E_2=(\beta+\alpha n_0{}^2)q_4{}^2-2\beta q_4p_4,\quad E_0=\beta\vec{q}^{\,2}-2\beta\vec{q}\cdot\vec{p}+\alpha(\vec{n}\cdot\vec{q})^2\,.$$

Instead of using formulas (A9) and (A10) in the second paper of Ref. 4, we integrate over q_4 and $\vec{q}$ space with the aid of the following integrals:

$$\int_{-\infty}^{+\infty}dq_4\exp(-E_2)=(\pi/A)^{1/2}\exp(+\beta^2p_4{}^2A^{-1}),\quad A\equiv\beta+n_0{}^2\alpha\,, \tag{11a}$$

$$\int_{-\infty}^{\infty}dq_4q_4\exp(-E_2)=\beta p_4(\pi)^{1/2}A^{-3/2}\exp(+\beta^2p_4{}^2A^{-1})\,, \tag{11b}$$

$$\int d^{2\omega-1}\vec{q}\exp(-E_0)=\pi^{\omega-1/2}\beta^{1-\omega}A^{-1/2}\exp\left[\beta\vec{p}^{\,2}-\frac{\alpha\beta(\vec{p}\cdot\vec{n})^2}{A}\right], \tag{11c}$$

$$\int d^{2\omega-1}\vec{q}\,\vec{q}\cdot\vec{n}\exp(-E_0)=\pi^{\omega-1/2}\beta^{2-\omega}\vec{p}\cdot\vec{n}A^{-3/2}\exp\left[\beta\vec{p}^{\,2}-\frac{\alpha\beta(\vec{p}\cdot\vec{n})^2}{A}\right]. \tag{11d}$$

Hence,

$$I=i\pi^\omega(\vec{p}\cdot\vec{n}+in_0p_4)\lim_{\mu\to 0}\int_0^\infty d\alpha\,d\beta\,\beta^{2-\omega}(\beta+\alpha n_0{}^2)^{-2}$$
$$\times\exp\left[-\alpha\mu^2-\beta p^2+\beta\vec{p}^{\,2}+\left[\frac{-\alpha\beta(\vec{p}\cdot\vec{n})^2+\beta^2p_4{}^2}{(\beta+\alpha n_0{}^2)}\right]\right].$$

Integrating over parameter space and keeping only the pole term as $\omega \to 2^+$, we readily obtain

$$I=\frac{n_0p_0+\vec{n}\cdot\vec{p}}{\vec{n}^2}\bar{I}[(\vec{n}\cdot\vec{p})^2+n_0{}^2p_4{}^2]^{\omega-2}\,,$$
$$I=\frac{n_0p_0+\vec{n}\cdot\vec{p}}{\vec{n}^2}\bar{I}\,, \tag{12}$$

where $\bar{I}$ denotes the divergent part of

$$\int\frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]}=\frac{i\pi^{\omega}\Gamma(2-\omega)(-p^2)^{\omega-2}[\Gamma(\omega-1)]^2}{\Gamma(2\omega-2)},\quad \epsilon\to 0\,.$$

The same result (12) is obtained in Minkowski space, as shown in Appendix A. The right-hand side of (12) may be rewritten as

$$I=\frac{2p\cdot n^*}{n\cdot n^*}\bar{I},\quad n^2=0 \tag{13}$$

by using the notation $n_\mu=(n_0,\vec{n})$ and $n_\mu^*=(n_0,-\vec{n})$. We note in passing that for the special value of the momentum $p_\mu=(p_0,0,0,p_3)$, with $n_\mu=(n_0,0,0,n_3)$, Eq. (13) actually reduces to $I=2p^2(p\cdot n)^{-1}\bar{I}$, which *is* Lorentz invariant. Similarly it may be shown that

$$\int\frac{d^{2\omega}q\,q_\mu}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}=\frac{n_\mu^*}{n\cdot n^*}\bar{I}\,, \tag{14a}$$

$$\int\frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}=0\,, \tag{14b}$$

for the divergent part of the integrals on the left. It is instructive to recall, at this stage, the results in the *axial* gauge:[4]

$$\int\frac{d^{2\omega}q}{[(q-p)^2+i\epsilon]q\cdot n}=\frac{2p\cdot n}{n^2}\bar{I},\quad n^2\neq 0\,, \tag{15}$$

$$\int\frac{d^{2\omega}q\,q\mu}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}=\frac{n_\mu}{n^2}\bar{I},\quad n^2\neq 0\,, \tag{16a}$$

$$\int\frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}=0,\quad n^2\neq 0\,. \tag{16b}$$

We see that there exists a remarkable similarity between the basic integrals in the light-cone gauge and those in the axial gauge.

V. THE YANG-MILLS SELF-ENERGY

Using the new prescription and the integrals (13) and (14), we find that the infinite part of the Yang-Mills (YM) self-energy in the light-cone gauge reads

$$\Pi_{\mu\nu}^{ab}(p)=c^{ab}\Gamma(2-\omega)\left[\tfrac{11}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)+\frac{2n\cdot p}{n\cdot n^*}(p_\mu n_\nu^*+p_\nu n_\mu^*)+\frac{2p\cdot n^*}{p\cdot n n\cdot n^*}[2p^2n_\mu n_\nu-p\cdot n(n_\nu p_\mu+n_\mu p_\nu)]-\frac{2p^2}{n\cdot n^*}(n_\mu n_\nu^*+n_\nu n_\mu^*)\right], \tag{17}$$

where $c^{ab}\equiv i\pi^2C_{\rm YM}g^2\delta^{ab}$ and $\delta^{ab}C_{\rm YM}=f^{acd}f^{bcd}$. Despite overall Lorentz noninvariance, the self-energy is seen to contain the traditional $\frac{11}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)$ term and the factor $[2p^2n_\mu n_\nu-p\cdot n(p_\mu n_\nu+p_\nu n_\mu)]$ which is reminiscent of the YM self-energy in the planar gauge.[4] Moreover, it is easy to see that (17) is transverse,

$$p_\mu\Pi_{\mu\nu}^{ab}(p)=0\,, \tag{18}$$

in agreement with the Ward identity

$$\frac{1}{\alpha}n_\mu n\cdot pG_{\mu\nu}'^{ab}(p)-\frac{i\delta^{ab}}{(2\pi)^4}p_\nu+\frac{gf^{abc}}{(2\pi)^4}B_\nu^c(p)=0\,, \tag{19}$$

which follows from the Lagrangian (1). Here $G_{\mu\nu}'^{ab}(p)$ denotes the bare propagator to one-loop order and $B_\nu^c(p)$ is the Fourier-transform vacuum expectation value of $A_\mu^c(y)$, and which is a tadpole term, vanishing in dimensional regularization.

Alternatively, the BRS identity gives[5]

$$\begin{aligned}p_\mu\Pi_{\mu\nu}^{ab}(p)&\propto\int d^{2\omega}q\frac{1}{n\cdot(p-q)}n_\mu G_{\mu\nu}^{ab}(q)\\&=-\alpha\int d^{2\omega}q\frac{1}{n\cdot(p-q)}\frac{1}{n\cdot q}q_\nu\delta^{ab}\\&=-\frac{\delta^{ab}\alpha}{n\cdot p}\int d^{2\omega}q\left[\frac{1}{n\cdot(p-q)}+\frac{1}{n\cdot q}\right]q_\nu\\&=+\delta^{ab}\alpha\frac{p_\nu}{n\cdot p}\int d^{2\omega}q'\frac{1}{n\cdot q'}=0\,,\end{aligned} \tag{20}$$

where $q'=q-p$.

VI. DISCUSSION

In summary, a prescription for $(q\cdot n)^{-1}$ in the light-cone gauge $n\cdot A^a=0$, $n^2=0$, has been suggested leading to exact and well-defined, albeit Lorentz-noninvariant, integrals. The Lorentz noninvariance manifests itself through the appearance of terms proportional to $p\cdot n^*$ and $n\cdot n^*$ as found, for example, in the one-loop Yang-Mills self-energy (17).

This Lorentz noninvariance, which may be traced back to the momentum-space integrals (13) and (14), is an unexpected peculiarity of the light-cone gauge. It is intimately connected with the fact that, in the light-cone gauge, expressions like $(p\cdot n)^2/n^2p^2$ cease to be meaningful invariants. As shown explicitly in this paper, the appropriate replacement, in the light-cone gauge, of a term like $p\cdot n/n^2$ is $p\cdot n^*/n\cdot n^*$.

We are the first to admit that a Lorentz-noninvariant quantity like the self-energy (17) may be somewhat unsettling. But if one insists on a Lorentz-*invariant* set of integrals (as assumed in the literature) and assumes, moreover, validity of the Ward (BRS) identity, one arrives at a self-energy $\Pi_{\mu\nu}^{ab}$ which is *nonlocal.* The details of this analysis are explained in Appendix C.

Let us also mention the following two technical features of the light-cone gauge. The first is that differentiation with respect to n_μ under the integral ceases to be a useful tool, because the components of n_μ are no longer independent. The second point concerns the parametrization of massless propagators. It seems that in Euclidean space exponential parametrization is superior to Feynman's conventional method of combining propagators. Although Euclidean- and Minkowski-space methods lead to the same results, our experience has been that calculations in Euclidean space [using the prescription (7)] are generally less complicated than those in Minkowski space, especially for integrals involving three or more propagators.

Finally, it remains to be seen what the implications of the present prescription will be for QCD, especially with respect to the evaluation of ladder graphs in the leading and next-to-leading log approximations.

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APPENDIX A

In this appendix we apply the new prescription (8b), in Minkowski space, to the integral

$$I=\int \frac{d^{2\omega}q}{[(q-p)^2+i\epsilon]q\cdot n}$$

$$=\lim_{\epsilon\to 0}\int \frac{d^{2\omega}q(\vec{q}\cdot\vec{n}+n_0q_0)}{[(q-p)^2+i\epsilon][n_0{}^2q_0{}^2-(\vec{n}\cdot\vec{q})^2+i\epsilon]} \tag{A1}$$

$$=\lim_{\epsilon\to 0}\int_0^1 dx\,A^{-2}\int d^{2\omega-1}\vec{q}\int_{-\infty}^{\infty}dq_0(\vec{n}\cdot\vec{q}+n_0q_0)\left[\left[q_0-\frac{B}{A}\right]^2-\frac{d}{A^2}+\frac{i\epsilon}{A}\right]^{-2}, \tag{A2}$$

where

$$A=x+(1-x)n_0{}^2,\quad B=xp_0,\quad d=B^2-AC\,,$$
$$C=xp_0{}^2-x(\vec{q}-\vec{p})^2-(1-x)(\vec{n}\cdot\vec{q})^2\,. \tag{A3}$$

The q_0 integral has double poles at $q_0^{\pm}=(B\pm\sqrt{d}\mp i\epsilon)/A$; it suffices, therefore, to evaluate the integral at the double pole q_0^-, closing the contour above the real q_0 axis. Hence,

$$\int_{-\infty}^{\infty}\frac{dq_0(n_0q_0+\vec{n}\cdot\vec{q})}{[(q_0-B/A)^2-d/A^2+i\epsilon/A]^2}=\frac{i\pi A^2}{2}\left[\frac{n_0B}{d^{3/2}}+\frac{\vec{n}\cdot\vec{q}A}{d^{3/2}}\right] \tag{A4}$$

so that

$$I=\frac{i\pi}{2}\int_0^1 dx\int d^{2\omega-1}\vec{q}\left[\frac{n_0B}{d^{3/2}}+\frac{\vec{n}\cdot\vec{q}A}{d^{3/2}}\right]. \tag{A5}$$

To integrate over $(2\omega-1)$-dimensional $\vec{q}$ space it is convenient to define $(\vec{q}-\vec{p})\equiv\vec{Q}$, in which case

$$I=\frac{i\pi}{2}\int_0^1 dx\int d^{2\omega-1}\vec{Q}\left[\frac{n_0B}{d^{3/2}}+\frac{\vec{n}\cdot(\vec{p}+\vec{Q})A}{d^{3/2}}\right]$$

$$\equiv\frac{i\pi}{2}\int_0^1 dx(J_1+J_2)\,, \tag{A6}$$

$$J_1=n_0B\int d^{2\omega-1}\vec{Q}\,d^{-3/2}\,,$$
$$J_2=A\int d^{2\omega-1}\vec{Q}\,\vec{n}\cdot(\vec{p}+\vec{Q})d^{-3/2}\,, \tag{A7}$$

and

$$d=a\vec{Q}^2+2b\vec{n}\cdot\vec{Q}+g(\vec{n}\cdot\vec{Q})^2+f\,;$$
$$a=Ax,\quad b=(1-x)A\vec{n}\cdot\vec{p},\quad g=(1-x)A\,,$$
$$f=(1-x)[A(\vec{n}\cdot\vec{p})^2-xn_0{}^2p_0{}^2]\,. \tag{A8}$$

We analyze the two J integrals separately.

In order to evaluate J_1 we employ the representation

$$d^{-3/2}=[\Gamma(\tfrac{3}{2})]^{-1}\int_0^\infty d\alpha\,\alpha^{1/2}\exp(-\alpha d)$$

and write

$$J_1=\frac{n_0B}{\Gamma(\frac{3}{2})}\int_0^\infty d\alpha\,\alpha^{1/2}\int d^{2\omega-1}\vec{Q}\,e^{-\alpha d}\,. \tag{A9}$$

The $\vec{Q}$ integration is then readily performed with the aid of the formula[4]

$$\int d^{2\omega}q\,\exp[-\alpha q^2-2\beta p\cdot q-\gamma(q\cdot n)^2]=\left[\frac{\pi}{\alpha}\right]^{\omega}\frac{\alpha^{1/2}}{(\alpha+\gamma n^2)^{1/2}}\exp\left[\frac{\beta^2p^2}{\alpha}-\frac{\gamma\beta^2(p\cdot n)^2}{\alpha(\alpha+\gamma n^2)}\right]$$

and yields

$$J_1=\frac{n_0B\pi^{\omega-1/2}a^{1-\omega}}{\Gamma(\frac{3}{2})(a+g\vec{n}^2)^{1/2}}\int_0^\infty d\alpha\,\alpha^{1-\omega}\exp\left[-\alpha f+\frac{\alpha b^2\vec{n}^2}{(a+g\vec{n}^2)}\right]$$

$$=\frac{2\pi^{\omega-1}n_0Ba^{1-\omega}}{A}\int_0^\infty d\alpha\,\alpha^{1-\omega}\exp(-\alpha G),\quad G=x(1-x)N^2,\quad N^2=(\vec{n}\cdot\vec{p})^2-n_0{}^2p_0{}^2\,, \tag{A10}$$

$$J_1=\frac{2n_0B\pi^{\omega-1}\Gamma(2-\omega)(N^2)^{\omega-2}(1-x)^{\omega-2}}{xA^{\omega}}\,. \tag{A11}$$

From (A6) integration over x gives

$$\frac{i\pi}{2}\int_0^1 dx\,J_1=in_0p_0\pi^{\omega}\Gamma(2-\omega)(N^2)^{\omega-2}\times\int_0^1\frac{dx(1-x)^{\omega-2}}{[x+(1-x)n_0{}^2]^{\omega}}=\frac{n_0p_0(N^2)^{\omega-2}i\pi^{\omega}\Gamma(2-\omega)}{n_0{}^2}\,. \tag{A12}$$

The computation of the second integral J_2 is similar to that of J_1 and yields

$$J_2=\frac{2\pi^{\omega-1}\Gamma(2-\omega)\vec{p}\cdot\vec{n}(N^2)^{\omega-2}}{xA^{\omega+1}}\times[A(1-x)^{\omega-2}-\vec{n}^2(1-x)^{\omega-1}]\,, \tag{A13}$$

so that

$$\frac{i\pi}{2}\int_0^1 dx\,J_2=\frac{i\pi^{\omega}\vec{n}\cdot\vec{p}\,\Gamma(2-\omega)(N^2)^{\omega-2}}{n_0{}^2}\,. \tag{A14}$$

From Eqs. (A6), (A12), and (A14) we finally obtain, for the divergent part of the integral (A1),

$$\int\frac{d^{2\omega}q}{(q-p)^2q\cdot n}=\frac{(n_0p_0+\vec{n}\cdot\vec{p})}{n_0{}^2}i\pi^{\omega}\Gamma(2-\omega)(N^2)^{\omega-2} \tag{A15}$$

$$=\frac{2p\cdot n^*}{n\cdot n^*}\bar{I},\quad \omega\to 2^+\,. \tag{A16}$$

This result is identical to the expression in the main text, Eq. (13), obtained with the Euclidean prescription.

APPENDIX B

The purpose of this appendix is to demonstrate that in the light-cone gauge $n^2=0$, naive use of the prescription

$$\frac{1}{q\cdot n}=\frac{1}{2}\lim_{\mu\to 0}\left[\frac{1}{q\cdot n+i\mu}+\frac{1}{q\cdot n-i\mu}\right],\quad \mu>0 \tag{B1}$$

leads to integrals which are internally inconsistent and, in addition, violate the Ward and BRS identities. It suffices to consider the three basic integrals

$$\int\frac{d^{2\omega}q}{[(q-p)^2+i\epsilon]q\cdot n}\,,$$

$$\int\frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}\,,$$

$$\int\frac{d^{2\omega}q\,q_\mu}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}\,.$$

We adopt the following notation, frequently employed in calculations in QCD:

$$\begin{aligned}&q^{\pm}=q_0\pm q_3,\quad n^{\pm}=n_0\pm n_3\,,\\&q^2=q^+q^--\vec{q}_\perp{}^2,\vec{q}_\perp{}^2=q_1{}^2+q_2{}^2\,,\\&q\cdot n=2^{-1}(n^+q^-+n^-q^+-2\vec{n}_\perp\cdot\vec{q}_\perp)\,,\\&n_0=2^{-1}(n^++n^-)\,,\\&n_3=2^{-1}(n^+-n^-)\,;\\&d^{2\omega}q=|J|\,dq^+dq^-d^{2\omega-2}\vec{q}_\perp,\quad |J|=\text{Jacobian}\,.\end{aligned} \tag{B2}$$

The choice $n_\mu\equiv(n_0,n_1,n_2,n_3)=(1,0,0,1)$ implies $n^-=0$, $n^+=2$, and $\vec{n}_\perp{}^2=0$, and will be used throughout this appendix.

1. The integral $\int d^{2\omega}q\{[(q-p)^2+i\epsilon](q\cdot n)\}^{-1}$

Applying the prescription (B1) we first write

$$\int\frac{d^{2\omega}q}{[(q-p)^2+i\epsilon]q\cdot n}\equiv I_1+I_2,\quad \epsilon\to 0^+\,, \tag{B3}$$

where

$$I_1=\frac{1}{2}\lim_{\epsilon,\mu\to 0}\int\frac{d^{2\omega}q}{[(p-q)^2+i\epsilon](q\cdot n+i\mu)}\,, \tag{B4a}$$

$$I_2=\frac{1}{2}\lim_{\epsilon,\mu\to 0}\int\frac{d^{2\omega}q}{[(q-p)^2+i\epsilon](q\cdot n-i\mu)}\,. \tag{B4b}$$

Hence,

$$I_1 = |J| \lim_{\epsilon,\mu\to 0} \int d^{2\omega-2}\vec{q}_\perp \int_{-\infty}^{\infty} dq^+ \int_{-\infty}^{\infty} \frac{dq^-}{[(q-p)^+(q-p)^- - (\vec{q}_\perp - \vec{p}_\perp)^2 + i\epsilon](n^+q^- + 2i\mu)}$$

$$= |J| \lim_{\epsilon,\mu\to 0} \int d^{2\omega-2}\vec{q}_\perp \int_{-\infty}^{\infty} dq^+ I_1^- \,, \tag{B5}$$

where

$$I_1^- = \int_{-\infty}^{\infty} \frac{dq^-}{[Q^+(q^- - p^-) - \vec{Q}_\perp{}^2 + i\epsilon](n^+q^- + 2i\mu)}, \quad Q^+ \equiv (q-p)^+ \,.$$

The last integral has poles at $q^- = -2i\mu/n^+$, and

$$q^- = \begin{cases} \dfrac{-\vec{Q}_\perp{}^2 + p^-|Q^+| + i\epsilon}{|Q^+|} & \text{if } Q^+ < 0, \\[2ex] \dfrac{+\vec{Q}_\perp{}^2 + p^-Q^+ - i\epsilon}{Q^+} & \text{if } Q^+ > 0. \end{cases} \tag{B6}$$

Closing the contour *above* the real q^- axis, we readily find

$$I_1^- = \frac{-2\pi i}{n^+(p^-|Q^+| - \vec{Q}_\perp{}^2 + i\lambda)}, \quad Q^+ < 0 \,,$$

where $\lambda = \epsilon + (2\mu|Q^+|/n^+) > 0$. Substitution of I_1^- into (B5) gives

$$I_1 = \frac{-2\pi i\,|J|}{n^+} \lim_{\lambda\to 0^+} \int d^{2\omega-2}\vec{Q}_\perp \int_{-\infty}^{0} dQ^+ (p^-|Q^+| - \vec{Q}_\perp{}^2 + i\lambda)^{-1}$$

$$= \frac{-2\pi i\,|J|}{n^+} \lim_{\lambda\to 0^+} \int d^{2\omega-2}\vec{Q}_\perp \int_0^\infty \frac{dx}{xp^- - \vec{Q}_\perp{}^2 + i\lambda} \,. \tag{B7}$$

The procedure is similar for I_2, except that we close the contour below the real q^- axis, evaluating the integral at the pole

$$q^- = (p^-Q^+ + \vec{Q}_\perp{}^2 - i\epsilon)/Q^+, \quad Q^+ > 0 \,,$$

and adding an overall minus sign for closing the contour in the opposite (clockwise) direction:

$$I_2 = \frac{-2\pi i\,|J|}{n^+} \lim_{\lambda\to 0^+} \int d^{2\omega-2}\vec{Q}_\perp \int_0^\infty \frac{dx}{xp^- + \vec{Q}_\perp{}^2 - i\lambda} \,. \tag{B8}$$

This integral, like (B7), is divergent. Nevertheless, we proceed, imposing a cutoff Λ. Hence, from (B3)

$$I_1 + I_2 = \frac{-2\pi i\,|J|}{n^+} \lim_{\lambda\to 0^+} \lim_{\Lambda\to\infty} \int d^{2\omega-2}\vec{Q}_\perp \int_0^\Lambda dx \left[\frac{1}{xp^- + \vec{Q}_\perp{}^2 - i\lambda} + \frac{1}{xp^- - \vec{Q}_\perp{}^2 + i\lambda} \right],$$

$$I_1 + I_2 = \frac{-2i\pi\,|J|}{n^+p^-} \lim_{\Lambda\to\infty} \int d^{2\omega-2}\vec{Q}_\perp \ln \left[\frac{(\vec{Q}_\perp{}^2)^2 - (p^-)^2\Lambda^2}{(\vec{Q}_\perp{}^2)^2} \right], \tag{B9}$$

where we have omitted the $i\lambda$. It is not necessary, for our subsequent discussion, to evaluate the integral (B9) explicitly.

2. The integral $\int d^{2\omega}q\{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n\}^{-1}$

Consider next the prescription (B1) for

$$\int \frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n} = \int_0^1 dx \int \frac{d^{2\omega}k}{(k^2+A^2+i\epsilon)^2 n\cdot[k+p(1-x)]} \equiv I_1 + I_2 \,,$$

$$I_2 = I_1|_{\mu\to-\mu}, \quad A^2 = x(1-x)p^2 \tag{B10}$$

with

$$I_1 = |J| \lim_{\epsilon,\mu\to 0} \int_0^1 dx \int d^{2\omega-2}\vec{k}_\perp \int_{-\infty}^{\infty} dk^+ \int_{-\infty}^{\infty} \frac{dk^-}{(k^2+A^2+i\epsilon)^2\{n^+[k^-+p^-(1-x)]+2i\mu\}} \,. \tag{B11}$$

The k^- integral, being characterized by a double pole at $k^- = (-\vec{k}_\perp^2 + A^2 + i\epsilon)/|k^+|$, $k^+ < 0$, has the value (contour is closed *above)*

$$\frac{-2\pi i}{n^+[\,|k^+|p^-(1-x)+A^2-\vec{k}_\perp^2+i\lambda]^2},$$

$$\lambda = \epsilon + \frac{2\mu|k^+|}{n^+}.$$

Substituting this expression into (B11) and remembering that only $k^+ < 0$ contributes to the integral, we get

$$I_1 = \frac{2\pi i\,|J|}{n^+p^-}\lim_{\lambda\to 0^+}\int_0^1 \frac{dx}{1-x}\int \frac{d^{2\omega-2}\vec{k}_\perp}{\vec{k}_\perp^2 - x(1-x)p^2 - i\lambda}$$

$$= \frac{2\pi i\,|J|}{n^+p^-}\int_0^1 \frac{dx}{1-x}\{\pi^{\omega-1}\Gamma(2-\omega)[-p^2(1-x)x]^{\omega-2}\},$$

where the $i\lambda$ is no longer needed and has been dropped. Hence,

$$I_1 = \frac{2i\pi^\omega|J|(-p^2)^{\omega-2}\Gamma(\omega-1)\Gamma(2-\omega)\Gamma(\omega-2)}{n^+p^-\Gamma(2\omega-3)}. \tag{B12}$$

Our next task is to evaluate I_2, for which we close the contour *below* the real k^- axis. At the double pole $k^- = (\vec{k}_\perp^2 - A^2 - i\epsilon)/k^+$, $k^+ > 0$, the k^- integral has the value

$$\frac{+2\pi i}{n^+[k^+p^-(1-x)+\vec{k}_\perp^2-A^2-i\lambda]^2}.$$

Thus,

$$I_2 = \frac{2\pi i\,|J|}{n^+p^-}\lim_{\lambda\to 0^+}\int_0^1 \frac{dx}{1-x}\int \frac{d^{2\omega-2}\vec{k}_\perp}{\vec{k}_\perp^2 - p^2x(1-x) - i\lambda}$$

$$= I_1, \tag{B13}$$

so that

$$\int \frac{d^{2\omega}q}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n} = \frac{4i\pi^\omega|J|\Gamma(2-\omega)\Gamma(\omega-2)\Gamma(\omega-1)(-p^2)^{\omega-2}}{n^+p^-\Gamma(2\omega-3)}. \tag{B14}$$

The curious double pole at $\omega = 2$ has already been mentioned in the literature.[2]

3. The integral $\int d^{2\omega}q\, q_\mu\{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n\}^{-1} \equiv I_\mu$

We proceed as in Sec. II, using again

$$\{(q^2+i\epsilon)[(q-p)^2+i\epsilon]\}^{-1} = \int_0^1 dx\,(k^2+A^2+i\epsilon)^{-2},$$

$$k_\mu + p_\mu(1-x) = q_\mu,\quad A^2 = x(1-x)p^2,$$

in which case

$$I_\mu = \int_0^1 dx \int \frac{d^{2\omega}k\,[k_\mu + p_\mu(1-x)]}{(k^2+A^2+i\epsilon)^2 n\cdot[k+p(1-x)]}$$

$$\equiv I_{1\mu} + I_{2\mu}, \tag{B15}$$

where

$$I_{1\mu} = p_\mu \int_0^1 dx\,(1-x)\int \frac{d^{2\omega}k}{(k^2+A^2+i\epsilon)^2 n\cdot[k+p(1-x)]},$$

$$I_{2\mu} = \int_0^1 dx \int \frac{d^{2\omega}k\,k_\mu}{(k^2+A^2+i\epsilon)^2 n\cdot[k+p(1-x)]}.$$

(a) *The integral $I_{1\mu}$*. Apart from the additional factor $p_\mu(1-x)$, this integral has already been evaluated in Sec. II above [cf. Eq. (B10)], with the result that

$$I_{1\mu} = \frac{4i\pi p_\mu|J|}{n^+p^-}\lim_{\lambda\to 0^+}\int_0^1 dx \int \frac{d^{2\omega-2}\vec{k}_\perp}{\vec{k}_\perp^2 - p^2x(1-x) - i\lambda} \tag{B16}$$

$$= \frac{4i\pi^\omega p_\mu|J|(-p^2)^{\omega-2}\Gamma(2-\omega)}{n^+p^-}\int_0^1 dx\, x^{\omega-2}(1-x)^{\omega-2}$$

$$= \frac{4i\pi^\omega|J|(-p^2)^{\omega-2}\Gamma(2-\omega)[\Gamma(\omega-1)]^2}{n^+p^-\Gamma(2\omega-2)}p_\mu. \tag{B17}$$

(b) *The integral $I_{2\mu}$*. Application of the principal-value prescription (B1) leads to two integrals:

$$I_{2\mu} = I_{21\mu} + I_{22\mu}, \tag{B18}$$

where

$$I_{21\mu} = \tfrac{1}{2}\lim_{\epsilon,\mu_0\to 0}\int_0^1 dx\int \frac{d^{2\omega}k\,k_\mu}{(k^2+A^2+i\epsilon)^2\{n\cdot[k+p(1-x)]+i\mu_0\}},\quad I_{22\mu} = I_{21\mu}|_{\mu_0\to-\mu_0},\quad A^2 = x(1-x)p^2.$$

We demonstrate the method of evaluation for $I_{21\mu}$. Recalling that

$$k_\mu=(k_0,k_1,k_2,k_3)=\left[\frac{k^+ + k^-}{2},\vec{k}_\perp,\frac{k^+ - k^-}{2}\right],$$

we define

$$I_{21\mu}\equiv(H_1,\vec{H}_\perp,H_3)\,, \tag{B19}$$

where

$$H_1=\tfrac{1}{2}\,|J|\lim_{\epsilon,\mu_0\to 0}\int_0^1 dx\int d^{2\omega-2}\vec{k}_\perp\int_{-\infty}^{\infty}dk^+\int_{-\infty}^{\infty}\frac{dk^-(k^+ + k^-)}{D}\equiv H_1^{(+)}+H_1^{(-)}\,,$$

$$\vec{H}_\perp=|J|\lim_{\epsilon,\mu_0\to 0}\int_0^1 dx\int d^{2\omega-2}\vec{k}_\perp\vec{k}_\perp\int_{-\infty}^{\infty}dk^+\int_{-\infty}^{\infty}\frac{dk^-}{D}\,,$$

$$H_3=\tfrac{1}{2}\,|J|\lim_{\epsilon,\mu_0\to 0}\int_0^1 dx\int d^{2\omega-2}\vec{k}_\perp\int_{-\infty}^{\infty}dk^+\int_{-\infty}^{\infty}\frac{dk^-(k^+ - k^-)}{D}\equiv H_1^{(+)}-H_1^{(-)}\,,$$

$$D\equiv(k^+k^- - \vec{k}_\perp^{\,2}+A^2+i\epsilon)^2\{n^+[k^- + p^-(1-x)]+2i\mu_0\}\,.$$

Proceeding as in Sec. II, we obtain

$$\int_{-\infty}^{\infty}\frac{dk^-}{D}=\frac{-2\pi i}{n^+[\,|k^+|p^-(1-x)+A^2-\vec{k}_\perp^{\,2}+i\lambda]^2},\quad k^+<0\,, \tag{B20a}$$

$$\int_{-\infty}^{\infty}\frac{dk^-k^-}{D}=\frac{2\pi i p^-(1-x)}{n^+[\,|k^+|p^-(1-x)+A^2-\vec{k}_\perp^{\,2}+i\lambda]^2},\quad k^+<0\,, \tag{B20b}$$

so that

$$H_1^{(+)}=\frac{i\pi|J|}{n^+(p^-)^2}\lim_{\lambda\to 0^+}\int_0^1\frac{dx}{(1-x)^2}\int d^{2\omega-2}\vec{k}_\perp[\ln(a\Lambda+b)-(\ln b)-1]_{\Lambda\to\infty}\,,$$

$$a=p^-(1-x),\quad b=A^2-\vec{k}_\perp^{\,2}+i\lambda,\quad \lambda=\epsilon+2\mu_0|k^+|/n^+\,, \tag{B21}$$

while

$$H_1^{(-)}=\frac{-i\pi^\omega|J|\Gamma(2-\omega)(-p^2)^{\omega-2}[\Gamma(\omega-1)]^2}{n^+\Gamma(2\omega-2)}\,. \tag{B22}$$

Adding Eqs. (B21) and (B22) and taking the limit $\omega\to 2^+$, we finally get

$$H_1=\frac{-i\pi^2|J|\Gamma(2-\omega)}{n^+}+M(b)\,, \tag{B23}$$

$$M(b)=\frac{i\pi|J|}{n^+(p^-)^2}\lim_{\lambda\to 0^+}\int_0^1\frac{dx}{(1-x)^2}\int d^{2\omega-2}\vec{k}_\perp[\ln(a\Lambda+b)-(\ln b)-1]_{\Lambda\to\infty}\,. \tag{B24}$$

The component $\vec{H}_\perp$ in (B19) vanishes, since

$$\int\frac{d^{2\omega-2}\vec{k}_\perp\vec{k}_\perp}{\vec{k}_\perp^{\,2}-p^2x(1-x)}=\vec{0}\,, \tag{B25}$$

while the computation of $H_3\equiv H_1^{(+)}-H_1^{(-)}$ is similar to that of H_1 and yields, as $\omega\to 2^+$,

$$H_3=\frac{+i\pi^2|J|\Gamma(2-\omega)}{n^+}+M(b)\,. \tag{B26}$$

Substituting (B23), (B25), and (B26) into (B19) we find

$$I_{21\mu}=(M(b)-|J|\bar{I}/n^+,0,0,M(b)+|J|\bar{I}/n^+)\,, \tag{B27a}$$

where, as usual, $\bar{I}\equiv\text{div}\int d^{2\omega}q[q^2(q-p)^2]^{-1}=i\pi^2\Gamma(2-\omega)$;

$$I_{22\mu}=(M(-b)-|J|\bar{I}/n^+,0,0,M(-b)+|J|\bar{I}/n^+)\,. \tag{B27b}$$

Finally from (B15), (B17), (B18), and (B27)

$$\int \frac{d^{2\omega}q\, q_\mu}{q^2(q-p)^2 q\cdot n} = I_{1\mu}+I_{2\mu} = \frac{4i\pi^2 |J|\Gamma(2-\omega)}{n^+p^-}p_\mu + (M-2|J|\bar{I}/n^+, 0, 0, M+2|J|\bar{I}/n^+)\,, \tag{B28}$$

where $M = M(b)+M(-b)$. We see that the right-hand side of (B28) is *not* Lorentz invariant.

The basic integrals (B9), (B14), and (B28) are internally inconsistent. This can be seen, for example, by multiplying (B28) by n_μ (recall that $n\cdot p = n^+p^-/2$, $n^+=2$, $n^-=0$):

$$\begin{aligned} n_\mu \int \frac{d^{2\omega}q\, q_\mu}{q^2(q-p)^2 q\cdot n} &= \int \frac{d^{2\omega}q}{q^2(q-p)^2} \\ &= \frac{4i\pi^2\Gamma(2-\omega)|J|n\cdot p}{n^+p^-} + n_0(M-2|J|\bar{I}/n^+) - n_3(M+2|J|\bar{I}/n^+) \\ &= 2\bar{I}|J| + n_0(-4|J|\bar{I}/n^+) = 2\bar{I}|J| - 2\bar{I}|J| = 0\,; \end{aligned} \tag{B29}$$

the answer "zero" contradicts the well-established value of

$$\int \frac{d^{2\omega}q}{q^2(q-p)^2} = \frac{i\pi^\omega\Gamma(2-\omega)[\Gamma(\omega-1)]^2(-p^2)^{\omega-2}}{\Gamma(2\omega-2)}\,.$$

Other inconsistencies emerge when some of the other integrals are computed exactly, such as $\int d^{2\omega}q\, q_\mu q_\nu[q^2(q-p)^2 q\cdot n]^{-1}$ and $\int d^{2\omega}q\, q^2[q^2(q-p)^2 q\cdot n]^{-1}$.

Another problem with the integrals (B9), (B14), and (B28) is that they lead to a Yang-Mills self-energy which no longer satisfies either the Ward identity or the BRS identity, as may be verified by explicit calculation.

APPENDIX C

Our aim in this note is to show, for $n^2=0$, that under the assumptions of Lorentz invariance of the momentum integrals and validity of the Ward (BRS) identity, the Yang-Mills self-energy to one loop becomes a nonlocal function of the noncovariant vector n_μ and the external momentum p_μ.

The basic integrals needed to evaluate the YM self-energy are ($dq \equiv d^{2\omega}q$)

$$\int \frac{dq}{[(q-p)^2+i\epsilon]q\cdot n}\,,\quad \int \frac{dq}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}\,,\quad \int \frac{dq\, q_\mu}{(q^2+i\epsilon)[(q-p)^2+i\epsilon]q\cdot n}\,, \tag{C1}$$

where the $i\epsilon$ terms will not be shown explicitly in the subsequent discussion. On dimensional grounds, i.e., in terms of the available vectors p_μ, n_μ, and under the assumption of general Lorentz invariance the above integrals must possess the form

$$\int dq[(q-p)^2 q\cdot n]^{-1} = A\frac{p^2}{p\cdot n}\,, \tag{C2a}$$

$$\int dq[q^2(q-p)^2 q\cdot n]^{-1} = D\frac{1}{p\cdot n}\,, \tag{C2b}$$

$$\int dq\, q_\mu[q^2(q-p)^2 q\cdot n]^{-1} = p_\mu E\frac{1}{p\cdot n} + n_\mu G\frac{p^2}{(n\cdot p)^2}\,. \tag{C2c}$$

Inserting the integrals (C2) into the one-loop self-energy we obtain

$$\Pi^{ab}_{\mu\nu}(p) = \tfrac{1}{2}C_{\rm YM}g^2\delta^{ab}\left[\tfrac{22}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)\bar{I} - 8p^2D\delta_{\mu\nu} + 8Ep_\mu p_\nu + \frac{2p^2}{n\cdot p}(p_\mu n_\nu + p_\nu n_\mu)(2G+3D-A-2E) + 4\left[\frac{p^2}{n\cdot p}\right]^2 n_\mu n_\nu(A-D-2G)\right]. \tag{C3}$$

Agreement with the Ward (BRS) identity implies that $\Pi^{ab}_{\mu\nu}(p)$ in (C3) must satisfy

$$p_\nu \Pi^{ab}_{\mu\nu}(p) = 0\,, \tag{C4}$$

which leads to the following condition among the four coefficients:

$$A + D - 2E - 2G = 0\,. \tag{C5}$$

The coefficient E may be determined by multiplying (C2c) by n_μ,

$$n_\mu \int \frac{dq\, q_\mu}{q^2(q-p)^2 q\cdot n} = E + n^2 G p^2/(n\cdot p)^2 = \bar{I}\,,$$

or

$$E = \bar{I}\quad (n^2=0)\,, \tag{C6}$$

so that (C5) becomes

$$2G = A + D - 2\bar{I}\,. \tag{C7}$$

As usual, $\bar{I}$ denotes the divergent part of the basic integral $\int dq[q^2(q-p)^2]^{-1}$. We note that multiplication of (C2c) by p_μ merely reproduces (C5) and so does not lead to a second condition among the four coefficients.

Replacing $2G$ in (C3) by the right-hand side of (C7) we readily find

$$\Pi^{ab}_{\mu\nu}(p)=\tfrac{1}{2}C_{\rm YM}g^2\delta^{ab}\left[\tfrac{22}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)\bar{I}-8(p^2D\delta_{\mu\nu}-\bar{I}p_\mu p_\nu)+8\frac{p^2}{n\cdot p}(D-\bar{I})\left[p_\mu n_\nu+p_\nu n_\mu-\frac{p^2}{n\cdot p}n_\mu n_\nu\right]\right]$$
$$=\tfrac{1}{2}C_{\rm YM}g^2\delta^{ab}\left\{\tfrac{22}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)\bar{I}-8p^2D\left[\delta_{\mu\nu}-\frac{1}{n\cdot p}(p_\mu n_\nu+p_\nu n_\mu)+\frac{p^2}{(n\cdot p)^2}n_\mu n_\nu\right]+8\bar{I}\left[p_\mu p_\nu-\frac{p^2}{n\cdot p}(p_\mu n_\nu+p_\nu n_\mu)+\left[\frac{p^2}{n\cdot p}\right]^2 n_\mu n_\nu\right]\right\} \tag{C8}$$

with $p_\nu\Pi^{ab}_{\mu\nu}=0$. The self-energy will be *local* only in the event that $D=\bar{I}$. On the other hand, $\Pi^{ab}_{\mu\nu}$ will be *nonlocal,* in the light-cone gauge, if the divergent part of (C2b) either vanishes, $D=0$, or if $D\neq\bar{I}$. The argument, therefore, hinges decisively on the integral (C2b).

As far as locality is concerned, it can only be achieved if $D=\bar{I}$. Unfortunately this value for D leads to the wrong expression for the YM self-energy, namely, $\Pi^{ab}_{\mu\nu}=-\frac{1}{3}(p^2\delta_{\mu\nu}-p_\mu p_\nu)\bar{I}C_{\rm YM}g^2\delta^{ab}$, according to (C8). Hence, $D=\bar{I}$ must be ruled out.

A value such as $D=0$ is certainly more plausible as the integral (C2b) appears to be both UV and IR convergent, at least by power counting. (It might be useful to recall in this connection that the corresponding integral in the axial gauge $n^2\neq 0$ has also been shown to have a vanishing divergent part.[4]) Of course the problem in the light-cone gauge is that invariants such as $(p\cdot n)^2/n^2p^2$ are no longer admissible, as has already been pointed out by Crewther[6] and Konetschny.[7]

To summarize, if we assume a Lorentz-invariant set of integrals as well as validity of the Ward (BRS) identity we are led to a nonlocal Yang-Mills self-energy. This would have rather unpleasant implications for the renormalization program in the light-cone gauge.

[1]J. M. Cornwall, Phys. Rev. D 10, 500 (1974); G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. B175, 27 (1980); D. J. Pritchard and W. J. Stirling, *ibid.* B165, 237 (1980).

[2]J. Kalinowski, K. Konishi, and T. R. Taylor, Nucl. Phys. B181, 221 (1981).

[3]S. Mandelstam, Nucl. Phys. B213, 149 (1983).

[4]D. M. Capper and G. Leibbrandt, Phys. Rev. D 25, 1002 (1982); 25, 1009 (1982).

[5]The author is grateful to Professor J. C. Taylor for providing the following analysis in terms of the BRS identity.

[6]R. J. Crewther, in *Weak and Electromagnetic Interactions at High Energies,* proceedings of the 1975 Cargèse Summer Institute, edited by M. Levy *et al.* (Plenum, New York, 1976), Vol. A, p. 345.

[7]W. Konetschny (private communication).