Acceleration radiation in interacting field theories

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(Received 21 November 1983)

Using the path-integral formulation of quantum field theory, we generalize the result that an accelerated observer sees a thermal spectrum to a large class of interacting field theories.

I. INTRODUCTION

It is widely believed that an observer accelerating uniformly in the vacuum of a relativistic quantum field theory behaves as if he were in a thermal bath at a temperature $T=a/2\pi$ where a is the observer's proper acceleration.[1] This result is easily shown for free field theories. For interacting field theories the proofs have relied on the claim that Green's functions which are periodic in imaginary time are thermal.[2] Although the converse is certainly true the proof that periodicity necessarily implies thermality is somewhat obscure to us. We will therefore present a more direct (although somewhat formal) proof for a large class of interacting scalar and fermionic field theories.

We consider a relativistic quantum field theory in flat space-time with coordinates (x,y,z,t). One can describe the physics of an accelerated observer by transforming to Rindler coordinates[3]

$$x=r\cosh a\eta,\quad t=r\sinh a\eta \tag{1.1}$$

defined on the Rindler wedge

$$0<r<\infty,\quad -\infty<\eta<\infty\ .$$

The line element in these coordinates is given by

$$ds^2=-a^2r^2d\eta^2+dr^2+dy^2+dz^2\ . \tag{1.2}$$

An observer at fixed r,y,z measures a proper time $d\tau=ar\,d\eta$ and has a proper acceleration $1/r$. The observer at $r=1/a$ measures $d\tau=d\eta$ and has a proper acceleration a. The Hamiltonian for a quantum field theory in Rindler coordinates will generate translations in η. Thus for the observer at $r=1/a$, it will generate translations in proper time. The Rindler Hamiltonian is thus a physical Hamiltonian for the observer at $r=1/a$ accelerating with a proper acceleration "a."

In Sec. II, we consider a scalar field theory and show that any Green's function calculated by the Rindler observer using the Rindler Hamiltonian at a temperature $T=a/2\pi$ is the same as the corresponding vacuum Green's function of the Minkowski observer at $T=0$. This result is extended to fermion field theories in Sec. III. In Sec. IV, we consider a theory with spontaneous symmetry breaking. We show that even if there exists a critical temperature above which the symmetry is restored, no phase transition is expected as a function of acceleration.

II. SCALAR FIELD THEORY

Let us start by considering a scalar field theory with an action

$$S=\int d^4x\left[\tfrac{1}{2}\dot\phi^2-\tfrac{1}{2}(\nabla\phi)^2-V(\phi)\right] \tag{2.1}$$

with $V(\phi)$ a polynomial function of ϕ. In Rindler coordinates for the half space $r>0$,

$$x_1=r\cosh a\eta,\quad t=r\sinh a\eta\ , \tag{2.2}$$

the action becomes

$$S_+=\int_{r>0}ar\,dr\,d\eta\,dx_\perp\left[\frac{1}{2(ar)^2}\left[\frac{\partial\phi}{\partial\eta}\right]^2-\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2-\tfrac{1}{2}(\nabla_\perp\phi)^2-V(\phi)\right],$$

where $x_\perp$ denotes the x_2, x_3 coordinates. A Legendre transformation of the action yields the momentum Π_ϕ^R canonically conjugate to ϕ in Rindler coordinates:

$$\Pi_\phi^R=\frac{1}{ar}\frac{\partial\phi}{\partial\eta} \tag{2.3a}$$

and the Rindler Hamiltonian

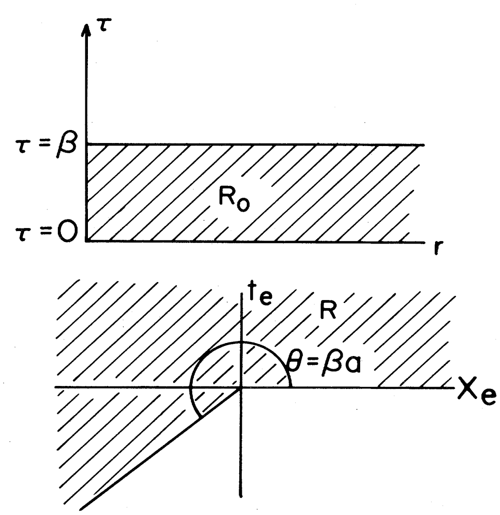

FIG. 1. Under the transformation (2.9) the region R_0 is transformed into the region R.

$$H^R=\int_{r>0}dr\,dx_\perp(ar)\left[\frac{(\Pi^R)^2}{2}+\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2+\tfrac{1}{2}(\nabla_\perp\phi)^2+V(\phi)\right]. \quad (2.3b)$$

Since η translation corresponds to a boost in Minkowski space-time, this Rindler Hamiltonian is simply a times the boost generator.

We wish to show that in the Minkowski observer's vacuum state, the Rindler observer sees a thermal spectrum with a temperature $T=a/2\pi$. We shall do this by showing that all vacuum Green's functions between space-time points within the same Rindler wedge in Minkowski coordinates are the same as the (real-time) Green's functions of the Rindler observer in thermal equilibrium at temperature $T=a/2\pi$, i.e., we shall show

$$\langle 0|(\phi(\vec{x}_1,t_1)\cdots\phi(\vec{x}_n,t_n))_t|0\rangle =\frac{\mathrm{Tr}[e^{-\beta H^R}(\phi(\vec{r}_1,\eta_1)\cdots\phi(\vec{r}_n,\eta_n))_\eta]}{\mathrm{Tr}(e^{-\beta H^R})} \quad (2.4)$$

with $\beta a=2\pi$. Here $(\ )_t$ and $(\ )_\eta$ denote time and η ordering, respectively. $(\vec{r}_m,\eta_m)$ represents the same space-time point as $(\vec{x}_m,t_m)$ but in Rindler coordinates, $|0\rangle$ denotes the Minkowski vacuum state, and Tr denotes a trace.

Let us start by considering the partition function

$$Z^R(\beta)=\mathrm{Tr}(e^{-\beta H^R}) \quad (2.5)$$

for arbitrary β. The functional integral form for Z is given by[4]

$$Z^R(\beta)=N_0\int_{\phi(\tau=0)=\phi(\tau=\beta)}D\Pi^R D\phi\exp\left[-\int_0^\beta d\tau\int_{r>0}dr\,dx_\perp\left\{(ar)\left[\frac{(\Pi^R)^2}{2}+\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2+\frac{1}{2}(\nabla_\perp\phi)^2+V(\phi)\right]+i\Pi^R\frac{\partial\phi}{\partial\tau}\right\}\right]. \quad (2.6)$$

Here N_0 is a normalization factor, and the functional integration is to be done over fields ϕ satisfying the periodic boundary conditions $\phi(\tau=0)=\phi(\tau=\beta)$. The Π^R integral is Gaussian and the result is obtained by setting

$$\Pi^R=\frac{-i}{ar}\frac{\partial\phi}{\partial\tau}. \quad (2.7)$$

Thus

$$Z^R(\beta)=N_1\int_{\phi(\tau=0)=\phi(\tau=\beta)}D\phi\exp\left[-\int_0^\beta d\tau\int_{r>0}dr\,dx_\perp\left\{\frac{1}{2}\frac{1}{(ar)}\left[\frac{\partial\phi}{\partial\tau}\right]^2+(ar)\left[\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2+\frac{(\nabla_\perp\phi)^2}{2}+V(\phi)\right]\right\}\right]$$
$$\equiv N_1\int D\phi\exp[-S_E^R(\beta)]. \quad (2.8)$$

We now perform a change of variables

$$x_e=r\cos a\tau,\quad t_e=r\sin a\tau. \quad (2.9)$$

We must emphasize that this is simply a change of integration variables and not to be interpreted as a change of coordinates. If we want the transformation to be single valued, we must have $\beta a\le 2\pi$. The region $r>0$, $0\le\tau\le\beta$ is transformed into the shaded region R shown in Fig. 1. One finds

$$Z^R(\beta)=N_1\int_{\phi(\tau=0)=\phi(\tau=\beta)}D\phi\exp\left\{-\int_R dx_e dt_e dx_\perp\left[\frac{(\nabla\phi)^2}{2}+\frac{1}{2}\left[\frac{\partial\phi}{\partial t_e}\right]^2+V(\phi)\right]\right\}. \quad (2.10)$$

When $\beta a=2\pi$, the region R is the full x_e, t_e, $x_\perp$ space, the periodic boundary conditions become consistency conditions at $a\tau=2\pi$, and we have

$$Z^R(\beta)=N_1\int D\phi\exp\left\{-\int d^4x\left[\frac{(\nabla_4\phi)^2}{2}+V(\phi)\right]\right\}\equiv N_1\int D\phi\exp[-S_E]. \quad (2.11)$$

This is simply the Euclidean generating functional for the theory in the inertial coordinates. We thus have

$$\mathrm{Tr}(e^{-(2\pi/a)H^R})=N_2\lim_{T\to\infty}\mathrm{Tr}(e^{-HT})\underset{T\to\infty}{\sim}N_2e^{-E_0T}, \quad (2.12)$$

where E_0 is the energy of the Minkowski vacuum. The free energy at $\beta=2\pi/a$ in Rindler coordinates is thus intimately related to the ground-state energy of the Minkowski vacuum.

We can now show the equality (2.4) of the Green's functions. Consider first

$$K=\mathrm{Tr}[e^{-\beta H^R}\Pi^R(\vec{r}_1)\cdots\Pi^R(\vec{r}_n)\phi(\vec{r}_{n+1})\cdots\phi(\vec{r}_m)] \quad (2.13)$$

with all the operators defined at $\eta=0$. Following the steps leading to (2.8), we find

$$K=N_0\frac{\delta^n}{\delta J(\vec{r}_1,0)\cdots\delta J(\vec{r}_n,0)}\int D\phi\, D\Pi^R \exp\left[-\int_0^\beta d\tau\int_{r>0}dr\,dx_\perp\left\{(ar)\left[\frac{(\Pi^R)^2}{2}+\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2+\frac{1}{2}(\nabla_\perp\phi)^2+V(\phi)\right]+i\Pi^R\frac{\partial\psi}{\partial\tau}-J(\vec{r},\tau)\Pi^R(\vec{r},\tau)\right\}\right]$$
$$\times\phi(\vec{r}_{n+1},0)\cdots\phi(\vec{r}_m,0)\Big|_{J=0}. \tag{2.14}$$

Upon performing the Gaussian integration over Π^R, this leads to

$$K=N_1\frac{\delta^n}{\delta J(\vec{r}_1,0)\cdots\delta J(\vec{r}_n,0)}\int D\phi \exp\left[-\int_0^\beta d\tau\int_{r>0}dr\,dx_\perp\left\{\frac{1}{2ar}\left[\frac{\partial\phi}{\partial\tau}+iJ\right]^2+ar\left[\frac{1}{2}\left[\frac{\partial\phi}{\partial r}\right]^2+\frac{1}{2}(\nabla_\perp\phi)^2+V(\phi)\right]\right\}\right]$$
$$\times\phi(\vec{r}_{n+1},0)\cdots\phi(\vec{r}_m,0)\Big|_{J=0}. \tag{2.15}$$

We now perform the change of variables (2.9) with $\beta a=2\pi$. We use the fact that J is only relevant at $\tau=0$ and that

$$\frac{\partial\phi}{\partial t_e}\bigg|_{t_e=0}=\frac{1}{ar}\frac{\partial\phi}{\partial\tau}\bigg|_{\tau=0}$$

to obtain

$$K=N_1\frac{\delta^n}{\delta J(\vec{r}_1,0)\cdots\delta J(\vec{r}_n,0)}\int D\phi\exp\left\{-d^4x\left[\frac{1}{2}\left[\frac{\partial\phi}{\partial t_e}+i\tilde{J}\right]^2+\frac{1}{2}\frac{(\nabla\phi)^2}{2}+V(\phi)\right]\right\}$$
$$\times\phi(\vec{r}_{n+1},0)\cdots\phi(\vec{r}_m,0)\Big|_{J=0} \tag{2.16}$$

with $\tilde{J}=J/ar$. Now

$$\frac{\delta}{\delta J(\vec{r},\eta)}=\left|\frac{\partial(x,t)}{\partial(r,\eta)}\right|\frac{\delta}{\delta J(x,y,z,t)},$$

where $|\partial(x,t)/\partial(r,\eta)|=ar$ is the Jacobian of the transformation to Rindler coordinates. Thus $\delta/\delta J(\vec{r},\eta)=\delta/\delta\tilde{J}(x,y,z,t)$. It then follows that

$$K\propto\int D\phi\,D\Pi\exp\left[-\int d^4x\left[\frac{\Pi^2}{2}+\frac{(\nabla\phi)^2}{2}+V(\phi)+i\Pi\dot{\phi}\right]\right]\Pi(\vec{r}_1,0)\cdots\Pi(\vec{r}_n,0)\phi(\vec{r}_{n+1},0)\cdots\phi(\vec{r}_m,0). \tag{2.17}$$

Dividing (2.14) by $\mathrm{Tr}\,e^{-\beta H^R}$ and (2.16) by $\mathrm{Tr}(e^{-HT})(T\to\infty)$, we conclude that

$$\frac{\mathrm{Tr}\left[e^{-\beta H^R}\Pi^R(\vec{r}_1)\cdots\phi(\vec{r}_m)\right]}{\mathrm{Tr}(e^{-\beta H^R})}\underset{\beta a=2\pi}{=}\langle 0|\Pi(\vec{r}_1)\cdots\phi(\vec{r}_m)|0\rangle. \tag{2.18}$$

Equivalently for any function $F(\Pi,\phi)$ which can be expanded in a Taylor series, we have

$$\frac{\mathrm{Tr}[e^{-\beta H^R}F(\Pi^R,\phi)]}{\mathrm{Tr}(e^{-\beta H^R})}\underset{\beta a=2\pi}{=}\langle 0|F(\Pi,\phi)|0\rangle \tag{2.19}$$

with Π^R, ϕ, and Π defined at $t=\eta=0$.

The next step is to prove that

$$\frac{\mathrm{Tr}[e^{-\beta H^R}(\phi(\vec{r}_1,\eta_1)\cdots\phi(\vec{r}_n,\eta_0))_\eta]}{\mathrm{Tr}(e^{-\beta H^R})}=\langle 0|(\phi(\vec{x}_1,t_1)\cdots\phi(\vec{x}_n,t_n))_t|0\rangle. \tag{2.20}$$

The left-hand side of Eq. (2.20) is equal to

$$\mathrm{LHS}=\frac{\mathrm{Tr}\left\{e^{-\beta H^R}(\eta\text{ ordered})\prod_{j=1}^{n}\left[e^{+iH^R(\Pi^R,\phi)\eta_j}\phi(\vec{r}_j,0)e^{-iH^R(\Pi^R,\phi)\eta_j}\right]\right\}}{\mathrm{Tr}(e^{-\beta H^R})}. \tag{2.21}$$

Using (2.19) this becomes

$$\mathrm{LHS}=\left\langle 0\left|(\eta\text{ ordered})\prod_{j=1}^{n}[e^{iH^R(\Pi,\phi)\eta_j}\phi(\vec{r}_j,0)e^{-iH^R(\Pi,\phi)\eta_j}]\right|0\right\rangle . \tag{2.22}$$

But $H^R(\Pi,\phi)$ is just a times the boost generator, so that $\phi(\vec{r}_j,0)$ is mapped to $\phi(\vec{x}_j,t_j)$ with $x_{j(1)}=r_j\times\cosh a\eta$, $t_j=r_j\sinh a\eta$. Thus

$$\mathrm{LHS}=\langle 0\,|\,(\phi(\vec{x}_1 t_1)\cdots\phi(\vec{x}_n,t_n))_\eta\,|\,0\rangle . \tag{2.23}$$

To complete the proof, we need to show that η ordering and t ordering are equivalent. The time ordering matters only if one space-time point lies within the future null cone of the other since the fields commute for spacelike separations. Since both η and t increase everywhere within the future, null cone of any point, as long as we restrict all points to lie within the same Rindler wedge, η ordering and t ordering will be the same. This completes the proof of (2.4).

We have thus shown the equivalence of the physics seen by a Minkowski observer in his vacuum state with that seen by a Rindler observer at temperature $T=a/2\pi$. Only an observer at position $r=1/a$ accelerates with proper acceleration a. And only for this observer is η the proper time. Thus the observer accelerating with acceleration a sees a thermal spectrum at $T=a/2\pi$.

III. FERMIONS

Fermion field theories require special treatment for two reasons. First of all, they have nontrivial Lorentz transformation properties (as, of course, do all theories of higher-spin particles). In addition, the functional integral representation for the fermion partition function [analogous to (2.6)] requires integration over fields with *antiperiodic* boundary conditions.[5] We must show that the transformation (2.9) still leads to consistency on the positive x_e axis.

Consider the free fermion field theory with an action

$$S=\int d^4x\,\bar{\psi}(x)(i\gamma\cdot\partial-m)\psi(x) . \tag{3.1}$$

The generalization to interacting theories is straightforward.

In Rindler coordinates for the half-space $r>0$ the action becomes

$$S_+=\int_{r>0}ar\,dr\,d\eta\,dx_\perp\bar{\psi}(x)\times\left[\frac{i}{ar}\Sigma^0\frac{\partial\psi}{\partial\eta}+i\Sigma^1\frac{\partial\psi}{\partial r}+i\gamma^\perp\cdot\nabla_\perp\psi-m\psi\right] \tag{3.2}$$

with

$$\begin{aligned}\Sigma^1&=\gamma^1\cosh a\eta-\gamma^0\sinh a\eta ,\\ \Sigma^0&=\gamma^0\cosh a\eta-\gamma^1\sinh a\eta .\end{aligned} \tag{3.3}$$

It is useful to change the spin basis by performing the transformation

$$\begin{aligned}\psi&\to\left[\cosh\frac{a\eta}{2}+\Sigma\sinh\frac{a\eta}{2}\right]\psi ,\\ \psi^\dagger&\to\psi^\dagger\left[\cosh\frac{a\eta}{2}+\Sigma\sinh\frac{a\eta}{2}\right] ,\end{aligned} \tag{3.4}$$

with $\Sigma=\gamma^0\gamma^1$. Equation (3.2) now becomes

$$S_+=\int_{r>0}ar\,dr\,d\eta\,dx_\perp\bar{\psi}(x)\left[\frac{i}{ar}\gamma^0\frac{\partial\psi}{\partial\eta}+i\gamma^1\frac{\partial\psi}{\partial r}+i\gamma^\perp\cdot\nabla_\perp\psi-m\psi+\frac{i}{2r}\gamma^1\psi\right] . \tag{3.5}$$

The $\bar{\psi}\gamma^1\psi$ term comes from the extra term in the derivative $\partial\psi/\partial\eta$ due to the transformation (3.4).

The momentum Π^R canonically conjugate to ψ in Rindler coordinates is

$$\Pi_R=i\psi^\dagger \tag{3.6}$$

leading to the Hamiltonian

$$H_R=-\int_{r>0}ar\,dr\,dx_\perp\bar{\psi}(x)\left[i\gamma^1\frac{\partial\psi}{\partial r}+i\gamma^\perp\cdot\nabla_\perp\psi-m\psi+\frac{i}{2r}\gamma^1\psi\right] \tag{3.7}$$

with the equal-time anticommutator

$$\{\psi_\alpha(x),\psi^\dagger_\beta(y)\}=\delta_{\alpha\beta}\delta^{(3)}(x-y) . \tag{3.8}$$

Now consider the partition function in Rindler coordinates:

$$Z^R(\beta)=\mathrm{Tr}(e^{-\beta H_R}) . \tag{3.9}$$

Using standard results on fermion path integrals,[5] (3.9) can be written as

$$Z^R(\beta)\sim\int_{\psi(\tau=0)=-\psi(\tau=\beta)}D\psi(\vec{x},\tau)D\bar{\psi}(\vec{x},\tau)\exp\left\{\int_0^\beta d\tau\left[\int_{r>0}dr\,dx_\perp\left[i\Pi^R\frac{\partial\psi}{\partial\tau}\right]+H_R\right]\right\} \tag{3.10}$$

with $\Pi^R = i\bar{\psi}\gamma^0$ and where $\sim$ denotes equality up to an overall normalization factor. Using (3.6) and (3.7), Eq. (3.10) becomes

$$Z^R(\beta) \sim \int_{\psi(\tau=0)=-\psi(\tau=\beta)} D\psi D\bar{\psi} \exp\left[-\int_0^\beta d\tau \int_{r>0} ar\, dr\, dx_\perp \bar{\psi}(x) \left[\frac{\gamma^0}{ar}\frac{\partial\psi}{\partial\tau} + i\gamma^1 \frac{\partial\psi}{\partial r} + i\gamma^\perp \cdot \nabla_\perp \psi - m\psi + \frac{i}{2r}\gamma^1\psi \right]\right]$$
$$\equiv \int_{\psi(\tau=0)=-\psi(\tau=\beta)} D\psi D\bar{\psi} \exp[-S_E^R(\beta)] \;, \tag{3.11}$$

where $S_E^R(\beta)$ is the Euclidean action for S_+ of Eq. (3.5) on the imaginary-time interval $(0,\beta)$. Note that ψ and $\bar{\psi}$ are independent variables. We can thus make a change of variables in the integral (3.11):

$$\psi(x) \to \left[\cos\frac{a\tau}{2} + i\Sigma \sin\frac{a\tau}{2} \right]\psi \;, \quad \bar{\psi}(x) \to \bar{\psi}(x) \left[\cos\frac{a\tau}{2} + i\Sigma \sin\frac{a\tau}{2} \right] . \tag{3.12}$$

Equation (3.11) then becomes

$$Z^R(\beta) \sim \int D\psi D\bar{\psi} \exp\left[-\int_0^\beta d\tau \int ar\, dr\, dx_\perp \bar{\psi}(x) \left[\frac{1}{ar}\bar{\Sigma}^0 \frac{\partial\psi}{\partial\tau} + i\bar{\Sigma}^1 \frac{\partial\psi}{\partial r} + i\gamma^\perp \cdot \nabla_\perp \psi - m\psi \right]\right] \tag{3.13}$$

with

$$\bar{\Sigma}^0 = \gamma^0 \cos a\tau - i\gamma^1 \sin a\tau, \quad \bar{\Sigma}^1 = \gamma^1 \cos a\tau - i\gamma^0 \sin a\tau \;, \tag{3.14}$$

and with boundary conditions

$$\psi(\tau=0) = -\left[\cos\frac{a\beta}{2} - i\Sigma \sin\frac{a\beta}{2} \right]\psi(\tau=\beta) \;. \tag{3.15}$$

When $\beta a = 2\pi$, the antiperiodic boundary conditions on ψ of Eq. (3.11) become periodic boundary conditions due to (3.15). This is crucial since the periodicity condition, in the Euclidean inertial path integral, is simply a consistency condition at $\tau a = 2\pi$. The properties of the field under rotations by 2π have combined with the antiperiodicity in the fermion path integral to give this consistency.

We now perform the coordinate transformation [as we did in Eq. (2.9)]:

$$x_e = r \cos a\tau, \quad t_e = r \sin a\tau \;. \tag{3.16}$$

When $\beta a = 2\pi$, we obtain

$$Z^R(\beta a = 2\pi) \sim \int D\psi D\bar{\psi} \exp\left[-\int_{-\infty < x^\mu < \infty} d^4x\, \bar{\psi}(x) \left[\gamma^0 \frac{\partial\psi}{\partial t_e} + i\gamma^1 \frac{\partial\psi}{\partial x_e} + i\gamma^\perp \cdot \nabla_\perp \psi - m\psi \right]\right]$$
$$\equiv \int D\psi d\bar{\psi} \exp[-S_E] \;, \tag{3.17}$$

where S_E is the Euclidean action for the action (3.1). Thus the partition function of the Rindler observer at temperature $T = a/2\pi$ is proportional to the Euclidean generating functional for the inertial observer so that

$$Z^R(\beta a = 2\pi) = \mathrm{Tr}\, e^{-\beta H_R} \underset{T\to\infty}{\propto} \mathrm{Tr}\, e^{-HT} \;, \tag{3.18}$$

where H is the Minkowski observer's Hamiltonian.

The arguments of the preceding section are easily generalized to relating the real-time thermal Green's functions in Rindler coordinates to the Green's functions in Minkowski coordinates. One finds as in (2.4)

$$\langle 0 | (\psi_{\alpha_1}(\vec{x}_1, t_1) \cdots \psi_{\alpha_n}(\vec{x}_n, t_n))_t | 0 \rangle = \frac{\mathrm{Tr}[e^{-\beta H_R}((U(\eta_1)\psi(\vec{r},\eta_1))_{\alpha_1} \cdots (U(\eta_n)\psi(\vec{r}_n,\eta_n))_{\alpha_n})_\eta]}{\mathrm{Tr}(e^{-\beta H_R})} \;, \tag{3.19}$$

where $(\vec{r}_j, \eta_j)$ is the same space-time point as $(\vec{x}_j, t_j)$ but in Rindler coordinates and where $U(\eta_j)$ is the 4×4 matrix representing a boost with rapidity $a\eta_j$.

We thus conclude for fermions as well, that an observer accelerating in the vacuum state of Minkowski space is in a thermal bath at temperature $T = a/2\pi$.

The generalization of this result to theories of interacting fermions is straightforward. Consider, for example, the action

$$S = S_F(\psi, \bar{\psi}) + S_B(\phi) - g\int d^4x\, \bar{\psi}(x)\psi(x)\phi(x) \;, \tag{3.20}$$

where S_F is the free fermion action of (3.1) and S_B is the boson action of (2.1). The Rindler Hamiltonian is given by

$$H_R = H_F(\psi,\Pi_\psi) + H_B(\phi,\Pi_\phi) + g\int d^3x\, \bar{\psi}(x)\psi(x)\phi(x) \tag{3.21}$$

with H_F given by (3.7) and H_B given by (2.3b). The partition function for the Rindler Hamiltonian is then

$$\mathrm{Tr}\, e^{-\beta H_R} \sim \int_{\substack{\psi(\tau=0)=-\psi(\tau=\beta)\\ \phi(\tau=0)=\phi(\tau=\beta)}} D\psi\, D\bar{\psi}\, D\phi \exp\left\{-\left[S_E^R(\beta)_F + S_E^R(\beta)_B + g\int_0^\beta d\tau \int_{r>0} ar\, dr\, dx_\perp \bar{\psi}(x)\psi(x)\phi(x)\right]\right\} \tag{3.22}$$

with $S_E^R(\beta)_F$ and $S_E^R(\beta)_B$ given by Eqs. (2.8) and (3.11), respectively. Now perform the transformations (3.12) and (3.16). When $\beta a = 2\pi$, one finds

$$\mathrm{Tr}(e^{-\beta H_R}) = \int_{\substack{\text{periodic}\\ \text{boundary conditions}}} D\psi\, D\bar{\psi}\, D\phi \exp\left\{-\left[(S_E)_F + (S_E)_B + g\int d^4x\, \bar{\psi}(x)\psi(x)\phi(x)\right]\right\}$$

$$\equiv \int D\psi\, D\bar{\psi}\, D\phi \exp[-S_E]$$

with S_E the Euclidean action corresponding to (3.20). The result can similarly be proven for the Green's functions. It is easy to check that interaction terms of the form $\bar{\psi}\gamma^5\psi\phi$, $(\bar{\psi}\psi)^2$, $\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi$, and $\bar{\psi}\gamma^\mu\psi\partial_\mu\psi$ also lead to the result that the accelerated observer in thermal equilibrium at $T = a/2\pi$ measures the same Green's functions as the Minkowski observer in his vacuum state. We expect the result to hold for any Lorentz-invariant interaction.

IV. THEORIES WITH MULTIPLE VACUA

In the previous sections, we have shown that in a large class of interacting field theories, an accelerated observer sees a thermal spectrum of radiation. An interesting question arises when we consider a theory in which spontaneous symmetry breaking is expected. For example, consider the scalar field theory with Hamiltonian

$$H = \int d^3x \left[\frac{\Pi^2}{2} + \frac{(\nabla\phi)^2}{2} + V(\phi)\right] \tag{4.1}$$

with $V(\phi) = -m^2\phi^2/2 + \lambda\phi^4/4$. The Hamiltonian H is symmetric under $\phi \to -\phi$. However, this symmetry is expected to be spontaneously broken. There are two vacua $|\pm\rangle$ and in the tree approximation

$$\langle \pm | \phi(x) | \pm \rangle = \pm \left[\frac{m^2}{\lambda}\right]^{1/2} . \tag{4.2}$$

It is believed that at high temperature this symmetry is restored so that for $T >$ some critical temperature T_{cr}

$$\langle \phi(x) \rangle \equiv \mathrm{Tr}[e^{-\beta H}\phi(x)]/\mathrm{Tr}\, e^{-\beta H} = 0 . \tag{4.3}$$

We would like to know whether the results of Secs. II and III apply in the case of multiple vacua. Furthermore, if an observer accelerates with an acceleration $a \sim 2\pi T_{\mathrm{cr}}$ does he see a phase transition?

A field theory which has multiple vacua by virtue of a symmetry such as $\phi \to -\phi$ can be dealt with by introducing a symmetry-breaking term (analogous to putting a spin system in a magnetic field) such as

$$H \to H + \int d^3x\, J(x)\phi(x) . \tag{4.4}$$

For fixed $J \neq 0$, the theory has a unique vacuum. The properties of the theory are studied for fixed J and then J is allowed to approach 0. In the tree approximation, one finds

$$\lim_{J\to 0_\pm} \langle 0 | \phi(x) | 0 \rangle = \mp \left[\frac{m^2}{\lambda}\right]^{1/2} \tag{4.5}$$

and, in fact

$$\lim_{J\to 0\pm} | 0,J \rangle = | \mp \rangle . \tag{4.6}$$

Now consider an observer accelerating in the vacuum $|0,J\rangle$. For fixed $J \neq 0$, the vacuum is unique and the observer will see a thermal spectrum. In fact, if $\bar{\phi}(J)$ is the vacuum expectation value of ϕ for fixed J, then the observer will find

$$\mathrm{Tr}[e^{-\beta H_R}\phi(x)]/\mathrm{Tr}(e^{-\beta H_R}) = \bar{\phi}(J) . \tag{4.7}$$

If we let $J \to 0^-$, for example, then the Minkowski vacuum approaches $|+\rangle$ and $\bar{\phi}(J) \to +(m^2/\lambda)^{1/2}$ (in the tree approximation). All the Green's functions of the Rindler observer approach the Minkowski Green's functions in the vacuum $|+\rangle$. (In particular $\langle\phi\rangle \neq 0$.) We conclude that the results of Secs. II and III apply to this case as well as that there is *no* phase transition as a function of acceleration, since

$$\lim_{J\to 0\pm} \mathrm{Tr}(e^{-\beta H_R}\phi)/\mathrm{Tr}\, e^{-\beta H_R} = \langle \mp | \phi | \mp \rangle \neq 0 \tag{4.8}$$

for all values of the acceleration. Recall that the accelerated observer sees himself not only in a thermal bath but also in a gravitational field. Furthermore, due to the gravitational field, the local temperature changes significantly over a distance of the order of the peak wavelength in the thermal distribution. Thus one does not expect the same behavior as in a thermal bath in flat space-time.

V. SUMMARY

We have used the path-integral formulation of field theory to show that for a large class of interacting scalar and spinor field theories, an accelerated observer sees a thermal spectrum of particles. It is straightforward to extend our method to prove the result for gauge theories. We believe that our method can be generalized to yield a proof for any Lorentz-invariant field theory.

From Eq. (2.4), one sees that we have proven equality of the Green's functions for the two observers. We have not shown that this implies equality of the density matrices $[e^{-\beta H^R}$ and $e^{-HT}(T\to\infty)]$ for the two observers. However, one can deduce from our proof that any uniformly accelerated detector which couples to the fields via ϕ, Π, or any of its derivatives will be thermally excited. This is certainly a reasonable property to expect of a detector. (In fact, we do not know how to construct a detector which does not couple in this way.)

Finally, we note that the results of this paper should not be affected by renormalization of the theory. The easiest way to see this is to note that the result holds in any number of spatial dimensions d. Thus any Feynman graph regulated using dimensional regularization in the Rindler system at temperature $T=a/2\pi$ will be equal to the corresponding regulated $T=0$ graph in Minkowski space. Since all counterterms are Lorentz invariant, the renormalized graphs should be equal as well.

ACKNOWLEDGMENT

We would like to thank NSERC of Canada for partial support of this research.

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