Continuous measurement: Watchdog effect versus golden rule

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Conditions which may lead to a freezing of the motion of a system under continuous observation (the so-called "Zeno paradox" or "watchdog effect") are examined. The measurement process is treated phenomenologically by the usual wave-packet reduction as well as in a more realistic way by including the measuring apparatus. For this purpose a model for an ideal measurement process is employed, following an example given by von Neumann. The resulting behavior varies between complete freezing and a mere suppression of interference terms and constant transition rates as represented by a master equation (rate equation). The most familiar example of the latter is Fermi's golden rule, with integration leading to exponential decay. Reviewing and extending the derivation of the Pauli master equation, the conditions leading to constant transition rates are discussed. The importance of the interaction with the natural environment for establishing a master equation is emphasized. Some consequences for the derivation of macroscopic equations of motion and for the physical foundations of superselection rules are pointed out.

I. INTRODUCTION

It seems to be a trivial observation that there exist no completely isolated objects in our world. Usually the interaction with the environment is thought either to be "controllable" or negligible. In particular, macroscopic systems are strongly coupled to their environment: macroscopic objects cannot avoid emitting (and absorbing) thermal radiation. Gravitation yields another long-range interaction, which strongly couples macroscopic objects to their surroundings. So in a sense the environment records the state of the system under consideration, as does a measuring apparatus. Therefore, macroscopic systems must be considered as "continuously measured" by their environment. An example is the position of a macroscopic body. It is inevitably measured by scattering of photons. In classical mechanics the back-reaction of such a measurement on the system can be thought of as being arbitrarily small. This is quite different in quantum mechanics, where the collapse of the state vector in general modifies the evolution of the measured system. This collapse describes the effect of measurement phenomenologically.

Extending the quantum-mechanical description by including the environment into the formalism instead leads to a "dislocalization of phases." In the example of photon scattering, interference terms connecting different positions become unobservable at the macroscopic body itself, though still existing in the whole system. This was first discussed by von Neumann in his theory of the measurement process.[1]

Owing to the nature of quantum correlations the dynamics of a system in interaction with its surroundings cannot be described by a Schrödinger equation for the system alone (not even a time-dependent one).[2] As a consequence, it is not realistic to use the Ehrenfest theorem on the time dependence of mean values to justify classical motion for macroscopic bodies, since it presupposes the validity of a Schrödinger equation. Instead one has to solve the Schrödinger equation for system plus environment and then "look at" the system of interest, which for this purpose is described by its density matrix.

Continuous measurements give rise to two different, albeit connected, effects.

The first is destruction of interference terms in *the* basis of the "observable" according to the usual collapse assumption. Repeated measurements thus lead to a permanent suppression of phases in this basis. On the other hand, the absence of interference terms plays a decisive role in the derivation of master equations in irreversible statistical mechanics. A special case of such a rate equation is Fermi's golden rule, where integration leads to the exponential decay law. Destruction of interference terms and constant transition rates are usually achieved by using the "random-phase approximation." The interaction with the environment may give a physical mechanism to explain the validity of this assumption and in particular may define the basis in which the density matrix becomes diagonal. The same mechanism can be employed to derive superselection rules. Because of the interaction with the environment some interference terms may become unobservable.[2,3]

The second, by now well-known, effect is the so-called "Zeno paradox" or "watchdog effect." It has been shown[4,5] that the motion of a continuously measured system may become frozen, i.e., its internal dynamics are (in the extreme case) totally suppressed. This apparently surprising effect is obviously not relevant for most systems under measurement. For example, decaying atoms or nuclei behave according to a master equation, i.e., their transition rates are independent of their environment (Fermi's golden rule).

The main subject discussed in this paper is the following: How are these two effects related to one another? Which types of systems show the watchdog effect and what distinguishes them from systems behaving according to some master equation?

The measurement scheme used in the following is the so-called "ideal measurement." The interaction Hamiltonian is chosen in such a way that the measured system influences the "apparatus" (environment) without significant back-reaction. Many interactions of simple systems with their environment are approximately of this type. For example, a scattered photon will carry away information on the location of the scattering object, or, if the scattering amplitude depends on polarization, on properties like handedness (chirality) of a molecule. Such measurementlike processes will lead to a destruction (or rather "dislocalization") of phase relations. Interference terms become locally unobservable. The general features of this mechanism can be sketched as follows.

An ideal measurement of some state $|\varphi_n\rangle$ of the measured object leads to a correlated state $|\phi_n\rangle$ of the measuring apparatus (with initial state $|\phi_0\rangle$),

$$|\varphi_n\rangle|\phi_0\rangle \rightarrow |\varphi_n\rangle|\phi_n\rangle . \tag{1.1}$$

While the state of the object has not changed (hence the term ideal measurement), information about the "quantum number n" has been transferred to the environment. Following von Neumann[1] a model for such an ideal measurement can easily be constructed by using an interaction Hamiltonian, which is diagonal in the basis of the $|\varphi_n\rangle$. This in turn defines *dynamically* the observable, which is is measured by the apparatus. Explicit examples are given in the following sections.

If the measured object is initially in a superposition of different states with amplitudes c_n, the superposition principle immediately leads to

$$\sum_n c_n|\varphi_n\rangle|\phi_0\rangle \rightarrow \sum_n c_n|\varphi_n\rangle|\phi_n\rangle . \tag{1.2}$$

The resulting state of the whole system still contains interference terms between different n. However, they are unobservable locally, that is, at the measured object, if the corresponding "pointer positions" $|\phi_n\rangle$ are orthogonal. This can be seen from the density matrix ρ_{nm} for the measured object,

$$\begin{aligned}\rho_{nm} &= c_n c_m^* \langle\phi_m|\phi_n\rangle \\ &= |c_n|^2\delta_{nm} \quad \text{if } \langle\phi_m|\phi_n\rangle=\delta_{nm} .\end{aligned} \tag{1.3}$$

Since the phase relations between different coefficients c_n are still present in the whole system, they may now be called "dislocalized." The assumption of ideal measurements, however, is by no means essential for the effect of dislocalization since all interactions will induce correlations into an initial product state. As will be discussed in the following sections, this may have different dynamical consequences depending on the strength of the coupling to the environment.

Section II summarizes the main features of the watchdog effect, including a simple model for the measurement process on a two-state system. Section III recalls the essentials of Pauli's derivation of a master equation and then extends the model of Sec. II in an appropriate manner. There the difference between the two situations can be clearly recognized. In Sec. IV the importance of the interaction with the environment is discussed. It also contains some remarks concerning the dynamical foundation of superselection rules and the measurement problem of quantum mechanics.

II. THE WATCHDOG EFFECT

What is the general behavior of a system which is repeatedly measured in short time intervals? The simplest argument runs as follows. Let $|\phi\rangle$ be the state of the system at $t=0$. The probability of not finding the state $|\phi\rangle$ at the (small) time Δt ("decay probability") is

$$\begin{aligned}P(\Delta t) &= 1-|\langle\phi|e^{-iH\Delta t}|\phi\rangle|^2 \\ &= \Lambda^2(\Delta t)^2+O((\Delta t)^3) ,\end{aligned} \tag{2.1}$$

where

$$\Lambda^2=\langle\phi|H^2|\phi\rangle-(\langle\phi|H|\phi\rangle)^2 . \tag{2.2}$$

Because of the quadratic time dependence a repetition of the measurement, e.g., n times during Δt, reduces $P(\Delta t)$ according to

$$P \rightarrow P' = n\left[\frac{\Delta t}{n}\right]^2\Lambda^2=\frac{(\Delta t)^2}{n}\Lambda^2 . \tag{2.3}$$

In the limit $n\rightarrow\infty$ one has $P'=0$. So the internal dynamics of the system under measurement is totally suppressed. Some authors called this "Zeno's paradox in quantum theory."[4,6] It was often regarded as contrary to experience. Thereupon unfounded conclusions on the completeness or interpretation of quantum theory were drawn.[7] But the effect can be shown to be present in simple situations, e.g., by repeated measurements of the polarization of a beam of light passing through an optically active medium.[6]

In this argument the measurement process has been treated phenomenologically. A more realistic approach incorporates the measuring apparatus into the quantum formalism. The following model is built in analogy to an example given by von Neumann[1] for an ideal measurement process.

The measured system is a two-state system with Hamiltonian

$$H_0=V(|1\rangle\langle 2|+|2\rangle\langle 1|) . \tag{2.4}$$

The measurement apparatus consists of a one-dimensional mass point, which is moved to the right or left, depending on the state of the measured system. The Hamilton operator of the whole system then reads

$$\begin{aligned}H &= H_0+W \\ &= V(|1\rangle\langle 2|+|2\rangle\langle 1|) \\ &\quad +\gamma\hat{p}(|1\rangle\langle 1|-|2\rangle\langle 2|) ,\end{aligned} \tag{2.5}$$

where $\hat{p}$ is the momentum operator of the pointer and γ a coupling parameter. A similar model was discussed by Kraus,[9] where the name "watchdog effect" was introduced. The model has the exact solution

$$e^{-iHt}=\frac{1}{2}\left\{\frac{e^{-i\Omega t}}{\Omega(\Omega-\gamma\hat{p})}[V^2|1\rangle\langle 1|+(\Omega-\gamma\hat{p})^2|2\rangle\langle 2|+V(\Omega-\gamma\hat{p})(|1\rangle\langle 2|+|2\rangle\langle 1|)]\right.$$
$$\left.+\frac{e^{i\Omega t}}{\Omega(\Omega+\gamma\hat{p})}[V^2|1\rangle\langle 1|+(\Omega+\gamma\hat{p})^2|2\rangle\langle 2|-V(\Omega+\gamma\hat{p})(|1\rangle\langle 2|+|2\rangle\langle 1|)]\right\}, \tag{2.6}$$

where

$$\Omega\equiv[V^2+(\gamma\hat{p})^2]^{1/2}. \tag{2.7}$$

For the initial state $|1\rangle|\phi\rangle$ one finds

$$|\psi(t)\rangle=\left[\cos\Omega t-\frac{i\gamma\hat{p}}{\Omega}\sin\Omega t\right]|1\rangle|\phi\rangle-\frac{iV}{\Omega}\sin\Omega t|2\rangle|\phi\rangle. \tag{2.8}$$

In view of the watchdog effect in this paper interest will concentrate on the case of strong coupling to the measuring apparatus. It is then appropriate to use the Born approximation with H_0 as "perturbation."

Let the wave function of the pointer at $t=0$ be $\phi(x)$ and the measured system be in the state $|1\rangle$. The Born approximation to lowest order yields then

$$|\psi(t)\rangle=\int dx\,|x\rangle\left[\phi(x-\gamma t)|1\rangle-\frac{iV}{2\gamma}\int_{x-\gamma t}^{x+\gamma t}dx'\phi(x')|2\rangle\right]. \tag{2.9}$$

The behavior of the measured system is given by its density matrix ρ_{ij}:[10]

$$\rho_{ij}=\langle\phi_j|\phi_i\rangle \tag{2.10}$$

with

$$|\psi(t)\rangle\equiv|1\rangle|\phi_1\rangle+|2\rangle|\phi_2\rangle. \tag{2.11}$$

The term $\rho_{22}=\langle\phi_2|\phi_2\rangle$ in this case represents the transition probability from $|1\rangle$ to $|2\rangle$. Writing

$$\rho_{22}(t)=\int dx\,|\phi_2(x,t)|^2, \tag{2.12}$$

with

$$\phi_2(x,t)=\frac{-iV}{2\gamma}\int_{x-\gamma t}^{x+\gamma t}dx'\phi(x'), \tag{2.13}$$

one easily verifies that ρ_{22} depends *linearly* on time as soon as $\phi(x-\gamma t)$ and $\phi(x+\gamma t)$ have negligible overlap, i.e., when $\gamma t>B$, where B is the width of $\phi(x)$. The main contribution to the integral (2.12) then comes from a region with constant height $(\pi V^2/2\gamma^2)|\tilde{\phi}(0)|^2$ [where $\tilde{\phi}(p)$ is the Fourier transform of $\phi(x)$] and width growing as $2\gamma t$. The Born approximation on the other hand requires $Vt\ll 1$. Consequently this linear behavior

$$\rho_{22}(t)\approx\frac{\pi V^2}{\gamma}|\tilde{\phi}(0)|^2 t \tag{2.14}$$

is restricted to time scales $B/\gamma \lesssim t\ll 1/V$ (hence $BV/\gamma\ll 1$ is required for consistency). For very small times one has of course

$$\rho_{22}(t)=V^2t^2+O(t^3). \tag{2.15}$$

The linear behavior of the transition probability in (2.14) in contrast to that in (2.15) will prove to be one of the central points in the following discussions, especially for the derivation of master equations in Sec. III.

The above result may be further illustrated by an explicit example. Choosing for simplicity a rectangular wave function with width B for the pointer

$$\phi(x)=\frac{1}{\sqrt{B}}\theta\left[\frac{B}{2}-x\right]\theta\left[\frac{B}{2}+x\right], \tag{2.16}$$

one finds

$$\rho_{22}(t)=\begin{cases}V^2\left[t^2-\frac{2\gamma}{3B}t^3\right], & t\leq\frac{1}{2}\frac{B}{\gamma}\\ V^2\left[\frac{1}{2}\frac{B}{\gamma}t-\frac{1}{12}\frac{B^2}{\gamma^2}\right], & t\geq\frac{1}{2}\frac{B}{\gamma}.\end{cases} \tag{2.17}$$

As can be seen from (2.14) or (2.17) the watchdog effect causes the transition probability to decrease with $1/\gamma$ for $t\gtrsim B/\gamma$ (note that B/γ characterizes the time between two measurements in the phenomenological approach). This result parallels the situation described by Eq. (2.3). It is important to realize that the watchdog effect is *not* produced by disturbing the measured system in a classical way, as is demonstrated by this model, which describes an ideal (passive) measurement.

However, the suppression of transition probabilities with $1/\gamma$ does not represent the general behavior of systems under continuous measurement, as can be seen by extending the above model.

In H_0 of Eq. (2.4) diagonal elements are absent in the measurement basis. To be more general H_0 is replaced by

$$H_0=V(|1\rangle\langle 2|+|2\rangle\langle 1|)+\epsilon|2\rangle\langle 2|. \tag{2.18}$$

Now the behavior of ρ_{22} is given in the linear region by

$$\rho_{22}(t)\approx\frac{\pi V^2}{\gamma}\left|\tilde{\phi}\left[\frac{\epsilon}{2\gamma}\right]\right|^2 t. \tag{2.19}$$

As a function of γ, ρ_{22} obviously need not decrease as $1/\gamma$ except in the limit $\gamma\to\infty$ (which is the extreme watchdog effect). Instead, depending on the special shape of $\tilde{\phi}(p)$ there exist regions, where an increase of the coupling γ leads to an enhancement of ρ_{22}.[11] This outcome can again be explicitly demonstrated by the example of a rectangular wave function for the pointer. Now the result is

$$\rho_{22}(t)=\begin{cases}4V^2\left[-\frac{2\gamma}{B\epsilon^3}\sin\epsilon t+\frac{1}{\epsilon^2}\sin^2\frac{\epsilon t}{2}+\frac{\gamma}{B\epsilon^2}t(1+\cos\epsilon t)\right], & t\le\frac{1}{2}\frac{B}{\gamma}\\ 4V^2\left[\frac{1}{\epsilon^2}\cos^2\frac{B\epsilon}{4\gamma}-\frac{2\gamma}{B\epsilon^3}\sin\frac{B\epsilon}{2\gamma}+\frac{2\gamma}{B\epsilon^2}t\sin^2\frac{B\epsilon}{4\gamma}\right], & t\ge\frac{1}{2}\frac{B}{\gamma}\,.\end{cases} \tag{2.20}$$

The crucial linear term shows very distinctly the nonmonotonic γ dependence as well as the limit $\rho_{22}\sim 1/\gamma$ for $\gamma\rightarrow\infty$.

The results of this simple model may be summarized as follows.

(a) Coupling of a two-state system to a measuring apparatus changes the transition rates of the system compared to that of the free evolution. In general, transitions can be enhanced or suppressed. In the limit of strong coupling the transitions are always inhibited by a factor $1/\gamma$ (watchdog effect—corresponding to Zeno's paradox in the phenomenological approach).

(b) The relevant parameter is γ/B, where B is the width of the pointer wave function. It is a measure for the "effectivity" of the measurement, or the inverse of time intervals between two measurements.

(c) For times $t\gtrsim B/\gamma$, as long as the Born approximation is valid, ρ_{22} is (approximately) proportional to time, i.e., the system has *constant transition rates,* but these are influenced by the measurement and for $\epsilon B/\gamma\ll 1$ suppressed by a factor $\sim B/\gamma$.

(d) Coupling to a measurement apparatus thus leads to a masterlike behavior (regarding time dependence), but—in contrast to the usual master behavior—the transition rates here depend on the dynamics of the system itself as well as on the coupling to the environment.

III. MASTER BEHAVIOR

A master equation in the context of quantum mechanics was first derived by Pauli.[12] A characteristic of master dynamics is (beside constant transition rates—see below) the permanent omission of nondiagonal elements of the density matrix in a given basis. Usually this is achieved by applying a random-phase assumption. As mentioned in the Introduction, this may find its justification in the interaction with the environment (see also Sec. IV). Pauli's work also contains some remarks which may be understood in this way:

"... um dagegen den zeitlichen Ablauf irgendeines Vorganges zu erfassen, muβ ein System stets als durch irgendwelche Apparate messend verfolgt, d.h. als unabgeschlossen angesehen werden." "... werden im allgemeinen die Beobachtungen selbst eine solche Regellosigkeit begünstigen."[13]

In view of the previous section one may ask whether for systems with many degrees of freedom the watchdog effect plays a similarly dominant role. This will be examined by extending the model of Sec. II. For comparison the central points of the usual derivation of a master equation will first be recalled.

Consider a many-state system with Hamiltonian

$$H=\sum_{\alpha E}E\,|\alpha E\rangle\langle\alpha E| + \sum_{\alpha E\neq\alpha' E'}V_{\alpha E,\alpha' E'}\,|\alpha E\rangle\langle\alpha' E'|\ , \tag{3.1}$$

where α is supposed to distinguish between macroscopically different properties. Microscopic degeneracy is neglected for simplicity.

The most general initial state with property α_0 is

$$|\psi(0)\rangle=\sum_{E_0}c_{\alpha_0E_0}(0)\,|\alpha_0E_0\rangle\ . \tag{3.2}$$

The amplitude at time t for a state $|\alpha E\rangle$ with $\alpha\neq\alpha_0$ is in the Born approximation given by

$$c_{\alpha E}(t)=\sum_{E_0\neq E}V_{\alpha E,\alpha_0E_0}\frac{e^{-iE_0t}-e^{-iEt}}{E_0-E}c_{\alpha_0E_0}(0) \tag{3.3}$$

and therefore the diagonal part of the density matrix is

$$\rho(\alpha E,\alpha E,t)=|c_{\alpha E}(t)|^2=\sum_{\substack{E_0\neq E\\ E_0'\neq E}}V_{\alpha E,\alpha_0E_0}V^*_{\alpha E,\alpha_0'E_0'}\frac{e^{-iE_0t}-e^{-iEt}}{E_0-E}\frac{e^{iE_0't}-e^{iEt}}{E_0'-E}c_{\alpha_0E_0}(0)c^*_{\alpha_0'E_0'}(0)\ . \tag{3.4}$$

At this stage usually a random-phase assumption for the coefficients $c_{\alpha_0E_0}(0)$ is applied in order to replace in (3.4) the initial density matrix $\rho(\alpha_0E_0,\alpha_0'E_0',0)=c_{\alpha_0E_0}(0)c^*_{\alpha_0'E_0'}(0)$ by a diagonal one. Because of the large number of nondiagonal terms, this procedure may only be justified by the presence of the resonance factors. The approximation can be improved by using a random mixture of many initial states or by summing over final states. For further discussion see also Sec. IV. Then

$$\rho(\alpha E,\alpha E,t)=4\sum_{E_0\neq E}|V_{\alpha E,\alpha_0E_0}|^2\frac{\sin^2(E-E_0)t/2}{(E-E_0)^2}|c_{\alpha_0E_0}(0)|^2\ . \tag{3.5}$$

Writing this sum as an integral, one gets

$$\rho(\alpha E,\alpha E,t)=4\int dE_0\sigma_{\alpha_0}(E_0)\,|\,V(\alpha E,\alpha_0E_0)\,|^2\frac{\sin^2(E-E_0)t/2}{(E-E_0)^2}\rho(\alpha_0E_0,\alpha_0E_0,0)\ , \tag{3.6}$$

where σ_α are level densities. If

$$\sigma_{\alpha_0}(E_0)\,|\,V(\alpha E,\alpha_0E_0)\,|^2\rho(\alpha_0E_0,\alpha_0E_0,0)$$

is approximately constant over a range of $E-E_0$ larger than the width $1/t$ of the resonance factor, then the latter may be replaced by a δ function

$$\frac{\sin^2(E-E_0)t/2}{(E-E_0)^2}\to\frac{\pi}{2}t\delta(E-E_0)\ . \tag{3.7}$$

If the width of this "range of constancy" is called κ, then (3.7) is valid for $t\gg 1/\kappa$. κ is roughly given by the inverse square root of the second derivative of $\sigma\,|\,V\,|^2\rho$ at E, as may be seen from the Taylor expansion.

Starting more generally with a mixture of different macroscopic values α leads to time "derivatives" (meaningful only for $t\gg 1/\kappa$), which are given by

$$\dot\rho(\alpha E,\alpha E,t)\approx\frac{\Delta\rho}{\Delta t}=\sum_{\alpha'}A_{\alpha\alpha'}(E)\rho(\alpha'E,\alpha'E,t)\ , \tag{3.8}$$

where

$$A_{\alpha\alpha'}(E)=2\pi\sigma_{\alpha'}(E)\,|\,V(\alpha E,\alpha'E)\,|^2\ \text{ for }\alpha\neq\alpha'\ . \tag{3.9}$$

Integration of this master equation requires the random-phase assumption to be valid repeatedly after time intervals $\Delta t\gg 1/\kappa$. In particular, interferences between different values of macroscopic properties α are assumed to be absent although they would have to occur according to (3.3), if there are transitions at all. Only then an autonomous dynamics for the diagonal part of the density matrix can be established. We shall return to this point later on.

It is important to realize that—in contrast to the situation with watchdog behavior—the linear time dependence of the transition probabilities is caused here by summation over many states with slightly different energy.

In Eq. (2.3) the watchdog effect was due to the quadratic time dependence for small times. This general argument is still valid. Therefore, if the absence of macroscopic interference terms, as implicitly assumed by the Pauli equation, is to be explained by continuous measurements, these have to occur within sufficiently large time intervals.

In these considerations the measurement was described phenomenologically by the collapse. In order to study the influence of measurements nonphenomenologically by a von Neumann-type interaction, the model of Sec. II will now be extended in a straightforward manner.

The Hamilton operator is in analogy to (2.5):

$$H=\sum_{\alpha E}E\,|\,\alpha E\,\rangle\langle\,\alpha E\,|+\sum_{\alpha E\neq\alpha'E'}V_{\alpha E,\alpha'E'}\,|\,\alpha E\,\rangle\langle\,\alpha'E'\,|+\sum_{\alpha E}\gamma(\alpha,E)\hat p\,|\,\alpha E\,\rangle\langle\,\alpha E\,|\ , \tag{3.10}$$

where the pointer acts as a measuring apparatus if $\gamma(\alpha,E)$ is a nonconstant function. Corresponding to the above treatment macroscopic measurement is supposed to discriminate only between different α, that is $\gamma=\gamma(\alpha)$. To simplify notation in the following calculations we will first deal with the case of an eigenstate $|\,\alpha_0E_0\,\rangle$ as the initial state. The extension to the general case [corresponding to Eq. (3.5)] is then straightforward. Assume $\gamma(\alpha_0)=0$. Let the wave function of the pointer be $\phi(x)$. Writing

$$|\,\psi(t)\,\rangle\equiv\sum_{\alpha E}|\,\alpha E\,\rangle\,|\,\phi_{\alpha E}(t)\,\rangle\ , \tag{3.11}$$

the initial state is

$$|\,\psi(0)\,\rangle=|\,\alpha_0E_0\,\rangle\,|\,\phi\,\rangle \tag{3.12}$$

and the Born approximation with respect to $V_{\alpha E,\alpha'E'}$ yields for $\alpha\neq\alpha_0$

$$\phi_{\alpha E}(x,t)=-i\frac{V_{\alpha E,\alpha_0E_0}}{\gamma(\alpha)}\exp\left[\frac{-i(E-E_0)}{\gamma(\alpha)}x-iE_0t\right]\int_{x-\gamma(\alpha)t}^{x}dz\exp\left[\frac{i(E-E_0)z}{\gamma(\alpha)}\right]\phi(z)\ . \tag{3.13}$$

The diagonal part of the density matrix is

$$\begin{aligned}\rho(\alpha E,\alpha E,t)&=\langle\,\phi_{\alpha E}(t)\,|\,\phi_{\alpha E}(t)\,\rangle\\&\approx 2\pi\frac{|\,V_{\alpha E,\alpha_0E_0}\,|^2}{\gamma(\alpha)}\left|\tilde\phi\left[\frac{E-E_0}{\gamma(\alpha)}\right]\right|^2t\ \text{ for }t\gg\frac{B}{\gamma(\alpha)}\ ,\end{aligned} \tag{3.14}$$

where again $\tilde\phi(p)$ is the Fourier transform of $\phi(x)$, which has width B [compare Eqs. (2.14) and (2.19)].

Hence the transition probability into a single state is again linear due to the measurement. This is identical with the corresponding result of Sec. II. The main difference here is the existence of a host of states with different energy.

Replacing the single initial state by a mixture of states with the same α_0 (again assuming random phases for different E_0) one gets

$$\rho(\alpha E,\alpha E,t)=\sum_{E_0}\langle\phi_{\alpha E}^{(E_0)}(t)\,|\,\phi_{\alpha E}^{(E_0)}(t)\rangle\rho(\alpha_0E_0,\alpha_0E_0,0)$$
$$\approx 2\pi t\int dE_0\sigma_{\alpha_0}(E_0)\frac{|V_{\alpha E,\alpha_0E_0}|^2}{\gamma(\alpha)}\left|\tilde{\phi}\left[\frac{E-E_0}{\gamma(\alpha)}\right]\right|^2\rho(\alpha_0E_0,\alpha_0E_0,0)$$
$$\approx 2\pi t\,|V_{\alpha E,\alpha_0E}|^2\sigma_{\alpha_0}(E)\ \text{ if }\ \frac{\gamma(\alpha)}{B}\ll\kappa\ , \tag{3.15}$$

where $\gamma(\alpha)/B$ is the width of $\tilde{\phi}$ in energy units and κ is again the range of constancy of $\sigma\,|\,V\,|^2\rho(E_0)$. Thus Pauli's result has been recovered while the coupling parameter $\gamma(\alpha)$ has disappeared due to the summation. The latter result was obtained because after the substitution $Z=(E-E_0)/\gamma(\alpha)$ in (3.15) only the slowly varying product $\sigma\,|\,V\,|^2\rho(E_0)$ would implicitly depend on $\gamma(\alpha)$. This dependence could be neglected in the last step of (3.15) by assuming a sufficiently broad "pointer" wave function ϕ.

Equation (3.15) was derived for the "linear watchdog range" $t\gg B/\gamma(\alpha)$ assuming $B/\gamma(\alpha)\gg 1/\kappa$. Pauli's derivation required only $t\gg 1/\kappa$. In order to discuss the whole range of linearity it is more appropriate to use the momentum representation, for which the Born approximation reads

$$|\,\psi(t)\rangle=\sum_{\alpha E}e^{-i[E+\gamma(\alpha)\hat{p}]t}\left[\delta_{\alpha E,\alpha_0E_0}-\frac{V_{\alpha E,\alpha_0E_0}(e^{i[E-E_0+\gamma(\alpha)\hat{p}]t}-1)}{E-E_0+\gamma(\alpha)\hat{p}}\right]|\,\alpha E\rangle\,|\,\phi\rangle\ . \tag{3.16}$$

The diagonal part of the density matrix is then [instead of (3.14), but without the additional assumption $t\gg B/\gamma(\alpha)$] for $\alpha\neq\alpha_0$ given by

$$\rho(\alpha E,\alpha E,t)=\langle\phi_{\alpha E}(t)\,|\,\phi_{\alpha E}(t)\rangle$$
$$=4\,|\,V_{\alpha E,\alpha_0E_0}|^2\int_{-\infty}^{\infty}dp\,|\,\tilde{\phi}(p)\,|^2\frac{\sin^2\{[E-E_0+\gamma(\alpha)p]t/2\}}{[E-E_0+\gamma(\alpha)p]^2}\ . \tag{3.17}$$

For the initial density matrix $\rho(\alpha_0E_0,\alpha_0E_0,0)$ the result is then

$$\rho(\alpha E,\alpha E,t)=4\int dE_0dp\;\sigma_{\alpha_0}(E_0)\,|\,V_{\alpha E,\alpha_0E_0}|^2\,|\,\tilde{\phi}(p)\,|^2\frac{\sin^2\{[E-E_0+\gamma(\alpha)p]t/2\}}{[E-E_0+\gamma(\alpha)p]^2}\rho(\alpha_0E_0,\alpha_0E_0,0)\ . \tag{3.18}$$

The questions to be discussed are as follows:

(1) Under what circumstances is $\rho(\alpha E,\alpha E,t)$ linear in time as is required for master behavior?

(2) When are the resulting constant transition rates independent of the coupling to the environment, that is, independent of $\gamma(\alpha)$ and B?

Substituting $z=E-E_0+\gamma(\alpha)p$ yields

$$\rho(\alpha E,\alpha E,t)=4\int dz\,dp\,|\,\tilde{\phi}(p)\,|^2\sigma\,|\,V\,|^2\rho(E-z+\gamma(\alpha)p)\frac{\sin^2(zt/2)}{z^2}\ . \tag{3.19}$$

One first notes that the time dependence is given by the z integral. Therefore, it is appropriate to write

$$\rho(\alpha E,\alpha E,t)=\int dz\,\Lambda(z)\frac{\sin^2(zt/2)}{z^2}\ , \tag{3.20}$$

$$\Lambda(z)=4\int dp\,|\,\tilde{\phi}(p)\,|^2\sigma\,|\,V\,|^2\rho(E-z+\gamma(\alpha)p)\ , \tag{3.21}$$

and to discuss the shape of the function $\Lambda(z)$.

To get linear behavior, the width of $\Lambda(z)$ must exceed the width of the resonance factor, that is

$$\text{width }(\Lambda)\gg\frac{1}{t}\ . \tag{3.22}$$

The width of $\Lambda(z)$ is given by the overlap of $|\,\tilde{\phi}(p)\,|^2$ which has width $1/B$, and $\sigma\,|\,V\,|^2\rho(E-z+\gamma p)$ which is shifted by $(E-z)/\gamma$ and compressed by a factor $1/\gamma$ when considered as a function of p. If the width of $\sigma\,|\,V\,|^2\rho(E_0)$ is κ, then the width of $\Lambda(z)$ is roughly given by

$$\text{width }(\Lambda)\approx\ \max\left[\kappa,\frac{\gamma(\alpha)}{B}\right]\ . \tag{3.23}$$

With these preliminaries the two extreme cases can be pictured as follows.

Case 1: $\kappa\ll\gamma(\alpha)/B$. One finds linearity for times $t\gg B/\gamma(\alpha)$ accompanied by a suppression of $\rho(\alpha E,\alpha E,t)$ by a factor $1/\gamma(\alpha)$ (because $\sigma V^2\rho$ is compressed by this factor). In this case the measurement by the environment has a great influence on the system (watchdog effect).

Case 2: $\kappa\gg\gamma(\alpha)/B$. $\rho(\alpha E,\alpha E,t)$ is now *independent* of $\gamma(\alpha)/B$. Linearity is obtained for times $t\gg 1/\kappa$. This corresponds to master behavior in accordance with the usual results.

So the question of whether a system shows master behavior or the watchdog effect can be decided by inspection of the properties of the system (in this model mainly characterized by the parameter κ) *and* the properties of the measurement by the environment (here given by γ/B).

The remarkable γ independence of the result (in case 2) is caused by the summation over the various states with different energy, which smears out the γ dependence of the individual transitions. The reason that cancellation can occur is that the coupling to the environment does not always lead to a suppression of the transitions as in the case of extreme watchdog effect or of degeneracy [compare (2.14) and (2.19)], but also can give rise to enhanced transition probabilities. This can again be illustrated by employing the example of the rectangular wave function (2.16) and considering the linear term [compare (2.20)]

$$\rho(\alpha E,\alpha E,t)\,|_{\text{linear}}=4\,|\,V_{\alpha E,\alpha_0 E_0}\,|^2\frac{\gamma(\alpha)t}{B(E-E_0)^2}\times\sin^2\frac{B(E-E_0)}{2\gamma(\alpha)}\ . \tag{3.24}$$

$\rho(\alpha E,\alpha E,t)$ does not depend monotonically on γ (except for $\gamma\rightarrow\infty$). Therefore, the summation over an ensemble of initial (or final) states may give a γ-independent result:

$$\begin{aligned}\rho(\alpha E,\alpha E,t)\,|_{\text{linear}}&=4\frac{\gamma(\alpha)}{B}t\int dE_0\sigma_{\alpha_0}(E_0)\,|\,V_{\alpha E,\alpha_0 E_0}\,|^2\frac{\sin^2[B(E-E_0)/2\gamma(\alpha)]}{(E-E_0)^2}\rho(\alpha_0E_0,\alpha_0E_0,0)\\&\simeq 2\pi t\sigma_{\alpha_0}\,|\,V_{\alpha E,\alpha_0 E}\,|^2 \ \text{ if } \kappa\gg\frac{\gamma(\alpha)}{B}\ .\end{aligned} \tag{3.25}$$

So far we were concerned with the diagonal part of the density matrix. But in order to justify a master equation for macroscopic properties α the absence of interference terms between different α is essential.

This suppression of nondiagonal elements with respect to α can easily be demonstrated in the model under consideration. Let again the initial state be $|\alpha_0E_0\rangle$ (a member of the ensemble with property α_0). Then for $\alpha\neq\alpha_0$ the density matrix is given by

$$\rho(\alpha E,\alpha_0E_0,t)=\langle\phi_{\alpha_0E_0}\,|\,\phi_{\alpha E}\rangle=V_{\alpha E,\alpha_0E_0}\int_{-\infty}^{\infty}dp\,|\,\tilde{\phi}(p)\,|^2\frac{e^{-i[E-E_0+\gamma(\alpha)p]t}-1}{E-E_0+\gamma(\alpha)p}\ . \tag{3.26}$$

For times $t\gg B/\gamma$, where master behavior for the transition probabilities was found, this expression is of the order

$$\begin{aligned}\rho(\alpha E,\alpha_0E_0,t)&\sim V_{\alpha E,\alpha_0E_0}\frac{1}{\gamma(\alpha)}\left|\tilde{\phi}\left[\frac{E-E_0}{\gamma(\alpha)}\right]\right|^2\\&\lesssim\frac{BV_{\alpha E,\alpha_0E}}{\gamma(\alpha)}\ll 1\ .\end{aligned} \tag{3.27}$$

So interference terms first grow and then approach a constant (and small) value in the linear (master) region. Therefore, the build-up of interference terms between macroscopically different properties is inhibited by the measurement as expected.

To sum up:

(1) Coupling of a many-state system to a measuring apparatus again yields constant transition rates.

(2) While the individual transitions are still influenced by the coupling to the surroundings, the existence of a very dense energy spectrum allows the cancellation of the effect on individual states, therefore leading to transition rates, which are independent of the coupling to the environment.

(3) The destruction of interferences between macroscopically different states allows integration of the resulting master equation.

(4) Properties of the system in combination with characteristics of the interaction with the environment define which properties of the system under consideration behave according to a master equation and therefore can be called "macroscopic."

IV. CONCLUDING REMARKS

The interaction with the environment, which is present for all physical systems, may have great influence on the behavior of some systems, as is exemplified by the watchdog effect. In addition, more common effects are explained by such mechanisms, as the discussion of Pauli's master equation has shown. Usually master equations are derived by considering only the system itself and applying suitable assumptions (such as random phases) and (inessential for our discussion) approximations. The substantiation of the additional assumptions which are necessary to derive irreversible equations of motion from reversible dynamics, is an old problem of statistical mechanics since Boltzmann used his "Stosszahlansatz" for deriving entropy increase. These difficulties are more fundamental in quantum mechanics. As is well known, a master equation is incompatible with the validity of a von Neumann equation for the system under consideration.[14] However, a system in interaction with its environment does not in general obey a von Neumann equation. Instead, as the model of Sec. III has shown, constant transition rates are possible, while interference terms are negligible because they are destroyed by the measurement. Then an initial random-phase assumption is necessary only for *microscopic* degrees of freedom. In most treatments this distinction is not made. However, assuming a Schrödinger equation necessarily leads to a build-up of phase relations between macroscopically different properties (provided there are transitions at all), so master dynamics would be impossible. At the same time the interaction with the natural environment *defines* the basis where interference terms become negligible. The corresponding properties of the system may then be called macroscopic.

The interaction with the surroundings can also have effects on systems which are not obviously macroscopic. For example, optically active molecules like sugar are always found in right- or left-handed configurations, never in eigenstates of the Hamilton operator of the molecule.[5,14] Interferences between these chiral states cannot be observed, because the scattering of photons (measuring of handedness) dislocalizes the connecting phases. In this case the dislocalization of phases has also been described as a time-independent correlation.[15] However, such correlations merely seem to describe a "dressing" of states by static fields that should be accompanied by a corresponding renormalization of observables so that superpositions could still be observed. The disappearance of interferences as discussed in this paper is an irreversible process. The unavoidable interaction with the radiation field also has the consequence that interferences between different positions of macroscopic objects, which necessarily emerge in measurementlike processes (compare Schrödinger's cat) are unobservable at the macroscopic body itself. This is a more realistic argument than merely to say that the interference pattern would be very minute due to the short wavelength. This in effect leads to a superselection rule, which in these cases need not be postulated, but can be *derived* from the dynamical behavior of the local density matrix.[2,3] A basic presupposition for this mechanism to work is a special (e.g., separating) initial state. Only then the phases are dislocalized "forever" because of the long Poincaré times. The relevant Poincaré times are not only those of the pointer interacting with the measured system; the pointer itself is coupled to its surroundings and so on. Hence the effective Poincaré times become those of the whole universe. This immediately leads to cosmological considerations and the supposed connection between the cosmological and the thermodynamical arrow of time.[16]

Superselection rules for macroscopic objects, which in a sense limit the validity of the superposition principle of quantum mechanics, may be a hint for solving the measurement problem. The disappearance of certain interferences is a basic consequence of the nonunitary collapse of the state vector. However, as long as the Schrödinger dynamics (hence the superposition principle) is accepted for the whole system, the nonlocality of quantum mechanics does in general not allow states of subsystems. Therefore, such a solution would require fundamental changes in the formalism. One can only hope to achieve further progress by a deeper understanding of the physical conceptions and mechanisms underlying the measurement process.

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[10] It should be stressed at this point that using density matrices for subsystems implicitly involves the collapse of the state vector, hence such a model cannot explain measurement itself.

[11] This result is valid for any $\phi(x)$. Because $\tilde{\phi}(p)$ is normalizable, $|\tilde{\phi}(p)|^2$ may decrease as $p^{-(1+\alpha)}$ with $\alpha>0$ for $p\to\infty$. Therefore $(1/\gamma)|\tilde{\phi}(\epsilon/2\gamma)|^2$ behaves as γ^α for small γ. This is an increasing function of γ.

[12] W. Pauli, *Collected Scientific Papers,* edited by R. Kronig and V. F. Weisskopf (Interscience, New York, 1964).

[13] ". . . in order to record the time dependence of some process, the system has to be considered as being kept track of by some measuring instruments, i.e., as open." ". . . will in general the observations themselves favor such an irregularity."

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