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Stretching cosmic strings

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The evolution of a network of strings produced at a grand-unification phase transition in an expanding universe is discussed, with particular reference to the processes of energy exchange between the strings and the rest of the universe. This is supported by numerical calculations simulating the behavior of strings in an expanding universe. It is found that in order that the energy density of the strings does not come to dominate the total energy density there must be an efficient mechanism for energy loss—the only plausible one being the production of closed loops and their subsequent decay via gravitational radiation.

I. INTRODUCTION

The idea, first due to Zeldovich,[1] that topologically stable strings which occur naturally in some grand unified theories[2] could seed the process of galaxy formation seems increasingly attractive. In particular, Vilenkin's scenario[3] in which long-lived closed loops with a mass per unit length set by the grand unified scale provide adequate density perturbations at decoupling to give rise to galaxies at recent epochs seems viable.

Strings are not predicted in the minimal SU(5) grand unified theory, but are predicted by SO(10) or E_6 models[2] in which the same Higgs field is used to break the symmetry and give the unobserved fermions a superheavy mass. In these theories the strings do not have ends—so must either form closed loops or be infinite.

In previous papers, it has been shown that there exists a class of long-lived, (i.e., non-self-intersecting) loop solutions[4] and that these may be produced by the collision of waves traveling on lengths of string.[5] In Ref. 6, it was shown that each galaxy could be the result of gravitational accretion around a single loop, whose effects might still be visible in galactic cores. It was also suggested that the occurrence of very large scale filamentary structure of the type recently observed by Giovanelli and Haynes[7] is predicted in the string theory.

However, crucial to the viability of this picture is a better understanding of how a network of strings formed at grand-unification phase transition would behave as the universe expanded. In this paper we make some progress towards this goal by obtaining a quantitative picture of the processes of energy exchange between the network of strings and the expanding universe they lie in.

In Sec. II, we present a first-order analysis of the energetics of lengths and loops of string in an expanding universe. In Sec. III, we describe our numerical results which confirm this analysis, and in Sec. IV we discuss the implications of our work for the consistency of the string picture. We end by discussing what seem to us to be the most important issues remaining.

II. STRETCHING STRING

The width of the string is negligible compared to the scales we are interested in, so its action is proportional simply to the area of the world sheet it sweeps out,[8]

$$S=-\mu\int dA$$
$$=-\mu\int d\sigma\,d\tau\left[\left(\frac{\partial x}{\partial\sigma}\cdot\frac{\partial x}{\partial\tau}\right)^2-\left(\frac{\partial x}{\partial\sigma}\right)^2\left(\frac{\partial x}{\partial\tau}\right)^2\right]^{1/2}, \tag{2.1}$$

where $x^{\mu}(\sigma,\tau)$ are spacetime coordinates, σ and τ the parameters describing the sheet, and μ has dimensions of mass per unit length. For the times we are interested in, before decoupling, spatial-curvature effects may be ignored and the universe described by the Robertson-Walker metric

$$ds^2=dt^2-R^2(t)d\vec{x}^{\,2}\,. \tag{2.2}$$

We may always choose a parametrization of the surface such that $t=x^0=\tau$ and

$$\frac{\partial x}{\partial\sigma}\cdot\frac{\partial x}{\partial\tau}=0\,.$$

Then (2.1) yields for the energy of the string

$$E=\int d^3x\sqrt{-g}\,T^{00}=\int d^3x\sqrt{-g}\left[\frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g_{00}}\right]$$
$$=\mu R\int d\sigma\,\epsilon(\sigma,t)\,, \tag{2.3}$$

where

$$\epsilon(\sigma,t)=[(\vec{x}'^2)/(1-\dot{\vec{x}}^2)]^{1/2}\,,\quad \vec{x}'=\frac{\partial\vec{x}}{\partial\sigma}\,,\quad \dot{\vec{x}}=\frac{\partial\vec{x}}{\partial\eta}\,,$$

and we have introduced a new "time" variable η defined by $dt=R\,d\eta$.

The equations of motion (2.1) yields are

$$\frac{\partial}{\partial\eta}(\epsilon\dot{\vec{x}})+2(\dot{R}/R)\epsilon\dot{\vec{x}}=\frac{\partial}{\partial\sigma}(\vec{x}\,'/\epsilon)\;, \tag{2.4}$$

where $\dot{R}=\partial R/\partial\eta$, from which it is found, using $\dot{\vec{x}}\cdot\vec{x}\,'=0$, that

$$\dot{\epsilon}=-2(\dot{R}/R)\epsilon\dot{\vec{x}}^{\,2}\;, \tag{2.5}$$

enabling us to write (2.4) as

$$\ddot{\vec{x}}+2(\dot{R}/R)\dot{\vec{x}}(1-\dot{\vec{x}}^{\,2})=\frac{1}{\epsilon}\frac{\partial}{\partial\sigma}(\vec{x}\,'/\epsilon)\;. \tag{2.6}$$

It is seen that the expansion of the universe has the effect of damping the strings motion. For a radiation-dominated universe $R\propto t^{1/2}\propto\eta$, and for a matter-dominated universe $R\propto t^{2/3}\propto\eta^2$. In either case, $\dot{R}/R\propto\eta^{-1}$, and for a given curvature scale on the string, the damping term dominates at very early times.

Equation (2.6) leads us to introduce the following terminology. We adopt as a measure of the length along the string the quantity $\int\epsilon\,d\sigma$. Any length of string may be described by coordinates $\vec{x}(\sigma,t)$ expanded as a Fourier series in $\int\epsilon\,d\sigma$, whose coefficients vary with t as the string evolves. We call the periodicity in $\int\epsilon\,d\sigma$ the "length in wave"—it is measured along the string, as opposed to a more conventional wavelength which would be defined as the periodicity in spatial coordinates. In what follows, coordinate wavelength or amplitude will mean the periodicity or amplitude in the comoving spatial coordinates in (2.2).

The spectrum of wavelengths and the spectrum of length in waves may in general be totally different. From the definition of ϵ it is seen that $|\partial\vec{x}/\partial\sigma|<\epsilon$, so the coordinate wavelength or amplitude of a given wave is always less than the length in wave.

Now, returning to (2.6) for a given length in wave l of amlitude $A(l)$, the curvature term is of order $(2\pi/l)^2A(l)$. At early times, damping will dominate and the velocity $|\dot{\vec{x}}|$ is of order

$$(2\pi)^2(R/\dot{R})\frac{A(l)}{l^2}\;,$$

which is less than $(R/\dot{R})(1/l)\sim\eta/l$, since $A(l)$ is less than l. If l is much greater than η, damping dominates and the terminal velocity is very small. For a radiation-dominated universe, $R\propto t^{1/2}\propto\eta$, so $|\dot{\vec{x}}|\ll 1$ when the length in wave l is greater than the coordinate distance to the horizon η. From (2.5), we see that the magnitude of $\dot{\epsilon}/\epsilon$ is much smaller than $\dot{R}/R$, and so from (2.3), the behavior of the energy is dominated by the expansion factor R. What happens is that the string is conformally stretched by the expansion of the universe, its energy growing like R.

At later times the string begins to move more freely. From (2.4) the curvature term is order $(2\pi)^2A(l)l^2$, and ignoring the damping term, we obtain an estimate of the typical velocity

$$\dot{\vec{x}}\sim 2\pi\frac{A(l)}{l}\;.$$

Now comparing the damping term with the curvature term in (2.6), and using $\dot{R}/R=1/\eta$, we see that damping ceases to be dominant when $l/2\pi<\eta$, i.e., when a given length in wave falls inside the horizon. Notice that this condition is irrespective of either wavelength or amplitude. We note that it is accurately observed (including the factor of 2π) in our numerical calculations.

Now we proceed to the regime where the length in wave has fallen well within the horizon. Here we may perform a perturbation analysis to first order in $\dot{R}/R$, the Hubble constant. From (2.5), to first order we may take $\epsilon=\text{constant}\equiv c$ since we are free[8] at any fixed time to choose σ so that $\epsilon'=0$. Then, to zeroth order, the equations of motion (2.4) and (2.5) are

$$\ddot{\vec{x}}=\frac{1}{c^2}\vec{x}\,'',\quad \dot{\vec{x}}^{\,2}+\frac{1}{c^2}\vec{x}\,'^2=1,\quad \dot{\vec{x}}\cdot\vec{x}\,'=0\;. \tag{2.7}$$

Then (2.3) and (2.5) give, to first order,

$$\frac{\partial}{\partial\eta}(E/R)=-2(\dot{R}/R)\mu c\int d\sigma\dot{\vec{x}}^{\,2}\;. \tag{2.8}$$

For periods of motion less than the "expansion time" η (i.e., length in waves smaller than the horizon distance), we may average this over a period T to obtain

$$\frac{\partial}{\partial\eta}(\bar{E}/R)\approx-2(\dot{R}/R)(\mu c/T)\int_0^T d\eta\int d\sigma\dot{\vec{x}}^{\,2}\;, \tag{2.9}$$

which can be evaluated using (2.7) and integration by parts to give

$$\frac{\partial}{\partial\eta}(\bar{E}/R)\approx-2(\dot{R}/R)(\mu c/2)\left[L-\frac{1}{T}c^2\int_0^T(\vec{x}\cdot\vec{x}\,')_0^L d\eta+\frac{1}{T}\int_0^L(\vec{x}\cdot\dot{\vec{x}})_0^T d\sigma\right]\;, \tag{2.10}$$

where L is the length of string (in σ) involved. For a closed loop the boundary terms vanish and to zeroth order in $\dot{R}$, $\bar{E}=\mu RLc$, so (2.10) yields

$$\frac{\partial}{\partial\eta}(\bar{E}/R)\approx-(\dot{R}/R^2)\bar{E}\;, \tag{2.11}$$

i.e., $\bar{E}=\text{const}$.

This is just as well—if protons are made of string (as some current theories suggest) they had better not expand with the universe. As a slightly more complex example, consider a zeroth-order solution to (2.7) in the form of a spiral standing wave,

$$\vec{x}=(\lambda c\sigma, a\alpha\cos(c\sigma/a)\cos(\eta/a), a\alpha\sin(c\sigma/a)\cos(\eta/a)),$$
$$-\infty<\sigma<\infty \quad (2.12)$$

with $\lambda^2+\alpha^2=1$. From (2.10), we find for the energy per wavelength

$$\frac{\partial}{\partial\eta}(\bar{E}/R)\approx-(\dot{R}/R)^2\bar{E}(1-\lambda^2)$$

or

$$\bar{E}\propto R^{\lambda^2}. \quad (2.13)$$

Equation (2.11) is clearly a limiting case of (2.13) when $\lambda=0$. Notice that for a straight length of string ($\alpha=0$, $\lambda=1$) the energy increases like R.

To find what happens to a wave such as (2.12) over a larger period, we must consider a more general solution. With initial conditions

$$\vec{x}(\sigma,0)=(\lambda_0\sigma, a_0\alpha_0\cos\sigma/a_0, a_0\alpha_0\sin\sigma/a_0),$$
$$-\infty<\sigma<\infty, \quad (2.14)$$
$$\alpha_0^2+\lambda_0^2=1,$$

the solution which preserves the spiral symmetry and periodicity in σ [as seen from (2.6), the evolution does preserve this] and obeys $\dot{\vec{x}}\cdot\vec{x}'=0$ is

$$\vec{x}(\sigma,\eta)=(\lambda_0\sigma, a_0\alpha(\eta)\cos\sigma/a_0, a_0\alpha(\eta)\sin\sigma/a_0), \quad (2.15)$$

where $\alpha(\eta)$ is the only time-varying parameter. Let us choose $\lambda_0\ll1$ so the spiral is initially very tightly bound. Then (2.13) tells us that initially the energy is almost constant, as in the case of a closed loop. Now $\alpha(\eta)$ oscillates, so consider the envelope of points where $\dot{\alpha}(\eta)=0$. Then $\epsilon=[\lambda_0^2+\alpha^2(\eta)]^{1/2}$ and this must decrease by (2.5) so the envelope of $\alpha(\eta)$ decreases. Now define $\lambda=\lambda_0/\epsilon$, $\alpha=\alpha(\eta)/\epsilon$, and $a=a_0\epsilon$ to bring (2.15) into the form (2.12) with $c=\epsilon$, $\lambda^2+\alpha^2=1$.

From this we see that λ increases, ultimately approaching unity as α goes to zero. As this happens, the energy increases more and more nearly in proportion to R. The transition between almost constant energy and energy scaling like R occurs when λ, the ratio of the comoving wavelength $2\pi a_0\lambda_0$ (i.e., the periodicity of the spiral measured along the x axis) to the length in wave $2\pi a_0\epsilon$ becomes nearly unity. Finally, when α goes to zero the energy per comoving wavelength approaches $R2\pi\mu a_0\lambda_0$.

III. NUMERICAL CALCULATIONS

We have solved Eq. (2.6) for $\vec{u}=\dot{\vec{x}}$ and $\vec{v}=\vec{x}'$ by a simple finite-difference algorithm using derivatives

$$\dot{\vec{u}}(t)=\frac{\vec{u}(\sigma,t+\Delta t)-\frac{1}{2}[\vec{u}(\sigma+\Delta\sigma,t)+\vec{u}(\sigma-\Delta\sigma,t]}{\Delta t},$$
$$\vec{u}'(\sigma)=\frac{\vec{u}(\sigma+\Delta\sigma,t)-\vec{u}(\sigma-\Delta\sigma,t)}{2\Delta\sigma}. \quad (3.1)$$

The wave equation

$$\ddot{\vec{x}}=(1/\epsilon^2)\vec{x}'' \quad (3.2)$$

can be checked to be stable when approximated by (3.1) for

$$\frac{\Delta\sigma}{\Delta t}\geq\frac{1}{\epsilon}.$$

This can be understood as a reflection of the need to supply adequate information to fill the backward light cone. Further, the lowest-order errors introduced in (3.2) (proportional to $\dot{\vec{x}}''$ and $\dddot{\vec{x}}$) can be shown to cancel when $\Delta t=\epsilon\Delta\sigma$. Now, in general, we expect the damping term in (2.6) to only add to the stability of the integration, but if ϵ varies with σ one needs to use the minimal value of ϵ to maintain stability, and so lose accuracy at other points. For this reason the cylindrically symmetric standing wave described in the previous section are the easiest to deal with—the symmetry guarantees $\epsilon'=0$ always. We present extensive results for them in Fig. 1. The initial conditions were chosen as in (2.14) with $\alpha_0=1$,

$$\vec{x}(\sigma,\eta=0)=(\lambda_0\sigma, \alpha_0\cos\sigma, \alpha_0\sin\sigma),$$
$$\lambda_0^2+\alpha_0^2=1. \quad (3.3)$$

Dimensional quantities have been scaled out in this equation—so that the length in wave was always 2π initially. The graph shows how the energy (the solid line) grows in proportion to R initially, the comoving length in wave l staying constant at 2π until $\eta\sim l/2\pi$, and the wave falls inside the horizon. The results for several values of λ_0 are plotted. Small-amplitude waves have $\lambda_0^2\sim1$ and so their energy grows rapidly, soon approaching the asymptote $E=2\pi\mu\lambda_0R$ (shown by a dashed line). Larger-amplitude waves ($\lambda^2\ll1$) initially have almost constant energy, but as $2\pi R\lambda_0$ (the proper wavelength) approaches E/μ, the energy rises faster, again approaching the asymptote $E=2\pi\mu\lambda_0R$. On shorter time scales it is seen that the energy oscillates. This is due to energy exchange between the string and the universe around it—when a wave is collapsing inwards it gains energy from the expansion damping, whereas when it is expanding outwards it loses energy.

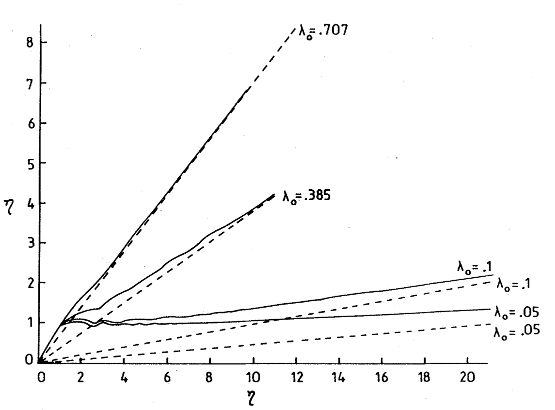

FIG. 1. Energy per wavelength ($\epsilon\eta$) versus time (η) for spiral waves of different shapes.

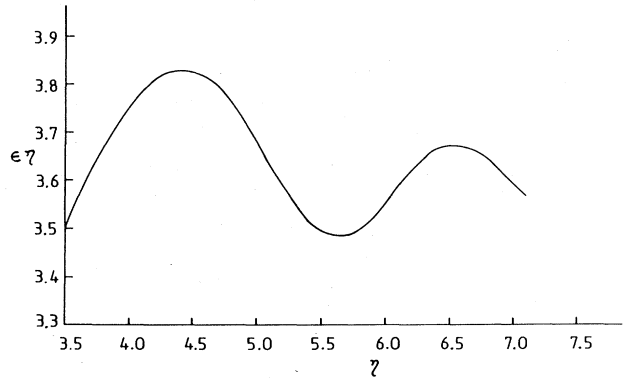

FIG. 2. Energy ($\epsilon\eta$) of an oscillating loop versus time (η).

We also followed the time evolution of closed loops such as those described in Ref. 4. Loops which are chosen "static" initially, i.e., with $\dot{\vec{x}}(\sigma,0)=0$, stretched by the expansion while they are larger than the particle horizon, shorter waves on the loop (i.e., higher Fourier modes) being stretched out by the expansion and the loops assuming a near-circular form as they fall within the particle horizon. Thereafter they begin to oscillate violently, approaching the velocity of light. This does not mean they necessarily intersect themselves, but it does mean ϵ becomes large and the error terms due to the right-hand side of (2.6) grow rapidly.

We also looked at loops started in an initially nonstatic configuration with radius of the order of the horizon distance (loops like these might be formed by the collision of waves traveling on the string[5]).

The loop whose energy $\epsilon\eta$ is plotted in Fig. 2 had initial conditions

$$\vec{x}(\sigma,0)=\tfrac{1}{2}((2-\alpha)\sin\sigma+\alpha/3\sin3\sigma\ , \quad -(2-\alpha)\cos\sigma-\alpha/3\cos3\sigma\ , \quad -2[\alpha(1-\alpha)]^{1/2}\cos\sigma)\ ,$$

$$\dot{\vec{x}}(\sigma,0)=\tfrac{1}{2}(\alpha\cos\sigma-\alpha\cos3\sigma,\alpha\sin\sigma-\alpha\sin3\sigma\ , \quad -2[\alpha(1-\alpha)]^{1/2}\sin\sigma)$$

with $\alpha=0.9$, $0\le\sigma<2\pi$.

As is seen from the graph, the loop energy initially grew like $R\propto\eta$, the scale factor, and as it fell inside the horizon it began to oscillate—growing as the loop moved inwards against the expansion and falling as the loop moved outwards faster than the expansion.

IV. ENERGETIC CONSISTENCY OF THE STRING PICTURE

We now want to discuss the significance of our results. The energetics of the network of strings in an expanding universe is crucial to the consistency of the string picture in the following way.

As we have seen, length in waves larger than the horizon are conformally stretched by the expansion. Naively, if this were true on all scales, the energy of a length of string would scale as R and the volume it lies in as R^3, so the string density would scale as R^{-2}, compared to matter scaling as R^{-3} and radiation as R^{-4}. It would thus rapidly come to dominate the energy density of the universe—a situation clearly incompatible with observation.

We have seen that things are not quite as bad as this—the energy of large-amplitude waves was almost constant. However, the analysis of Sec. II did show that the time-averaged energy per wavelength[9] *always increases,* albeit slowly [e.g., Eq. (2.13)], because the damping effect always reduces the average velocity squared below that for a free loop so

$$\frac{1}{T}\int_0^T d\eta \int d\sigma\, \dot{\vec{x}}^2 \le \frac{L}{2}$$

in (2.9). This simple fact allows us to put a lower bound on the total energy density of string in the universe. *In the absence of any mechanism of energy loss or string interactions, the density of string scales more slowly than that of matter* (the string energy increases, and the volume increase like R^{-3}). This is an important result—for it shows that if there were no mechanism for energy loss the string density would rapidly (in the radiation-dominated era) come to dominate the total energy density of the universe. It is independent of the density spectrum of strings at formation.

The most obvious mechanism of energy loss is via gravitational radiation. When string starts to move it radiates away its mass M via gravitational waves at a constant rate $dM/dt\sim -G\mu^2$. This gives loops a finite lifetime $\sim l/G\mu$, where l is their length when they start to move freely, i.e., fall inside the horizon. Recent numerical simulations have confirmed the assumption[10] of a scale-free initial spectrum for strings.[11,14] The meaning of this is as follows. There is one length scale in the problem, the correlation length at the time the strings are formed. The assumption is that the large-scale distribution of string (i.e., the distribution observed at lower resolution) should be independent of this length and is intuitively reasonable. Now consider the number density of loops $n(a)$ of a "radius" between a and $a+da$. Since radius is a large-scale (low-resolution) property, $n(a)$ should be independent of the correlation length.

By dimensions, we find that[12]

$$n(a)da \sim \frac{da}{a^4}\ . \qquad (4.1)$$

The spectrum cannot in itself be used to find the total energy density of string. To do this we need to know the length l of a loop of radius a. A first guess would be that the string describes an approximately Brownian trajectory, so $l\propto a^2/\xi$, where ξ is the correlation length.

Now let us make the most optimistic assumption about the behavior of string beneath the horizon—we shall assume that waves are straightened out up to the horizon distance $\sim t$ so that the correlation length grows as t and the length of any loop of radius larger than the horizon is given by $l\propto a^2t$—$n(a)$ is unchanged by the expansion. Then the density contribution from loops of radius greater than the horizon is approximately (ignoring all dimensionless constants)

$$\rho(a \gtrsim t) \sim \mu \int_t^\infty \frac{da}{a^4}\frac{a^2}{t} = \mu t^{-1} , \tag{4.2}$$

which scales as radiation in a radiation-dominated universe.

However, this estimate is only a lower bound—and we can see that it is indeed too low for the following reason. As we saw in Sec. II, the length of string always grows, albeit slowly [Eq. (2.13)], while $L \propto a^2/t$ suggests it remains constant (the radius a would grow as the scale factor $R \propto t^{1/2}$ in a radiation-dominated universe). Thus we should have

$$L \propto a^2 \Big/ \left[\xi\left[\frac{t}{\xi}\right]\right]^\alpha$$

with ξ the original correlation length and $\alpha < 1$. Of course the strings are not really Brownian—they describe trajectories which are straighter (i.e., have larger radius for given total length) than Brownian ones because they never turn back on themselves. Thus we should have $l \propto a(a/c)^\beta$, with c the coherence length $\sim \xi(t/\xi)^\alpha$, and $\beta \lesssim 1$. These two formulas yield

$$\rho_{(a \gtrsim t)} \propto t^{\beta(1-\alpha)-2} , \tag{4.3}$$

and since $\alpha < 1$ and β is positive, we see the energy density of string scales slower than that of radiation. For $\rho \propto t^{-2}$, we need $\alpha = 1$—it does not matter exactly what β is. It is easily checked that including the effects of gravitational radiation means subtracting from l a term $G\mu(t-t_0)$, where t_0 is the time at formation. This term then scales as t^{-2} for $t \gg t_0$ and so cannot cancel the original term discussed above if $\alpha \neq 1$.

It is clear from the above analysis that some energy-loss mechanism is needed which increases the coherence length of the string to scale with the horizon distance $\sim t$(so $\alpha = 1$). The only plausible mechanism would seem to be the self-intersection of these large loops and the exchange of partners to form more smaller closed loops. This would have to happen on a scale $\sim t$ and would thus give rise to loops of radius of order the horizon distance. To yield $\rho \propto t^{-2}$ it would have to occur at the maximum possible rate, producing a constant number of such loops per horizon volume per expansion time, i.e.,

$$\frac{dn}{dt} \sim \frac{1}{t^4} ,$$

where n is the number of loops per unit volume.

Loops of radius smaller than the horizon never present an energy problem; due to energy loss via gravitational radiation they have a finite lifetime $\sim l/G\mu$, where l is their length when they begin moving freely, in the above picture of the order of the horizon distance. Their radius and length as we saw in Sec. I remain constant after they fall inside the horizon. For loops falling inside the horizon, or those produced by self-intersection as discussed above, the energy-density spectrum obeys $[\rho = \int da\,\nu(a)]$

$$\nu(a) \sim \mu \left[\frac{t_h}{t}\right]^{3/2} \frac{da_h}{{a_h}^4} t_h , \tag{4.4}$$

where we have used the scale factor $\sim t^{1/2}$ and the fact that their length is of order t_h, when they fall inside the horizon. a_h is their radius at this time, and $t_h \sim a_h$, and $a \sim \text{const} \sim a_h$ thereafter, so

$$\rho_{\text{loops}\,(a \lesssim t)} \sim \frac{\mu}{t^{3/2}} \int_{G\mu t}^{t} \frac{da}{a^{3/2}} \sim \frac{\mu}{\sqrt{G\mu}} t^{-2} , \tag{4.5}$$

thus scaling like radiation.

We are left with the conclusion that in order to render the string picture energetically consistent, there must occur production of closed loops of radius $\sim t$ at a rate $dn/dt \sim 1/t^4$. As shown in Ref. 5, this is indeed possible although more detailed studies are needed to establish whether it actually occurs.

In fact, it is not as outlandish a requirement as it might seem at first sight—for what determines the production of such loops on this scale is the spectrum of waves on the same scale along larger lengths of string. Since the spectrum at least on scales larger than the horizon is "scale-free", i.e., independent of the coherence length, the only possible scale entering the problem must be the scale on which waves start to move, the horizon scale $\sim t$.

It should be stressed that the above argument only dealt with the form of $\rho(t)$ and not its exact value. It is unlikely but conceivable that some dimensionless constants occur like π or 2 in such a way as to make the density contribution of string in (3.2) or (3.5) much greater than all other matter. This would also rule out the string picture as a realistic model for galaxy formation.

In conclusion, more detailed numerical calculations are needed to check whether closed loops are formed by a network of strings in an expanding universe at a rate $dn/dt \sim 1/t^4$. Knowledge of the actual rate could be used to give a better value for the density perturbation spectrum produced by strings and to give predictions of the morphology[6] of and spatial distribution correlation functions[13] of galaxies.

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