Leptonic decays of τ^- lepton: Finite ν_τ mass and effects of mass mixing

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A reexamination of the leptonic decays of the τ^- lepton with the inclusion of mass mixing of the hierarchical type provides ν_τ mass limits as 389.98 ± 135.36 and 329.34 ± 172.07 MeV from the electronic and muonic decays, respectively. Slightly different values are obtained for Kobayashi-Maskawa mixing, but with no mass mixing, the limits are substantially higher. The decay probabilities are sensitive to mass mixing, whereas Michel spectra remain practically unaffected.

In this paper we report our investigations about the effects of mass mixing on the leptonic decays of the τ^- lepton. The mixings considered are hierarchical[1] and Kobayashi-Maskawa[2] (KM) types and the neutrinos are treated as massive Dirac particles.

Shrock[3] has given a detailed analysis of some weak decays involving considerations of finite neutrino masses and the associated mass mixing. But in the case of leptonic decays of leptons he has not shown explicitly the effect of mass mixing on Michel spectra and decay probabilities. This work, therefore, may be considered as a supplement to his work involving mass mixing explicitly. It turns out that mass mixing has substantial influence, in particular, on mass limits obtained with and without mass mixing as also on total decay probabilities. But the effects on Michel spectra inclusive of the contributions from electromagnetic (EM) radiative corrections are insignificant except at the tail end of the energy spectrum.

The matrix element for the processes $\tau^- \to l^- \bar{\nu}_i \nu_j$ with the inclusion of the mixing matrix is given by

$$M = -\frac{iG}{\sqrt{2}} \sum_{i,j} U_{li} U^\dagger_{\tau j} \bar{u}_1 \gamma_\lambda (1-\gamma_5) V_{\nu_i} \bar{u}_{\nu_j} \gamma^\lambda (1-\gamma^5) u_\tau \ , \qquad (1)$$

where U_{li} and $U_{\tau j}$ are the elements of the neutrino mass-mixing matrix with $l = e, \mu$ and ν_i, ν_j the mass eigenstates, with i,j taking values 1, 2, 3.

The expression for the Michel spectrum for polarized τ^- is obtained directly from Eqs. (2.3)–(2.13) of Ref. 3 with the replacements $U^\dagger_{ai} \to U^\dagger_{\tau j}$, $U_{bj} \to U_{li}$, $m_a \to m_\tau$, $l_b \to l$, and the subscript $b \to l$.

The resulting expression shows that in general the Michel spectrum for $l = e$ or μ will be a sum of nine different spectra for the cases $i,j = 1, 2, 3$. We confine our attention to the three-neutrino world, and, in order to have an order-of-magnitude estimate, we adopt the convention that the neutrino masses m_i are in ascending order of values, i.e., $m_1 < m_2 < m_3$. Further, for the case of nondegenerate neutrinos we take ν_1, ν_2, and ν_3 to be, respectively, ν_e, ν_μ, and ν_τ. Using the present experimental bounds on the masses of various neutrino species,[4–6] $14 \text{ eV} < m(\nu_e) < 46$ eV, $m(\nu_\mu) < 0.52$ MeV, and $m(\nu_\tau) < 250$ MeV, we obtain $\delta_1 < 2.57 \times 10^{-8}$, $\delta_2 < 2.91 \times 10^{-4}$, and $\delta_3 < 0.14$, where $\delta_i = m_i/m_\tau$. Thus the dominant contribution comes from δ_3 only. The expressions for the spin-independent $(dW/dx\, dZ_e)_{\rm SI}$ and the spin-dependent $(dW/dx\, dZ_e)_{\rm SD}$ parts for the electron are obtained from the aforementioned resulting expression in the usual way by retaining only the dominant part δ_3, and with the substitution $x = 2E_e/m_\tau$, $Z_e = \cos\theta_e$, and $l = e$. For hierarchical mixing,[7]

$$|U_{13}|^2 = 0.0003 \ ,$$
$$|U_{23}|^2 = 0.059 \ , \qquad (2)$$
$$|U_{33}|^2 = 0.94 \ .$$

For KM mixing, following Barger[8] with his solution (C), we have

$$U_{13} = 0.41 \ ,$$
$$U_{23} = 0.02 \ , \qquad (3)$$
$$U_{33} = 0.91 \ .$$

The Michel spectra for e^- with $m(\nu_\tau) = 0$ and $m(\nu_\tau) = 250$ MeV, without and with hierarchical and KM mixing, are shown in Figs. 1(a) and 1(b) for the spin-independent and spin-dependent parts, respectively. It is seen that the contributions from finite mass start becoming manifest at $x > 0.2$, but mixing has a significant contribution towards the high-energy tail of the spectra when $x > 0.7$. This requires inclusion of contributions from EM radiative corrections. Using the recent work of Kalyniak and Ng,[9] we calculate these corrections for $\tau^- \to e^- \bar{\nu}_i \nu_j$, where we use Eqs. (7)–(10d) of Ref. 9, and Eqs. (2.8)–(2.9c) of Ref. 7, with the replacements $m_\mu \to m_\tau$, $\delta \to m(\nu_\tau)/m_\tau$, $U_{\mu i} \to U_{\tau i} = U_{3i}$, $i = 1, 2, 3$, $\delta_e \to m_e/m_\tau$, and $\vec{\sigma}_\mu \to -\vec{\sigma}_\tau$. The corrected Michel spectra so obtained are given in Figs. 1(c) and 1(d) corresponding to Figs. 1(a) and 1(b), respectively. The inclusion of radiative correction makes the nature of the Michel spectra for $m(\nu_\tau) = 0$ and $m(\nu_\tau) = 250$ MeV almost identical, thus rendering the detection of finite-mass effects difficult. With mixing included, the qualitative nature of the Michel spectra with and without inclusion of contributions from radiative correction remain almost identical with minor quantitative differences which are slightly predominant for KM mixing.

The total decay probability for this decay with mass mixing ($W_{\rm M}$) is given by

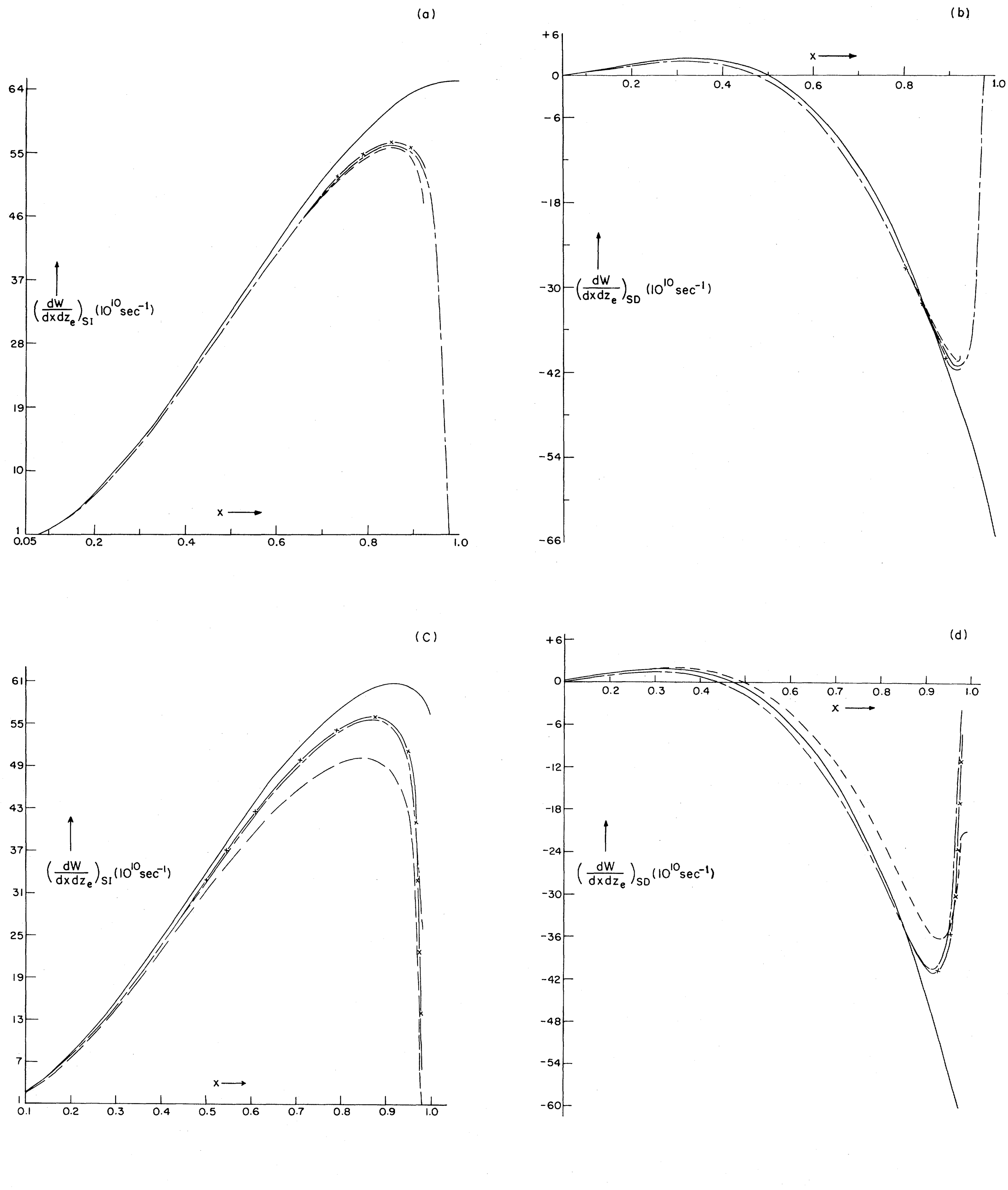

FIG. 1. (a) Spin-independent part of Michel spectra of e^- in the decay $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$. The solid curve is for $m(\nu_\tau)=0$. The dot-dash curve, the dashed curve, and the crossed curve are for $m(\nu_\tau)=250$ MeV without mixing, with KM mixing, and hierarchical mixing, respectively, with $m(\nu_e)=0$, and mixing elements used according to Eqs. (2) and (3). (b) Spin-dependent part of Michel spectra of e^- in the decay $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$. Description of the curves is identicall to that given in (a). (c) Spin-independent part of Michel spectra corresponding to (a) with radiative corrections included. (d) Spin-dependent part of Michel spectra corresponding to (b) with radiative corrections included.

$$W_{\mathrm{M}}(\tau^- \to e^- + \bar{\nu}_i + \nu_j) = \frac{G^2 m_\tau^5}{96\pi^3} \int_{x_{\min}}^{x_{\max}} (x^2 - B)^{1/2} \times \Bigg[(U_{23}^2 + U_{33}^2 U_{13}^2)[x(k-x) + (2kx - x^2 - B)]$$
$$+ \frac{U_{33}^2(1-U_{13}^2)}{(k-x)^3}(k - \delta_3^2 - x)\{(k-\delta_3^2-x)^2 x(k-x) + [(k-x)^2 + \delta_3^2(k-x) - 2\delta_3^4](2kx - x^2 - B)\}$$
$$+ \frac{U_{13}^2(1-U_{33}^2)}{(k-x)^3}[(k+\delta_3^2-x)^2 - 4\delta_3^2(k-x)]^{1/2}$$
$$\times\{[(k+\delta_3^2-x)^2 - 4\delta_3^2(k-x)]x(k-x) + [(k-x)^2 + \delta_3^2(k-x) - 2\delta_3^4](2kx - x^2 - B)\}$$
$$+ \frac{U_{13}^2 U_{33}^2}{(k-x)^{3/2}}(k - x - 4\delta_3^2)^{1/2}[(k - x - 4\delta_3^2)x(k-x) + (k - x + 2\delta_3^2)(2kx - x^2 - B)]\Bigg] dx \ , \qquad (4)$$

with

$$k = 1 + \frac{m_e^2}{m_\tau^2}, \quad B = \frac{4m_e^2}{m_\tau^2}, \quad x_{\max} = \frac{m_\tau^2 + m_e^2 - [m(\nu_e) + m(\nu_\tau)]^2}{m_\tau^2}, \quad \text{and} \quad x_{\min} = \frac{2m_e}{m_\tau} \ .$$

The corresponding expression for this decay without mass mixing (W_{NM}) is obtained from Eq. (1) with $U_{li} = U_{\tau j} = 1$:

$$W_{\mathrm{NM}}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = \frac{G^2 m_\tau^5}{96\pi^3} \int_{x_{\min}}^{x_{\max}} \frac{(x^2 - B)^{1/2}}{(k-x)^3} [(k - x + \delta_1^2 - \delta_3^2)^2 - 4\delta_1^2(k-x)]^{1/2}$$
$$\times\{[(k - x + \delta_1^2 - \delta_3^2)^2 - 4\delta_1^2(k-x)]x(k-x)$$
$$+ [(k-x)^2 + (k-x)(\delta_1^2 + \delta_3^2)^2 - 2(\delta_1^2 - \delta_3^2)^2](2kx - x^2 - B)\} dx \ . \qquad (5)$$

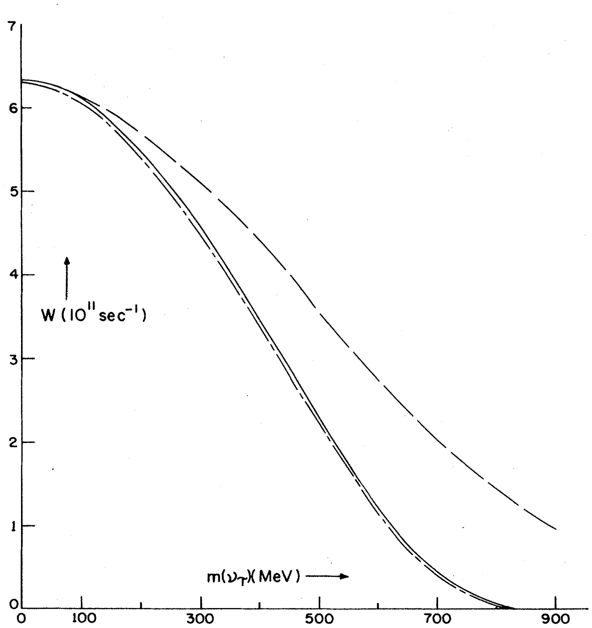

FIG. 2. Variation of decay probability with $m(\nu_\tau)$ in the decay $\tau^- \to e^- \bar{\nu}_e \nu_\tau$. Solid curve, dot-dash curve, and dashed curve are for hierarchical mixing, KM mixing, and without mixing, respectively, with $m(\nu_e) = 0$ and mixing elements used according to Eqs. (2) and (3).

The variation of total decay probability without and with mass mixing as a function of ν_τ mass are shown in Fig. 2. We notice that mass-mixing effects start becoming manifest appreciably from $m(\nu_\tau) > 100$ MeV.

The experimental value of this decay rate, with the use of the τ^- decay time[10] as $(4.6 \pm 1.9) \times 10^{-13}$ sec, and the branching ratio equal to $(16.2 \pm 1) \times 10^{-2}$, is found to be

$$W_{\mathrm{expt}}(\tau^- \to e^- + \bar{\nu}_e + \nu_\tau) = (3.5217 \pm 1.4707) \times 10^{11} \text{ per sec} . \qquad (6)$$

We calculate theoretical values of W_{M} and W_{NM}, giving different values of $m(\nu_\tau)$ in numerical integration, with the use of Eqs. (2) and (3) for mixing and $m(\nu_e) = 0$ in the expressions for W_{M} and W_{NM}. The corresponding value of $m(\nu_\tau)$ for which theoretical values of W_{M} and W_{NM} are equal to W_{expt} given by Eq. (6) are the values of ν_τ masses in this decay for the two cases. They are given in Table I.

TABLE I. Values of ν_τ mass in the leptonic decays of τ^- lepton.

Decay modes of τ^-	$m(\nu_\tau)$ (MeV) Hierarchical mixing	KM mixing	Without mixing
$e^- \bar{\nu}_e \nu_\tau$	389.98 ± 135.36	388.19 ± 131.06	502.89 ± 195.23
$\mu^- \bar{\nu}_\mu \nu_\tau$	329.34 ± 172.07	339.91 ± 175.22	421.62 ± 238.67

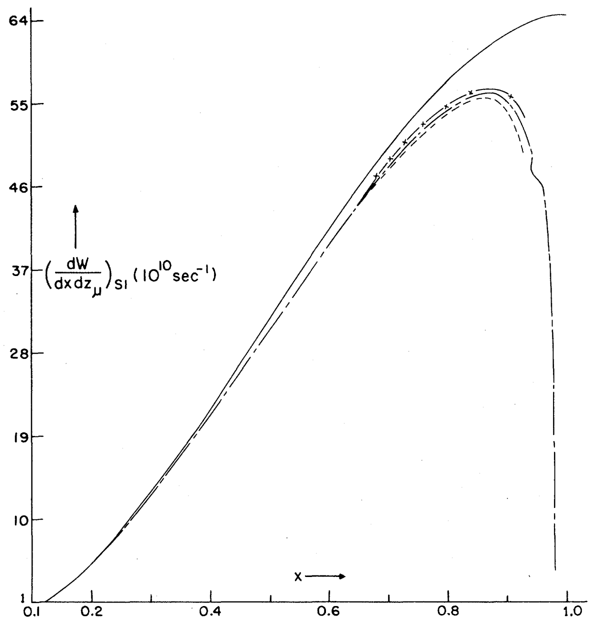

FIG. 3. Spin-independent part of Michel spectrum of μ^- in the decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$. Solid curve is for $m(\nu_\tau) = 0$. Dashed curve, crossed curve, and dot-dash curve are hierarchical mixing, KM mixing, and without mixing, respectively, with $m(\nu_\tau) = 250$ MeV, $m(\nu_\mu) = 0.52$ MeV, and mixing elements used according to Eqs. (2) and (3).

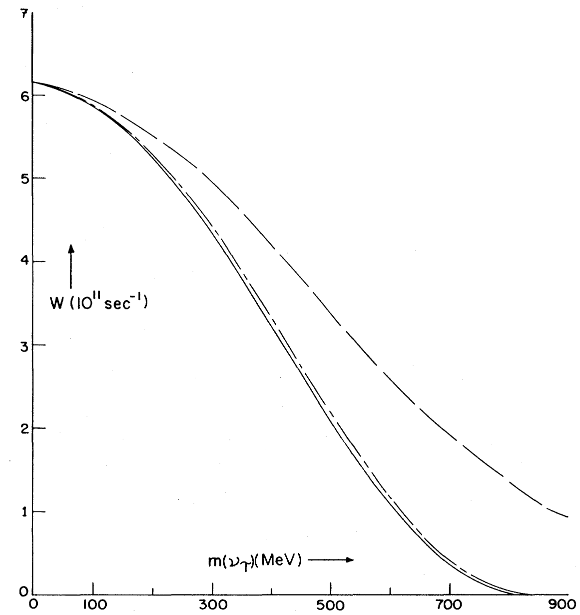

FIG. 4. Variation of decay probability with $m(\nu_\tau)$ in the decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$. Description of the curves is identical to that given in Fig. 2 with $m(\nu_\mu) = 0.52$ MeV.

For the decay $\tau^- \rightarrow \mu^- \bar{\nu}_i \nu_j$, the expressions for Michel spectrum and total decay probability are obtained from the corresponding expressions of $\tau^- \rightarrow e^- \bar{\nu}_i \nu_j$, with the replacements $e \rightarrow \mu$, $U_{13} \rightarrow U_{23}$, and $U_{23} \rightarrow U_{13}$.

The spin-independent part of the Michel spectra for μ^- with the inclusion of hierarchical and KM mixing are plotted in Fig. 3, which again manifest significant contributions from mixing at the high-energy end. The nature of the spin-dependent part of the μ^- Michel spectra is nearly identical to that of e^- given in Fig. 1(b). EM-radiative-correction contributions can be incorporated for this case too in the same way as done for e^- earlier, with almost identical results.

The variation of the total decay probability as a function of $m(\nu_\tau)$ is shown in Fig. 4. The mass-mixing effects start becoming manifest appreciably for $m(\nu_\tau) > 50$ MeV.

The experimental value of the total decay rate obtained with the use of τ decay time[10] = $(4.6 \pm 1.9) \times 10^{-13}$ sec and branching fraction $= (18.5 \pm 1.2) \times 10^{-2}$ is found to be

$$W_{\text{expt}}(\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau) = (4.0217 \pm 1.6815) \times 10^{11} \text{ per sec}. \tag{7}$$

Using again the procedure given following Eq. (6), with the corresponding use of the total decay probability with $m(\nu_\mu) = 0.52$ MeV, $m(\nu_\tau) =$ finite, and neutrino mass mixing of hierarchical and KM types, we obtain the masses of ν_τ which are given in Table I.

We notice that $m(\nu_\tau)$ values obtained with mixing are much nearer the experimental bounds[6] and are substantially lower as compared with those when no mixing is involved. Finally, this investigation leads to the conclusion that these decays, with the present values of the relevant parameters, point toward a finite mass for ν_τ. The large statistical errors in the values for $m(\nu_\tau)$ are because of the large errors in the experimentally predicted parameters such as decay time, branching ratios, and mixing angles. As such, the discrepancy between the experimental and theoretical values of $m(\nu_\tau)$ does not require any serious consideration about the structure of theory until the experimentally relevant parameters are reasonably refined and stabilized.

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