PHYSICAL REVIEW D VOLUME 29, NUMBER 7 1 APRIL 1984

SU(8) theory of multigenerational grand unification

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(Received 6 July 1983; revised manuscript received 23 January 1984)

As a concrete example of SU(N) theory of grand unification, an SU(8) model which contains three generations of light fermions and three new generations of heavy fermions (one of which lacks a toplike quark) are presented. The model consists of $\overline{8}_L$, $\overline{28}_L$, and 56_L representations of fermions, and it requires 63 gauge bosons with new generation vector bosons and an extra boson $Z^{0'}$ in addition to the γ, W, Z^0, X, and Y bosons in the standard SU(5) model. It satisfactorily embeds all features of the SU(5) model extended to three generations of light fermions, and predicts new interactions such as quantum generation dynamics, which changes generation. This interaction can cause various mixing angles among fermions. The complete gauge interactions and the masses of gauge bosons as well as fermions are derived. The masses of neutral vector bosons Z^0 and $Z^{0'}$ are predicted to be, by crude estimates, $92.3 \geq M_Z \geq 91.5$ GeV and >200 GeV, respectively. The mass of Z^0 is practically the same as, but slightly less than, that predicted by the standard SU(2)$\times$U(1) model.

I. INTRODUCTION

The idea of grand unification of the strong and the electroweak interactions is widely accepted as a promising step to pursue after confidence was gained in the SU(2)$\times$U(1) electroweak interaction[1] and SU(3)^c quantum chromodynamics (QCD) through experimental verification. The minimal gauge group that can contain SU(3)$^c\times$SU(2)$\times$U(1) is SU(5), and the SU(5) model[2] has enjoyed the most favorable attention. Its simplicity and interesting features that naturally explain charge quantization and nucleon decay, which is being tested in recent years, present a reasonable prototype model of grand unification theory (GUT).

The SU(5) model is essentially a one-generation (or family) model, and the experimental observations indicate that there exist at least two and a half generations or possibly three generations if the generation structure repeats itself. Since, in the standard SU(5) minimal model, the generations are put in by hand, there are no definitive intergenerational interactions prescribed by the theory. As more phenomenological data concerning heavier quarks accumulates, it is desirable to explain the relations among the various fermion masses, and have intergenerational interactions that can predict the transitions among various generations and the possible mixing angles among fermions in different generations.

We present here a concrete model based on $\overline{8}_L$, $\overline{28}_L$, and 56_L representations of SU(8) which contains the SU(5) model and extends to three generations of light fermions. It requires three additional generations of heavy fermions (one of which lacks a toplike quark). The model predicts generation-changing interactions and an extra neutral $Z^{0'}$ boson in addition to all features of the SU(5) model.

In Sec. II we present the model based on $\overline{8}_L+\overline{28}_L+56_L$ representations, and the complete gauge interactions are derived in Sec. III. Symmetry breaking and gauge-boson masses are discussed in Sec. IV. Section V deals with the masses of fermions, and the discussion is given in Sec. VI.

II. OUR $\overline{8}_L+\overline{28}_L+56_L$ MODEL of SU(8)

There are many models[3] that attempt to unify generations. The simple group we would like to consider in detail here is SU(N). Our criteria for GUT will be the following. (i) The SU(N) gauge group embeds the SU(5) model and contains a generation (G) group, i.e., SU(N)$\supset$SU($N-5$)$^G\times$SU(5)$\times$U(1)$'$ for generation unification with at least three generations of $\overline{5}_L$ and 10_L representations of SU(5). (ii) All fermions belong to the smallest number of complex, antisymmetric representations without repetition which contain SU(3)^c triplets (3), antitriplets ($\overline{3}$), and singlets. (iii) The model is anomaly free. (iv) The model is asymptotically free at high energy.

In the subgroup decomposition SU(N)$\supset$SU($N-5$)^G $\times$SU(5)$^F\times$U(1)$'$, the three lowest antisymmetric representations contain the following representations of the SU($N-5$)^G generation group and SU(5)^F family group, where on the left-hand side we give the dimension of the SU(N) representations and on the right-hand side within parentheses the dimensions of the SU($N-5$)^G and SU(5)^F representations:

$$\square : N=(N-5,1)+(1,5) ,$$

$$\begin{array}{c}\square\\\square\end{array} : \frac{N(N-1)}{2}=(\tfrac{1}{2}(N-5)(N-6),1)+(N-5,5)+(1,10) , \quad (1)$$

$$\begin{array}{c}\square\\\square\\\square\end{array} : \tfrac{1}{6}N(N-1)(N-2)= (\tfrac{1}{6}(N-5)(N-6)(N-7),1)+(\tfrac{1}{2}(N-5)(N-6),5)+(N-5,10)+(1,\overline{10}) .$$

The anomalies[4] A for these representations are

$$A(\square)=1\,,$$
$$A\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}\right)=N-4\,, \qquad (2)$$
$$A\left(\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\right)=\tfrac{1}{2}(N-3)(N-6)\,.$$

The simplest model for three generations of light fermions of $(\overline{5}_L+10_L)$ representations in SU(5) is the SU(8) model[5] with $\overline{8}_L+\overline{28}_L+56_L$ representations which satisfy all the criteria mentioned above. For a concrete example, we will pursue the details of this simplest $\overline{8}_L+\overline{28}_L+56_L$ model of SU(8) multigenerational GUT. For more than three light-fermion generations, it is a straightforward generalization for $N>8$, and a similar method can be applied.

For SU(8), Eqs. (1) and (2) provide

$$(\mathscr{Y}_L)_a;\ \overline{8}_L=(\overline{3},1)(5)+(1,\overline{5})(-3),\quad A(\square)=-1\,,$$
$$(\chi_L)_{ab};\ \overline{28}_L=(3,1)(10)+(1,\overline{10})(-6)+(\overline{3},\overline{5})(2),\quad A\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}\right)=-4\,, \qquad (3)$$
$$(\psi_L)^{abc};\ 56_L=(1,1)(-15)+(1,\overline{10})(9)+(\overline{3},5)(-7)+(3,10)(1),\quad A\left(\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\right)=5\,,$$

where the number in the second set of parentheses in each term indicates the quantum number[3] for U(1)′, and $a,b,c=1,2,\ldots,8$ are the SU(8) indices. The generation antitriplet $(\overline{3},\overline{5})$ and the generation triplet (3,10) in Eq. (3) can be identified as the three generations of light fermions, and the rest are new heavy fermions. The fact that $\overline{5}$ belongs to generation antitriplet while 10 belongs to generation triplet is an important ingredient in embedding the SU(5) model extended to the three generations of light fermions on an equal footing as shown in Sec. III. The fermion particle assignments are given in the Appendix in matrix form.

We note that in addition to the familiar three generations of light fermions (u_i,d_i,e,ν_e), (c_i,s_i,μ,ν_μ), and (t_i,b_i,τ,ν_τ), there are two generations of heavy fermions $(\widetilde{u}_i,\widetilde{d}_i,\widetilde{e},\widetilde{\nu}_e)$ and $(\widetilde{c}_i,\widetilde{s}_i,\widetilde{\mu},\widetilde{\nu}_\mu)$, plus one lacking a toplike quark, $(\widetilde{b}_i,\widetilde{\tau},\widetilde{\nu}_\tau)$, and neutral fermions N_α and N_0. In terms of tensor notations, these fermions can be expressed in the following manner, where $\text{SU}(3)^G$ indices $\alpha,\beta,\gamma=1,2,3$, $\text{SU}(3)^c$ indices $i,j,k=4,5,6$, SU(2) indices $l,m=7,8$, and SU(5) indices $r,s,t=4,\ldots,8$ are used:

$$(\mathscr{Y}_L)_\alpha=(N_L^c)_\alpha,\quad (\mathscr{Y}_L)_i=(\widetilde{b}^{\,c}_{Li}),\quad (\mathscr{Y}_L)_l=\begin{bmatrix}\widetilde{\tau}\\ -\widetilde{\nu}_\tau\end{bmatrix}_L,$$
$$(\chi_L)_{\alpha\beta}=\frac{1}{\sqrt{2}}\epsilon_{\alpha\beta\gamma}(N_L)^\gamma,\quad (\chi_L)_{ij}=\frac{1}{\sqrt{2}}\epsilon_{ijk}(\widetilde{u}_L)^k\,,$$
$$(\chi_L)_{\alpha i}=\frac{1}{\sqrt{2}}(D^c_{Li})_\alpha,\quad (\chi_L)_{\alpha l}=\frac{1}{\sqrt{2}}\begin{bmatrix}E_\alpha\\ -\nu_\alpha\end{bmatrix}_L,$$
$$(\chi_L)_{il}=\frac{1}{\sqrt{2}}\begin{bmatrix}-\widetilde{u}^{\,c}_i\\ -\widetilde{d}^{\,c}_i\end{bmatrix}_L,\quad (\chi_L)_{78}=\frac{1}{\sqrt{2}}(-\widetilde{e}_L)\,,$$

$$(\psi_L)^{\alpha\beta i}=\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}(\widetilde{D}^{\,i}_L)_\gamma,\quad (\psi_L)^{\alpha\beta l}=\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}\begin{bmatrix}\widetilde{E}^{\dagger}\\ -\widetilde{\nu}^{\,c}\end{bmatrix}_{L,\gamma},$$
$$(\psi_L)^{i78}=\frac{1}{\sqrt{6}}(\widetilde{c}^{\,i}_L),\quad (\psi_L)^{ijl}=\frac{1}{\sqrt{6}}\epsilon^{ijk}\begin{bmatrix}\widetilde{s}^{\,c}_k\\ -\widetilde{c}^{\,c}_k\end{bmatrix}_L, \qquad (4)$$
$$(\psi_L)^{ijk}=\frac{1}{\sqrt{6}}\epsilon^{ijk}(-\widetilde{\mu}_L)\,,$$
$$(\psi_L)^{\alpha ij}=\frac{1}{\sqrt{6}}\epsilon^{ijk}(U^c_{Lk})^\alpha\,,$$
$$(\psi_L)^{\alpha il}=\frac{1}{\sqrt{6}}\begin{bmatrix}-U^i\\ -D^i\end{bmatrix}_{L,\alpha},$$
$$(\psi_L)^{\alpha 78}=\frac{1}{\sqrt{6}}(-E^{\dagger}_L)^\alpha\,,$$
$$(\psi_L)^{\alpha\beta\gamma}=\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}(N_{0L})\,.$$

Here the three generations of light fermions are expressed as generation triplets,

$$(U_{Li})_\alpha\equiv\begin{bmatrix}u_i\\ c_i\\ t_i\end{bmatrix}_L,\quad (D_{Li})_\alpha\equiv\begin{bmatrix}d_i\\ s_i\\ b_i\end{bmatrix}_L,$$
$$(E_L)_\alpha\equiv\begin{bmatrix}e\\ \mu\\ \tau\end{bmatrix}_L,\quad (\nu_L)_\alpha\equiv\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}_L, \qquad (5a)$$

and for new heavy fermions,

$$(N_L)_\alpha\equiv\begin{bmatrix}N_1\\ N_2\\ N_3\end{bmatrix}_L,\quad (\widetilde{D}_{Li})_\alpha\equiv\begin{bmatrix}\widetilde{d}_i\\ \widetilde{s}_i\\ \widetilde{b}_i\end{bmatrix}_L,$$
$$(\widetilde{E}_L)_\alpha\equiv\begin{bmatrix}\widetilde{e}\\ \widetilde{\mu}\\ \widetilde{\tau}\end{bmatrix}_L,\quad (\widetilde{\nu}_L)_\alpha\equiv\begin{bmatrix}\widetilde{\nu}_e\\ \widetilde{\nu}_\mu\\ \widetilde{\nu}_\tau\end{bmatrix}_L \qquad (5b)$$

are used. The electric charges for $(U_{i\alpha},D_{i\alpha},E_\alpha,\nu_\alpha)$ are $(\frac{2}{3},-\frac{1}{3},-1,0)e$, and similarly for ($\widetilde{u}_i$ and $\widetilde{c}_i$, $\widetilde{D}_{i\alpha}$, $\widetilde{E}_\alpha$, $\widetilde{\nu}$, N_α and N_0) are $(\frac{2}{3},-\frac{1}{3},-1,0,0)e$.

The 63 gauge bosons belonging to the adjoint representation of SU(8) are

$$(\mathscr{A}_\mu)^b_a:\ 63=(1,1)+(8,1)+(3,\overline{5})+(\overline{3},5)+(1,24)\,, \qquad (6)$$

where 38 of them are generation bosons and the rest are the generationless bosons including the familiar 24 bosons in the standard SU(5) model. We identify them as

$$(\mathscr{A}_\mu)^b_a=\left[\begin{array}{cccccccc} g^1_1 & g^1_2 & g^1_3 & \bar{A}^1_4 & \bar{A}^1_5 & \bar{A}^1_6 & \bar{B}^1 & \bar{C}^1 \\ g^2_1 & g^2_2 & g^2_3 & \bar{A}^2_4 & \bar{A}^2_5 & \bar{A}^2_6 & \bar{B}^2 & \bar{C}^2 \\ g^3_1 & g^3_2 & g^3_3 & \bar{A}^3_4 & \bar{A}^3_5 & \bar{A}^3_6 & \bar{B}^3 & \bar{C}^3 \\ A^4_1 & A^4_2 & A^4_3 & G^4_4 & G^4_5 & G^4_6 & \bar{X}^4 & \bar{Y}^4 \\ A^5_1 & A^5_2 & A^5_3 & G^5_4 & G^5_5 & G^5_6 & \bar{X}^5 & \bar{Y}^5 \\ A^6_1 & A^6_2 & A^6_3 & G^6_4 & 6^6_5 & G^6_6 & \bar{X}^6 & \bar{Y}^6 \\ B_1 & B_2 & B_3 & X_4 & X_5 & X_6 & W^7_7 & W^7_8 \\ C_1 & C_2 & C_3 & Y_4 & Y_5 & Y_6 & W^8_7 & W^8_8 \end{array}\right] \begin{array}{l} \left.\right\} \mathrm{SU}(3)^G \text{ (rows 1–3)} \\ \left.\right\} \mathrm{SU}(3)^c \text{ (rows 4–6)} \\ \left.\right\} \mathrm{SU}(2) \text{ (rows 7–8)} \\ \left.\right\} \mathrm{SU}(5)^F \text{ (rows 4–8)} \end{array} , \tag{7}$$

where the new vector bosons are

g^β_α = colorless generation bosons belonging to (8,1) and (1,1) ,

A^i_α = colored generation triplet bosons belonging to $(3,\bar{5})$,

B_α and C_α = colorless generation triplet bosons belonging to $(3,\bar{5})$,

G^j_i = gluons ,

and others are the familiar vector bosons appearing in the $\mathrm{SU}(5)^F$ family model.

The traceless diagonal elements are normalized as

$$g^1_1=\left[\frac{g^3}{\sqrt{2}}+\frac{g^8}{\sqrt{6}}\right]-\frac{5}{2}\frac{1}{\sqrt{30}}C ,$$

$$g^2_2=\left[-\frac{g^3}{\sqrt{2}}+\frac{g^8}{\sqrt{6}}\right]-\frac{5}{2}\frac{1}{\sqrt{30}}C ,$$

$$g^3_3=-\frac{2}{\sqrt{6}}g^8-\frac{5}{2}\frac{1}{\sqrt{30}}C ,$$

$$G^4_4=\left[\frac{G^3}{\sqrt{2}}+\frac{G^8}{\sqrt{6}}\right]-\frac{2}{\sqrt{30}}B+\frac{3}{2}\frac{1}{\sqrt{30}}C , \tag{8}$$

$$G^5_5=\left[-\frac{G^3}{\sqrt{2}}+\frac{G^8}{\sqrt{6}}\right]-\frac{2}{\sqrt{30}}B+\frac{3}{2}\frac{1}{\sqrt{30}}C ,$$

$$G^6_6=-\frac{2}{\sqrt{6}}G^8-\frac{2}{\sqrt{30}}B+\frac{3}{2}\frac{1}{\sqrt{30}}C ,$$

$$W^7_7=\frac{W^3}{\sqrt{2}}+\frac{3}{\sqrt{30}}B+\frac{3}{2}\frac{1}{\sqrt{30}}C ,$$

$$W^8_8=-\frac{W^3}{\sqrt{2}}+\frac{3}{\sqrt{30}}B+\frac{3}{2}\frac{1}{\sqrt{30}}C ,$$

and, as usual, $W^7_8=W^+$, $W^8_7=W^-$, $W^\pm=(1/\sqrt{2})(W^1 \mp iW^2)$. In Eq. (8), B belongs to $\mathrm{U}(1)_B$ in $\mathrm{SU}(5)\to\mathrm{SU}(3)^c\times\mathrm{SU}(2)\times\mathrm{U}(1)_B$, and C belongs to $\mathrm{U}'(1)_c$ in $\mathrm{SU}(8)\to\mathrm{SU}(3)^G\times\mathrm{SU}(5)^F\times\mathrm{U}'(1)_c$ in the symmetry-breaking stages discussed in Sec. IV. The electric charges of new gauge bosons $(g^\beta_{\mu\alpha},A^i_{\mu\alpha},B_{\mu\alpha},C_{\mu\alpha})$ are $(0,-\frac{1}{3},1,0)e$.

III. GAUGE INTERACTIONS

The relevant SU(8)-invariant gauge interaction Lagrangian for fermions is

$$\mathscr{L}_f=g_8\left[(\overline{\mathscr{Y}}{}^c_R)_a\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^a_b(\mathscr{Y}^c_R)^b+2(\bar{X}{}^c_R)_{ab}\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^a_c(X^c_R)^{cb}+3(\bar{\psi}_L)_{abc}\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^a_d(\psi_L)^{dbc}\right] , \tag{9}$$

where g_8 is the SU(8) coupling constant, and

$$\begin{aligned} &\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^\beta_\alpha=g^k\frac{\lambda^k}{2}-(\tfrac{3}{5})^{1/2}\tfrac{5}{12}\mathbb{C}\delta^\beta_\alpha,\quad k=1,\ldots,8,\quad g^k\equiv g^k_\mu\gamma^\mu \\ &\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^j_i=\mathbb{G}^k\frac{\lambda^k}{2}-(\tfrac{3}{5})^{1/2}\tfrac{1}{3}\mathbb{B}\delta^j_i+(\tfrac{3}{5})^{1/2}\tfrac{1}{4}\mathbb{C}\delta^j_i,\quad \mathbb{G}^k\equiv G^k_\mu\gamma^\mu \\ &\frac{1}{\sqrt{2}}(\mathscr{A}_\mu)^m_l=\mathbb{W}^{k'}\frac{\tau^{k'}}{2}+(\tfrac{3}{5})^{1/2}\tfrac{1}{2}\mathbb{B}\delta^m_l+(\tfrac{3}{5})^{1/2}\tfrac{1}{4}\mathbb{C}\delta^m_l,\quad k'=1,2,3,\quad \mathbb{W}^{k'}\equiv W^{k'}_\mu\gamma^\mu . \end{aligned} \tag{10}$$

λ^k are 3×3 matrices and $\tau^{k'}$ are 2×2 matrices. The gauge interaction participated by the colorless octet generation bosons (g^k_μ) is, according to Eq. (9),

$$\mathcal{L}_f(g_\mu^k)=g_8\left[\left[\bar{U}_{Li}g^k\frac{\lambda^k}{2}U_L^i+\bar{U}_L^{ci}g^k\frac{\lambda^k}{2}U_{Li}^c+\bar{D}_i g^k\frac{\lambda^k}{2}D^i+\bar{E}^+g^k\frac{\lambda^k}{2}E^+-\bar{\nu}_L g^k\frac{\lambda^k}{2}\nu_L\right]\right.$$
$$\left.+\left[\bar{N}g^k\frac{\lambda^k}{2}N-\bar{\tilde{D}}_{Li}g^k\frac{\lambda^k}{2}\tilde{D}_L^i-\bar{\tilde{E}}_L^+g^k\frac{\lambda^k}{2}\tilde{E}_L^+-\bar{\tilde{\nu}}_L^c g^k\frac{\lambda^k}{2}\tilde{\nu}_L^c\right]\right]. \tag{11}$$

This $SU(3)^G$-symmetric quantum generation dynamics (QGD) is a new interaction predicted by this theory among the different generations. It has chiral (L-R) symmetry for D_i, E, and N, but not for other fermions. This interaction can create the mixing of generations, and may be responsible for the generation-mixing angles such as the Cabibbo angle. Since the interactions among the light-down-quark generations (D_i) and the light-lepton generations (E) have the same form, the mixing angles arising from this interaction must have the same structures except for the effects due to their masses

The interaction due to the colored octet gluons in Eq. (9) is the chiral-symmetric and $SU(3)^c$-symmetric QCD extended to new particles given by

$$\mathcal{L}_f(G_\mu^k)=g_8\left[\left[\bar{U}_\alpha\mathcal{G}^k\frac{\lambda^k}{2}U^\alpha+\bar{D}_\alpha\mathcal{G}^k\frac{\lambda^k}{2}D^\alpha\right]+\left[\bar{\tilde{u}}\mathcal{G}^k\frac{\lambda^k}{2}\tilde{u}+\bar{\tilde{c}}\mathcal{G}^k\frac{\lambda^k}{2}\tilde{c}+\bar{\tilde{D}}_\alpha\mathcal{G}^k\frac{\lambda^k}{2}\tilde{D}^\alpha\right]\right]. \tag{12}$$

The first term in Eq. (12) corresponds to the standard $SU(5)^F$ part of QCD extended naturally to the three generations of light fermions, and the second term is for the new heavy fermions.

The SU(2)-symmetric $W_\mu^{k'}$ boson interaction in Eq. (9) is given by

$$\mathcal{L}_f(W_\mu^{k'})=g_8\left\{\left[(\bar{U}_{i\alpha},\bar{D}_{i\alpha})_L W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}U^{i\alpha}\\D^{i\alpha}\end{bmatrix}+(\bar{\nu}_\alpha\bar{E}_\alpha)_L W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}\nu^\alpha\\E^\alpha\end{bmatrix}_L\right]\right.$$
$$+\left[(\bar{\tilde{u}}_i,\bar{\tilde{d}}_i)_R W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}\tilde{u}_i\\\tilde{d}_i\end{bmatrix}_R+(\bar{\tilde{c}}_i,\bar{\tilde{s}}_i)_R W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}\tilde{c}^i\\\tilde{s}^i\end{bmatrix}_R\right.$$
$$\left.\left.+(\bar{\tilde{\nu}}_\alpha,\bar{\tilde{E}}_\alpha)_R W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}\tilde{\nu}^\alpha\\\tilde{E}^\alpha\end{bmatrix}_R+(\bar{\tilde{\nu}}_\tau,\bar{\tilde{\tau}})_L W^{k'}\frac{\lambda^{k'}}{2}\begin{bmatrix}\tilde{\nu}_\tau\\\tilde{\tau}\end{bmatrix}_L\right]\right\}. \tag{13}$$

The first term in Eq. (13) is the SU(5) model extended to the three light generations, and the second term is the predicted new weak interactions dealing with heavy fermions. The interaction involving B_μ belonging to $U(1)_B$ in Eq. (9) is

$$\mathcal{L}_f(B_\mu)=g_8(\tfrac{3}{5})^{1/2}\{[\tfrac{1}{6}(\bar{U}_{Li\alpha}\mathcal{B}U_L^{i\alpha}+\bar{D}_{Li\alpha}\mathcal{B}D_L^{i\alpha})+\tfrac{2}{3}\bar{U}_{Ri\alpha}\mathcal{B}U_R^{i\alpha}-\tfrac{1}{3}\bar{D}_{Ri\alpha}\mathcal{B}D_R^{i\alpha}-\bar{E}_{R\alpha}\mathcal{B}E_R^\alpha-\tfrac{1}{2}(\bar{E}_{L\alpha}\mathcal{B}E_L^\alpha+\bar{\nu}_{L\alpha}\mathcal{B}\nu_L^\alpha)]$$
$$+[\tfrac{1}{6}(\bar{\tilde{u}}_{Ri}\mathcal{B}\tilde{u}_R^i+\bar{\tilde{d}}_{Ri}\mathcal{B}\tilde{d}_R^i)+\tfrac{2}{3}\bar{\tilde{u}}_{Li}\mathcal{B}\tilde{u}_L^i-\tfrac{1}{3}\bar{\tilde{d}}_{Li}\mathcal{B}\tilde{d}_L^i-\bar{\tilde{e}}_L\mathcal{B}\tilde{e}_L-\tfrac{1}{2}(\bar{\tilde{e}}_R\mathcal{B}\tilde{e}_R+\bar{\tilde{\nu}}_{eR}\mathcal{B}\tilde{\nu}_{eR})$$
$$+\tfrac{1}{6}(\bar{\tilde{c}}_{Ri}\mathcal{B}\tilde{c}_R^i+\bar{\tilde{s}}_{Ri}\mathcal{B}\tilde{s}_R^i)+\tfrac{2}{3}\bar{\tilde{c}}_{Li}\mathcal{B}\tilde{c}_L^i-\tfrac{1}{3}\bar{\tilde{s}}_{Li}\mathcal{B}\tilde{s}_L^i-\bar{\tilde{\mu}}_L\mathcal{B}\tilde{\mu}_L-\tfrac{1}{2}(\bar{\tilde{\mu}}_R\mathcal{B}\tilde{u}_R+\bar{\tilde{\nu}}_{\mu R}\mathcal{B}\tilde{\nu}_{\mu R})$$
$$-\tfrac{1}{3}\bar{\tilde{b}}_i\mathcal{B}\tilde{b}^i-\tfrac{1}{2}(\bar{\tilde{\tau}}\mathcal{B}\tilde{\tau}+\bar{\tilde{\nu}}_\tau\mathcal{B}\tilde{\nu}_\tau)]\}. \tag{14}$$

The first term in Eq. (14) is the same as the one in the SU(5) model with three light generations. The second term is the new interaction involving heavy fermions, which has the same form as the first term if $L\leftrightarrow R$, except for the interaction involving $(\tilde{b},\tilde{\tau},\tilde{\nu}_\tau)$, which has a chiral symmetry.

The new $U'(1)_C$ interaction involving C_μ in Eq. (9) is

$$\mathcal{L}_f(C_\mu)=g_8(\tfrac{3}{5})^{1/2}\tfrac{1}{4}\{[\tfrac{1}{3}(\bar{U}_{Li\alpha}\mathcal{C}U_L^{i\alpha}+\bar{D}_{Li\alpha}\mathcal{C}D_L^{i\alpha})-\tfrac{1}{3}U_{Ri\alpha}\mathcal{C}U_R^{i\alpha}$$
$$-\tfrac{2}{3}\bar{D}_{Ri\alpha}\mathcal{C}D_R^{i\alpha}-\tfrac{1}{3}\bar{E}_{R\alpha}\mathcal{C}E_R^\alpha+\tfrac{2}{3}(\bar{E}_{L\alpha}\mathcal{C}E_L^\alpha+\bar{\nu}_{L\alpha}\mathcal{C}\nu_L^\alpha)]$$
$$+[-\tfrac{5}{3}(\bar{N}_\alpha\mathcal{C}N^\alpha-3\bar{N}_{L\alpha}\mathcal{C}N_L^\alpha+3\bar{N}_{0L}\mathcal{C}N_{0L})+2(\bar{\tilde{u}}_{Ri}\mathcal{C}\tilde{u}_R^i+\bar{\tilde{d}}_{Ri}\mathcal{C}\tilde{d}_R^i)-2\bar{\tilde{u}}_{Li}\mathcal{C}\tilde{u}_L^i$$
$$+\bar{\tilde{d}}_{Li}\mathcal{C}\tilde{d}_L^i-2\bar{\tilde{e}}_L\mathcal{C}\tilde{e}_L-(\bar{\tilde{e}}_R\mathcal{C}\tilde{e}_R+\bar{\tilde{\nu}}_{eR}\mathcal{C}\tilde{\nu}_{eR})-3(\tilde{c}_R^i\mathcal{C}\tilde{c}_R^i+\bar{\tilde{s}}_{Ri}\mathcal{C}\tilde{s}_R^i)+3\bar{\tilde{c}}_{Li}\mathcal{C}\tilde{c}_L^i$$
$$+\bar{\tilde{s}}_{Li}\mathcal{C}\tilde{s}_L^i+3\bar{\tilde{\mu}}_L\mathcal{C}\tilde{\mu}_L-(\bar{\tilde{\mu}}_R\mathcal{C}\tilde{\mu}_R+\bar{\tilde{\nu}}_{\mu R}\mathcal{C}\tilde{\nu}_{\mu R})+\bar{\tilde{b}}_i\mathcal{C}\tilde{b}^i-(\bar{\tilde{\tau}}\mathcal{C}\tilde{\tau}+\bar{\tilde{\nu}}_\tau\mathcal{C}\tilde{\nu}_\tau)]\}. \tag{15}$$

Equation (15) gives a new interaction of this SU(8) theory involving C_μ, which has quite similar terms for both light and heavy fermions as in the case of the $U(1)_B$ interaction in Eq. (14) except for the coefficients. For the new interaction involving the colored generation triplet bosons $A_{\mu\alpha}^i$ in Eq. (9), we get

$$\mathscr{L}_f(A^i_{\mu\alpha})=g_8\frac{1}{\sqrt{2}}\bar{A}^{\alpha}_{\mu i}[\bar{N}_{R\alpha}\gamma^\mu\tilde{b}^i_R+\bar{N}_{0L}\gamma^\mu\tilde{D}^i_{L\alpha}-\bar{E}^+_{R\alpha}\gamma^\mu\tilde{u}^i_R+\tilde{\bar{\nu}}^c_R\gamma^\mu\tilde{d}^i_R-\bar{U}^{ci}_{L\alpha}\gamma^\mu\tilde{\mu}_L-\bar{E}^+_{L\alpha}\gamma^\mu\tilde{C}^i_L$$
$$+\epsilon_{\alpha\beta\gamma}(\bar{N}^{c\beta}_R\gamma^\mu D^{i\gamma}_R-\bar{\tilde{E}}^{+\beta}_L\gamma^\mu U^{i\gamma}_L-\bar{\tilde{\nu}}^{c\beta}_L\gamma^\mu D^{i\gamma}_L+\epsilon^{ijk}\bar{\tilde{D}}^{\beta}_{Lj}\gamma^\mu U^{c\gamma}_{Lk})$$
$$+\epsilon^{ijk}(\bar{D}_{Rj\alpha}\gamma^\mu\tilde{u}^c_{Rk}-\bar{U}_{Lj\alpha}\gamma^\mu\tilde{s}^c_{Lk}+\bar{D}_{Lj\alpha}\gamma^\mu\tilde{c}^c_{Lk})]+\text{H.c.} \tag{16}$$

This new interaction predicted by this model contains a color-conserving but generation-changing interaction, and also provides the interactions responsible for the transitions from heavy fermions to light fermions.

The new interaction involving the colorless generation triplet bosons $B_{\mu\alpha}$ is given by

$$\mathscr{L}_f(B_{\mu\alpha})=g_8\frac{1}{\sqrt{2}}\bar{B}^{\alpha}_{\mu}[\bar{N}_{R\alpha}\gamma^\mu\tilde{\tau}^+_R+\bar{N}_{0L}\gamma^\mu\tilde{E}^+_{L\alpha}+\bar{D}_{Ri\alpha}\gamma^\mu\tilde{u}^i_R+\tilde{\bar{\nu}}^c_R\gamma^\mu\tilde{e}^+_R+\bar{U}^{ci}_L\gamma^\mu\tilde{s}^c_{Li}+\bar{D}_{Li}\gamma^\mu\tilde{c}^i_L$$
$$+\epsilon_{\alpha\beta\gamma}(\bar{N}^{c\beta}_R\gamma^\mu E^{+\gamma}_R+\bar{\tilde{D}}^{\beta}_{Li}\gamma^\mu U^{i\gamma}_L+\bar{\tilde{\nu}}^{c\beta}_L\gamma^\mu E^{+\gamma}_L)]+\text{H.c.} \tag{17}$$

This contains also color-conserving but generation-changing interactions, and predicts the heavy-to-light-fermion transitions. The interaction involving the other generation triplet bosons $C_{\mu\alpha}$ is found to be

$$\mathscr{L}_f(C_{\mu\alpha})=g_8\frac{1}{\sqrt{2}}\bar{C}^{\alpha}_{\mu}[-\bar{N}_{R\alpha}\gamma^\mu\tilde{\nu}^c_{\tau R}-\bar{N}_{0L}\gamma^\mu\tilde{\nu}^c_{L\alpha}+\bar{D}_{Ri\alpha}\gamma^\mu\tilde{d}^i_R+\bar{E}^+_{R\alpha}\gamma^\mu\tilde{e}^+_R-\bar{U}^{ci}_{L\alpha}\gamma^\mu\tilde{c}^c_{Li}-\bar{U}_{Li\alpha}\gamma^\mu\tilde{c}^i_L$$
$$+\epsilon_{\alpha\beta\gamma}(-\bar{N}^{c\beta}_R\gamma^\mu\nu^{c\gamma}_R+\bar{\tilde{D}}^{\beta}_{Li}\gamma^\mu D^{i\gamma}_L+\bar{E}^{+\beta}_L\gamma^\mu E^{+\gamma}_L)]+\text{H.c.} \tag{18}$$

Equation (18) also contains the heavy-to-light-fermion transitions. This interaction is also color-changing and generation-changing, and it consists of flavor-changing neutral currents.

The baryon-number- and lepton-number-violating and generation-conserving leptoquark interaction involving the SU(5) superheavy boson $X_{\mu i}$ is

$$\mathscr{L}_f(X_{\mu i})=g_8\frac{1}{\sqrt{2}}\bar{X}^i_\mu\{(\bar{D}_{Ri\alpha}\gamma^\mu E^{+\alpha}_R)+(\bar{D}_{Li\alpha}\gamma^\mu E^{+\alpha}_L)+\bar{\tilde{D}}_{Ri\alpha}\gamma^\mu\tilde{E}^{+\alpha}_R+\bar{\tilde{D}}_{Li\alpha}\gamma^\mu\tilde{E}^{+\alpha}_L$$
$$+\epsilon_{ijk}[(\bar{U}^{ck}_{L\alpha}\gamma^\mu U^{j\alpha}_L)+\bar{\tilde{u}}^{ck}_R\gamma^\mu\tilde{u}^j_R+\bar{\tilde{c}}^{ck}_L\gamma^\mu\tilde{c}^j_L]\}+\text{H.c.} \tag{19}$$

Similarly, for the SU(5) superheavy boson $Y_{\mu i}$,

$$\mathscr{L}_f(Y_{\mu i})=g_8\frac{1}{\sqrt{2}}\bar{Y}^i_\mu\{-(\bar{D}_{Ri\alpha}\gamma^\mu\nu^{c\alpha}_R)-(\bar{U}_{Li\alpha}\gamma^\mu E^{+\alpha}_L)-\bar{\tilde{D}}_{Li\alpha}\gamma^\mu\tilde{\nu}^{c\alpha}_L-\bar{\tilde{u}}_{Ri}\gamma^\mu\tilde{e}^+_R-\bar{\tilde{c}}_{Ri}\gamma^\mu\tilde{\mu}^+_R-\bar{\tilde{b}}_{Ri}\gamma^\mu\tilde{\nu}^c_{\tau R}$$
$$+\epsilon_{ijk}[(\bar{U}^{ck}_{L\alpha}\gamma^\mu D^{j\alpha}_L)+\bar{\tilde{u}}^{ck}_R\gamma^\mu\tilde{d}^j_R+\bar{\tilde{s}}^{ck}_L\gamma^\mu\tilde{c}^j_L]\}+\text{H.c.} \tag{20}$$

Equations (19) and (20) contain the usual terms (in parentheses) in the SU(5) model extended to the three-light-fermion generations and similar terms for heavy fermions.

Thus, the standard SU(5) model extended to the three light generations is neatly embedded in our model as shown in Eq. (11) through Eq. (20). In addition there are new interactions involving generations and color.

IV. SYMMETRY BREAKING AND GAUGE-BOSON MASSES

The hierarchy of symmetry breaking to be considered is the following four stages:

$$\text{SU}(8)\underset{M_A\gtrsim M_X}{\overset{(\Phi_1)^b_a}{\longrightarrow}}\text{SU}(3)^G\times\text{SU}(5)^F\times\text{U}'(1)_C$$
$$\underset{M_X\sim 10^{14}\text{ GeV}}{\overset{(\Phi_2)^b_a}{\longrightarrow}}\text{SU}(3)^G\times\text{SU}(3)^C\times\text{SU}(2)\times\text{U}(1)_B\times\text{U}'(1)_C$$
$$\underset{M_g>200\text{ GeV}}{\overset{(H_\alpha)^a}{\longrightarrow}}\text{SU}(3)^C\times\text{SU}(2)\times\text{U}(1)$$
$$\underset{M_W\sim 10^2\text{ GeV}}{\overset{H^a}{\longrightarrow}}\text{SU}(3)^C\times\text{U}(1)\;. \tag{21}$$

The vacuum expectation value of the adjoint representation of Higgs particles $\langle(\Phi_1)^b_a\rangle = v_1(\delta^b_a - \frac{8}{5}\delta^r_a\delta^b_r)$ in the first stage gives extrasuperheavy masses to gauge bosons $A_{\mu\alpha}$, $B_{\mu\alpha}$, and $C_{\mu\alpha}$, and the second adjoint representation $\langle(\Phi_2)^b_a\rangle = v_2(\delta^b_a - 2\delta^i_a\delta^b_i - \delta^l_a\delta^b_l)$ gives superheavy masses to $A^i_{\mu\alpha}$, $B_{\mu\alpha}$, $C_{\mu\alpha}$, $X_{\mu\alpha}$, and $Y_{\mu\alpha}$ in the second stage. In the third stage the three different ($\alpha=1,2,3$) eight-dimensional representations $\langle(H_\alpha)^a\rangle = (v_0'/\sqrt{2})\delta^a_\alpha$ give medium-heavy masses to 39 bosons $g^\beta_{\mu\alpha}$, $A^i_{\mu\alpha}$, $B_{\mu\alpha}$, and $C_{\mu\alpha}$. The fourth stage gives masses to $C_{\mu\alpha}$, and 9 more bosons $Y_{\mu i}$, $W^\pm$, and Z^0_μ due to another eight-dimensional representation $\langle H^a\rangle = (v_0/\sqrt{2})\delta^a_8$. Thus, all vector bosons acquire masses except the octet gluons and photon (A_μ).

The Lagrangian relevant to the masses of the gauge bosons $A^b_{\mu a}$ gives their masses in the amount of

$$M_{A^b_a}{}^2(\Phi^b_a) = \frac{g_8{}^2}{2}(\langle\Phi^a_a\rangle - \langle\Phi^b_b\rangle)^2 , \tag{22}$$

due to Higgs-scalar particles Φ^b_a, and gives masses to them due to the vector (eight-dimensional) representations $(H_\alpha)^a$ in the amount of

$$M_{A^a_\alpha}{}^2((H_\alpha)^a) = \frac{g_8{}^2}{2}\left[\frac{v_0'}{\sqrt{2}}\right]^2 . \tag{23}$$

The gauge bosons A^8_a acquire their masses due to H^a, and they are

$$M_{A^8_a}{}^2(H^a) = \frac{g_8{}^2}{2}\left[\frac{v_0}{\sqrt{2}}\right]^2 . \tag{24}$$

In view of Eqs. (22)—(24), the masses of gauge bosons are obtained as

$$\begin{aligned}
M_{A^i_a}{}^2 &= \frac{g_8{}^2}{2}\left[(\tfrac{8}{5}v_1)^2 + (2v_2)^2 + \left[\frac{v_0'}{\sqrt{2}}\right]^2\right] ,\\
M_{B_a}{}^2 &= \frac{g_8{}^2}{2}\left[(\tfrac{8}{5}v_1)^2 + (v_2)^2 + \left[\frac{v_0'}{\sqrt{2}}\right]^2\right] ,\\
M_{C_a}{}^2 &= \frac{g_8{}^2}{2}\left[(\tfrac{8}{5}v_1)^2 + (v_2)^2 + \left[\frac{v_0'}{\sqrt{2}}\right]^2 + \left[\frac{v_0}{\sqrt{2}}\right]^2\right] ,\\
M_X{}^2 &= \frac{g_8{}^2}{2}(v_2)^2 ,\\
M_Y{}^2 &= \frac{g_8{}^2}{2}\left[(v_2)^2 + \left[\frac{v_0}{\sqrt{2}}\right]^2\right] ,\\
M_{g^\beta_\alpha}{}^2 &= \frac{g_8{}^2}{2}\left[\frac{v_0'}{\sqrt{2}}\right]^2 ,\\
M_W{}^2 &= \frac{g_8{}^2}{2}\left[\frac{v_0}{\sqrt{2}}\right]^2 ,\\
M_{Z,Z'}{}^2 &= \tfrac{1}{2}(\,[a(1+b^2) + \tfrac{1}{4}b^2c]\\
&\qquad \pm\{[a(1+b^2) - \tfrac{1}{4}b^2c]^2 + a^2b^2(1+b^2)\}^{1/2}) ,
\end{aligned} \tag{25}$$

where $a \equiv M_W{}^2$, $b \equiv \tan\theta_W$, and $c \equiv \frac{25}{3}M_{g^\beta_\alpha}{}^2 + M_W{}^2$, or

$$\begin{aligned}
M_Z{}^2 &= \frac{M_W{}^2}{\cos^2\theta_W}\left[1 - \frac{1}{c/a - 4/\sin^2\theta_W}\right] ,\\
M_{Z'}{}^2 &= M_Z{}^2\rho\left[1 - \frac{\sin^2\theta_W}{4}\frac{\rho}{1-\rho}\right], \quad \rho \equiv \frac{M_W{}^2}{M_Z{}^2\cos^2\theta_W} ,\\
M_{g^\beta_\alpha}{}^2 &= \frac{12}{25}\frac{M_Z{}^2}{\tan^2\theta_W}\left[1 - \frac{\sin^2\theta_W}{4}\frac{\rho}{1-\rho}\right]
\end{aligned}$$

and

$$\begin{aligned}
M_{g^3}{}^2 &= M_{g^8}{}^2 = 2M_{g^\beta_\alpha}{}^2 ,\\
M_{G^j_i}{}^2 &= 0, \quad M_{A_\mu}{}^2 = 0 ,
\end{aligned}$$

where

$$\begin{aligned}
A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu ,\\
Z_\mu &= -\cos\theta'(-\cos\theta_W W^3_\mu + \sin\theta_W B_\mu) - \sin\theta' C_\mu ,\\
Z'_\mu &= \sin\theta'(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu) + \cos\theta' C_\mu ,\\
\tan\theta' &= \frac{-(\sin\theta_W)/2}{1/\rho' - 1}, \quad \rho' \equiv \frac{M_W{}^2}{M_{Z'}{}^2\cos^2\theta_W} .
\end{aligned} \tag{26}$$

The small mixing (θ') of C_μ with W^3_μ and B_μ can be neglected for the case of $M_{g^\alpha_\beta} \gg M_W$, and

$$\begin{aligned}
g^3 &= \frac{1}{\sqrt{2}}(g^1_1 - g^2_2) ,\\
g^8 &= \frac{1}{\sqrt{6}}(g^1_1 + g^2_2 - 2g^3_3) ,\\
G^3 &= \frac{1}{\sqrt{2}}(G^4_4 - G^5_5) ,\\
G^8 &= \frac{1}{\sqrt{6}}(G^4_4 + G^5_5 - 2G^6_6) ,\\
B &= (\tfrac{2}{15})^{1/2}[\tfrac{3}{2}(W^7_7 + W^8_8) - (G^4_4 + G^5_5 + G^6_6)] ,\\
(\tfrac{3}{8})^{1/2}C &= \frac{1}{\sqrt{5}}[(W^7_7 + W^8_8) + (G^4_4 + G^5_5 + G^6_6)] ,\\
W &= \frac{1}{\sqrt{2}}(W^7_7 - W^8_8) .
\end{aligned} \tag{27}$$

In the special case of no C_μ, Eq. (26) reproduces the result of the SU(5) model. The Weinberg angle in this SU(8) model is also $\sin^2\theta_W = \frac{3}{8}$ just as the SU(5) model if the renormalization effects are neglected. The Z and Z' acquire their masses in the third and fourth stages of symmetry breaking in Eq. (21), and $M_{Z'} \simeq M_{g^\beta_\alpha} \simeq 10^2$ TeV (Ref. 5) is expected. The $U(1)_B$, $U'(1)_C$, and SU(2) coupling constants are related by $g_1' = 2g_1'' = g_2\tan\theta_W$. The predictions of $M_{g^\beta_\alpha}$ and $M_{Z'}$ are sensitive to the values of M_W and $M_{Z'}$, and they are, for example, $M_{g^\beta_\alpha} = 37$ (218) GeV, $M_{Z'} = 28$ (173) GeV for UA1 (UA2) results. Here UA1 results $M_W = 80.9$ GeV, $M_Z = 95$ GeV[8], and UA2 results $M_W = 81$ GeV, $M_Z = 91.2$ GeV are used. However, if we use $\rho = 1.018 \pm 0.045$ from neutrino neutral-current reactions [P. Q. Hung and J. J. Sakurai, Annu. Rev. Nucl.

Part. Sci. 31, 432 (1981)], $M_Z=195$ GeV and $M_{Z'}=245$ GeV with large uncertainties. This is much less than the expected $\sim 10^2$ TeV. Thus, it seems reasonable to say that $M_{Z'}$ and $M_{g_\alpha^\beta}$ are >200 GeV. Thus, more accurate data on M_W and M_Z are needed to predict reliable values for $M_{g_\alpha^\beta}$ and $M_{Z'}$ in this theory. In the limit of $M_{g_\beta^\alpha} \gg M_W$, we recover the results of SU(2)×U(1) theory. In Eq. (25), M_Z is practically the same as the SU(2)×U(1) result. In any case this SU(8) theory predicts, by crude estimates, the 92.3 GeV $\gtrsim M_Z \gtrsim 91.5$ GeV (Ref. 6) when the experimental values $\sin^2\theta_W=0.229+0.009$ (Ref. 3) and $M_W=81$ GeV (Ref. 7) and $M_{g_\alpha^\beta}\gtrsim 245$ GeV are used. Thus, the experimental test of the masses of g_β^α and Z'_μ are important tests[8] of the present SU(8) theory in the future. The mass relations among vector bosons are, by Eq. (25),

$$\begin{aligned} M_{C_\alpha}{}^2 &= M_{B_\alpha}{}^2+M_W{}^2 , \\ M_{A_\alpha^i}{}^2 &= M_{B_\alpha}{}^2+3M_X{}^2 , \\ M_Y{}^2 &= M_X{}^2+M_W{}^2 . \end{aligned} \tag{28}$$

The mass of the colorless generation boson octet, $M_{gk'}$ ($k'=1,\ldots,8$), is expected to be $\sim 10^2$ TeV (Ref. 5) from the limit set by the $\mu\to e$ decay. Thus, the generation group $SU(3)^G$ is a broken symmetry. The masses of the extrasuperheavy bosons $A_{\mu\alpha}^i$, $B_{\mu\alpha}$, and $C_{\mu\alpha}$ are expected to be greater than the superheavy bosons $X_{\mu i}$ and $Y_{\mu i}$ ($\sim 10^{14}$ GeV) in the SU(5) model. The running coupling constants g_2, g_3^c, and g_3^G associated with the groups SU(2), $SU(3)^C$, and $SU(3)^G$ are $g_2=g_3^c=g_3^G=g_8=(\frac{8}{3})^{1/2}e$ at the SU(8)-symmetry level by Eqs. (11)–(13).

V. FERMION MASSES

The Yukawa interaction relevant to the masses of the fermions D^α and E^α is of the form $f_{\overline{28}_L}\times f_{56_L}\times\Phi_{\overline{8}}$,

$$\begin{aligned} \mathcal{L}_{f\Phi_{\overline{8}}} &= \gamma_{\overline{8}}(\overline{\chi}_R^c)_{\alpha\beta}(\psi_L)^{\alpha\gamma 8}\langle\Phi_{\overline{8}}^\dagger\rangle_{\gamma 8}^\beta+\text{H.c.} \\ &= -\left[\frac{1}{\sqrt{3}}v_{\overline{8}}\right]\gamma_{\overline{8}}[(\overline{D}_{Ri\alpha}D_L^{i\alpha}+\overline{E}_{R\alpha}^+E_L^{+\alpha}) \\ &\quad +\overline{N}_R^{c\alpha}\tilde{\nu}_{L\alpha}^c]+\text{H.c.} , \end{aligned} \tag{29}$$

where $\langle\Phi_{\overline{8}}^\dagger\rangle_{\gamma 8}^\beta=(v_{\overline{8}})\delta_\gamma^\beta$, and the rest of the antitriplet Higgs scalars $\langle\Phi_{\overline{8}}\rangle_{b8}^b$ are zero. Equation (29) gives masses to the light-down-quark triplet D^α and the light-lepton triplet E^α,

$$m(D^{i\alpha})=m(E^\alpha)=\frac{1}{\sqrt{3}}v_{\overline{8}}\gamma_{\overline{8}} , \tag{30}$$

and the off-diagonal mass term for N^α and $\tilde{\nu}^\alpha$. The interaction of the form $f_{\overline{28}_L}\times f_{56_L}\times\Phi_{\overline{216}}$ gives masses to the heavy quark $\tilde{d}$ and the heavy lepton $\tilde{e}$ as well as the off-diagonal mass term for ν^α, N_0, and others. We have

$$\begin{aligned} \mathcal{L}_{f\Phi_{\overline{216}}} &= \gamma_{\overline{216}}(\overline{\chi}_R^c)_{ab}(\psi_L)^{acd}\langle\Phi_{\overline{216}}^\dagger\rangle_{cd}^b+\text{H.c.} \\ &= -\left[\frac{1}{\sqrt{3}}v_{\overline{216}}^\alpha\right]\gamma_{\overline{216}}[(\overline{\tilde{d}}_{Ri}\tilde{D}_{L\alpha}^i+\overline{\tilde{e}}_R^+\tilde{E}_{L\alpha}^+) \\ &\quad +\overline{\nu}_{R\alpha}^cN_{0L}]+\text{H.c.} , \end{aligned} \tag{31}$$

where $\langle\Phi_{\overline{216}}^\dagger\rangle_{\beta\gamma}^8=\epsilon_{\beta\gamma\alpha}v_{\overline{216}}^\alpha$. The masses of $\tilde{d}$ and $\tilde{e}$ are

$$m(\tilde{d}_i)=m(\tilde{e})=\frac{1}{\sqrt{3}}v_{\overline{216}}^1\gamma_{\overline{216}} . \tag{32}$$

The interaction of the form of $f_{56_L}\times f_{56_L}\times\Phi_{28}$ gives masses to the heavy quark $\tilde{s}$ and the heavy lepton $\tilde{\mu}$ as well as some off-diagonal mass terms for heavy leptons and generation-mixing off-diagonal mass terms for the generation triplet U^d and others:

$$\begin{aligned} \mathcal{L}_{f\Phi_{28}} &= \gamma_{28}\epsilon_{abcdefgh}(\overline{\psi}_R^c)^{abc}(\psi_L)^{def}\langle\Phi_{28}\rangle^{gh}+\text{H.c.} \\ &= -6v_{28}^\gamma\gamma_{28}[\epsilon_{\alpha\beta\gamma}(\overline{U}_{Ri}^\alpha U_L^{i\beta}-\overline{U}_R^{ci\alpha}U_{Li}^{c\beta})+(-\overline{\tilde{D}}_{R\gamma}^{ci}\tilde{s}_{Li}^c+\overline{\tilde{s}}_{Ri}\tilde{D}_{L\gamma}^i-\overline{\tilde{E}}_{R\gamma}\tilde{\mu}_L+\overline{\tilde{\mu}}_R^+\tilde{E}_{L\gamma}^+)]+\text{H.c.} \end{aligned} \tag{33}$$

Thus, the masses of $\tilde{s}$ and $\tilde{\mu}$ are

$$m(\tilde{s}_i)=m(\tilde{\mu})=6v_{28}{}^2\gamma_{28} , \tag{34}$$

where $\langle\Phi_{28}\rangle^{\gamma 8}=v_{28}^\gamma$. The mass for $\tilde{c}_i$ is given by $f_{56_L}\times f_{56_L}\times\Phi_{420}$,

$$\mathcal{L}_{f\Phi_{420}}=\gamma_{420}\epsilon_{abcdeghh'}(\overline{\psi}_R^c)^{abc}(\psi_L)^{def}\langle\Phi_{420}\rangle_f^{ghh'}=-12v_{420}\gamma_{420}(\overline{\tilde{c}}_{Ri}\tilde{c}_L^i+\overline{\tilde{c}}_R^{ci}\tilde{c}_{Li}^i)+\text{H.c.} \tag{35}$$

with the mass

$$m(\tilde{c}_i)=12v_{420}\gamma_{420} , \tag{36}$$

where $\langle\Phi_{420}\rangle_8^{\alpha\beta\gamma}=\epsilon^{\alpha\beta\gamma}v_{420}$.

The mass for $\tilde{u}$ is acquired by $f_{\overline{28}_L}\times f_{\overline{28}_L}\times\Phi_{70}$,

$$\mathcal{L}_{f\Phi_{70}}=\gamma_{70}(\overline{\chi}_R^c)_{ab}(\chi_L)_{cd}\langle\Phi_{70}\rangle^{abcd}+\text{H.c.}=-v_{70}\gamma_{70}(\overline{\tilde{u}}_R^{ci}\tilde{u}_{Li}^c+\overline{N}_R^{c\alpha}v_{L\alpha})+\text{H.c.} , \tag{37}$$

where $\langle\Phi_{70}\rangle^{ijk7}=\epsilon^{ijk}v_{70}$. The mass for $\tilde{u}$ is

$$m(\tilde{u}_i)=v_{70}\gamma_{70}\ , \tag{38}$$

and there is an off-diagonal mass term. The mass for N^α is given by $f_{\bar{8}_L}\times f_{\overline{28}_L}\times\Phi_{56}$,

$$\begin{aligned}\mathscr{L}_{f\Phi_{56}}&=-\gamma_{56}(\overline{\mathscr{Y}}{}^c_R)_a(\chi_L)_{bc}\langle\Phi_{56}\rangle^{abc}+\text{H.c.}\\&=-\sqrt{2}\gamma_{56}[v_{56}\overline{N}_{R\alpha}N^\alpha_L-v^8_{56\gamma}(\epsilon^{\alpha\beta\gamma}\overline{N}_{R\alpha}N_{L\beta}+\overline{\tilde{\nu}}{}^c_{\tau R}N^\gamma_L)]+\text{H.c.}\ ,\end{aligned} \tag{39}$$

where $\langle\Phi_{56}\rangle^{\alpha\beta\gamma}=\epsilon^{\alpha\beta\gamma}v_{56}$ and $\langle\Phi_{56}\rangle^{\alpha\beta8}=\epsilon^{\alpha\beta\gamma}v^8_{56\gamma}$. Thus, the mass for N^α is

$$m(N^\alpha)=\sqrt{2}v_{56}\gamma_{56}\ , \tag{40}$$

and there are generation-changing off-diagonal mass terms.

As for $\tilde{b}$, $\tilde{\tau}$, and $\tilde{\nu}_\tau$, the interaction by $f_{\bar{8}_L}\times f_{56_L}\times\Phi_{\overline{28}}$ is involved, and

$$\begin{aligned}\mathscr{L}_{f\Phi_{\overline{28}}}&=-\gamma_{\overline{28}}(\overline{\mathscr{Y}}{}^c_R)_a(\psi_L)^{abc}\langle\Phi^\dagger_{\overline{28}}\rangle_{bc}+\text{H.c.}\\&=-(\tfrac{2}{3})^{1/2}\gamma_{\overline{28}}[v^\alpha_{\overline{28}}(\overline{\tilde{b}}_{Ri}\tilde{D}^i{}_{L\alpha}+\overline{\tilde{\tau}}{}^+_R\tilde{E}^+_{L\alpha}+\overline{\tilde{\nu}}{}^c_{\tau_R}\tilde{\nu}^c_{L\alpha}+\overline{N}_{R\alpha}N_{0L})+v_{\overline{28}\alpha}(\overline{\tilde{b}}_{Ri}D^{i\alpha}_L+\overline{\tilde{\tau}}{}^+_RE^{+\alpha}_L+\epsilon^{\alpha\beta\gamma}\overline{N}_{R_\beta}\tilde{\nu}^c_{L\gamma})]+\text{H.c.}\end{aligned} \tag{41}$$

with $\langle\Phi^\dagger_{\overline{28}}\rangle_{\alpha\beta}=\epsilon_{\alpha\beta\gamma}v^\gamma_{\overline{28}}$ and $\langle\Phi^\dagger_{\overline{28}}\rangle_{\beta8}=v_{\overline{28}\beta}$. This gives masses for $\tilde{b}$, $\tilde{\tau}$, and $\tilde{\nu}_\tau$ as

$$m(\tilde{b})=m(\tilde{\tau})=m(\tilde{\nu}_\tau)=(\tfrac{2}{3})^{1/2}v^3_{\overline{28}}\gamma_{\overline{28}} \tag{42}$$

and other off-diagonal mass terms. Similarly $f_{\bar{8}_L}\times f_{\bar{8}_L}\times\Phi_{28}$ gives some off-diagonal mass terms such as

$$\begin{aligned}L'_{f\Phi_{28}}&=-\gamma'_{28}(\mathscr{Y}^c_R)_a(\mathscr{Y}_L)_b\langle\Phi_{28}\rangle^{ab}\\&=-\gamma'_{28}[\,v^\alpha_{28}(\overline{\tilde{\nu}}{}^c_{\tau_R}N^c_{L\alpha}-\overline{N}_{R\alpha}\tilde{\nu}_{\tau L})\\&\qquad+v_{28\alpha}\epsilon^{\alpha\beta\gamma}\overline{N}_{R\beta}N_{L\alpha}]+\text{H.c.}\ ,\end{aligned} \tag{43}$$

where $\langle\Phi_{28}\rangle^{\alpha\beta}=\epsilon^{\alpha\beta\gamma}v_{28\gamma}$ and $\langle\Phi_{28}\rangle^{\alpha8}=v^\alpha_{28}$.

In view of Eqs. (29)—(43), it is possible that all heavy fermions acquire heavier masses (possibly $\sim$ TeV) than light fermions. The SU(5) mass relations for light generations are given in Eq. (30), and there is no mass for U^α for light generations unless perhaps we go to a higher-dimensional representation or higher orders.

VI. DISCUSSION

We discussed a model of grand unification based on the representations of $\bar{8}_L$, $\overline{28}_L$, and 56_L as a concrete example of SU(N) theory. This model contains three generations of light fermions, three generations of heavy fermions (one of which lacks a toplike quark), and neutral fermions. It was shown that this theory reproduces the features of the standard SU(5) model with the extensions to the three generations of light fermions such as the gauge interactions, the masses of gauge vector bosons and fermions, and the Weinberg angle. $\sin^2\theta_W(M_W)$ in this model is the same as in the SU(5) model, since there are no extra light fermions and no additional light Higgs bosons. The values of $\sin^2\theta_W(M_W)$ increases by $\simeq 0.0015$ for an additional light Higgs doublet according to Marciano.[3] Since the coupling constants g_8 and g_5 are the same in the SU(8) limit, the SU(8) unification scale $M_A\simeq M_X$. The mass of M_X (therefore proton lifetime) will be increased due to the additional heavy fermions. This may be a blessing to overcome the apparent difficulties of the SU(5) prediction of proton lifetime. In addition to the embedding of the SU(5) model in this theory, naturally extended to the three light generations, it contains and predicts new interactions such as a generation-changing interaction (QGD) which may be related to the mixing angles of fermions like the Cabibbo angle and more generalized mixing angles among various particles. There are also interactions that connect the new heavy fermions and the light fermions. The branching ratios of the rare processes due to this interaction are dependent on the mixing angles among the fermion generations. The rich structure of the new interaction presented by this theory, the renormalization effects, and the details of asymptotic freedom are worth further continued investigation. Work is underway along these lines. The first step for this SU(8) theory to confront with experiment is to check the mass of the vector boson Z^0 which is predicted to be $92.3\gtrsim M_Z\gtrsim 91.5$ GeV, somewhat less than that predicted by the SU(2)×U(1) model, and also the mass of the vector boson $Z^{0'}$, which is expected to be $\sim 10^2$ TeV but as small as >200 GeV. Similar methods can be applied to general SU(N) theory.

ACKNOWLEDGMENTS

The author would like to thank W. A. Bardeen, S. Nandi, and B. R. Zou for helpful discussions.

APPENDIX

The fermion particle assignments are shown below in matrix form. The representation of 56_L in Eq. (A3) is symbolic in nature for convenience.

$$\overline{8}_L:\ (\mathcal{Y}_L)_a=\left[\begin{array}{c} N_1^c \\ N_2^c \\ N_3^c \\ \tilde{b}_4^c \\ \tilde{b}_5^c \\ \tilde{b}_6^c \\ \tilde{\tau} \\ -\tilde{\nu}_\tau \end{array}\right]_L \begin{array}{l} \\ \mathrm{SU}(3)^G \\ \\ \\ \mathrm{SU}(3)^C \\ \\ \mathrm{SU}(2) \\ \\ \end{array} \tag{A1}$$

$$\overline{28}_L:\ (\chi_L)_{ab}=\frac{1}{\sqrt{2}}\left[\begin{array}{cccccccc} 0 & N^3 & -N^2 & d_4^c & d_5^c & d_6^c & e & -\nu_e \\ -N^3 & 0 & N^1 & s_4^c & s_5^c & s_6^c & \mu & -\nu_\mu \\ N^2 & -N^1 & 0 & b_4^c & b_5^c & b_6^c & \tau & -\nu_\tau \\ -d_4^c & -s_4^c & -b_4^c & 0 & -\tilde{u}^6 & -\tilde{u}^5 & -\tilde{u}_4^c & -\tilde{d}_4^c \\ -d_5^c & -s_5^c & -b_5^c & -\tilde{u}^6 & 0 & \tilde{u}_4 & -\tilde{u}_5^c & -\tilde{d}_5^c \\ -d_6^c & -s_6^c & -b_6^c & \tilde{u}^5 & -\tilde{u}^4 & 0 & -\tilde{u}_6^c & -\tilde{d}_6^c \\ -e & -\mu & -\tau & \tilde{u}_4^c & \tilde{u}_5^c & \tilde{u}_6^c & 0 & -\tilde{e} \\ \nu_e & \nu_\mu & \nu_\tau & \tilde{d}_4^c & \tilde{d}_5^c & \tilde{d}_6^c & \tilde{e} & 0 \end{array}\right]_L , \tag{A2}$$

$$56_L:\ (\psi_L)^{abc}=\frac{1}{\sqrt{6}}\left[\begin{array}{cccccccc} 0 & & & \tilde{d}^4 & \tilde{d}^5 & \tilde{d}^6 & \tilde{e}^+ & -\tilde{\nu}_e^c \\ & 0 & & \tilde{s}^4 & \tilde{s}^5 & \tilde{s}^6 & \tilde{\mu}^+ & -\tilde{\nu}_\mu^c \\ & & 0 & \tilde{b}^4 & \tilde{b}^5 & \tilde{b}^6 & \tilde{\tau}^+ & -\tilde{\nu}_\tau^c \\ -\tilde{d}^4 & -\tilde{s}^4 & -\tilde{b}^4 & 0 & \tilde{c}^6 & -\tilde{c}^5 & -\tilde{c}_4^c & -\tilde{s}_4^c \\ -\tilde{d}^5 & -\tilde{s}^5 & -\tilde{b}^5 & -\tilde{c}^6 & 0 & \tilde{c}^4 & -\tilde{c}_5^c & -\tilde{s}_5^c \\ -\tilde{d}^6 & -\tilde{s}^6 & -\tilde{b}^6 & \tilde{c}^5 & -\tilde{c}^4 & 0 & -\tilde{c}_6^c & -\tilde{s}_6^c \\ -\tilde{e}^+ & -\tilde{\mu}^+ & -\tilde{\tau}^+ & \tilde{c}_4^c & \tilde{c}_5^c & \tilde{c}_6^c & 0 & -\tilde{\mu} \\ \tilde{\nu}_e^c & \tilde{\nu}_\mu^c & \tilde{\nu}_\tau^c & \tilde{s}_4^c & \tilde{s}_5^c & \tilde{s}_6^c & \tilde{\mu} & 0 \\ 32t & 13t & 21t & & & (\psi_L)^{rst} & & \end{array}\right]_L \begin{array}{l} 23t \\ 31t \\ 12t \\ \\ \\ \\ \\ \\ \\ \end{array}$$

$$+\frac{1}{\sqrt{6}}\left[\begin{array}{cccccccc} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & u_6^c & -u_5^c & -u^4 & -d^4 \\ & & & -u_6^c & 0 & u_4^c & -u^5 & -d^5 \\ & & & u_5^c & -u_4^c & 0 & -u^6 & -d^6 \\ & & & u^4 & u^5 & u^6 & 0 & -e^+ \\ & & & d^4 & d^5 & d^6 & e^+ & 0 \end{array}\right]_L^{\alpha=1}$$

$$+\frac{1}{\sqrt{6}}\left[\begin{array}{cccccccc} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & c_6^c & -c_5^c & -c^4 & -s^4 \\ & & & -c_6^c & 0 & c_4^c & -c^5 & -s^5 \\ & & & c_5^c & -c_4^c & 0 & -c^6 & -s^6 \\ & & & c^4 & c^5 & c^6 & 0 & -\mu^+ \\ & & & s^4 & s^5 & s^6 & \mu^+ & 0 \end{array}\right]_L^{\alpha=2}$$

$$+\frac{1}{\sqrt{6}}\begin{bmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & t_6^c & -t_5^c & -t^4 & -b^4 \\ & & & -t_6^c & 0 & t_4^c & -t^5 & -b^5 \\ & & & t_5^c & -t_4^c & 0 & -t^6 & -b^6 \\ & & & t^4 & t^5 & t^6 & 0 & -\tau^+ \\ & & & b^4 & b^5 & b^6 & \tau^+ & 0 \end{bmatrix}_L^{\alpha=3}$$

$$+\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}(N_0)_L \ . \tag{A3}$$

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); H. Georgi and S. L. Glashow, *ibid.* 1, 561 (1973).

[2]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978).

[3]For review, see R. Slansky, Phys. Rep. 79, 1 (1981); P. Langacker, *ibid.* 72, 185 (1981), and references therein.

[4]J. Banks and H. Georgi, Phys. Rev. D 14, 1159 (1976).

[5]A similar model is discussed by J. Chakrabarti, M. Popovic, and R. N. Mohapatra, Phys. Rev. D 21, 3212 (1980). This paper came to the author's attention after the major part of this work was done. For neutral generation-changing gauge bosons, see also R. N. Cahn and H. Harari, Nucl. Phys. B176, 135 (1980).

[6]This is a rough estimate and may need further corrections.

[7]UA1 Collaboration, G. Arnison *et al.*, Phys. Lett. 122B, 103 (1983). The Weinberg-Salam-Glashow model gives $M_W=82 \pm 2.4$ GeV. See, for example, A. Sirlin, Phys. Rev. D 22, 971 (1980); W. J. Marciano and A. Sirlin, *ibid.* 22, 2695 (1980); C. H. Llewellyn Smith and J. A. Wheater, Phys. Lett. 105B, 486 (1981).

[8]It is also reported that the mass of Z^0 is observed to be 95 ± 2 GeV (private communication from W. A. Bardeen).