Critical properties of a one-dimensional nonlinear lattice and hadron physics

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The statistical properties of a one-dimensional system are studied in the Ginzburg-Landau framework, where the most general free-energy density allowing for scale invariance is introduced. The grand partition function is expressed as a functional integral over the order-parameter function space and leads to an analytically soluble model. Near the critical point only the constant functions contribute to the thermodynamic potential, the system is simulated by a classical nonlinear lattice, and Kadanoff scaling is shown to be equivalent to Koba-Nielsen-Olesen scaling. The relevance of this lattice to hadron physics is established and several measurable quantities, such as multiplicities and correlations, are calculated. It is argued that, in the framework of this model, certain aspects of observable quantities could naturally be attributed to the past hadronization transition of a quark-gluon plasma.

I. INTRODUCTION

The study of critical phenomena within the Ginzburg-Landau (GL) framework[1] has provided insight in understanding the physics of complicated systems (ferromagnetism, superconductivity) at both the phenomenological and the microscopic levels. Although the modern microscopic treatment of superconductivity[2] has replaced the phenomenological GL theory, the latter approach is still powerful since it helps to understand, at the phenomenological level, the behavior of complicated systems for which a soluble microscopic theory is missing. Analogous methods have been introduced in particle physics[3-5] to cope with the complexity of many-hadron production problems.

In this work we adopt the point of view that complicated hadronic phenomena, for which several signals of an underlying phase transition exist,[6] may be analyzed at the phenomenological level in terms of an order parameter φ, related to the density of the produced hadrons and reflecting the as-yet-unsoluble fundamental quark-gluon interaction in quantum chromodynamics (QCD) for small momentum transfers, in analogy to the order parameter of the GL theory which is related to the electron-pair (Cooper-pair) density and reflects the properties of the microscopic electron-phonon interaction introduced in the Bardeen-Cooper-Schrieffer theory of superconductivity. We shall study a class of one-dimensional soluble models with the GL methodology, introducing a critical temperature T_c and imposing explicit scale invariance in the free energy of the system. For this purpose, in the rest of this section we briefly review some elements of the GL theory of superconductivity.

The basic quantity in this treatment is the free-energy density of the ordered phase, $f(\varphi(x),T)$, which is a function of the (small) order parameter and the temperature of the system. The total free energy of the one-dimensional system with length L is given by the sum

$$F(L,T)=F_n+\int_0^L f(\varphi(x),T_c-T)dx \ , \tag{1.1}$$

where F_n is the free energy of the normal state of the system, corresponding to $\varphi=0$. The GL model follows by approximating the free-energy density near the critical point $T\approx T_c$ by a few power terms of φ and $d\varphi/dx$, namely,

$$f(\varphi,T_c-T)\cong\frac{\hbar^2}{2m}\left[\frac{d\varphi}{dx}\right]^2+a(T-T_c)\varphi^2+\tfrac{1}{2}b\varphi^4 \ , \tag{1.2}$$

where a and b are constants.

In this model the phase transition is due to the shift of the stability point for $T<T_c$. In fact, the coordinate-independent order parameter, which minimizes the free energy for $T<T_c$, is

$$\varphi_{\min}{}^2=\frac{a}{b}(T_c-T) \ . \tag{1.3}$$

The superconducting electron density (1.3) as a function of the temperature decreases linearly to zero towards the transition point.

The whole GL program is based on the assumption that the regular expansion (1.2) exists. In this work we shall, instead, study a class of models in which scale invariance near $T=T_c$ is *imposed* and hence a singularity of the form φ^{2k} in the free-energy density is tolerated. Within this class of models several properties of hadron production at high energies and, in particular, Koba-Nielsen-Olesen (KNO) scaling[7] can be understood if the variable x is identified with the center-of-mass rapidity y^* of the produced particle.

The plan of this paper is as follows: In Sec. II we consider the most general free-energy density allowing for scale invariance in one dimension. We set up our mathematical formalism and show how scale invariance leads to the critical exponents. In Sec. III we show that Kadanoff scaling at the critical sector of our model is equivalent to KNO scaling. We calculate particle multiplicities, etc., and establish their connection with previous results obtained in the framework of the Feynman-Wilson fluid. In Sec. IV we turn to the space-time interpretation of the scaling properties of our model. We argue that within its framework one can possibly interpret certain properties of measurable quantities as signatures of the hadronization of a partonic plasma. Finally, our conclusions are given in Sec. V.

II. THE φ^{2k} MODEL

The most general form of the free-energy density which allows for scale invariance is

$$f(\varphi,T_c-T)=c^2(T_c-T)^{\lambda}\left[\frac{d\varphi}{dx}\right]^2+b^2\varphi^{2k}\,, \tag{2.1}$$

where b and c are constants. Introducing the ordering field (chemical potential) μ_χ, the thermodynamic potential density is found to be

$$\tilde{\Omega}(\varphi,z,T)=c^2(T_c-T)^{\lambda}\left[\frac{d\varphi}{dx}\right]^2+b^2\varphi^{2k}-(z-1)\varphi^2\,, \tag{2.2}$$

where z is the fugacity. Near the critical point $\mu_{\chi c}=0$, we have $\mu_\chi\equiv\ln z\cong z-1$. The grand partition function of the theory is given by an integral over the function space, namely,[3,8,9]

$$Q(z,L,T)=\frac{1}{Q_0(L,T)}\int[d\varphi]\exp\left[-\int_0^L\tilde{\Omega}(\varphi,z,T)dx\right]\,, \tag{2.3}$$

where the normalization factor is given by

$$Q_0(L,T)=\int[d\varphi]\exp\left[-\int_0^L\tilde{\Omega}(\varphi,1,T)dx\right]\,. \tag{2.4}$$

The density of particles is given by the average[3,8,9]

$$\langle\varphi(x)^2\rangle=\frac{1}{Q_0(L,T)}\int[d\varphi]\varphi^2\exp\left[-\int_0^L\tilde{\Omega}\,dx\right]\,. \tag{2.5}$$

Because of scale invariance, the critical exponents in our model can be expressed in terms of the parameters λ and k of the density (2.1). We remind the reader that the critical exponents $\alpha,\beta,\gamma,\delta,\epsilon,\mu,\nu,\eta$ are defined in terms of the thermodynamic quantities near the critical point ($T=T_c$ or $\mu_\chi=0$) through the limits[10,11]

$$C_p\underset{T=T_c}{\sim}(z-1)^{-\epsilon},\quad C_p\underset{z=1}{\sim}(T_c-T)^{-\alpha}\,, \tag{2.6a}$$

$$\rho\underset{T=T_c}{\sim}(z-1)^{1/\delta},\quad \rho\underset{z=1}{\sim}(T_c-T)^{\beta}\,, \tag{2.6b}$$

$$x_c\underset{T=T_c}{\sim}(z-1)^{-\mu},\quad x_c\underset{z=1}{\sim}(T_c-T)^{-\nu}\,, \tag{2.6c}$$

$$\chi\underset{z=1}{\sim}(T_c-T)^{\gamma}\,, \tag{2.6d}$$

$$G(x_2-x_1)\underset{T=T_c}{\sim}|x_2-x_1|^{-(d-2+\eta)}\,, \tag{2.6e}$$

where

$$C_p\sim\frac{\partial^2\tilde{\Omega}}{\partial T^2} \tag{2.7}$$

is the specific heat,

$$\rho\sim\frac{1}{L}\int_0^L\langle\varphi(x)^2\rangle dx \tag{2.8}$$

is the density,

$$\chi=\frac{\partial\rho}{\partial z} \tag{2.9}$$

is the susceptibility, and

$$G(x_2-x_1)\sim\langle\varphi(x_2)^2\varphi(x_1)^2\rangle\sim\exp\left[-\frac{|x_2-x_1|}{x_c}\right] \tag{2.10}$$

is the correlation function with x_c the correlation length.

Introducing the scale transformations[1,10]

$$x\to xu^{-1}\,, \tag{2.11a}$$

$$T_c-T\to(T_c-T)u^{\Delta_t}\,, \tag{2.11b}$$

$$z-1\to(z-1)u^{\Delta_z}\,, \tag{2.11c}$$

$$\varphi^2\to\varphi^2u^{\Delta_\varphi}\,, \tag{2.11d}$$

where Δ_t, Δ_z, and Δ_φ are the dimensions of the corresponding quantities, scale invariance, namely,

$$\tilde{\Omega}(\varphi^2u^{\Delta_\varphi},(z-1)u^{\Delta_z},(T_c-T)u^{\Delta_t})=u\tilde{\Omega}(\varphi^2,z-1,T_c-T)\,, \tag{2.12}$$

leads to

$$\Delta_\varphi=\frac{1}{k},\quad \Delta_z=\frac{k-1}{k},\quad \Delta_t=-\frac{k+1}{k\lambda}\,. \tag{2.13}$$

Since the correlation length x_c has dimension -1 [see scale transformation (2.11a)], the definition (2.6c) gives

$$\mu=\frac{1}{\Delta_z}=\frac{k}{k-1},\quad \nu=\frac{1}{\Delta_t}=-\frac{k\lambda}{k+1}\,. \tag{2.14}$$

Equations (2.14) and (2.6c) show that $\lambda<0$, since for $T\to T_c$ we must have $x_c\to\infty$. This remark illustrates the ordering mechanism in the theory, since for $T=T_c$ and $\lambda<0$ only the class of constant functions contributes to the integral (2.3).

The rest of the critical exponents follow from the unique scaling form,[10]

$$\Omega(z,T)=(z-1)^{1/\Delta_z}f_0((T_0-T)/(z-1)^{\Delta_t/\Delta_z}) \qquad (2.15)$$

of the thermodynamic function

$$\Omega(z,T)=-\lim_{L\to\infty}\frac{1}{L}\ln Q_0(L,T)Q(z,L,T) . \qquad (2.16)$$

The scaling form (2.15) follows from the requirement of scale invariance, Eq. (2.12), and is useful for $T=T_c$ and $z\to 1$, whereas for $z=1$ and $T\to T_c$ we can write

$$\Omega(z,T)=(T_c-T)^{1/\Delta_t}\tilde{f}_0((z-1)/(T_c-T)^{\Delta_z/\Delta_t}) . \qquad (2.15')$$

With the solution (2.13) we obtain

$$\Omega(z,T)=(z-1)^{k/(k-1)} \times f_0((T_c-T)/(z-1)^{(k+1)/\lambda(1-k)}) \qquad (2.17a)$$

or

$$\Omega(z,T)=(T_c-T)^{-k\lambda/(k+1)} \times \tilde{f}_0((z-1)/(T_c-T)^{\lambda(1-k)/(k+1)}) . \qquad (2.17b)$$

Similarly, we obtain the scaling law which satisfies the density (2.8) of the system ($\Delta_\rho=1/k$):

$$\rho(z,T)=(z-1)^{1/(k-1)} \times F_0((T_c-T)/(z-1)^{(k+1)/\lambda(1-k)}) , \qquad (2.18a)$$

$$\rho(z,T)=(T_c-T)^{-\lambda/(k+1)} \times \tilde{F}_0((z-1)/(T_c-T)^{\lambda(1-k)/(k+1)}) . \qquad (2.18b)$$

The relations (2.17) and (2.18) are typical Kadanoff scaling laws and lead immediately to the critical exponents

$$\alpha=[k(\lambda+2)+2]/(k+1) , \qquad (2.19a)$$

$$\beta=-\lambda/(k+1) , \qquad (2.19b)$$

$$\gamma=\lambda(2-k)/(k+1) , \qquad (2.19c)$$

$$\delta=k-1 , \qquad (2.19d)$$

$$\epsilon=[k(\lambda+2)+2]/\lambda(1-k) . \qquad (2.19e)$$

Finally, for $T=T_c$ the correlation function (2.10) has the asymptotic behavior

$$G(x_2-x_1)\sim(x_2-x_1)^{-2/k} , \qquad (2.20)$$

as follows from dimensional arguments. Hence, the exponent η [Eq. (2.6e)] in one dimension is found to be

$$\eta=\frac{k+2}{k} . \qquad (2.21)$$

For completeness, we note that the solution (2.14), (2.19), and (2.21) for the critical exponents satisfies the well-known relations established in the theory of phase transitions,[10] namely,

$$\begin{aligned}&\alpha+2\beta+\gamma=2 ,\\ &\beta\delta=\beta+\gamma=\alpha/\epsilon=\nu/\mu ,\\ &\nu(2-\eta)=\gamma .\end{aligned} \qquad (2.22)$$

As expected, our solution satisfies the general relations

$$\begin{aligned}&\nu d=2\beta+\gamma=2-\alpha ,\\ &\mu d=1+1/\delta=2\mu/\nu-\epsilon=2-2\mu+\eta\mu\end{aligned} \qquad (2.23)$$

due to scale invariance in d dimensions.

In this section we have studied the statistical mechanics of the model defined by (2.1) in an abstract way, without reference to the physical system it describes. In Sec. III we mark out the possible relevance of the model to particle physics.

III. THE CRITICAL SECTOR OF THE MODEL

We now proceed to study the critical sector $T=T_c$, where the functions $\varphi(x)=$constant characterize the system. In this case the functional integrals (2.3) and below become ordinary integrals and the order parameter φ is a global quantity, namely, $\varphi^2=N/L$. The grand partition function (2.3) has the representation

$$Q(z,L,T_c)=\frac{1}{Q_0(L,T_c)}\int_0^\infty N^{1/2}dN\,z^N\exp(-b^2N^kL^{1-k}) , \qquad (3.1)$$

where the normalization factor is given by

$$Q_0(L,T_c)=\int_0^\infty N^{1/2}dN\exp(-b^2N^kL^{1-k}) . \qquad (3.2)$$

The approach to the critical point ($L\to\infty$, $z\to 1$) is again determined by the scaling properties of this representation. In fact, taking into account the dimensions of the quantities N and $\ln z$, we obtain the scaling form

$$Q(z,L,T_c)=h(L^{(k-1)/k}\ln z) \qquad (3.3a)$$

or

$$Q(z,L,T_c)=\tilde{h}(L(\ln z)^{k/(k-1)}) . \qquad (3.3b)$$

From Eq. (3.3a) we find that the average multiplicity

$$\langle N\rangle=\frac{\partial}{\partial\ln z}\ln Q(z,L,T_c) \qquad (3.4)$$

of particles in the system obeys the scaling law

$$\langle N\rangle=L^{(k-1)/k}h_l^{(1)}(L^{(k-1)/k}\ln z) , \qquad (3.5)$$

where we have defined

$$h_l^{(q)}(\tau)=\frac{1}{h(\tau)}\frac{d^q}{d\tau^q}h(\tau) . \qquad (3.6)$$

More generally, the moments of the particle multiplicity are given by

$$\langle N^q\rangle=L^{q(k-1)/k}h_l^{(q)}(L^{(k-1)/k}\ln z) . \qquad (3.7)$$

At the critical point $z=1$, we have

$$\langle N^q\rangle\underset{L\to\infty}{\sim}L^{q(k-1)/k}h_l^{(q)}(0) , \qquad (3.8)$$

whence the quantity

$$\langle N^q\rangle/\langle N\rangle^q\underset{L\to\infty}{\sim}h_l^{(q)}(0)/[h_l^{(1)}(0)]^q \qquad (3.9)$$

is found to be independent of L. Equation (3.9) is the condition for asymptotic KNO scaling,[7] a particular scaling law which is theoretically expected and experimentally confirmed[12] to hold to a good approximation for hadron production in hadronic as well as in e^+e^- collisions over a wide range of energies.

This remark provides the link of the critical sector $T=T_c$ of the model studied in Sec. II to hadron physics. In fact, hadron production with KNO scaling in one-dimensional (rapidity) space has been studied[13] in the framework of the Feynman-Wilson (FW) fluid.[4] One finds that the solution is given in terms of a critical exponent k, which can be equivalently defined by Eq. (3.8). The present approach through the GL expansion (2.1) is thus equivalent to the approach through the FW fluid. In the rest of this section we summarize, for completeness, some results of this study.[13]

The hadronic system corresponds to an analog statistical system through the identifications

$$L \to \ln s \quad \text{(rapidity space)}\ , \tag{3.10}$$

$$Z(N,T_c,L) \to \sigma_N(\ln s) \quad (N\text{-particle-production cross section})\ . \tag{3.11}$$

The form of the KNO scaling function is found to be

$$\langle N\rangle \sigma_N(\ln s)/\sigma_t(\ln s) \equiv \psi(W) \sim \exp(-b^2 W^k) \tag{3.12}$$

for $W \to \infty$, where

$$W = N/\langle N\rangle\ . \tag{3.13}$$

The asymptotic behavior of the total cross section and the average multiplicity are given by

$$\sigma_t \underset{s\to\infty}{\sim} (\ln s)^{-1/k},\ \ \langle N\rangle \underset{s\to\infty}{\sim} (\ln s)^{1-1/k}\ . \tag{3.14}$$

Note that in the GL approach the canonical partition function of the system is found from Eq. (3.1) to be

$$Z(N,T_c,L) \underset{N\to\infty}{\sim} \frac{1}{L}\exp(-b^2 L^{1-k} N^k)\ . \tag{3.15}$$

At the classical level it is shown[13] that we are uniquely led to (3.15) by the nearest-neighbor potential

$$V(x_2-x_1) = \frac{a}{|x_2-x_1|^{k-1}} + \frac{k+1}{2}\ln|x_2-x_1|\ . \tag{3.16}$$

Thus, the system described by (2.1) in the limit $T\to T_c$ reduces to a one-dimensional nonlinear chain with an effective interaction of the form (3.16). One can argue[14] that in hadron physics the effective potential (3.16) is a manifestation of hard-scattering subprocesses in the framework of QCD.

The statistical mechanics of our nonlinear chain leads to the one-particle density

$$\langle \varphi(x)^2\rangle \sim x^{-1/k}(L-x)^{-1/k}L^{1/k} \tag{3.17}$$

for $x \sim L-x \sim L$. The exact form of (3.17) for all x is given in Ref. 13. Ignoring the end effects we have

$$\langle \varphi(x)^2\rangle \sim L^{-1/k} \sim \rho = \text{const in } x\ , \tag{3.18}$$

as expected from the general discussion of Sec. II.

Similarly, the two-particle correlation function for $x_2-x_1 \sim L$, $x_1 \sim L$, $L-x_2 \sim L$ is given by

$$G(x_1,x_2) \sim L^{1/k} x_1^{-1/k}(x_2-x_1)^{-1/k}(L-x_2)^{-1/k}\ , \tag{3.19}$$

which exhibits the general behavior (2.20) expected from dimensional analysis at $T=T_c$.

IV. φ^{2k} MODEL AND HADRONIZATION TRANSITION OF PARTONIC PLASMA

The properties of the model discussed in the previous sections motivate one to interpret the order parameter near the critical temperature, namely,

$$\langle N\rangle/L \sim (T_c-T)^\beta,\ \ \beta=-\lambda/(k+1)>0\ , \tag{4.1}$$

as the density of hadrons at an early stage of their formation within an expanding quark-gluon plasma generated by a highly relativistic hadronic reaction at the initial time $t=0$ ($T=T_0$) and undergoing a phase transition at a later time t_c, when the temperature reaches the critical value T_c. The significance of establishing a critical mechanism for the hadronization process has been recognized, since the early development of QCD, in connection to the as-yet-unsolved problem of the long-distance behavior of the theory.[15] Recently, there has been growing interest[6] in a systematic phenomenological study of the properties of the quark-gluon plasma formed in highly relativistic heavy-ion collisions in the central region. Within this spirit, in this section we qualitatively argue that our model, based on scale invariance of the free energy in the rapidity space, Eq. (2.1), is suitable for interpreting certain aspects of the behavior of measurable quantities in hadronic reactions (average multiplicity, average transverse momentum, KNO function) as the phenomenological signal for hadronization of the quark-gluon plasma.

In a highly relativistic heavy-ion collision, the center-of-mass rapidity y^* of the produced particles is the most suitable variable for describing the space-time evolution of hadronic matter. In fact the hadronic fluid during its evolution is distributed on space-time hyperbolas characterized by temperature T, which varies from the initial value $T=T_0$ corresponding to a hot quark-gluon plasma to the critical value $T=T_c$ corresponding to the appearance of the hadronic phase.[16] The distribution of hadrons on the hyperbolas near the critical point ($T \lesssim T_c$) corresponds to a definite hierarchy in space-time through $t=\tau\cosh y^*$, $x=\tau\sinh y^*$, where τ is the proper time common for all hadrons on a definite hyperbola during the evolution process. Scale invariance of the free energy, when the system reaches the hyperbola $T \simeq T_c$, is the basic hypothesis in our model leading uniquely to the form (2.1) with all its consequences discussed in the previous sections. In the space-time picture this hypothesis is supported by the fact that the scale transformation $y^* \to y^*/u$ does not affect the hierarchy of large-scale correlations, which control the critical behavior of the one-dimensional system on the hyperbola $T=T_c$ (critical FW fluid). Note that this is a well-known argument for Kadanoff scaling valid for any system near the critical point. It is therefore plausible to conjecture that although the free energy of our model may not be suitable to describe the evolution of the system far from the critical temperature, it describes correctly the distribution of hadrons at an early stage of their formation for $T \lesssim T_c$.

It is now interesting to examine qualitatively whether our present knowledge on the hadronic processes encourages a further systematic study of the phenomenological implications of our model. For this purpose we consider the dependence of the order parameter on the temperature, near the critical point, Eq. (4.1), as a typical prediction of our model. As a working hypothesis, we identify the temperature T with the average transverse momentum $\langle p_T \rangle$ involved in the hadronic process. Furthermore, we assume that near the critical point the dependence of $\langle p_T \rangle$ on the "energy" $L = \ln s/s_0$ can be evaluated by using perturbative QCD since the coexistence of hadrons with the quark-gluon plasma requires large momentum transfer. In our qualitative discussion we may simplify our approach using the simple parton model, which predicts a limiting behavior of the large-p_T spectrum for $s \to \infty$ of the form[17]

$$f(p_T,s) \underset{\substack{s\to\infty \\ p_t \text{ fixed}}}{\sim} \frac{1}{p_T{}^4} \ln\left[\frac{\sqrt{s}}{2p_T}\right], \quad p_T \geq m_0 \;, \tag{4.2}$$

where the threshold m_0 may be taken comparable to the parameter Λ of QCD. With the form (4.2) we have for $s \to \infty$

$$\langle p_T \rangle = 3m_0/2 - (m_0/4)L^{-1}, \quad L = \ln(\sqrt{s}/2m_0) \;, \tag{4.3}$$

and thus obtain the critical temperature $T_c = 3m_0/2$, which is identified with the highest $\langle p_T \rangle$ involved within the hadronic phase. The fact that the parton model predicts a limiting value of $\langle p_T \rangle$ for $L \to \infty$ supports the conjecture for the existence of a finite critical point beyond which the hadronic matter exists only in the state of the quark-gluon plasma.

It is worth noting that recent CERN-collider data[18] are consistent with a slow increase of the average transverse momentum according to Eq. (4.3) towards a critical value $T_c \sim 500$ MeV.

V. CONCLUSIONS

Since it has not yet been shown how QCD leads to quark and gluon confinement at large distances, in this paper we have adopted the Ginzburg-Landau philosophy of an effective free energy in order to describe in a phenomenological way the statistical mechanics of a system of many interacting partons. Such an idea had been brought, though in a different context, into hadron physics several years ago by Scalapino and Sugar.[3] We formalize the problem in the language of the grand canonical ensemble, where the grand partition function is given as a functional integral over the whole order-parameter function space. We have modified the GL free-energy density, which is a function of the order parameter, explicitly imposing scale invariance. This leads to an analytically soluble model. Our model free energy involves two exponents as basic parameters in terms of which the critical exponents follow as a result of Kadanoff scaling.

We show that near the critical point Kadanoff scaling is equivalent to Koba-Nielsen-Olesen scaling, a scaling law known to be obeyed to a very good approximation by hadroproduction data for a variety of processes ($pp, \bar{p}p, e^+e^-$) within an impressively wide energy range ($\sqrt{s} \sim 1-540$ GeV). This property establishes the relevance of the critical sector of our model to hadron physics. We show that the GL approach is equivalent to a description of hadron production in the framework of the Feynman-Wilson fluid, if in the latter treatment KNO scaling is imposed as an extra constraint. We give the predictions of our model for several measurable quantities at the hadronic phase, such as total cross section, average multiplicity, and density of produced hadrons, correlations, scaling functions, etc.

Since the center-of-mass rapidity of a produced particle plays the role of a classical position coordinate in the statistical system, our approach, and in particular scale transformations, have definite reflections in space-time. More precisely, a (rapidity) scale transformation near the critical point preserves the hierarchy of the produced plasma (in a heavy-ion collision) on constant proper-time hyperbolas in space-time, only affecting its distribution on a definite hyperbola. If the abstract temperature variable is related to the average transverse momentum $\langle p_T \rangle$ of the emerging hadrons, simple parton considerations, leading to a limiting $\langle p_T \rangle$ value for $s \to \infty$, suggest the existence of a critical temperature. Such a limiting $\langle p_T \rangle$ behavior is consistent with present experimental data. In the future we plan to study more quantitative consequences of these ideas.

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