

## Soliton bag model with chiral dynamics: Colored Higgs mechanism

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We describe a compact scheme which allows gluons to acquire large masses outside a bag, and only outside the bag, via the Higgs mechanism of a specific kind ("colored"). The proposed mechanism does not give rise to "unwanted" massless particles and yet maintains color gauge invariance. Accordingly, we demonstrate the feasibility of constructing a renormalizable soliton bag model with (or without)  $SU(2) \times SU(2)$  chiral dynamics explicitly built in by hand.

### I. INTRODUCTION

One of the most important questions in intermediate-energy physics is to properly take into account manifestations of hadron substructure; any such attempt is in principle to be guided by the standard wisdom, as established during the past two decades by high-energy physicists, that hadrons consist of quarks and gluons interacting among themselves via quantum chromodynamics (QCD). As suggested by the asymptotic-free nature of QCD, interactions among quarks at small distances are very weak so that the three-quark picture for a baryon and the quark-antiquark picture for a meson may be regarded as a plausible zeroth-order approximation. What remains mysterious in this zeroth-order approximation can be phrased as two questions: (1) Why are quarks and gluons not seen in reactions at ordinary energies, e.g., a few GeV? (2) Why does the description of pions in this picture appear to be not very successful? The first question is generally known as the confinement problem while the second is related to the phenomenon of chiral-symmetry breaking. Granting that QCD is the underlying theory of strong interactions, both phenomena, i.e., confinement and chiral-symmetry breaking, are believed to be nonperturbative and thus can be attributed to the large-distance behavior of QCD. In popular bag models,<sup>1-5</sup> either confinement alone or both confinement and chiral-symmetry breaking are built in by hand. However, none of these models appears to be renormalizable so that higher-order effects can hardly be addressed in a systematic manner. To incorporate hadron substructure into intermediate-energy physics, we need to search for a model in which higher-order effects can be characterized systematically.

In a preceding note,<sup>6</sup> I described in some detail an extended version of the soliton bag model which incorporates explicitly  $SU(2) \times SU(2)$  chiral dynamics. It was hoped that a renormalizable model can be constructed explicitly so that higher-order effects may be treated systematically. However, the gluon outside a bag acquires a large mass via its coupling to the soliton field  $\chi$  of Friedberg and Lee.<sup>5</sup> Such coupling spoils the standard proof of renormalizability<sup>7</sup> since Ward-Takahashi identities due to color gauge invariance are no longer valid. (This fact should not be taken to indicate nonrenormalizability of the model but, rather, the renormalization property cannot

be determined via the standard procedure.<sup>7</sup>) Fortunately, this gluon-mass problem is the only feature which forbids one to follow the standard proof of renormalizability. It is utterly obvious that a resolution to this problem represents a major breakthrough in bag models, since renormalizability allows one to treat higher-order effects in a well-defined manner.

The situation is similar to what occurred during the course of constructing the standard  $SU(2) \times U(1)$  gauge theory of electroweak interactions. The weak-boson-mass problem, as faced previously by Glashow,<sup>8</sup> was eventually resolved by Weinberg<sup>9</sup> and Salam<sup>10</sup> by resorting to the Higgs mechanism.<sup>11</sup> However, we need to resolve simultaneously two difficulties essential for the present problem, viz., (1) gluons acquire heavy masses only outside the bag (i.e.,  $\chi = \chi_\infty$ ), and (2) any massless particle, if it exists outside the bag, is "unwanted." The purpose of the present note is to describe a resolution to the gluon-mass problem via the Higgs mechanism of a specific kind ("colored"). The major obstacle in constructing a renormalizable bag model is thereby removed.

### II. THE MODEL

We begin with a brief summary of the model proposed earlier.<sup>6</sup> Quarks  $\psi^a(x)$  and gluons  $G_\mu^a(x)$  are described by the QCD Lagrangian:

$$L_{\text{QCD}} = -\frac{1}{4} G_{\mu\nu} G_{\mu\nu} - \psi^\dagger \gamma_4 \gamma_\mu (\partial_\mu - ig \frac{1}{2} \lambda_a G_\mu^a) \psi - m \psi^\dagger \gamma_4 \psi, \quad (1)$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f_{abc} G_\mu^b G_\nu^c.$$

Here  $\lambda_a$  and  $f_{abc}$  are, respectively, the well-known generating matrices and structure coefficients associated with the (color)  $SU(3)$  group.<sup>12</sup> (We suppress color indices wherever possible and Faddeev-Popov ghosts<sup>13</sup> due to gauge fixing are also not treated explicitly.) We describe only the case that  $\psi^a(x)$  is an isodoublet consisting of the  $u$  and  $d$  quarks. Accordingly,  $\frac{1}{2}(1 + \gamma_5)\psi$  and  $\frac{1}{2}(1 - \gamma_5)\psi$  transform like the left-handed and right-handed doublet irreducible representations  $[\frac{1}{2}, 0]$  and  $[0, \frac{1}{2}]$ , respectively, under the chiral  $SU(2) \times SU(2)$  group.

Four scalar fields  $[\sigma, \vec{\pi}]$  which transforms like a  $[\frac{1}{2}, \frac{1}{2}]$  irreducible representation under  $SU(2) \times SU(2)$  respect the

Lagrangian of the well-known  $\sigma$  model:<sup>14</sup>

$$L_{\sigma\pi} = -\frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2] - \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) - \frac{1}{4}\lambda^2(\sigma^2 + \vec{\pi}^2)^2 + \epsilon\sigma. \quad (2)$$

The "soliton" field of Friedberg and Lee will be designated by  $\chi$ . The  $\chi$  field is a singlet under color SU(3) and chiral SU(2)  $\times$  SU(2) and is described by

$$L_\chi = -\frac{1}{2}(\partial_\mu\chi)^2 - U(\chi), \quad (3)$$

$$U(\chi) = p + \frac{1}{2}a\chi^2 + \frac{1}{6}b\chi^3 + \frac{1}{24}c\chi^4, \quad p > 0.$$

Here  $U(\chi)$  has two local minima which are situated at  $\chi=0$  and  $\chi=\chi_\infty$ . Further, we assume  $U(0) > U(\chi_\infty) = 0$  so that  $p$  can be identified with the bag constant.<sup>5</sup>

The couplings among these fields are determined by their transformation properties under color SU(3) and chiral SU(2)  $\times$  SU(2). We write

$$L_{\text{int}}^{(1)} = -\psi^\dagger \gamma_4 f \chi \psi, \quad (4a)$$

$$L_{\text{int}}^{(2)} = -\psi^\dagger \gamma_4 g_\pi (\sigma + i\vec{\pi} \cdot \vec{\tau} \gamma_5) \psi - \frac{1}{2}h_2^2 [1 - (\chi/\chi_\infty)^2] (\sigma^2 + \vec{\pi}^2), \quad (4b)$$

$$L_{\text{int}}^{(3)} = -\frac{1}{2}h_1^2 \chi^2 G_\mu^a G_\mu^a. \quad (4c)$$

Here the coupling constants  $f$ ,  $h_1$ , and  $h_2$  are "large" in the scale of a typical hadron mass such as  $m_p$ , i.e.,  $f\chi_\infty \gg m_p$ ,  $h_1\chi_\infty \gg m_p$ , and  $h_2 \gg m_p$ . The model proposed previously<sup>6</sup> is defined by

$$L = L_{\text{QCD}} + L_\chi + L_{\sigma\pi} + L_{\text{int}}^{(1)} + L_{\text{int}}^{(2)} + L_{\text{int}}^{(3)}. \quad (5)$$

We may use the proton to illustrate the main feature of this model. To the lowest order in the strong coupling constant  $\alpha_s = (4\pi)^{-1}g^2$ , the proton consists mainly of two  $u$  quarks and one  $d$  quark in a color-singlet state. To attain the lowest-energy solution, we need to choose<sup>5,6</sup>

$$\chi(\vec{r}) = \chi_\infty \quad \text{for all } \vec{r} \text{ except inside a volume } V, \quad (6)$$

$$\simeq 0 \quad \text{for } \vec{r} \text{ inside } V.$$

The quarks are confined inside  $V$  since, otherwise, the coupling specified by  $L_{\text{int}}^{(1)}$  [Eq. (4a)] yields a quark-mass energy proportional to  $f\chi_\infty$ . These quarks act as a source to generate the  $(\sigma, \vec{\pi})$  cloud which, in view of the "mass" in  $L_{\text{int}}^{(2)}$  exist primarily outside the bag volume  $V$ . Beyond the lowest order in  $\alpha_s$ , the quarks also act as a source for the gluon fields which, in view of  $L_{\text{int}}^{(3)}$ , exists only inside the bag. Nevertheless,  $L_{\text{int}}^{(3)}$  violates color gauge invariance such that it affects, albeit indirectly, the "definability" of certain higher-order effects. The purpose of the present note is to simulate  $L_{\text{int}}^{(3)}$  via a colored Higgs mechanism without introduction of "unwanted" massless particles so that color gauge invariance is preserved and the salient features of the model [Eq. (5)] remain intact.

I propose to choose two triplets of complex scalar fields,

$$\Phi_+ = \begin{pmatrix} \Phi_+^1 \\ \Phi_+^2 \\ \Phi_+^3 \end{pmatrix}, \quad \Phi_- = \begin{pmatrix} \Phi_-^1 \\ \Phi_-^2 \\ \Phi_-^3 \end{pmatrix}, \quad (7)$$

which transform under color SU(3) as follows:

$$\Phi'_+ = \exp \left[ -\frac{i}{2} \lambda_a \zeta^a(x) \right] \Phi_+, \quad (8)$$

$$\Phi'_- = \exp \left[ -\frac{i}{2} \lambda_a \zeta^a(x) \right] \Phi_-.$$

It is useful to note that, for arbitrary  $\eta_a$ ,

$$\eta_a \lambda_a \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} = \begin{pmatrix} v(\eta_1 - i\eta_2) + u \left[ \eta_3 + \frac{1}{\sqrt{3}}\eta_8 \right] \\ u(\eta_1 + i\eta_2) - v \left[ \eta_3 + \frac{1}{\sqrt{3}}\eta_8 \right] \\ u(\eta_4 + i\eta_5) + v(\eta_6 + i\eta_7) \end{pmatrix}. \quad (9)$$

Accordingly, we assume that it is possible to choose  $\{\zeta_0^a(x)\}$  such that

$$\Phi_+ = \exp \left[ \frac{i}{2} \lambda_a \zeta_0^a(x) \right] \begin{pmatrix} u_+ + \rho_+(x) \\ v_+ + \eta_+(x) \\ 0 \end{pmatrix}, \quad (10a)$$

$$\Phi_- = \exp \left[ \frac{i}{2} \lambda_a \zeta_0^a(x) \right] \begin{pmatrix} u_- + \rho_-(x) \\ v_- + \eta_-(x) \\ 0 \end{pmatrix}. \quad (10b)$$

Here  $u_\pm$  and  $v_\pm$  are four numbers which will be shown to be real. In the proposed Higgs mechanism, the eight real fields  $\zeta_0^a(x)$  are absorbed by eight gluons and the remaining four fields  $\rho_\pm(x)$  and  $\eta_\pm(x)$  must be correlated in a nontrivial fashion. The Higgs Lagrangian is chosen to be

$$L_H = -[(D_\mu \Phi_+)^\dagger (D_\mu \Phi_+) + (D_\mu \Phi_-)^\dagger (D_\mu \Phi_-)] - V_\Phi, \quad (11a)$$

$$V_\Phi = \frac{v^2}{2} \left\{ [2(\chi/\chi_\infty)^2 - 1] [(\Phi_+^\dagger \Phi_+) + (\Phi_-^\dagger \Phi_-)] + \frac{\eta^2}{4} [(\Phi_+^\dagger \Phi_+)^2 + (\Phi_-^\dagger \Phi_-)^2 + 2(\Phi_+^\dagger \Phi_-)(\Phi_-^\dagger \Phi_+)] \right\}, \quad (11b)$$

with

$$D_\mu \equiv \partial_\mu - ig \frac{\lambda_a}{2} G_\mu^a,$$

$$v^2 \equiv -v^2/\eta^2 \gg m_p^2,$$

$$\eta^2 > 0.$$

We note that spontaneous symmetry breaking takes place only outside the bag since we have

$$\frac{1}{2}v^2[2(\chi/\chi_\infty)^2 - 1] = \frac{1}{2}v^2 < 0 \quad \text{outside } V, \\ \cong -\frac{1}{2}v^2 > 0 \quad \text{inside } V. \quad (12)$$

The presence of the  $\chi$  field allows us to devise a Higgs mechanism which is a function of  $\vec{r}$ . The situation is similar to what the second term in  $L_{\text{int}}^{(2)}$  [Eq. (4b)] does for

us—chiral-symmetry breaking takes place only outside the bag.

We now consider what happens outside the bag. Equations (10a), (10b), and (11b) yield the following relations for vacuum expectation values:

$$\begin{aligned} |u_+|^2 + |v_+|^2 &= |u_-|^2 + |v_-|^2 = v^2, \\ u_+ u_- + v_+ v_- &= 0. \end{aligned} \quad (13a)$$

Accordingly, we write

$$\begin{aligned} u_+ &= v_- = v \cos\theta, \\ v_+ &= -u_- = v \sin\theta \end{aligned} \quad (13b)$$

with  $\theta$  an arbitrary angle. The gluon-mass terms as given by Eq. (11a) can be cast into a simple form:

$$\begin{aligned} \frac{1}{2}(M^2)_{ab} G_\mu^a G_\mu^b &= \frac{1}{2} g^2 v^2 \{ G_\mu^1 G_\mu^1 + G_\mu^2 G_\mu^2 + G_\mu^3 G_\mu^3 + \frac{1}{3} G_\mu^8 G_\mu^8 + \frac{1}{2} [(\cos\theta G_\mu^4 + \sin\theta G_\mu^6)^2 + (-\sin\theta G_\mu^4 + \cos\theta G_\mu^6)^2 \\ &\quad + (\cos\theta G_\mu^5 + \sin\theta G_\mu^7)^2 + (-\sin\theta G_\mu^5 + \cos\theta G_\mu^7)^2] \}. \end{aligned} \quad (14a)$$

Thus, we have

$$M_{1,2,3} = gv, \quad M_8 = gv/\sqrt{3}, \quad M_{4,5,6,7} = gv/\sqrt{2}. \quad (14b)$$

All the eight gluons acquire heavy masses outside the bag and the mass spectrum [Eq. (14b)] is extremely simple. The fact that  $u_\pm$  and  $v_\pm$  are real numbers follows from the requirement that the mixing among real gluon fields does not give rise to any unphysical phase.

Do we get any “unwanted” massless particles? Using Eqs. (9) and (13a), we extract from Eq. (11a) those terms which are both linear in  $G_\mu^a$  and linear in any of  $\rho_\pm$  and  $\eta_\pm$ :

$$\begin{aligned} B &= \frac{1}{2} igv \left\{ \cos\theta \left[ \partial_\mu \eta_+ (G_\mu^1 - iG_\mu^2) + \partial_\mu \rho_+ \left[ G_\mu^3 + \frac{1}{\sqrt{3}} G_\mu^8 \right] \right] + \sin\theta \left[ \partial_\mu \rho_+ (G_\mu^1 + iG_\mu^2) - \partial_\mu \eta_+ \left[ G_\mu^3 - \frac{1}{\sqrt{3}} G_\mu^8 \right] \right] \right. \\ &\quad - \sin\theta \left[ \partial_\mu \eta_- (G_\mu^1 - iG_\mu^2) + \partial_\mu \rho_- \left[ G_\mu^3 + \frac{1}{\sqrt{3}} G_\mu^8 \right] \right] \\ &\quad \left. + \cos\theta \left[ \partial_\mu \rho_- (G_\mu^1 + iG_\mu^2) - \partial_\mu \eta_- \left[ G_\mu^3 - \frac{1}{\sqrt{3}} G_\mu^8 \right] \right] \right\} + \text{complex conjugates}. \end{aligned} \quad (15)$$

$B$  must vanish identically since there are already eight real fields  $\zeta_0^a(x)$  which are absorbed by the eight gluons and the remaining Higgs fields  $\rho_\pm$  and  $\eta_\pm$  must act like independent objects. In other words, a direct conversion of a gluon field into any of  $\rho_\pm$  and  $\eta_\pm$  is strictly forbidden. This “orthogonality” condition yields

$$\begin{aligned} \rho_+ &= \sin\theta(\eta_1 + i\eta_2) + \cos\theta \left[ \eta_3 + \frac{1}{\sqrt{3}} \eta_8 \right], \\ \eta_+ &= \cos\theta(\eta_1 - i\eta_2) - \sin\theta \left[ \eta_3 - \frac{1}{\sqrt{3}} \eta_8 \right], \\ \rho_- &= \cos\theta(\eta_1 + i\eta_2) - \sin\theta \left[ \eta_3 + \frac{1}{\sqrt{3}} \eta_8 \right], \\ \eta_- &= -\sin\theta(\eta_1 - i\eta_2) - \cos\theta \left[ \eta_3 - \frac{1}{\sqrt{3}} \eta_8 \right], \end{aligned} \quad (16)$$

with  $\eta_1, \eta_2, \eta_3$ , and  $\eta_8$  four real scalar Higgs particles. The mass matrix as determined by Eq. (11b) is also extremely simple:

$$(m^2)_{ij} \eta_i \eta_j = 2\eta^2 v^2 \left[ \eta_1^2 + \eta_2^2 + \eta_3^2 + \left[ \frac{1}{\sqrt{3}} \eta_8 \right]^2 \right]. \quad (17)$$

Using Eq. (16), we rewrite the “kinetic-energy” term as

implied by Eq. (11a):

$$\begin{aligned} & - [(\partial_\mu \rho_+)^{\dagger} (\partial_\mu \rho_+) + (\partial_\mu \eta_+)^{\dagger} (\partial_\mu \eta_+) \\ & \quad + (\partial_\mu \rho_-)^{\dagger} (\partial_\mu \rho_-) + (\partial_\mu \eta_-)^{\dagger} (\partial_\mu \eta_-)] \\ & = -2[(\partial_\mu \eta_1)^2 + (\partial_\mu \eta_2)^2 + (\partial_\mu \eta_3)^2 + \frac{1}{3} (\partial_\mu \eta_8)^2]. \end{aligned} \quad (18)$$

Comparing the normalization factors, we conclude that all four Higgs particles have the same mass  $M_H$ :

$$M_H = \eta v > 0. \quad (19)$$

Therefore, the proposed scheme allows gluons to acquire large masses outside a bag and only outside the bag. It does not give rise to “unwanted” massless particles and yet maintains color gauge invariance. It should be useful to stress the uniqueness property of the above scheme. That is, the gluon and Higgs-particle mass spectrum [Eqs. (14b) and (19)] is independent of which and how eight scalar particles get absorbed by gluons—three gluons are of mass  $gv$ , one is  $gv/\sqrt{3}$ , the other four must be of mass  $gv/\sqrt{2}$ , and the four Higgs particles must have the same mass  $M_H$ . One may start with a choice which looks different from Eqs. (10a) and (10b). But this only amounts to relabeling the various entities and the structure of the theory is not altered.

Nevertheless, the proposed mechanism differs from the conventional Higgs mechanism (as in Glashow-Weinberg-

Salam theory) in several respects, viz., (1) the proposed spontaneous symmetry breaking takes place only outside the bag [Eq. (12)], (2) colored, but electrically neutral, scalar particles are allowed to develop nonzero vacuum expectation values, and (3) the nonzero vacuum expectation values are extremely large (i.e.,  $\sim f\chi_\infty \gg m_p$ ) so that, at ordinary energies ( $\sim m_p$ ), gluons are confined. Accordingly, theorems proven for the conventional Higgs mechanism need to be reexamined for this new scheme.

Accordingly, we may define, instead of Eq. (5),

$$L = L_{\text{QCD}} + L_\chi + L_{\sigma\pi} + L_{\text{int}}^{(1)} + L_{\text{int}}^{(2)} + L_H, \quad (20)$$

or, by deleting the  $(\sigma, \vec{\pi})$  degrees of freedom,

$$L' = L_{\text{QCD}} + L_\chi + L_{\text{int}}^{(1)} + L_H. \quad (21)$$

Note that both models remain asymptotically free.

In our opinion, it is desirable to work with  $L$  unless chiral-symmetry breaking, as a nonperturbative phenomenon, can be demonstrated to be of observed characteristics from a model specified by  $L'$ . Further, we also find<sup>15</sup> that, in view of the distinct roles played, respectively, by  $\chi$  and  $\sigma$ , it is very difficult to conceive a compact model in which a scalar field assumes simultaneously both the roles played by  $\chi$  and  $\sigma$ .

### III. DISCUSSION

The models proposed here are of potential interest for many reasons, among which we choose to list only the following:

(A) To incorporate manifestations of hadron substructure in nuclear or intermediate-energy physics, it has become obvious that corrections to the leading approximation in any of popular bag models are in general of numerical significance and so must be included. For instance, recoil corrections to baryon magnetic moments are found to be substantial.<sup>16</sup> In addition, pionic corrections<sup>17</sup> and gluon-exchange-current contributions<sup>18</sup> have also been shown to be of numerical importance. Without a *renormalizable* soliton bag model as a guideline, it is just impossible to formulate most of these problems in a clear-cut manner.

(B) The models proposed here contain key ingredients in the standard picture: Quarks and gluons are building blocks of low-lying hadrons, these constituents interact among themselves primarily via QCD, they are confined

at ordinary energies ( $\sim m_p$ ), and perhaps chiral dynamics dictates the physics in the "outside" region. However, any such model is only an *effective* theory. It is by no means a new story to demand an effective theory to be renormalizable—Weinberg and Salam demanded the  $SU(2) \times U(1)$  electroweak theory, as an effective theory to some grand unification scheme, to be renormalizable. Since only renormalizable theories can be made quantitative by the present technique, their predictions can be checked against experiment to see if there is any new physics missing from the picture. To the least, the models proposed here will remain as interesting mathematical toy models.

(C) The models proposed here, if taken seriously, represent a departure from the orthodox belief that confinement can be deduced from QCD [Eq. (1)]. The scenario one may have in mind for such models is that the physics responsible for confinement takes place at the scale characterized by  $f\chi_\infty$  ( $\gg m_p$ ) and, at about the same scale, chiral dynamics starts dictating the physics in the outside region. It will of course be interesting to ask whether a grand unified theory can contain any such model as an effective theory. On the other hand, one can attempt to eliminate the  $\chi$  field, at least at ordinary energies ( $\sim m_p$ ), and thereby check whether boundary conditions proposed in popular bag models can arise naturally.<sup>19</sup> Efforts along the last line already indicate that popular bag models appear as limiting cases of the present model [Eq. (20)]. Moreover, one may attempt to formulate corrections to these limiting cases in a well-defined manner.

In summary, we point out in this note that, using a colored Higgs mechanism specified by Eqs. (7), (11a), and (11b), one can construct a renormalizable soliton bag model such as Eq. (20) or Eq. (21). The model holds the promise that all higher-order effects can be characterized explicitly.

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