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Nearly massless pion in a modified bag model

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The properties of the pions are studied in a modified bag model. The pion wave function is approximated by the projection of the $q\bar{q}$ wave function in the bag that has zero center-of-mass motion. The parameters of the model are determined by the usual stability condition and the additional requirement that the pion mass m_π approach zero in a manner consistent with the hypothesis of partially conserved axial-vector current (PCAC). This fixes the bag volume energy to be $m_\pi/4$. Static pion properties are in good agreement with data provided that quarks in the pion are correlated by a modulating function characterized by a correlation momentum $p_c \sim 300$ MeV/c. The bag-model and Nambu-Goldstone views of the pion are reconciled as a distinction is made between constituent and current quarks. Assuming their fields to be related by a transformation analogous to that of Nambu and Jona-Lasinio we calculate the vacuum expectation value of the current quark density to be 0.04 GeV^3, corresponding to a current quark mass of 5 MeV. We test our ideas by calculating, to first order of the gluon-exchange interaction, the pion decay constant and the vector and axial-vector form factors for pion radiative decay. In our model the pion decay constant satisfies the PCAC requirement that it has a finite value in the limit of zero-mass pion.

I. INTRODUCTION

The abnormally small mass of the pion compared to the masses of all other hadrons and the considerable success of the hypothesis of partially conserved axial-vector current (PCAC) provide strong motivation for identifying the pion as the approximate Nambu-Goldstone boson associated with the spontaneous breaking of the chiral symmetry[1,2] of the Lagrangian for strong interaction, which we assume to be given by quantum chromodynamics (QCD). Recent computer simulations of lattice QCD have shown[3] that chiral symmetry is indeed spontaneously broken and the PCAC pion realized when the strong coupling constant exceeds a certain critical strength. On the other hand, a full description of the PCAC pion in terms of the fundamental fields of QCD is not yet available. The QCD Lagrangian is chirally invariant when the current quark mass m_0 is zero. Chiral symmetry of the physical system can, however, be spontaneously broken by a scalar, such as the quark density $\bar{\psi}_0\psi_0$, acquiring a nonzero vacuum expectation value, which then gives rise to a massless Nambu-Goldstone boson that we identify as the pion. The simplest explanation for the nonvanishing mass of the physical pion is that the chiral invariance of the QCD Lagrangian is explicitly broken by a small quark-mass term $m_0\bar{\psi}_0\psi_0$. Thus, when $\langle\bar{\psi}_0\psi_0\rangle\neq 0$ the pion mass approaches zero only in the limit $m_0\to 0$. Another important element of PCAC is that the pion decay constant f_π, which is defined as proportional to the transition amplitude between the pion and the vacuum, remains finite in the limit of zero pion mass, $m_\pi\to 0$. The four quantities we have thus far introduced, m_0, m_π, f_π, and $\langle\bar{\psi}_0\psi_0\rangle$, are tied together in a PCAC sum rule[4]:

$$m_0\langle\bar{\psi}_0\psi_0\rangle=\tfrac{1}{2}m_\pi{}^2 f_\pi{}^2 \ .$$

The above view of the pion naturally leads to the problem of how to reconcile it with the quark-model description of the pion. By quark model we mean any model in which a meson is characterized as a quark-antiquark state, and a baryon as a three-quark state. We will be concerned with one of the most successful quark models, the MIT bag model,[5] in which hadrons are composites of relativistic quarks confined in a spherical cavity by an unspecified mechanism but represented phenomenologically by an inward pressure acting on the cavity.

Its many successes notwithstanding, the bag model (by which we mean the MIT version[6]) has several shortcomings with regard to the pion:

(a) The bag hadron wave functions formed as products of single-quark wave functions contain spurious center-of-mass (c.m.) motions. This defect is expected to affect most seriously the description of the pion since it is the lightest hadron.

(b) Because the bag states are not eigenstates of momentum, the decay constant cannot be properly defined.

(c) The pion in the bag model is not necessarily a Nambu-Goldstone boson in the sense that the symmetry limits $m_\pi\to 0$ and $f_\pi\to$a nonzero constant as $m_0\to 0$ cannot be realized in a natural way.

(d) Finally the model is silent on the value of the quark condensate $\langle\bar{\psi}_0\psi_0\rangle$; the implicit assumption is that as far as the spectrum of the low-energy hadrons is concerned, the bag is not in the Nambu-Goldstone mode, but in the Wigner-Weyl mode such that chiral symmetry is not spontaneously broken and that $\langle \bar{\psi}_0 \psi_0 \rangle = 0$.

Recently, Donoghue and Johnson[7] proposed an ansatz which could rectify (a) and (b); however, their procedure for removing the c.m. motions has the weakness of being too closely tied to an unrelated quantity, the decay constant. As an alternative, Wong[8] put forward the Peierls-Yoccoz projection method, but his calculated decay constant has the undesirable limit $f_\pi \propto 1/\sqrt{m_\pi} \rightarrow \infty$ as $m_0 \rightarrow 0$. A very different approach based on a model by Nambu and Jona-Lasinio[9] was taken by Goldman and Haymaker,[10] who sought an approximate solution of the appropriate inhomogeneous Bethe-Salpeter equation with a four-point quark-quark interaction. The special feature of their model is that chiral symmetry is dynamically broken before confinement and confining conditions are imposed on the $q\bar{q}$ relative coordinate, preserving translational invariance in the total coordinate.

In our view it is desirable to retain features of the MIT bag model that render it simple and amenable to calculations. Therefore, without altering the main premises of the model, we examine to what extent the bag-model pion can be reconciled with the Nambu-Goldstone boson. For this purpose we construct a modified bag model which at least partially rectifies the shortcomings (a)—(d) and test it by calculating some important properties of the pion.

The outline of the rest of this paper is as follows. In Sec. II we use a projection method to deal with the problem of the center-of-mass motion and calculate the different components of the pion energy in momentum representation. We propose an ansatz for determining the stationary configurations of the bag in a manner consistent with the PCAC constraints. In Sec. III the decay constant is calculated to lowest order of the static gluon-exchange interaction, the result of which strongly suggests the presence of quark correlations in the pion and the importance of making the distinction between current and constituent quarks. This distinction is central to the reconciliation of the apparently contradicting pictures of the quark-model pion and the Nambu-Goldstone pion. As a result, the pion decay constant has the correct symmetry limit. The vacuum expectation value for the massless quark condensate is then calculated, from which a value for the current quark mass is deduced. The vector and axial-vector form factors of the pion radiative decay, which are sensitive to details of the meson structure, are calculated in Sec. IV. In Sec. V we draw together our main results and conclusions.

II. PION WAVE FUNCTION AND PION MASS

In this section we define our model of the pion in terms of its wave function and proceed to calculate the various components of its energy and to determine the parameters of the model in its stationary configurations. The center-of-mass motion of a pair of bound relativistic particles is intimately tied to the interaction binding the system. Unless a completely Lorentz covariant solution is found for the interacting system, it is in general impossible to write down its momentum eigenstates. In the bag model, the mechanism of confinement is not sufficiently specified to admit the construction of eigenstates of the center-of-mass momentum. One must resort to approximations. In the approximation we shall adopt, we use as the pion wave function the component of the quark-antiquark wave function of the bag that has zero center-of-mass momentum relative to the center of the bag. Because this center is fixed in space, the wave functions are not covariant. Thus, although some of the operators that appear later, and for which expectation values shall be taken, are Lorentz invariant or covariant, the corresponding expectation values are not.

We begin by expanding the static bag wave function for the pion with the cavity centered at $\vec{X}$, $|\pi_B(\vec{X})\rangle$, in terms of the momentum components $|\pi(\vec{P})\rangle$,

$$|\pi_B(\vec{X})\rangle = \int \frac{d\vec{P}}{2E_P} |\pi(\vec{P})\rangle e^{i\vec{P}\cdot\vec{X}} , \tag{2.1}$$

with its inverse transform

$$|\pi(\vec{P})\rangle = \frac{2E_P}{(2\pi)^3} \int d\vec{X}\, |\pi_B(\vec{X})\rangle e^{-i\vec{P}\cdot\vec{X}} . \tag{2.2}$$

As a quark-antiquark ($q\bar{q}$) composite $|\pi(\vec{P})\rangle$ can also be expressed as [with $\vec{P}=\vec{p}_1+\vec{p}_2$, $\vec{p}=(\vec{p}_1-\vec{p}_2)/2$]

$$|\pi(\vec{P})\rangle = 2E_P \int d\vec{p}\, F(P,p) \sum_{s_1 s_2} \bar{u}(p_1 s_1)\gamma_5 v(p_2 s_2) \times b^\dagger_{p_1 s_1} d^\dagger_{p_2 s_2} |0\rangle , \tag{2.3}$$

where the integral over $\vec{p}$ is a shorthand notation for

$$\int d\vec{p}_1 d\vec{p}_2 \delta(\vec{p}_1+\vec{p}_2-\vec{P}) .$$

Here $b_1^\dagger$ creates a quark of momentum p_1, mass m_1, and polarization s_1 while $d_2^\dagger$ creates an antiquark (p_2,m_2,s_2). For the $q\bar{q}$ wave function we postulate

$$F(\vec{P},\vec{p}) = f(\vec{p}_1) f(\vec{p}_2) \xi(p) , \tag{2.4}$$

where $f(p_i)$ is the Fourier transform of the s-wave orbital wave function of the ith quark in the lowest cavity mode of the bag:

$$\begin{aligned} f(p_i) &= R^3 f(x_i,z_i) \\ &= R^3[j_0(x_i-z_i)-j_0(x_i+z_i)]/2x_i z_i . \end{aligned} \tag{2.5}$$

R is the bag radius, $z_i = p_i R$, and x_i is an eigensolution[5] of

$$x_i - [1-\mu_i-({x_1}^2+{\mu_i}^2)^{1/2}]\tan x_i = 0 \ , \mu_i = m_i R .$$

Because we are working with Dirac spinors in momentum space, it is not necessary to specify the p-wave (small) components; their effect is automatically included. By construction (2.3) has definite $\vec{P}$. On the other hand, because the $f(p_i)$ are not plane waves, the internal wave function F depends explicitly on $\vec{P}$ and so is not translationally invariant. This reflects the fact that the wave function is expanded about the center of the cavity fixed in space. In the bag model confined quarks move freely within the cavity. However, the very large energy gap separating the pion from the other hadrons could be interpreted as a manifestation of a quark-antiquark interaction. To simulate any eventual finite-range effects of such an interaction we have introduced in (2.4) a correlation function $\xi(p)$ which acts to suppress the wave function for momenta greater than some correlation momentum p_c, and for lack of a truly dynamical calculation have chosen it to have a simple form:

$$\xi(p)=[1+(p/p_c)^2]^{-1} . \tag{2.6}$$

In the limit $p_c \to \infty$, the correlations vanish. In this limit, the wave function (2.3), when integrated over the c.m. momentum, becomes identical to that of the usual bag model.

In (2.3) the bilinear spinor product $\bar{u}_1\gamma_5 v_2$ serves as a convenient projection operator in spin space. The advantage of working in momentum representation is that trace techniques can be applied on Dirac matrices and spinors. For example, after summing over spins and integrating over relative momenta we obtain the norm of $|\pi(\vec{P})\rangle$:

$$\langle \pi(\vec{P}\,')\,|\,\pi(\vec{P})\rangle=\delta(\vec{P}\,'-\vec{P})2E_P N^2(\vec{P}) ,$$
$$N^2(\vec{P})=2E_P \int d\vec{p}\,|F(\vec{P},\vec{p})|^2 \times\frac{(E_1+E_2)^2-\vec{P}^2-(m_1-m_2)^2}{2m_1m_2} . \tag{2.7}$$

Although there are in principle no obstacles in formulating the model for an arbitrary $\vec{P}$, it is simpler to require from the start that the pion be at rest, $\vec{P}=0$. Then $\vec{p}_1=-\vec{p}_2=\vec{p}$, and the $q\bar{q}$ wave function becomes a function of a one-dimensional variable $p=|\vec{p}|$. After an angular interaction in (2.7) we obtain

$$N^2(0)=(4\pi R^8\omega_0/\mu_1\mu_2)I_N ,$$
$$I_N=\int_0^\infty dz\, z^2\phi^2(z)N_f^{\,2}(z) , \tag{2.8}$$

where we have defined the dimensionless variables $z=pR$, $\omega_0=RE_{\vec{P}=0}$, $\mu_i=Rm_i$, and $\omega_i=RE_i$, the radial wave function

$$\phi(z)=f(x_1,z)f(x_2,z)\xi(z) ,$$

and the energy factor

$$N_f^{\,2}(z)=(\omega_1+\omega_2)^2-(\mu_1-\mu_2)^2 .$$

The total energy of the pion includes on the one hand the zero-point energy $E_Z=-Z/R$ and the pressure energy $E_B=(4\pi/3)BR^3$ with the same parametrization as in the original bag model and, on the other hand, the kinetic energy E_K and the interaction energy E_g which are to be recalculated with the new wave function. The kinetic energy E_K is derived from the usual free Hamiltonian

$$H_0=\sum_i \gamma_0(\vec{\gamma}\cdot\vec{p}_i+m_i)$$

with the result

$$E_K=\langle\pi(0)\,|\,H_0\,|\,\pi(0)\rangle/\langle\pi(0)\,|\,\pi(0)\rangle$$
$$=2x/R , \tag{2.9}$$

where $2x=I_K/I_N$ is the ratio of the energy and normalization integrals. I_K is defined as

$$I_K=\int dz\, z^2\phi^2(z)N_f^{\,2}(z)(\omega_1+\omega_2) . \tag{2.10}$$

The interaction energy E_g arises from an effective gluon-exchange quark-quark interaction whose primary role in any quark model is to split the pseudoscalar- and vector-meson masses as well as the nucleon and Δ masses. It is derived from the color-electromagnetic energy of the gluon fields:

$$H_g=-2\pi\alpha_s\int d\vec{x}\,\partial_\nu A_\mu{}^a\partial^\nu A^{a\mu} , \tag{2.11}$$

where α_s is the effective strong coupling constant, and

$$A_\mu{}^a(\vec{x})=\int d\vec{y}\,\bar{\psi}(\vec{y},x_0)\frac{\lambda^a}{2}\gamma_\mu\psi(\vec{y},x_0)\frac{1}{4\pi\,|\,\vec{x}-\vec{y}\,|} . \tag{2.12}$$

Here a static approximation has been explicitly applied because H_g will act only on quarks in the lowest eigenmode; λ^a $(a=1,\ldots,8)$ are generators of the SU(3) color group. In momentum space the parts of H_g that contribute to the pion energy are given by

$$H_g=\frac{\alpha_s}{4\pi^2}\int\left[\prod_{i=1}^{4}d^3p_i\left[\frac{m_i}{E_i}\right]^{1/2}\right]\delta(\vec{p}_1-\vec{p}_2-\vec{p}_3-\vec{p}_4)\delta(\vec{p}_1-\vec{p}_3-\vec{q}_1)\delta(\vec{p}_2-\vec{p}_4-\vec{q}_2)$$
$$\times\frac{\vec{q}_1\cdot\vec{q}_2}{q_1{}^2q_2{}^2}\left[\bar{u}_1\frac{\lambda^a}{2}\gamma_\mu u_3\right]\left[\bar{v}_4\frac{\lambda_a}{2}\gamma^\mu v_2\right]2b_1^\dagger d_2^\dagger b_3 d_4+\cdots , \tag{2.13}$$

where $+\cdots$ indicates terms irrelevant to our calculation. We may at this point proceed to calculate $\langle\pi\,|\,H_g\,|\,\pi\rangle$ with (2.13). It proves, however, to be more fruitful to introduce an additional simplification by applying a Fierz transformation to the spin product in (2.13) and dropping all but the pseudoscalar term:

$$(\bar{u}_1\gamma_\mu u_{1'})(\bar{v}_2\gamma^\mu v_{2'})=-(\bar{u}_1\gamma_5 v_{2'})(\bar{v}_2\gamma_5 u_{1'})+\cdots . \tag{2.14}$$

The approximation is akin to the ladder approximation[9] in which the intermediate states are restricted to pseudoscalar states and should be valid in the pion case. Since we treat α_s as an effective coupling constant, we compute the effect of H_g only to lowest order. We then obtain (Appendix A)

$$E_g=\langle\pi(0)\,|\,H_g\,|\,\pi(0)\rangle/\langle\pi(0)\,|\,\pi(0)\rangle$$
$$=-\alpha_s G/R\ , \tag{2.15}$$

$$G=\frac{2}{3\pi I_N}\int dz\,J(z)\bar{K}(z)\ , \tag{2.16}$$

where we have defined

$$J(z)=z\phi(z)N_f^{\,2}(z)/\sqrt{\omega_1\omega_2}\ , \tag{2.17a}$$

$$\bar{K}(z)=\int dz' J(z')L(z,z')\ , \tag{2.17b}$$

$$L(z,z')=\ln(\,|z+z'|\,/\,|z-z'|\,)\ . \tag{2.17c}$$

The quantities x and G defined above are functions of μ_i $(i=1,2)$ and $K=p_cR$; they become independent of R when $\mu_i\to 0$ and $K\to\infty$.

Finally the total energy of the pion is

$$E_\pi=\frac{1}{R}\left[2x-\alpha_s G-Z+\frac{4\pi}{3}BR^4\right]\ , \tag{2.18}$$

where Z and B are constants, x and G depend on R as seen above, and we assume α_s to have a logarithmic scale dependence[7] characteristic of QCD:

$$\alpha_s(R)=\alpha_0/\ln(1+1/\Lambda R)\ , \tag{2.19}$$

where empirically[11] $\Lambda=0.1-0.5$ GeV with small values being favored by the more recent data. We use $\Lambda=0.2$ GeV and $\alpha_0=0.5$; at the pion radius ($R\sim 0.6$ fm) this gives $\alpha_s\sim 0.5$ and will yield about the right amount of gluon energy ($E_g\sim 0.6$ GeV) needed for the π and ρ mass splitting. Stationary solutions for the pion are then attained by requiring that

$$E_\pi=m_\pi=140\text{ MeV} \tag{2.20}$$

and

$$dE_\pi/dR=0\ . \tag{2.21}$$

For equal quark masses ($\mu_1=\mu_2=\mu=mR$) the above equations become

$$2x-\alpha_s G-Z+\frac{4\pi}{3}BR^4-\mu_\pi=0\ , \tag{2.22}$$

$$\mu(2x'-\alpha_s G')-\frac{G\alpha_s^{\,2}}{\alpha_0(1+\lambda)}+\frac{16\pi}{3}BR^4-\mu_\pi=0\ , \tag{2.23}$$

where $\mu_\pi=m_\pi R$, $\lambda=\Lambda R$, $x'=dx/d\mu$, and $G'=dG/d\mu$. The parameters yet unspecified are B, Z, m, R, and p_c. Within our model there seems to be no dynamical mechanism to drive the pion mass to zero. However, assuming the zero-mass pion to be a Nambu-Goldstone boson, whatever mechanism we choose must leave m, R, and p_c essentially unchanged. The only parameters that can be changed are therefore B and Z. From (2.22) and (2.23), we see that if $E_\pi(B,Z)=m_\pi$ is a stationary solution, then so is $E_\pi(B_0,Z_0)=0$, provided that

$$B_0=B-(3/16\pi)m_\pi R^{-3} \tag{2.24}$$

and

$$Z_0=Z+3m_\pi R/4\ . \tag{2.25}$$

Since the pressure explicitly violates chiral invariance it should vanish in the symmetry limit, i.e., $B_0=0$, from which it follows that $B=(3/16\pi)m_\pi R^{-3}$ or

$$E_B=\tfrac{1}{4}m_\pi\ . \tag{2.26}$$

This remarkable result reconciles the bag pion with the PCAC pion and, at the same time, fixes the parameter B. At $R=0.6$ fm it yields $B^{1/4}=131$ MeV, which is surprisingly close to the value (~ 140 MeV) determined phenomenologically for the MIT bag.[5,7] The derivation of (2.26) depends on the implicit assumption that the mechanism for confinement, which controls B, and that for spontaneous symmetry breaking, which controls m and $\langle\bar{\psi}_0\psi_0\rangle$, can be separately treated. This seems reasonable since the latter are realized in models in which the former does not occur. The fundamental consequence of the pion being a Nambu-Goldstone boson is that its properties, excluding its mass, are on the whole determined by the nature of spontaneous chiral-symmetry breaking, but not by the nature of confinement. Within the context of the bag model, relations (2.24)—(2.26) guarantee this eventuality.

Of the remaining four undetermined parameters Z, m, R, p_c, if any two are chosen then the other two will be determined by Eqs. (2.22)—(2.23). We chose $R=0.6$ fm to correspond to the rms radius of the pion,[12] vary p_c in the range from 150 MeV to ∞, and solve (2.22) and (2.23) for m and Z. In this range of p_c these equations happen not to admit solutions with $R\gtrsim 0.9$ fm. Table I presents the main results of our work. We first give four sets of parameters p_c, m, Z, and R that satisfy Eqs. (2.22) and (2.23), and in the following part of the table the different energy components corresponding to these four solutions. The quark mass and the interaction energy rapidly saturate their respective magnitudes as p_c increases, whereas the kinetic energy and the zero-point energy grow in opposite directions. The zero-point fluctuations and the gluon exchange are essential in lowering the pion state. The best agreement with empirical values ($m\sim 350$ MeV, $E_g\sim -600$ MeV) is reached by the set of parameters $p_c=300$ MeV, $Z=1.26$, $m=352$ MeV, and $R=0.6$ fm.

III. PION DECAY CONSTANT AND QUARK CONDENSATE

Since from the energy scale of the bag model it is expected that the simplest estimate of the decay constant will be large, we calculate the next-order correction using a naive perturbation formula:

$$f_\pi=f(0)+f_g(0)\ , \tag{3.1}$$

where

$$f(P)P_\mu=\langle 0\,|\,J_{\mu 5}\,|\,\pi(P)\rangle/N(P)\ , \tag{3.2}$$

$$f_g(P)P_\mu=\left\langle 0\left|J_{\mu 5}\frac{\mathrm{P}}{E_0-H_0}H_g\right|\pi(P)\right\rangle\Big/N(P)\ . \tag{3.3}$$

TABLE I. Some properties of the pion calculated with four sets of parameters satisfying the stationary conditions in the modified bag model. Energies (momenta) are given in MeV (MeV/c). The bag constant is constrained such that $E_B = m_\pi/4$. The coupling is $\alpha_s = \alpha_0/\ln(1+1/R\Lambda)$, $\Lambda = 200$ MeV. We assume $m_u = m_d$, except in the calculation of the axial-vector form factor for which the value $m_d - m_u = 3$ MeV is used (Ref. 20). The value for R is chosen to agree with the measured root-mean-square radius of the pion.

p_c	150	300	500	∞
Z	0.842	1.26	1.34	2.37
m	283	352	386	518
R (fm)	0.6	0.6	0.6	0.6
E_K	925	1157	1323	1787
E_g	-543	-639	-777	-903
E_Z	-277	-413	-442	-780
E_B	35	35	35	35
$\langle\bar\psi\psi\rangle$ (10^{-2} GeV^3)	2.0	3.6	4.5	9.7
f	638	748	873	1050
f_g	-554	-622	-636	-778
$f_\pi = f + f_g$	84	126	237	272
v	0.066	0.053	0.052	0.036
v_g	-0.023	-0.016	-0.007	-0.005
$v_\pi = v + v_g$	0.043	0.037	0.045	0.031
a (10^{-3})	-0.884	-0.571	-0.516	-0.265
a_g (10^{-3})	0.882	0.527	0.358	0.183
$a_\pi = a + a_g$ (10^{-3})	-0.0016	-0.044	-0.158	-0.982
$\gamma_\pi = a_\pi/v_\pi$ (10^{-3})	-0.37	-1.19	-3.51	-2.64

Here H_0 is the Hamiltonian associated with $E_0 = E_\pi - E_g$, P denotes the principal part, and $J_{\mu 5} = \bar\psi_0 \gamma_\mu \gamma_5 \psi_0$ is the axial-vector current in terms of current quark fields. Since both the vacuum and pion states refer to constituent quarks, the current operator must be reexpressed in terms of the constituent quark fields ψ and $\psi^\dagger$. However, as quarks acquire masses in an as yet unknown dynamical fashion the exact transformation between ψ and ψ_0 remains unknown. We simply use an ansatz analogous to that proposed by Nambu and Jona-Lasinio.[9] Let $U_{p,s}^{(0)}$ and $V_{p,s}^{(0)}$ be the positive- and negative-energy components of the current quark field, and $U_{p,s}$ and $V_{p,s}$ the corresponding components of the constituent quark field. Then we postulate the following unitary transformation:

$$\begin{bmatrix} U_{p,s}^{(0)} \\ V_{-p,s}^{(0)} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} U_{p,s} \\ V_{-p,s} \end{bmatrix}, \tag{3.4}$$

where $\theta = \tan^{-1}[(1-\beta)/(1+\beta)]^{1/2}$ with $\beta = p^2/(p^2+m^2)$. This transformation contains the main features of the correct transformation,[13] namely, it deviates maximally from 1 as $(p/m) \to 0$ and approaches 1 at large (p/m). We show in Appendix B how to derive the two terms of the decay constant:

$$f(0) = (\Omega/R) \int dz\, z c_f(z) \phi(z)\,, \tag{3.5}$$

$$f_g(0) = \frac{-2\alpha_s}{3\pi} \frac{\Omega}{R} \int dz\, c_f(z) \frac{K(z)}{\sqrt{\omega_1\omega_2}} \frac{\mathrm{P}}{\omega_1 + \omega_2 - \omega_K}\,, \tag{3.6}$$

where we have used the notation

$$c_f(z) = \tfrac{1}{2}(\mu_1 + \mu_2) z \eta(z) \cos(\theta_1 + \theta_2)/\sqrt{\omega_1\omega_2}\,, \tag{3.7}$$

$$\Omega = (16\pi R E_0 / I_N)^{1/2} \tag{3.8}$$

and $\omega_K = R E_K$. Since $E_0 = E_\pi - E_g$, f_π does not vanish with $E_\pi = m_\pi$ but is rather proportional to $\sqrt{E_0} = \sqrt{-E_g}$ as $m_\pi \to 0$. Thus, the pion in our model satisfies PCAC. We also note that f_π is proportional to m rather than to m_0. In the above, explicitly in (3.7), we have made a dynamical assumption on the vacuum state by assigning to it a cutoff function η. This is necessary because without some cutoff the vacuum expectation value of the current quark density would diverge logarithmically. We simply assume a sharp cutoff at Λ_c, fully aware of the fact that it is an oversimplification of a complex situation. Summing over two flavors, three colors, and the positive- and negative-energy states, we obtain the current quark condensate

$$\langle \bar\psi_0 \psi_0 \rangle = \frac{12}{(2\pi)^3} \int d^3p \frac{m}{E} \sin^2\theta_p \eta(p) = \frac{3m^3}{\pi^2} \left[\ln\left[\frac{2\Lambda_c}{m}\right] - 1 + O\left[\frac{m}{\Lambda_c}\right] \right]. \tag{3.9}$$

Thus, to within a logarithm the condensate is proportional to the third power of the constituent quark mass when the latter is small compared to the ultraviolet cutoff. Up to the logarithm term, η can have any other functional form, e.g., ξ in (2.6), without substantially altering this result.

Table II shows that the pion decay constant (and the radiative-decay form factors to be discussed below) be-

TABLE II. Dependence of pion decay constant and radiative form factors on the ultraviolet cutoff Λ_c.

Λ_c (GeV)	5	6	7	8
f (MeV)	747	747	747	747
f_g (MeV)	−644	−631	−622	−622
$f_\pi=f+f_g$ (MeV)	103	116	125	125
v	0.053	0.053	0.053	0.053
v_g	−0.018	−0.017	−0.016	−0.016
$v_\pi=v+v_g$	0.035	0.036	0.037	0.037
a (10^{-3})	−0.57	−0.57	−0.57	−0.57
a_g (10^{-3})	0.55	0.53	0.53	0.53
$a_\pi=a+a_g$ (10^{-3})	−0.02	−0.042	−0.042	−0.042

come essentially independent of Λ_c for $\Lambda_c \gtrsim 7$ GeV; we therefore use this cutoff in our calculation. We show in Table II the decay constant for some sets of parameters consistent with stationary bag configurations. Again the set at $p_c=300$ MeV gives the best value, $f_\pi=126$ MeV, compared to the experimental value[14] of 132 MeV. As in the case of the energy, the gluon contribution plays an essential role in bringing the final result down to this relatively small value.

In the range $p_c=150$—300 MeV the quark condensate is calculated to be $(2$—$4)\times 10^{-2}$ GeV^3, which is not inconsistent with the recently reported result $\langle \bar\psi\psi\rangle \sim 3\times 10^{-2}$ GeV^3 obtained in a Monte Carlo calculation[3] in lattice QCD. If we use the current-algebra sum rule[4]

$$m_0\langle \bar\psi_0\psi_0\rangle = \tfrac{1}{2} f_\pi{}^2 m_\pi{}^2 \ , \qquad (3.10)$$

then we deduce $m_0 \sim 5$ MeV for the current quark mass, a value which is in agreement with other phenomenological estimates.

IV. PION-RADIATIVE-DECAY FORM FACTORS

We further test our model by calculating the radiative decay $\pi\to\gamma e\nu_e$. A dynamical study of this decay is in itself interesting; its energy distribution and rate are under certain kinematical conditions sensitive to the two so-called structure-dependent form factors, vector and axial vector.[15] A detailed calculation of these two quantities would provide a good test of several assumptions of strong-interaction physics and would give information on the dynamics governing the bound quarks. For example, if the conserved-vector-current (CVC) hypothesis holds, the vector form factor v_π can be related to the lifetime of the neutral pion. On the other hand, while v_π depends on the relative $q\bar q$ wave function at the origin, as does the decay constant, the axial-vector form factor a_π carries new and more sensitive information[16] on the wave function, $a_\pi \sim \nabla^2\psi(0)$. Further, because of its dependence on the u-d quark-mass difference, a_π provides a measure of strong isospin-breaking effects.

The structure-dependent part of the amplitude for $\pi\to\gamma e\nu_e$ is given by[15,16]

$$M_{\rm SD}(\pi\to\gamma e\nu) \sim \frac{1}{m_\pi}[iv_\pi p^\alpha k^\beta \epsilon_{\lambda\alpha\beta\mu} + a_\pi(P\cdot k g_{\lambda\mu} - P_\lambda k_\mu)]\epsilon^\lambda L^\mu \ , \qquad (4.1)$$

where P is the pion momentum, ϵ and k are the polarization and momentum of the photon, and L^μ is the electron-neutrino weak current. Under the same assumptions as used above to obtain f_π and applying the approach of Ref. 16 we derive the form factors to $O(\alpha_s)$ as follows:

$$v_\pi = v + v_g \ , \qquad (4.2)$$

$$a_\pi = a + a_g \ , \qquad (4.3)$$

where the zero-order terms $\delta = v,a$ are given by

$$\delta = \Omega \int dz\, z c_\delta(z)\phi(z) \qquad (4.4)$$

and the first-order terms $\delta_g = v_g, a_g$ by

$$\delta_g = -\frac{2\alpha_s}{3\pi}\Omega \int dz \frac{K(z)}{\sqrt{\omega_1\omega_2}} \frac{\rm P}{\omega_1+\omega_2-\omega_K} c_\delta(z) \ . \qquad (4.5)$$

Here Ω, K, and ϕ are as defined previously, and

$$c_v(z) = \tfrac{1}{4}(\mu_1+\mu_2)(e_1L_1+e_2L_2)\eta(z)\cos^2(\theta_1+\theta_2)/\sqrt{\omega_1\omega_2} \ ,$$

$$c_a(z) = (\mu_2-\mu_1)(e_1+e_2)(L_1+L_2)\eta(z)\cos^2(\theta_1+\theta_2)(z_0{}^2-z^2)/[(\zeta^2+2z^2)\sqrt{\omega_1\omega_2}] \ , \qquad (4.6)$$

$$\zeta^2 = \tfrac{1}{2}\mu_\pi{}^2 + \mu_1{}^2 + \mu_2{}^2 - 2z_0{}^2 \ , \qquad (4.7)$$

$$L_i = \ln\{[(z+\mu_\pi/2)^2+\mu_i{}^2-z_0{}^2]/[(z-\mu_\pi/2)^2+\mu_i{}^2-z_0{}^2]\} \ , \qquad (4.8)$$

in which $z_0=\omega_1-\omega_2$ and e_1 and e_2 indicate the quark and antiquark charges. Through Ω, both v_π and a_π are proportional to $\sqrt{E_0}$, and hence remain finite as $m_\pi \to 0$. But whereas the vector form factor has a similar dependence on the quark masses and on the relative momentum as the decay constant, the axial-vector form factor varies as $(\mu_2-\mu_1)(z_0{}^2-z^2)$; the first factor indicates an isospin-invariance-violation effect and makes this form factor small, and the second is approximately related to the Laplacian of the relative wave function. We also note that the presence of the factor (e_1+e_2) guarantees the vanishing of c_a in the case of a two-photon emission.

The results of our calculations of v_π a_π, and $\gamma_\pi = a_\pi / v_\pi$ are presented in the last part of Table I. It is recalled that the value of the vector form factor is deduced,[15] up to a sign and under the CVC hypothesis, from the pion lifetime τ_{π^0}. Using the most recent value,[17] $\tau_{\pi^0}=(0.828\pm0.057)\times10^{-16}$ sec, we can infer that $v_\pi=0.0265$. With this information, an analysis of the $\pi \to \gamma e \nu$ data then yields $(1+\gamma_\pi)^2$ or two solutions for γ_π; Depommier *et al.*[18] obtain $\gamma_\pi=0.26$ or -1.98 (no errors quoted) while Stetz *et al.*[19] conclude that $\gamma_\pi=0.44\pm0.12$ or -2.32 ± 0.12.

Our result for $v_\pi \sim 0.037$ is in fair agreement with the measured value (as well as with our previous result 0.035 ± 0.011 derived in a similar but cruder approach[16]). This agreement is not unexpected in view of our success in predicting the decay constant. As for the axial-vector form factor, our result favors the smaller experimental solution. It has been argued[16] that since a_π varies as the mean square of the momentum, $-\langle p_\mu{}^2 \rangle$, it must be small and negative in any model that treats quarks as essentially free particles; this clearly holds true as shown by the smallness of the zeroth-order estimate a, but it is also incomplete because it ignores a_g. Gluon corrections are important; they almost cancel out the lowest-order estimate and could easily have reversed its sign for a larger coupling. Our present view is that a_π is small but its sign remains difficult to predict.

V. SUMMARY AND CONCLUSIONS

We have achieved the two goals that we set out to accomplish: (i) to remove spurious center-of-mass motions in the $q\bar{q}$ system in the bag model so that the properties of the pion, in particular its decay constant, can be properly calculated, and (ii) to reconcile the bag-model description of the pion with a Nambu-Goldstone or PCAC pion. Since an important characteristic of the PCAC pion is the finiteness of f_π in the limit $m_\pi \to 0$, (i) is really a prerequisite of (ii).

We achieved our goals by modifying the MIT bag model in a way that does not significantly impair its simplicity and its amenability to calculations. The major modifications are first removing the spurious center-of-mass contribution by projection so that wave function has zero center-of-mass momentum, and second including lowest-order gluon corrections to the calculated quantities. Otherwise all the important features of the original model, in particular the boundary conditions, are retained. We demonstrated that this program is very easy to implement in momentum representation. We emphasize that because the constraint that the bag is fixed in space has not been removed, our results are not Lorentz invariant.

We showed that the bag-model and Nambu-Goldstone views of the pion can be made compatible provided a clear distinction is made between the massive constituent quarks and the nearly massless current quarks. Whereas bag states, including the vacuum state, refer to the former, Lagrangian densities and current densities are defined in terms of the latter. To connect these two types of quarks we used a unitary transformation analogous to that proposed by Nambu and Jona-Lasinio.[9] This, among other things, allowed us to compute the quark condensate $\langle \bar{\psi}_0 \psi_0 \rangle$ to be ~ 0.04 GeV^3, corresponding to a current quark mass of ~ 5 MeV. We argued that in order to restore the chiral invariance of the Lagrangian in the limit of zero pion mass, it is required that the volume energy E_B also vanishes in that limit. This condition ineluctably fixed the value of E_B to be $m_\pi/4$.

The effects of our modification to the bag model are best exemplified by the qualitative features of f_π that we calculated. In our model f_π is proportional to $(m_\pi - E_g)^{1/2}$ and to the constituent quark mass m, but not to the current quark mass m_0. It follows that in the chiral-symmetry limit, i.e., when $m_0 \to 0$ and $m_\pi \to 0$, $f_\pi \propto m\sqrt{-E_g}$ is nonzero thereby satisfying the PCAC constraint. On the other hand, if the gluon contribution to f_π were not included then $f_\pi \propto \sqrt{m_\pi}$. Similarly if no distinction were made between m and m_0 then $f_\pi \propto m_0$. In either case f_π would have the unacceptable property that it vanishes in the chiral-symmetry limit. We feel that these considerations make it clear that the constituent quarks in the bag, or indeed in any quark model (in the sense set down in Sec. I), cannot be massless or nearly massless.

We found that in order to obtain sensible numerical results, the $\bar{q}q$ wave function specified by the bag boundary condition must be quite strongly modulated such that its high-momentum components be suppressed. We achieved this effect by using a Yukawa form factor with a correlation momentum $p_c \sim 300$ MeV. We interpret the necessity of these correlations as a reflection of the presence of finite-range gluon-exchange effects in the confining cavity, a conclusion previously reached by Goldman and Haymaker[10] via a different approach. With these correlations the typical energy scale in the modified bag model is still $\gtrsim 0.5$ MeV (without correlations it is $\gtrsim 1$ GeV), and the small physical values of both m_π and F_π (~ 0.1 GeV) are results of cancellations between bag and gluon contributions.

We further tested our model by calculating the structure-dependent vector and axial-vector form factors for the pion radiative decay. Our result for v_π is in fair agreement with experiment. Owing to the small value of the factor $(m_d - m_u)/(m_d + m_u)$ and an almost complete cancellation between the bag and gluon contributions, the ratio $\gamma_\pi = a_\pi / v_\pi$ is nonzero but very small; it is nevertheless in agreement with the smaller of the two possible experimental values.

Our model can be easily and naturally applied to the consideration of other mesons. The adaptation of the model to baryons should be straightforward, but there the removal of spurious center-of-mass motions from the three-quark system is expected to be numerically more involved.

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APPENDIX A

We show here the calculation of the matrix element $\langle \pi | H_g | \pi \rangle$. From (2.11) and (2.12), H_g is expressed in the second-quantized form

$$H_g = \frac{\alpha_s}{4\pi^2} \int \prod_{i=1}^{4} \left[d^3p_i \left[\frac{m_i}{E_i} \right]^{1/2} \right] \delta(\vec{p}_1 - \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta(\vec{p}_1 - \vec{p}_3 - \vec{q}_1) \delta(\vec{p}_2 - \vec{p}_4 - \vec{q}_2)$$
$$\times \frac{\vec{q}_1 \cdot \vec{q}_2}{q_1^2 q_2^2} \left[\bar{u}_1 \frac{\lambda^a}{2} \gamma_\mu u_3 \right] \left[\bar{v}_4 \frac{\lambda_a}{2} \gamma^\mu v_2 \right] 2 b_1^\dagger d_2^\dagger b_3 d_4 + \cdots , \tag{A1}$$

where the factor 2 represents the 2 terms, out of a total of 16, that contribute to the interaction energy calculated in a ladder approximation. The subscripts in the spinors and operators stand for both spins and momenta. Applying a Fierz transformation on the spin product and dropping all but the pseudoscalar term, as in (2.14), we obtain the matrix element of H_g at $\vec{P} = \vec{P}' = 0$ (therefore, $\vec{p}_1 = -\vec{p}_2 = \vec{p}$, $\vec{p}_3 = -\vec{p}_4 = \vec{p}\,'$):

$$\langle \pi(0) | H_g | \pi(0) \rangle = -\frac{4}{3} \frac{\alpha_s}{4\pi^2} \frac{(2E_0)^2}{2\mu_1\mu_2} \int d^3p \frac{F(0,\vec{p}) N_f^2}{(\omega_1\omega_2)^{1/2}} \int d^3p' \frac{F(0,\vec{p}\,') N_f^2}{(\omega_1'\omega_2')^{1/2}} \frac{1}{|\vec{p} - \vec{p}\,'|^2} , \tag{A2}$$

where the factor $-\frac{4}{3}$ comes from a color summation. To avoid $\delta(\vec{0})$ we have used normalization in a finite volume. Recalling that we have s-wave states, $F(0,\vec{p}) = R^6 \phi(z)$, we integrate over all angles and obtain

$$\langle \pi(0) | H_g | \pi(0) \rangle = -\frac{2\alpha_s}{3\pi} \frac{(2E_0)^2}{2\mu_1\mu_2} 4\pi R^8 \int dz\, z \phi(z) N_f^2(z) / (\omega_1\omega_2)^{1/2}$$
$$\times \int dz' z' \phi(z') N_f^2(z') \ln(|z + z'| / |z - z'|) / (\omega_1'\omega_2')^{1/2} . \tag{A3}$$

Since the normalization is $\langle \pi(0) | \pi(0) \rangle = 4\pi R^9 I_N (2E_0)^2 / 2\mu_1\mu_2$ we obtain the results (2.15)—(2.17).

APPENDIX B

We give here a derivation of the decay constant to first order in H_g. The vector and axial-vector form factors for the pion decay are derived along the same lines. Writing the quark field as a superposition of positive- and negative-energy solutions

$$\psi(x) = \sum_{p,s} (U_{p,s} + V_{p,s}) e^{-ip\cdot x} , \tag{B1}$$

where we have, as usual, $U_{p,s} = (m/E)^{1/2} b(p,s) u(p,s)$ and $V_{p,s} = (m/E)^{1/2} d^\dagger(p,s) v(p,s)$. We apply a Nambu—Jona-Lasinio—type transformation to relate the current quarks $(U^{(0)}, V^{(0)})$ to the constituent quarks (U, V):

$$\begin{bmatrix} U_{p,s}^{(0)} \\ V_{-p,s}^{(0)} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} U_{p,s} \\ V_{-p,s} \end{bmatrix} . \tag{B2}$$

Then after the transformation the axial-vector current becomes $[N_{12} = (m_1 m_2 / E_1 E_2)^{1/2}]$:

$$J_{\mu 5}(x) = \bar{\psi}_0(x) \gamma_\mu \gamma_5 \psi_0(x)$$
$$= \sum_{p_i s_i} (\bar{U}_{p_1 s_1}^{(0)} + \bar{V}_{-p_1 s_1}^{(0)}) \gamma_\mu \gamma_5 (U_{p_2 s_2}^{(0)} + V_{-p_2 s_2}^{(0)}) e^{-i(p_1 + p_2)\cdot x}$$
$$= \sum_{p_i s_i} (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) \bar{V}_{-p_1 s_1} \gamma_\mu \gamma_5 U_{p_2 s_2} e^{-i(p_1 + p_2)\cdot x} + \cdots$$
$$= \sum_{p_i s_i} \cos(\theta_1 + \theta_2) N_{12} \bar{v}(-p_1, s_1) \gamma_\mu \gamma_5 u(p_2, s_2) d(-p_1, s_1) b(p_2, s_2) e^{-i(p_1 + p_2)\cdot x} + \cdots , \tag{B3}$$

where $+\cdots$ indicates terms that do not contribute to our calculation. Then using the wave function

$$|\pi(P)\rangle = 2E_0 \sum_{s_1 s_2} \int d^3p\, F(P,p)\bar{u}_{p_1}\gamma_5 v_{p_2} b_1^\dagger d_2^\dagger |0\rangle \tag{B4}$$

we obtain the transition amplitude

$$\langle 0 | J_{\mu 5} | \pi(P)\rangle = 2E_P \int d^3p\, \eta(p) F(P,p) N_{12} \cos(\theta_1+\theta_2)[(m_1+m_2)P_\mu - 2(m_1-m_2)p_\mu]/2m_1 m_2$$
$$= P_\mu 4\pi E_P \frac{m_1+m_2}{\sqrt{m_1 m_2}} \int dp\, p^2 F(P,p)\eta(p)\cos(\theta_1+\theta_2)/\sqrt{E_1 E_2}\ , \tag{B5}$$

where we have assumed s-wave for the wave function. Dividing this matrix element by the normalization factor $N(P)$ defined in (2.7), and assuming the pion at rest we obtain $f(0)$ as given in (3.5).

The first-order correction as defined in (3.3) is calculated exactly as above, except that the wave function $|\pi(0)\rangle$ is now replaced by

$$|\pi(0)\rangle_g = \frac{\mathrm{P}}{E_0 - H_0} H_g |\pi(0)\rangle\ . \tag{B6}$$

We obtain ($\vec{z} = R\vec{p}$)

$$|\pi(0)\rangle_g = 2E_0 R^3 \int d^3z\, \phi_g(z) \sum_{s_1 s_2} (\bar{u}_1 \gamma_5 v_2) b_1^\dagger d_2^\dagger |0\rangle\ , \tag{B7}$$

where

$$\phi_g(z) = -\frac{2\alpha_s}{3\pi} \frac{\mathrm{P}}{\omega_1+\omega_2-\omega_k} \frac{1}{\sqrt{\omega_1 \omega_2}} \times \int dz' \frac{z'\phi(z') N_f^{\,2}(z')}{z\sqrt{\omega_1' \omega_2'}} L(z,z')\ . \tag{B8}$$

Thus the correction term f_g has the same form as f except for the substitution of the radial wave function $\phi(z)$ by $\phi_g(z)$.

APPENDIX C

We used the two algorithms given below to calculate the integrals in (2.16), (2.17), (3.6), and (4.5) which involve singular integrands.

(1) To calculate the integral

$$K(z) = \int_a^b dx\, J(x) \ln(|z+x|/|z-x|)\ , \tag{C1}$$

we divide the range of z into three intervals and treat the singularity explicitly in each:

$$K(z) = \begin{cases} K_>(z) + K_<(z) - 2\epsilon J(z)[1-\ln(2z/\epsilon)], & a+\epsilon < z < b-\epsilon \\ K_>(z) - J(z)\{\epsilon[1-\ln(2z/\epsilon)] + \eta[1-\ln(2a/\eta)]\}, & z-a=\eta<\epsilon \\ K_<(z) - J(z)\{\epsilon[1-\ln(2z/\epsilon)] + \eta[1-\ln(2b/\eta)]\}, & b-z=\eta<\epsilon, \end{cases} \tag{C2}$$

$$K_>(z) = \int_{z+\epsilon}^b dx\, J(x)\ln(|z+x|/|z-x|)\ , \tag{C3}$$

$$K_<(z) = \int_a^{z-\epsilon} dx\, J(x)\ln(|z+x|/|z-x|)\ . \tag{C4}$$

This gives a result accurate to $O(\epsilon)$.

(2) To calculate the principal-value integral

$$I = \mathrm{P}\int_a^b dz\, G(z)(\omega_+ - \omega_0)^{-1}\ , \tag{C5}$$

we write $\omega_+(z)$ in the form

$$\omega_+(z) = \omega_0 + c_0^{-1}(z-z_0) + O((z-z_0)^2)\ , \tag{C6}$$

then

$$I = c_0 G(z_0) \ln\left|\frac{(b-z_0)(a+z_0)}{(b+z_0)(a-z_0)}\right| + \int_a^b dz \left[\frac{G(z)}{\omega_+ - \omega_0} - \frac{2z_0 c_0 G(z_0)}{z^2 - z_0^{\,2}}\right]. \tag{C7}$$

The denominator $z^2 - z_0^{\,2}$ is chosen over $z - z_0$ to effect a faster convergence of the second integrand in the large-z region.

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