Hadron-hadron collisions at extreme energies: Light-cone QCD with an axial-vector anomaly current and an infrared fixed point

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We argue that QCD with the maximum number of fermions allowed by asymptotic freedom provides a "parton-model" description of *soft* high-energy collisions. That is, infinite-momentum quantization can be based on the perturbative vacuum and yet produce confinement and chiral-symmetry breaking. A first-stage infrared construction gives SU(2) gauge invariance and confinement. An infrared fixed point produces transverse-momentum scaling and associated infrared divergences which couple to an anomaly-current component of a Fock-space wave function. The divergences factor on to color-zero states allowing the *wave-function zeros* needed by transverse gauge invariance. Parton interactions are dominated by fermion-loop anomalies coupled to the divergences. As a result a pion has a *vector* valence-quark component. The infrared limit giving SU(3) gauge invariance is argued to be accompanied by critical-Pomeron high-energy behavior and spontaneous chiral-symmetry breaking, but is only briefly discussed in this paper.

I. INTRODUCTION

Feynman's original motivation[1] for the parton model was that the simplicity of field theory at infinite momentum ought to naturally describe the universal high-energy "soft"-physics phenomena that he saw emerging from experiment. Ironically, while over a decade of experimental results, reaching up to $\bar{p}$-p collider energies, has strongly reinforced this motivation, parton-model ideas are now motivated theoretically by QCD and are almost exclusively applied to high-energy hard-physics phenomena. In fact, QCD provides its own strong motivation for a "parton" description of high-energy soft physics in that the vector nature of the parton (quark-gluon) interaction should ultimately explain the fundamental twenty-year-old mystery of the approximate energy independence of high-energy hadronic total cross sections.

This paper began as an attempt to translate my recent work[2] on diffraction scattering in QCD, which has been based on multidispersion—Regge-theory methods, into a more widely accessible field-theoretic language. For reasons I shall explain, I gradually realized that my central goal of determining under what conditions (if any) critical-Pomeron scaling[3] is the high-energy limit of QCD could be translated as determining under what conditions (if any) Feynman's original formulation of the parton model can be applied to study high-energy soft physics in QCD. Effectively then this latter goal can be thought of as the central theme of the paper.

It is an old story that vacuum structure simplifies in field theory at infinite momentum.[4] What Feynman wanted to exploit was the consequent appearance of a well-defined (parton) wave function in this limit. The physics of high-energy hadron collisions was to be a consequence of simple natural properties of such wave functions. In a sense the aim of this paper is to try to realize this idea as closely as possible in deriving the high-energy behavior of QCD. However, since the partons of QCD are quarks and gluons, to introduce a parton wave function we must somehow quantize the theory at infinite momentum using the "perturbative" quark-gluon vacuum. With our present ideas on how the hadronic solution of QCD emerges, this may seem an extraordinary goal to pursue.

In a gauge theory, such as QCD, there is nevertheless an immediate simplification[5,6] that seems to ideally suit our purpose. Choosing the special light-cone gauge

$$A_{+}=0 \tag{1.1}$$

and using light-cone coordinates, it is straightforward to show that the perturbatively defined vacuum state is an eigenstate of the complete light-cone Hamiltonian P_{-}. For a non-Abelian gauge theory this is in sharp contrast to the behavior of the perturbative vacuum with respect to the complete Hamiltonian in all other gauges.[7] Light-cone quantization is therefore used to define[8] the simple Fock-space hadronic wave functions which dominate much of perturbative QCD. But can such wave functions be those required by Feynman to describe soft high-energy collisions? In fact, if they are not, can the use of such wave functions be justified in any application of perturbative QCD?

Clearly, if we can obtain a simplified vacuum by a special choice of gauge, then either all the complexities normally associated with the vacuum must be transferred to the description of the states, or else, as seems more likely, the vacuum we have found is not the true vacuum and it must be unstable. In a conventional quantization of QCD it is nonperturbative properties of the vacuum which are thought to produce both confinement and chiral-symmetry breaking. It is natural therefore to anticipate that effects outside of light-cone perturbation theory will simply destabilize the light-cone vacuum. Thorn[9] has argued that at large N [for SU(N) gauge theory], large closed "strings" of flux produce tachyons which are indeed responsible for such a destabilization. Presumably the conventional confinement "bag" with an external nonperturbative vacuum is then reinstated around the quarks inside a hadron.

In fact, we believe we can only possibly succeed with a light-cone-gauge quantization of QCD based on the perturbative vacuum if we limit ourselves to infinite momentum. We shall first discuss this from a heuristic parton-model point of view. Later in this introduction we shall briefly discuss how the deep problems of quantization[7] raised originally by Mandelstam (and Gribov) are encountered and sufficiently resolved at infinite momentum. The issue is discussed much more extensively in Sec. II.

From a heuristic point of view it is perhaps plausible that if a hadron's cross section at infinite momentum is infinite (as experiment suggests), then effectively the bag has expanded to infinite radius and the nonperturbative vacuum outside of it can be ignored. Perhaps then asymptotically rising total cross sections are actually a necessary condition for consistency between confinement and an infinite-momentum quantization based on the perturbative vacuum.

Even supposing that consistency with confinement is possible, spontaneous chiral-symmetry breaking seems to pose an almost insurmountable problem. Since the light-cone perturbative vacuum is simply ignorant of the chiral direction chosen for symmetry breaking, it seems impossible for the constructed states to carry all the consequent properties.

The problem of chiral-symmetry breaking was raised by Kogut and Susskind[10] in the original parton model. They emphasized the necessity of wee partons[1] for its resolution as well as the special properties the wee partons have to carry. Essentially the wee partons must behave just like "vacuum-produced" particles. Feynman was at least partially aware of this since he made[1] the "bold assumption" that "the wee-parton distribution is the same for all hadrons." Since (by definition) groups of wee partons can remain at rest when a hadron travels with infinite momentum, *if they are present and if they are universal,* they will have all the properties of vacuum-produced particles and can carry information, such as the direction of chiral-symmetry breaking, normally carried by the vacuum.

Feynman envisaged[1] the wee partons as manifest in the central region of the rapidity axis of hadron-hadron collisions. Hadrons produced in this region arise directly from the collisions of the wee partons in both initial hadrons (in the center of mass). The presence of wee partons therefore implies that the cross section for producing central-region particles certainly does not go to zero at infinite energy. In addition, if the total cross section rises, then so will the central plateau. That is, for N central-region particles with rapidities $y_1, \ldots, y_N$,

$$\left[\frac{d^N\sigma}{dy_1 dy_2 \cdots dy_N}\right]_{y_i \sim y/2} \underset{y\to\infty}{\longrightarrow} \infty\ , \tag{1.2}$$

where y is the total rapidity. *If the wee partons are truly universal,* then in addition there should be a complete factorization of the differential cross section giving (1.2). That is, if $P_T^1, \ldots, P_T^N$ are the corresponding transverse momenta, then

$$\frac{d^{3N}\sigma}{dy_1 d^2P_T^1 \cdots dy_N d^2P_T^N} \underset{y\to\infty}{\longrightarrow} f_A f_B y^\gamma D(y_2-y_1, \ldots, y_N-y_{N-1}, P_T^1, \ldots, P_T^N)\ , \tag{1.3}$$

where γ is some universal power and D is completely independent of the scattering hadrons A and B. Only the overall normalization factor $f_A f_B$ depends on A and B and even this clearly must factorize.

The requirements (1.2) and (1.3) are so strong as exact asymptotic statements that there is only one known high-energy theory which satisfies them. They are not satisfied by any known model (either directly based on field theory[11] or Reggeon-field-theory based[12]) which gives Froissart-bound behavior for the total cross section. Equations (1.2) and (1.3) can actually be regarded as defining properties for the critical-Pomeron solution of the Reggeon field theory.[3] The factorization property (1.3) requires *single* Regge-pole behavior (the Pomeron) for diffraction scattering, while (1.2) requires intercept one—the critical Pomeron.

In the above we have directly argued that (1.2) and (1.3) are necessary conditions for the wee partons of QCD to allow light-cone quantization based on the perturbative vacuum. We conclude therefore that the conditions for the light-cone derivation of the parton model in QCD may actually be the same conditions that produce critical-Pomeron high-energy behavior. We wish to make a direct case for this result using only field-theoretic arguments and beginning from the description of the limiting wave function for a hadron at infinite momentum.

As will become clear, the present paper is devoted more to developing and formulating language, ideas, and arguments than to presenting the completed case. We particularly wish to communicate how many different ingredients of modern field theory can be seen coming together to provide a logical basis for attacking the (at first sight perhaps awesome) task of understanding high-energy soft physics in QCD. We believe the framework we are setting up promises that a combination of sophisticated high-energy dispersion theory[2,13] and all the power of still-developing field-theory techniques[14] will allow the subject to be brought firmly and indisputably under control in the foreseeable future.

If we can establish that the critical Pomeron and the infinite-momentum perturbative vacuum can be brought together in QCD (under the conditions we shall describe) then many exciting results will follow. The argument we shall develop, that under certain conditions we can move confinement and chiral-symmetry breaking out of the vacuum at infinite momentum, will also imply that a further infinitesimal variation of parameters will (partially) remove these properties from the S matrix. This amounts to showing that the phase-transition phenomenon associated with the critical Pomeron in QCD is actually a combination of deconfinement and loss of spontaneous chiral-symmetry breaking. That gluons are therefore "almost" in the high-energy S matrix *as physical particles* will be the deep explanation we are seeking of energy-independent (and ultimately rising) total cross sections. Critical-Pomeron scaling will, of course, also explain all the logarithmic scaling properties observed at the $\bar{p}$-p collider.[2]

Clearly, achieving a parton description of soft high-energy physics should also maximize the validity of the parton-model description of high-energy hard-physics phenomena in QCD. On a technical level, establishing a parton-model result in perturbative QCD amounts to proving a factorization of infrared-sensitive quantities. Establishing the factorization property (1.3) in the highly infrared-sensitive Regge region goes far beyond the factorization required for a parton description of such phenomena as jet cross sections, higher-twist contributions to the Drell-Yan process, large-angle elastic cross sections, etc. We suspect therefore that if the standard assumptions of perturbative QCD do not lead to the necessary factorization, it may nevertheless follow if the hadronic states, which we shall show lead to (1.3), are used.

We shall argue that the conditions necessary for critical-Pomeron behavior in QCD are not only that the gauge group should be SU(3), but in addition the quark content must be very restricted—in a way that could be of significance also for the electroweak theory. It is actually necessary to saturate (in QCD) the asymptotic-freedom constraint on the number of fermions. By far the most plausible way to do this is to have three generations of color-triplet quarks and one generation of color-sextet quarks. The Goldstone bosons produced by the chiral condensation of the sextet quarks can then be responsible for the Higgs mechanism of the electroweak theory.[15] This possibility has many virtues simply from the point of view of electroweak theory.[16] From the QCD viewpoint we are discussing, its virtue is clearly that it justifies a true parton model of quarks and gluons, which we should point out also has the property (virtue?) that the "running" of the coupling constant (a phenomenon so far unobserved experimentally) would be essentially absent.

The field-theoretic derivation of the above results, which this paper will be a starting point for, involves many subtleties, as we shall now describe. The basic demand that we begin with a high-energy formalism that is as close to perturbation theory as possible leads us both to build up the gauge invariance by stages and also to prevent the uncontrollable (from our point of view) infrared growth of coupling constants by infrared fixed points (of the corresponding massless theory). In building up the gauge invariance we implement the general principle of complementarity derived from lattice gauge theory,[17] since we use the Higgs mechanism to regulate the theory in the infrared. Complementarity states that if we use fundamental representation Higgs scalars, then we will not encounter a phase transition in going from the perturbative Higgs region of parameter space to the confinement region. Initially therefore we have two scalar triplets of SU(3) color. The decoupling of one triplet produces an SU(2) gauge symmetry and can be regarded as a substantial infrared problem in itself. Indeed, it is the construction of this SU(2) theory which is really the central concern of this paper. In our formalism SU(2) gauge invariance produces *confinement but not chiral-symmetry breaking*. We should emphasize that this is not a general result but depends specifically on our addition (from the outset) of the full quark structure discussed above. This structure produces[18] *asymptotic freedom and infrared fixed points for both the gauge coupling and the Higgs-boson coupling*. As a result we obtain a theory which is surely as close to the perturbation expansion as is possible for a theory with an unbroken non-Abelian gauge invariance. Not surprisingly, the dominant perturbative problem, at infinite momentum, of such a theory is the analysis of infrared transverse-momentum singularities.

Nonperturbative properties of the theory nevertheless play an important role. The essence of our construction of the high-energy SU(2) theory is the introduction of an axial-vector anomaly current describing the "nontrivial topological" part of the "classical field" in the wave function of an infinite-momentum hadron. This current then couples to quark-loop anomalies in the vertices of transverse momentum diagrams describing high-energy scattering. (Such "anomalous" interactions are, we believe, the infinite-momentum analog of instanton-produced interactions at finite energy.) In this way the full phenomenon of fermion-loop anomalies[19] for axial-vector currents enters our analysis. We do not yet have a comprehensive analysis of all possible anomalous interactions that occur, but we understand their general significance as follows.

If there is an infrared fixed-point (of the corresponding massless fermion theory), anomalous interactions produce a class of transverse-momentum infrared divergences which do not exponentiate but rather factorize into a single infinity for each color-zero state. Consequently, the S matrix for color-zero states (with an anomaly-current component) is infinite relative to other amplitudes and this is what produces confinement in our formalism. (It is possible that the infinity can be traced to the sum over topological charges in the classical field of a hadron, although we are not confident of this.) The effect of the infinity is to allow us to introduce a normalization zero for each external color-zero state. This is an infrared zero which is actually the wave-function zero which Mandelstam[6] has emphasized is necessary to achieve the additional gauge invariance remaining after a linear gauge condition such as (1.1) is imposed. It is a special property of infinite-momentum quantization that this zero can be regarded as a normalization factor for individual Fock-space states. Its effect is to produce zero S matrix except for those states whose S matrix initially contains infrared divergences produced by the anomaly current. Thus we are arguing that at infinite momentum and in the special case of an infrared fixed point, the anomaly current produced by the topological properties of gauge fields sufficiently resolves the ambiguities of quantization to produce confinement of SU(2) color in the high-energy S matrix. The nontrivial topological effects are, however, transferred from the vacuum to the states and so there is no ambiguity as to how to develop a (high-energy) unitary theory.

The anomaly current is, for our purposes, a complete description of the nonperturbative aspects of SU(2) gauge invariance, provided there are additional massive vector gluons in the theory. The high-energy behavior of confined hadrons is given by the exchange of the *even-*

signature combination of the anomaly current and an SU(2)-singlet (perturbatively) Reggeized gluon. The infrared divergences pick out the zero-transverse-momentum part of the anomaly current. Consequently, the singlet gluon Regge-pole trajectory gives the high-energy t dependence of amplitudes. That is, there is a Pomeron—an even-signature Regge pole with intercept below one—in the "vacuum" channel. At this stage there is no rising cross section, but *the factorization property (1.3) is satisfied.* Since the intercept of the Pomeron is below one, it is possible to systematically develop the complete set of Reggeon diagrams describing multiple-Pomeron exchange with Pomeron interactions included. A complete categorization of anomalous interactions is certainly required for this and so in this paper we shall do it only in outline.

The second-stage construction of SU(3) gauge invariance, which is achieved by removing the second triplet of Higgs scalars, is also something we shall do in outline only. It is this infrared limit, enlarging the gauge invariance from SU(2) to SU(3), which (when the full fermion structure is present) we believe simultaneously produces critical-Pomeron high-energy behavior and chiral-symmetry breaking. In the SU(2) theory, there are parity partners for all hadron bound states—including the pion. To study the effects of the SU(3) limit on hadrons we need to revert to the Regge language of our previous analysis[2] since our hadron bound states interact as "Reggeons" with the Pomeron. Such interactions become vital as the Pomeron intercept moves to one. In particular, such interactions produce a branch point in all hadronic Regge trajectories at zero momentum transfer which is well known[20] to be able to hide parity-partner particles. We believe this is the technical realization of the breaking of chiral symmetry by wee partons discussed heuristically above, although we will not attempt to make this connection explicit in this paper.

If the gauge symmetry were larger than SU(3) there would be more than one Pomeron Regge pole and the factorization property (1.3) would be violated. The wee-parton distribution would therefore be nonuniversal. We have no idea whether this implies the instability of the light-cone vacuum, but it may well do. We have emphasized that in order to build a high-energy parton theory we are forced to stick as closely as possible to perturbation theory. With the *minimum* gauge symmetry of SU(2) and all couplings protected from blowing up by infrared and ultraviolet fixed points, we are just able to handle the nonperturbative part of the theory at infinite momentum using the anomaly current. At this stage (assuming our analysis goes through in full) we are able to obtain confinement without having to simultaneously handle chiral-symmetry breaking. Finally, we anticipate that we are just able to reach SU(3) gauge symmetry and chiral-symmetry breaking with the full weaponry of critical-Pomeron theory. Beyond this we cannot go. We therefore believe that QCD "saturated with fermions" is the unique theory able to produce a parton (quark-gluon) description of high-energy scattering and yet be consistent with confinement and chiral-symmetry breaking. All the universal scaling features of critical-Pomeron behavior[2] which are emerging from high-energy experiments are the outcome of a very special field theory at infinite momentum.

Section II begins with a qualitative description of the relation between classical fields and line integrals and discusses how the ambiguity of intersecting line integrals at infinite momentum can produce an anomaly-current contribution in a hadronic wave function. We then go through conventional light-cone-gauge quantization and show how it can be modified to introduce the anomaly current. As a result we define "generalized" light-cone Fock-space states at infinite momentum which have a conventional perturbative quark and gluon content but also contain additional zero-transverse-momentum gluons produced by the anomaly current. An important consequence of the presence of such gluons is that the valence-quark and -antiquark component of a pion forms a vector rather than a pseudoscalar state.

Section III is devoted to describing how the anomaly-current component of generalized Fock-space states scattering at infinite momentum generates transverse-momentum diagrams whose vertices contain fermion-loop axial-vector anomalies. We also discuss some particular transverse-momentum diagrams which will contribute to hadron-hadron scattering. The infrared analysis of transverse-momentum diagrams is carried out in Sec. IV. We discuss how the exponentiation of Reggeization in fact suppresses most infrared regions of transverse-momentum diagrams. The anomaly current, however, couples to divergences which persist if transverse-momentum interaction kernels have either the scaling properties of leading-logarithm perturbation theory or the more general scaling properties produced by an infrared fixed point. We discuss how such divergences factorize onto external states and produce confinement with the wave-function zeros discussed above.

Section V is devoted to discussing how the results of the previous sections allow the construction of the complete set of Reggeon diagrams for high-energy scattering in the SU(2) theory [that is, QCD "saturated" with fermions and with the gauge symmetry initially broken to SU(2) by one triplet of Higgs scalars]. Finally, we briefly describe how restoring the gauge symmetry to SU(3) produces the critical Pomeron and discuss the relationship of confinement and chiral-symmetry breaking to general phase-transition analysis in Reggeon field theory.

II. INFINITE-MOMENTUM WAVE FUNCTIONS WITH AN ANOMALY CURRENT

We begin with a semiheuristic discussion of how we can hope to construct a hadronic state at infinite momentum using the perturbative vacuum. The discussion is implicitly functional-integral based and has only an indirect relation to the formal analysis which follows.

In QED it is well known that the simplest gauge-invariant quantity which can describe an electron is the nonlocal operator

$$\Psi(x)\exp\left[\int_{P(x)} dx' A(x')\right], \tag{2.1}$$

where $P(x)$ is some path from the point x to infinity. The path-ordered exponential (when averaged over paths) gives an approximation to the coherent state of photons[21] or "asymptotic classical field"[22] which surrounds an isolated electron due to the long-range photon force. Equivalently it can be regarded as an approximation to the exponentiation of infrared divergences[23] which must be factored out of (pointlike) electron-scattering amplitudes to give finite results for the scattering of physical electrons. We therefore have three equivalent ways of viewing the origin of (2.1), that is gauge invariance, a classical field or infrared divergences in perturbation theory.

In a non-Abelian theory it is known that there are indeed infrared effects which can be absorbed by attaching line integrals of the form (2.1) to free quarks. In sophisticated perturbative calculations it is actually very important to unravel such effects in defining quark-distribution functions.[24] However, it is clear that (2.1) cannot be an approximation to a true asymptotic field of a quark in analogy with QED. At the classical level non-Abelian gauge fields carry topological properties which cannot be adequately represented by the free gluons created by (2.1). Such an expression could at best approximate the trivial topological classical sector.

In practice we expect that summing perturbation theory is ambiguous because of infrared effects associated with the vacuum.[25] Therefore we expect the relation of perturbation theory to expressions such as (2.1), and indeed the very concept of an asymptotic quark field, to be ambiguous at distance scales where confinement and the associated nonperturbative vacuum properties are important. In a conventional "baglike" picture of a hadron, as illustrated in Fig. 1, there is a region of "perturbative vacuum" containing the quarks which is separated from the nonperturbative vacuum by the bag surface. The common belief is that fields with nontrivial topological properties are irrelevant in this picture. In the nonperturbative vacuum they are believed to be completely suppressed. Inside the bag gauge invariance presumably requires an "effective classical field" for a quark far from the surface. However, asymptotic freedom is believed to keep the gauge coupling so small that the effects of nontrivial field topologies are suppressed (by e^{-1/g^2}, of course) except possibly near the bag surface. Therefore it is sufficient to satisfy gauge invariance by attaching line integrals constructed perturbatively to the free quarks inside the bag. From the perturbative viewpoint it should be equivalent to take single line integrals disappearing into the bag surface as illustrated in Fig. 2(a) or line integrals joining a quark and antiquark as illustrated in Fig. 2(b).

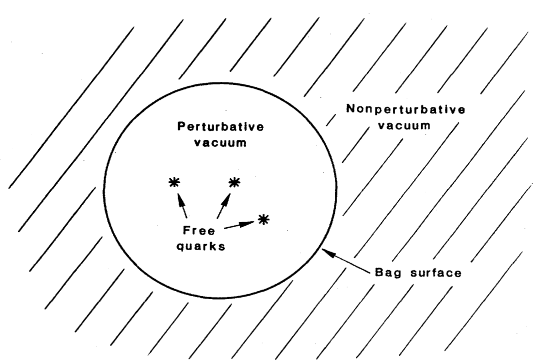

FIG. 1. The conventional "bag model" of a hadron.

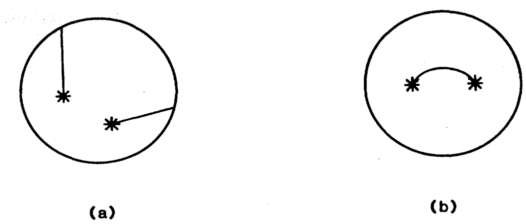

FIG. 2. Paths of line integrals inside the bag.

If it is possible, under some circumstances, to avoid the nonperturbative vacuum at infinite momentum, then presumably we can suppose that the bag expands to infinite radius in the above picture. (Infinite cross sections at infinite energy are clearly suggestive of this). If we are to then obtain anything besides simple perturbation theory at infinite momentum the "classical field" inside the bag must acquire some significant complexity—with the nontrivial topology of gauge fields perhaps playing a more important role. We can understand this in terms of line integrals as follows.

Lorentz contraction as we pass to infinite momentum compresses the three-dimensional interior of the bag onto the two-dimensional transverse plane. As a consequence the set of paths $P(x)$ which are attached to distinct quarks and yet intersect increases from being effectively a set of zero measure to being a finite fraction of the complete set of paths. In particular, some fraction of the straight-line paths (for which the line integrals are straightforward to extract from perturbation theory) which do not intersect in three dimensions will necessarily intersect when projected on two dimensions. Consequently, perturbation theory at infinite momentum potentially acquires a new and more substantial ambiguity since it cannot distinguish between paths that cross and paths that do not.

Consider two paths, as shown in Fig. 3(a), which do not intersect at finite momentum but give a crossing at infinite momentum as shown in Fig. 3(b). Perturbation theory at infinite momentum cannot distinguish the crossed paths from the uncrossed paths shown in Fig. 3(c). To convert the finite-momentum paths of Fig. 3(a) to paths giving the uncrossed paths of Fig. 3(c) at infinite momentum, we should insert line integrals of the form shown in Fig. 4. Approximating such integrals by infinitesimals, we will be inserting

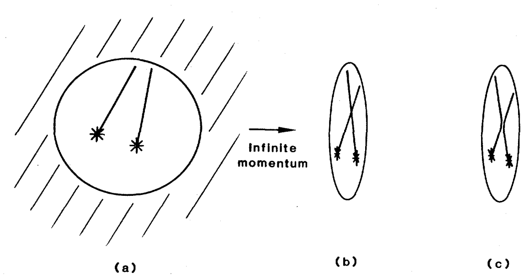

FIG. 3. Lorentz contraction of line integrals at infinite momentum.

$$[1+dx_1\cdot A(x)][1+dx_2\cdot A(x)][1+dx_3\cdot A(x)]-[1+dx_3\cdot A(x)][1+dx_2\cdot A(x)][1+dx_1\cdot A(x)] \quad (2.2)$$

$$\sim dV\,\epsilon_{0\mu 2\nu}A_\mu(x)A_2(x)A_\nu(x)+\cdots\ , \quad (2.3)$$

where dV is a three-dimensional volume differential since dx_1, dx_2, and dx_3 are (partially) orthogonal differentials.

If (2.3) could not give a gauge-invariant contribution when summed over all relevant paths, that is in effect when integrated over a three-dimensional volume, then clearly it could not give a significant effect. In fact, the nontrivial topologies of non-Abelian gauge fields can indeed give a significant effect in just this way, as we now discuss.

The topological content of a gauge-field configuration is measured by a winding-number operator constructed from the gauge-dependent axial-vector anomaly current[26]

$$K^\mu(x)=\frac{g^2}{8\pi^2}\epsilon_{\mu\alpha\beta\gamma}\mathrm{Tr}\left[A_\alpha\partial_\beta A_\gamma-\frac{2ig}{3}A_\alpha A_\beta A_\gamma\right]\ . \quad (2.4)$$

This current has the well-known property

$$\partial_\mu K_\mu=F\widetilde{F}=\partial_\mu J_\mu\ , \quad (2.5)$$

where J_μ is the usual U(1) axial-vector current. If K_μ is integrated over a three-dimensional volume, we obtain a *gauge-invariant* zero-momentum component of K_μ which defines a winding number. If the field A is pure gauge, then only the second term in K_μ contributes to the winding number. But this term has precisely the form of (2.3) apart from the volume differential. Therefore, pure gauge fields *with nontrivial topology* do give a gauge-invariant contribution of the form (2.3) to a three-dimensional volume integral. Consequently topological gauge fields will contribute distinctively to (averages over) crossed and uncrossed line integrals. We therefore introduce the operator

$$K_-(x_+)=\frac{-ig^3}{12\pi^2}\epsilon_{+\alpha\beta\gamma}\int d\vec{x}_\perp dx_-\,\mathrm{Tr}\,\epsilon^{ijk}A^i_\alpha A^j_\beta A^k_\gamma \quad (2.6)$$

$$=K_-(x_+,P_+=0,\vec{P}_\perp=\vec{0}) \quad (2.7)$$

as a potential measure of the contribution of topological gauge fields to the infinite-momentum classical field. A nonzero value of K_- is obtained for gauge fields with nontrivial topological properties in the three dimensions orthogonal to x_+.

The conclusion of our semiheuristic discussion is then that perturbation theory at infinite momentum will not distinguish crossed and uncrossed line integrals of the form shown in Fig. 3. Consequently, a complete description of the infinite-momentum limit of the full set of line integrals inside a bag requires the addition of the anomaly-current operator (2.7) to perturbative operators used to create states directly at infinite momentum. By keeping only the effects of pure gauge fields we will be making the minimal modification of infinite-momentum perturbation theory. Note that if line integrals are directed, as they would be if we consider SU(N) gauge theory with $N\geq 3$, then the manipulation of Fig. 4 and hence (2.3) is not sufficient to account for the ambiguity of intersections. It is very important therefore that we first consider SU(2) gauge invariance only. (It is possible that rather than discussing the relationship between intersections of line integrals and the anomaly current we should instead consider windings of line integrals around quarks. Such windings can also be viewed as a consequence of Lorentz contraction and are indeed well defined only in the transverse plane. As illustrated in Fig. 5, there clearly is a close relationship between windings and intersections for transverse line integrals connecting two quarks.)

We will clearly want to bring K_- into our light-cone-quantization procedure. To discuss this we first give a brief resumé of conventional light-cone quantization[5,6,9] as follows. Having imposed the gauge condition (1.1), we choose x_+ as the evolution parameter for the "Hamiltonian" P_- defined in terms of gluon and quark operators as

$$P_-=-\tfrac{1}{2}\int dx_-d\vec{x}_\perp[\mathrm{Tr}F_{+-}{}^2+\mathrm{Tr}F_{12}{}^2]+P_{-\,\mathrm{quarks}}\ , \quad (2.8)$$

where $F_{+-}=\partial A_-/\partial x_-$ is a function of the transverse operators A_r ($r=1,2$) satisfying

$$\frac{\partial F_{+-}}{\partial x_-}=-\frac{\partial}{\partial x_-}\nabla\cdot\vec{A}-g[A_r,\partial_-A_r]+j_{+\,\mathrm{quarks}}\ . \quad (2.9)$$

A linear gauge condition such as (1.1) does not fix the gauge sufficiently in a non-Abelian theory. The generator of x_--independent gauge transformations which leave (1.1) unchanged is the $P_+=0$ component of (2.9), that is

$$G(\vec{P}_\perp)=[P_+{}^2A_-(P_+,P_\perp)]_{P_+=0}\ . \quad (2.10)$$

The light-cone equivalent of the infrared problems found by Mandelstam and Gribov in other gauges,[7] is to invert (2.9) to define A_- while satisfying

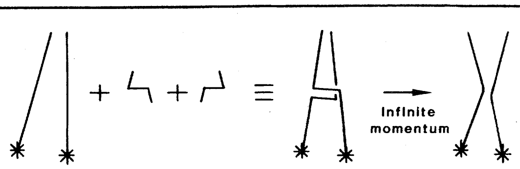

FIG. 4. Conversion of infinite-momentum crossed paths to uncrossed paths.

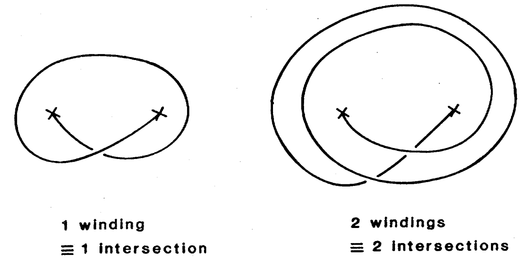

FIG. 5. Illustration of the relationship between intersections and windings for line integrals in a plane.

$$G(P_\perp)\,|\,S\rangle=0 \tag{2.11}$$

for all physical states $|\,S\rangle$.

The Hamiltonian quantization proceeds by introducing creation and destruction operators through the decomposition

$$A_r(\vec{x},x_-)=\frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{dP_+}{\sqrt{2P_+}}[\,a_r(\vec{x},P_+)e^{-ix_-P_+} - a_r^\dagger(\vec{x},P_+)e^{ix_-P_+}] \tag{2.12}$$

and the commutation relations

$$[a_r(\vec{x},P_+),a_s^\dagger(\vec{y},Q_+)]=\delta_{rs}\delta(\vec{x}-\vec{y})\delta(P_+-Q_+)\ . \tag{2.13}$$

(We have left implicit the color-matrix character of the A_r and the a_r.)

Note that there is a particular danger of an infrared divergence at $P_+=0$ arising from (2.12). The vacuum defined by

$$a_r(\vec{x},P_+)\,|\,0\rangle=0,\quad r=1,2 \tag{2.14}$$

satisfies (2.11) and is also an eigenstate of P_- (from which "energies" can be measured) due to the particular feature that

$$\{[A_r,\partial_-A_r]\}_{P_+=0}=\int_{-\infty}^{\infty} dx^-[A_r,\partial_-A_r] \tag{2.15}$$

has no terms quadratic in its creation operators. When acting *on a general Fock state* (2.15) will give a finite result and hence *(2.11) will not be satisfied.*

Even when (2.11) is satisfied it is not sufficient to ensure that P_- is finite. Suppose that for some matrix elements (2.11) is satisfied by

$$\langle S'\,|\,A_-\,|\,S\rangle \underset{P_+\to 0}{\sim} P_+{}^{-\delta},\quad \delta<2\ , \tag{2.16}$$

then since

$$P_-\sim\int dx_-\left[\frac{\partial A_-}{\partial x_-}\right]^2 , \tag{2.17}$$

$$\langle S\,|\,P_-\,|\,S\rangle\sim\sum_{S'}\int\frac{dP_+}{\sqrt{P_+}}\frac{dQ_+}{\sqrt{Q_+}} \times\delta(P_+-Q_+)\langle S\,|\,P_+A_-(P_+)\,|\,S'\rangle \times\langle S'\,|\,Q_+A_-(Q_+)\,|\,S\rangle \tag{2.18}$$

$$\sim\int\frac{dP_+}{P_+{}^{2\delta-1}}<\infty \text{ only for } \delta<1\ . \tag{2.19}$$

Conventionally[6,9] all problems with $P_+=0$ are eliminated by strongly overimposing (2.11) in the form

$$A_-(P_+=0,P_\perp)=0\ . \tag{2.20}$$

That is, $A_-(P_+=0,P_\perp)$ is removed altogether as a degree of freedom in the theory. Technically this can be done[6,9] by discretizing the $P_\perp$ axis and removing the point $P_+=0$, with the continuum limit taken at the end of a calculation. This procedure is argued to lead to conventional perturbation theory for gauge-invariant quantities and so can be regarded as good or as bad as any other derivation of the perturbation expansion. Presumably the "perturbative" vacuum defined by (2.14) is, in general, the wrong vacuum even though it is an exact energy eigenstate. The work of Thorn[9] at large N indicates that the nonperturbative states associated with large closed strings of flux will produce tachyons and so destabilize the vacuum as we expect. In the language of our earlier heuristic discussion this is, of course, the reinstatement of the nonperturbative vacuum outside the bag that is associated with confinement.

In our above resumé of light-cone quantization we have made no mention of infinite momentum. The light-cone gauge (1.1) is simply a special (particularly singular) gauge choice. The positivity condition on P_+ does, however, imply that any state with $P_+=0$ also has $P_+=0$ for all constituents. Since any relativistically invariant state with mass M has

$$P_+=\frac{P_\perp{}^2+M^2}{P_-}\underset{P_-\to\infty}{\longrightarrow}0 \tag{2.21}$$

states with infinite momentum have $P_+=0$ for all constituents. Consequently handling the constraint (2.11) is of crucial importance in defining states at infinite momentum.

Considering again K_-, we can define

$$K_-^\infty\equiv K_-(P_-=\infty) =\epsilon_{+\alpha\beta\gamma}\epsilon^{ijk}(A_\alpha^iA_\beta^jA_\gamma^k)_{P_-=\infty,P_+=0,\vec{P}_\perp=0} \tag{2.22}$$

in terms of the on-shell operators satisfying (2.9), (2.12), and (2.13). If we wish to have states $|\,S\rangle$ in our quantization for which

$$\langle S\,|\,K_-^\infty\,|\,S\rangle\neq 0\ , \tag{2.23}$$

as we would expect for the intersecting line-integral configurations discussed above, then

$$\epsilon^{ijk}\epsilon_{+\alpha\beta\gamma}\int_0^\infty\frac{dP_+^i}{\sqrt{P_+^i}}\frac{dP_+^j}{\sqrt{P_+^j}}\frac{dP_+^k}{\sqrt{P_+^k}}\int d^2\vec{P}_\perp^i d^2\vec{P}_\perp^j d^2\vec{P}_\perp^k\delta(P_+^i+P_+^j+P_+^k)\delta^2(\vec{P}_\perp^i+\vec{P}_\perp^j+\vec{P}_\perp^k) \times\langle S\,|\,A_\alpha^i(P_+^i,\vec{P}_\perp^i)A_\beta^j(P_+^j,\vec{P}_\perp^j)A_\gamma^k(P_+^k,\vec{P}_\perp^k)\,|\,S\rangle\neq 0\ . \tag{2.24}$$

Since

$$P_+^i+P_+^j+P_+^k=0 \text{ and } P_+^i,P_+^j,P_+^k\geq 0\ , \tag{2.25}$$

we must have

$$P_+^i=P_+^j=P_+^k=0\ . \tag{2.26}$$

A detailed discussion of behavior at this point shows that to get a nonzero result for (2.23) we need behavior for matrix elements at least as singular as

$$\langle S'|A^i_\alpha|S\rangle \underset{P^i_+\to 0}{\sim} (P^i_+)^{-1/2-2\nu} ,$$
$$\langle S'|A^j_\beta|S\rangle \underset{P^j_+\to 0}{\sim} (P^j_+)^{\nu} , \qquad (2.27)$$
$$\langle S'|A^k_\gamma|S\rangle \underset{P^k_+\to 0}{\sim} (P^k_+)^{\nu} .$$

In addition, the $\epsilon_{+\alpha\beta\gamma}$ factor in K^∞_- implies that one of α,β,γ must be $-$. Therefore if we choose $\alpha=-$, then β and γ will be transverse suffices in (2.27). If the transverse operators for a single gluon disappear from our theory at infinite momentum, as we anticipate, then presumably $\nu>0$. However, it is clear that whether $\nu>0$ or $\nu<0$ we cannot satisfy (2.23) by imposing (2.20).

Since (2.20) gives standard perturbation theory it is no surprise that it is incompatible with a nonzero value of K^∞_-. Conversely as soon as we attempt to satisfy (2.11) in a more complicated way than (2.20) we go beyond perturbation theory. We shall do this in the simplest possible way. We shall consider "generalized" Fock-space states

$$|S_F\rangle = \prod_{p,r,s} a^\dagger_p(\vec{k}_p) b^\dagger_r(\vec{k}_r) \bar{b}^\dagger_s(\vec{k}_s) K^\infty_- |0\rangle , \qquad (2.28)$$

where the $a^\dagger_p$, $b^\dagger_r$, and $\bar{b}^\dagger_s$ are, respectively, gluon, quark, and antiquark creation operators. K^∞_- is defined in terms of creation and destruction operators by using (2.9) to define

$$A_-(P_+=0,\vec{P}_\perp)=\left[\frac{1}{P_+{}^2}\{-P_+(\vec{P}_\perp\cdot\vec{A})-[A_r,\partial_-A_r] + j^+_{\text{quarks}}\}\right]_{P_+=0} . \qquad (2.29)$$

We have noted that as soon as we allow $A_-(P_+=0,\vec{P}_\perp)$ to be nonzero, then a simple Fock-space state, containing a finite number of quarks and gluons, will not satisfy (2.11). After a finite order of perturbation theory we will, of course, have a general mixture of quarks, antiquarks and gluons, and so although K^∞_- is completely gauge invariant the perturbative content of our Fock-space state will in general not satisfy the transverse gauge-invariance condition (2.11). However, *if we consider only Fock-space states with strictly infinite momentum,* or equivalently $P_+=0$, we can regard (2.11) as effectively a normalization condition. That is, whereas

$$G(\vec{P}_\perp)|S_F\rangle \neq 0 , \qquad (2.30)$$

we formally have

$$[G(\vec{P}_\perp)(P_+{}^{2-\delta/2}|S_F\rangle)]_{P_+=0}=0, \quad \delta<4 . \qquad (2.31)$$

From (2.19) it seems likely that if $|S_F\rangle$ has a finite overlap with a physical state defined from a smooth limit of states with P_- finite, then we will actually have $\delta\le 1$. However, we shall not attempt to seriously evaluate δ.

Apparently we are arguing that if $A_-(P_+=0)$ plays a nontrivial role in our quantization, then infinite-momentum Fock-space states should be normalized to zero and removed from our space of states. In fact, this fate will be avoided by Fock-space states only if they have an infinite S matrix to compensate for the zero normalization factor in (2.30). The presence of K^∞_- in $|S_F\rangle$ will be vital for this. Our semiheuristic understanding of this point based on our earlier discussion, is that states with topological contributions from intersecting line-integral configurations dominate at infinite momentum. Simple Fock-space states *with no topological content* have no overlap with states in the physical Hilbert space.

Our states $|S_F\rangle$ will be parametrized by the usual quark variables at infinite momentum,[8]

$$x_i=\lim(P_{i-}/P_-)$$
$$=\text{longitudinal-momentum fraction} ,$$
$$\vec{k}_i=\text{transverse momentum} , \qquad (2.32)$$
$$S_i=\text{helicity component} .$$

The states $|S_F\rangle$ will differ from conventional Fock-space quark states in that they contain at least one triplet of K_- gluons. Clearly by discussing states with multiple line-integral intersections we could have motivated replacing K^∞_- by $(K^\infty_-)^N$ with N arbitrary, in (2.28). In fact, we shall discover in Sec. IV that it is irrelevant what value of N we choose initially (except that we shall discover that N should be odd).

A vital feature of the states $|S_F\rangle$ follows by noting that although K^∞_- has zero transverse momentum it is the longitudinal component of a vector. Therefore the full Lorentz properties of $|S_F\rangle$ result from both the spin properties of the quarks and the vector character of K^∞_-. In particular, in a pion the valence-quark and antiquark helicities will be aligned rather than opposite as they would be if K^∞_- were zero.

We should also emphasize that our states $|S_F\rangle$ are not the complete Regge pole bound states discussed in Ref. 2. They are Fock-space states which have an overlap with true bound states in the same way that simple Fock-space states have an overlap with positronium.[8]

We go on now to discuss how infinite-momentum states of the form (2.28) can scatter. After this we will be able to discuss how $K^\infty_-\neq 0$ imposes confinement on the quark content of $|S_F\rangle$.

III. TRANSVERSE-MOMENTUM DIAGRAMS WITH ANOMALIES

In the last section we suggested that nonperturbative topological properties of vacuum fields can be transferred to properties of states at infinite momentum. If this is the case, then we might well expect nonperturbative vacuum-produced interactions of quarks (for example, instanton-produced interactions) to be similarly transferred as new interactions between infinite-momentum states. In fact, the K^∞_- gluons in our states will produce infinite-momentum interactions which are directly due to the fermion-loop anomalies[19] of axial vectors. We believe, therefore, that at infinite momentum instanton-produced interactions, etc., can be replaced by anomaly interactions describing how the quarks in one infinite-momentum hadron interact with the anomaly-current component of a second such hadron. Such interactions will dominate our description of high-energy hadron scattering, although our discussion in this section will be only a preliminary analysis of what we believe to be a very extensive phenomenon.

We need to discuss the interactions of two infinite-momentum hadron states traveling in opposite directions. This interaction takes place through wee partons with small or zero rapidity y ($\ln P_-$ in the notation of the last section). While it may eventually be possible to use the language of light-cone Hamiltonian quantization to describe an infinite-momentum wave function "evolving" to small rapidity, we shall not attempt that here. The states $|S_F\rangle$ of the previous section have $P_- = \infty$ strictly and so contain no wee partons. In fact Feynman himself emphasized that the concept of wee partons in a wave function is not strictly well defined. Only the Lorentz-invariant consequences of wee-parton interactions are well defined.

Having defined the infinite-momentum states $|S_F\rangle$ by a Hamiltonian quantization procedure we shall go over to the Lagrangian language of Feynman diagrams for describing the interactions of constituents. This is essentially a matter of convenience since it is well known how to derive high-energy transverse-momentum diagrams[14,27] from such Feynman diagrams. From the parton-model viewpoint transverse-momentum diagrams should be thought of as a technology for simultaneously describing the evolution to zero longitudinal momentum of the parton wave function and the interaction between two such parton systems. In effect, we shall assume that the *only* interactions that can take place over a large rapidity interval are those described by transverse-momentum diagrams. We give first a brief general description of such diagrams before going on to describe the origin of some rather special diagrams needed for our purpose.

Transverse-momentum diagrams for non-Abelian gauge theories are now well understood[14,27] when gluons have a mass provided by the Higgs mechanism. The diagrams (with which we shall be concerned in this paper) describe high-energy gluons propagating and interacting in a single transverse-momentum plane as a function of the rapidity y [$\equiv \ln(P_+P_-)$ in the center of mass for the scattering]. In effect, the remaining longitudinal-momentum variable is integrated out of the underlying Feynman diagrams. In general, many distinct Feynman diagrams generate the same transverse-momentum diagram. Let us first discuss the diagrams for a triplet of gluons with mass M resulting from the breaking of SU(2) local gauge invariance to SU(2) global invariance (referred to below as color) by a Higgs doublet of scalars.

There are three components to the diagrams. First the gluon propagator

$$\Gamma(\vec{k}_\perp{}^2) = \frac{1}{\vec{k}_\perp{}^2 + M^2}\ , \tag{3.1}$$

secondly the kernels

$$A^I_{NN'}(\vec{k}^{\,1}_\perp, \ldots, \vec{k}^{\,N}_\perp, \vec{k}^{\,N+1}_\perp, \ldots, \vec{k}^{\,N+N'}_\perp, M^2, g) \tag{3.2}$$

for a transition of N gluons with total color I to N' gluons with the same total color. We shall not need the $A^I_{NN'}$ in

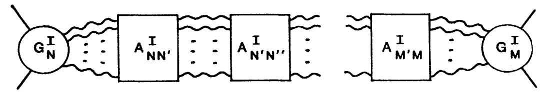

FIG. 6. The general form of transverse-momentum integrals.

this section, although some general properties of these kernels will be very important in the next section. In this section we shall be particularly concerned with the couplings

$$G^I_N \equiv \delta^2(\vec{k}_\perp - \vec{k}_{1\perp} \cdots - \vec{k}_{N\perp})\tilde{G}^I_N(\vec{k}_{1\perp}, \ldots, \vec{k}_{N\perp}, M^2, g)$$

of N gluons, with color I, to one infinite-momentum system.

The complete set of transverse-momentum diagrams for scattering of one fast state traveling with large P_- say, off a state traveling fast with large P_+ is given by putting Γ's, A^I's, and G^I's together in all possible ways, as illustrated in Fig. 6 (with Γ's denoted by wavy lines). We shall define the kernels $A^I_{NN'}$ to be dimensionless so that they include all possible transverse-momentum-conserving δ functions. In this case there is a transverse-momentum integration $\int d^2\vec{k}_\perp$ for each gluon propagator. There is also a factor of y for each propagation of gluons between interactions—apart from the initial propagation which gives a factor $e^y \equiv S$.

Consider first the coupling G^I_1 for a single fast quark to couple to a single slow gluon as illustrated in Fig. 7. A fast quark propagator gives

$$\frac{\gamma P + m}{P^2 - m^2} \underset{P_-\to\infty}{\sim} \frac{\gamma_+ P_- [1 + O(1/P_-)]}{(P^2 - m^2)} \tag{3.3}$$

$$\equiv \frac{\gamma_+}{\left[P_+ - \dfrac{P_\perp{}^2 - m^2}{P_-}\right]}\ . \tag{3.4}$$

For a quark initially and finally on shell we simply remove the $(P^2 - m^2)^{-1}$ factor from (3.3) and so in lowest-order perturbation theory we obtain, before inserting external spinor factors,

$$G^I_{1\mu} \sim C_I (P_-)^2 \gamma_+ \gamma_\mu \gamma_+ \sim \gamma_+ (P_-)^2 \ \text{ if } \gamma_\mu = \gamma_- \tag{3.5}$$

$$= 0 \text{ otherwise}\ , \tag{3.6}$$

where C_I is a color factor which will not concern us for the moment. Note that the spin structure of the scattering quark is conserved, that is there is helicity conservation. Inserting external spinors allows the replacement of one $\gamma_+ P_-$ factor in (3.5) by m giving, finally,

$$G^I_{I\mu} \sim C_I P_- m \delta_{\mu -}\ . \tag{3.7}$$

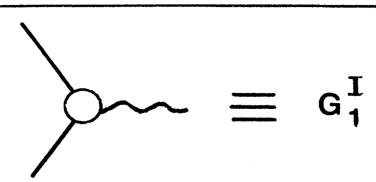

FIG. 7. The coupling of a fast quark to a transverse-momentum gluon.

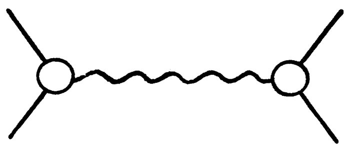

FIG. 8. The "Born" transverse-momentum diagram.

We wish to understand the spin structure of transverse-momentum diagrams without calculating all the relevant Feynman diagrams. For this purpose it will be sufficient to consider gluon exchange via a simple Feynman propagator. Additional gauge-dependent terms will not affect the general points we shall describe. First we note that

$$\frac{g_{\mu\nu}}{k^2+m^2} \sim \frac{g_{\mu\nu}}{\vec{k}_\perp{}^2+m^2} \tag{3.8}$$

provided that

$$k_+k_- \sim 0 \; . \tag{3.9}$$

The simplest of all transverse diagrams, that of Fig. 8 can be derived by combining (3.7) and (3.8) giving

$$C_I{}^2 P_- \delta_{-\mu} \left[\frac{g_{\mu\nu}}{k_\perp{}^2+m^2} \right] \delta_{\nu+} P_+ \sim \frac{P_+P_-m^2}{(k_\perp{}^2+m^2)} \equiv \frac{Sm^2}{(k_\perp{}^2+m^2)} \; . \tag{3.10}$$

Consider next the coupling G_N^I for N gluons in a transverse-momentum diagram to couple to an on-shell fast quark with momentum P_-. We can suppose that (in leading-order perturbation theory) the transverse-momentum diagram has been produced by using P_+ integrations to put each intermediate quark propagator on shell as illustrated in Fig. 9. The denominator is thus removed from (3.4) for each intermediate quark, giving in analogy with (3.5)

$$\tilde{G}^I_{N\mu_1\mu_N} \sim C_N^I \gamma_+ \gamma_{\mu_1} \gamma_+ \cdots \gamma_+ \gamma_{\mu_N} \gamma_+ P_-{}^2 \tag{3.11}$$

$$\sim \gamma^+ P_-{}^2 \; (\equiv mP_-) \text{ if } \mu_1 = \cdots \mu_N = - \tag{3.12}$$

$$= 0 \text{ otherwise} \; . \tag{3.13}$$

So again the quark spin structure is preserved, and using the propagator (3.7) the Feynman diagram of Fig. 10 will generate the transverse-momentum diagram of Fig. 11 (we

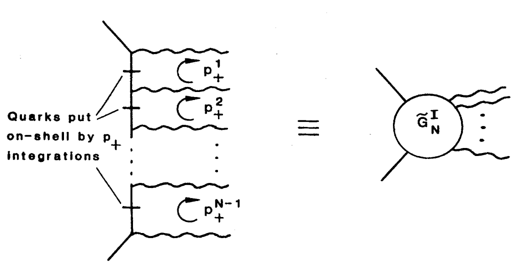

FIG. 9. The leading-order coupling of a fast quark to N transverse-momentum gluons.

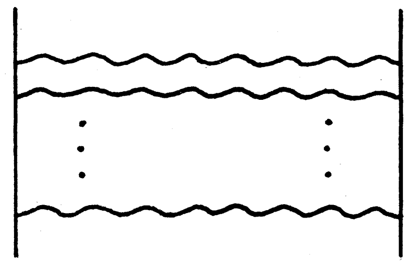

FIG. 10. A Feynman diagram involving the exchange of N gluons between two fast quarks.

emphasize that other Feynman diagrams will also generate the same transverse-momentum diagram), which we can write as

$$\Gamma_N^I = S(C_N^I)^2 \int d^2\vec{k}_{1\perp} \cdots d^2\vec{k}_{N\perp} \delta^2(\vec{k}_\perp - \vec{k}_{1\perp} \cdots - \vec{k}_{N\perp}) \times \frac{1}{\vec{k}_{1\perp}{}^2+M^2} \cdots \frac{1}{\vec{k}^{\,2}_{N\perp}+M^2} \; . \tag{3.14}$$

Consider next the possibility of coupling three transverse-momentum gluons (massive for the moment) with the spin structure of those in K_-, to a fast quark. From (3.11) it is clear that the antisymmetry of the gluon Lorentz indices is incompatible with a nonzero result for G_3^I.

The mass-shell condition imposes $P_+ \ll P_-$ for a quark with $P_- \to \infty$. Suppose instead that $P_+ \sim P_-$ for the initial and final quarks so that a γ_- spin factor is possible for each (P_+ integrations can still be used to put the intermediate quark states on-shell). We then obtain for the spin structure

$$G_3^I \sim \gamma_- \gamma_{\mu_1} \gamma_+ \gamma_{\mu_2} \gamma_+ \gamma_{\mu_3} \gamma_- \tag{3.15}$$

$$\sim \gamma_- \gamma_{\mu_1} \gamma_- \gamma_{\mu_3} \gamma_- \text{ if } \mu_2 = - \tag{3.16}$$

$$\sim \gamma_5 \gamma_- \text{ if also } \mu_1 = 1,2, \;\; \mu_3 = 2,1 \tag{3.17}$$

$$= 0 \text{ otherwise} \; . \tag{3.18}$$

If the gluons have propagated via (3.8), then their initial spin structure must be

$$(\nu_1,\nu_2,\nu_3) = (g_{\nu_1\mu_1}\mu_1, g_{\nu_2\mu_2}\mu_2, g_{\nu_3\mu_3}\mu_3) \; . \tag{3.19}$$

$$= (1,+,2) \tag{3.20}$$

or

$$\epsilon_{-\nu_1\nu_2\nu_3} \neq 0 \; . \tag{3.21}$$

This implies that gluons having propagated in a

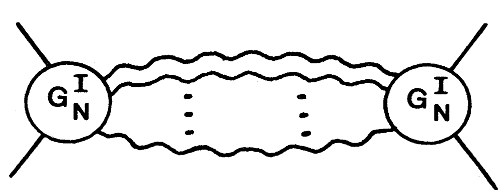

FIG. 11. The transverse momentum diagram given by the Feynman diagram of Fig. 10.

transverse-momentum diagram from a state $|S_F\rangle$ moving with $P_+ = \infty$ can couple to a fast quark with $P_- = \infty$ if the coupling (3.15) is possible.

But to obtain (3.15), we required $P_- \sim P_+$ for the quark and so in addition to $P_- = \infty$ we must also have $P_+ = \infty$. In practice, this implies that such a quark must be involved in a high-momentum divergence of a loop integral. (This will also ensure there is no suppression by the off-shell quark propagator.) The only possibility for such a divergence is a fermion loop with an axial-vector anomaly. In fact (3.17) is an axial-vector coupling and so if our scattering states had a vector character we could encounter such an anomaly. K_μ was originally introduced into anomaly theory[26] to reproduce the triangle and box-diagram anomalies. Not surprisingly, therefore, the three K_- gluons will couple, with the coupling (3.17), into that part of the quark box-diagram shown in Fig. 12, which generates the axial-vector anomaly at the K_- vertex. The couplings V_1, V_2, and V_3 at the other vertices must also be due to vectors. (Two couplings could be axial vectors—this will be important when we discuss chiral-symmetry properties in Sec. V. Note also that when we consider infrared divergences in the next section the K_- gluons will carry zero momentum and so will couple strictly to the anomaly in the quark loop of Fig. 12.)

At this point we see the significance of the helicity alignments for the quarks in our states $|S_F\rangle$. Such a state can be a scalar (pion) and yet the quark-antiquark part can have a vector structure. Thus the V_1 and V_2 couplings in Fig. 12 can be provided by the quark-antiquark pairs in the initial and final states, respectively. The third vector coupling V_3 can be provided by an additional exchanged gluon. It will be important that the exchanged gluon produce the usual high-energy behavior typical of a gauge vector boson. However, it is well known that fermion-loop anomalies can be regularized[19] in such a way that vector Ward identities are maintained. With such a regularization exchanged vectors will couple to our scattering states $|S_F\rangle$ via the anomaly produced by K_- gluons and still satisfy the Ward identities necessary to give conventional high-energy vector behavior. The basic coupling will be that illustrated in Fig. 13 with each of the triplets of gluons carrying the quantum numbers of

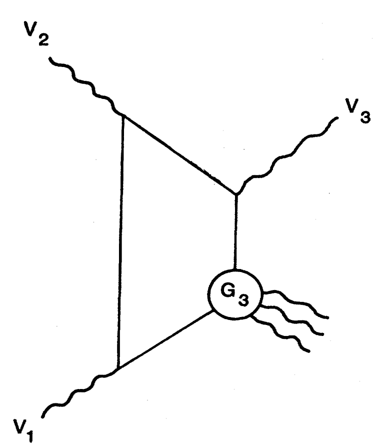

FIG. 12. A fermion box diagram with an anomaly coupling K_- gluons to three vectors.

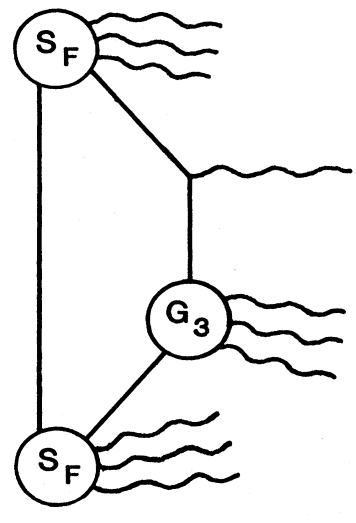

FIG. 13. The coupling of initial and final generalized Fock states to K_- gluons and an additional gluon.

K_- (although at this stage they are technically still massive gluons). The simplest elastic scattering diagram is that shown in Fig. 14, where now each dotted line represents a triplet of K^- gluons propagating via a transverse-momentum integral Γ_3 of the form (3.14), and the exchanged gluon carries transverse momentum which is integrated over. The quark loops are four-dimensional integrals.

The diagram of Fig. 14 will not, however, be amongst the most infrared divergent when the gluon mass goes to zero, as we shall discuss in the next section. The relevant diagrams involve further fermion-loop anomalies. Clearly we need a systematic analysis of fermion-loop anomalies that can arise in transverse-momentum gluon diagrams once the K_- gluons are present in the external states. We shall not attempt such a comprehensive task in this paper. However, it is clear from the above argument that many anomalous diagrams will be generated. We shall find extensive roles for both the triangle and box anomalies and possibly even the pentangle anomaly. It is important, however, that we do not generate the full set of anomalous diagrams that would be produced by starting from a Lagrangian theory with interacting axial-vector currents. This would lead to all the ultraviolet problems of a non-

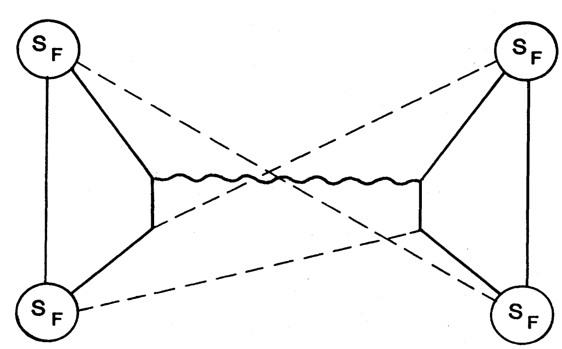

FIG. 14. The simplest elastic-scattering diagram for $|S_F\rangle$ states.

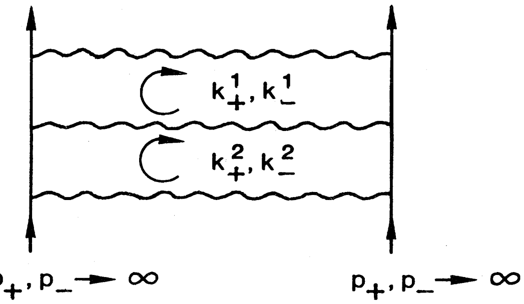

FIG. 15. A set of K_- gluons coupled to fast quarks.

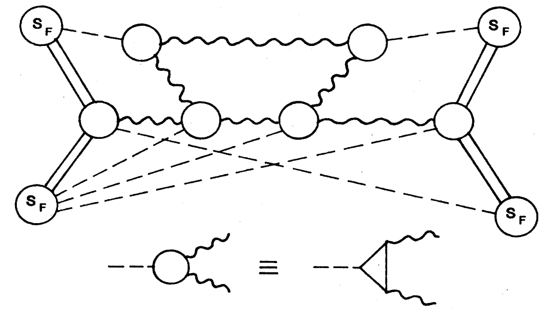

FIG. 17. An elastic-scattering diagram involving triangle anomaly couplings.

renormalizable field theory. The anomalous transverse-momentum diagrams generated by our K_- gluons are much more constrained.

For example, a very important constraint is that a set of K_- gluons cannot simply couple two anomalous fermion loops. The relevant (sub) Feynman diagram is shown in Fig. 15. To obtain anomalous couplings for both quarks we would require both P_+ and P_- large for both quarks. To put intermediate quark lines on-shell would then require both k_+ and k_- large in the loop integrals. However, Eq. (3.8) would then not hold for the gluon propagators and we would not obtain a transverse-momentum integral capable of describing the propagation over a large rapidity interval. The K_- gluons must therefore originate from an external state if they are to couple to an anomalous fermion loop. This constraint is the most important in the avoidance of an effective nonrenormalizable field theory.

Let us briefly describe some of the basic diagrams that are important in the next sections. The box anomaly can provide each of the couplings for the diagram of Fig. 16, as illustrated. Again each dotted line indicates a triplet of K_- gluons. The triangle anomaly can produce a coupling of K_- to two further gluons producing diagrams of the kind illustrated in Fig. 17. Each of Figs. 16 and 17 will give maximally divergent diagrams in the sense discussed in the next section.

Finally, we come to what may be an intriguing role for the pentangle anomaly. To maintain gauge-invariance for the vectors in a non-Abelian theory this anomaly must accompany the triangle and box anomalies.[19] We suspect the pentangle will provide the baryon coupling in our theory as the box, via Fig. 13, will provide the pion coupling. This is illustrated in Fig. 18. The pentangle coupling has the potential to produce the phenomenological requirements of both the additive quark model and helicity conservation.

IV. INFRARED DIVERGENCES AND AN INFRARED FIXED POINT

The argument we wish to make in this section is perhaps the most difficult for a reader not familiar with transverse-momentum diagrams to follow. We shall be concerned with infrared-divergence properties of infinite classes of transverse-momentum diagrams. We shall be particularly interested in infrared divergences which occur in circumstances under which the kernels $A^I_{NN'}$ are scale-invariant functions of the transverse momenta.

When the kernels $A^I_{NN'}$ are calculated in leading-order perturbation theory[14,27] (in effect leading-logarithm calculations), the gauge coupling appears as a dimensionless constant

$$g(\vec{k}_i,\Lambda)=g_0 \; . \tag{4.1}$$

That is, the renormalization scale Λ does not appear. Since the $A^I_{NN'}$ are defined to be dimensionless, when the infrared limit $M^2 \to 0$ of the leading-order kernels is considered either we must obtain a scale-invariant function or the limit cannot exist. In fact, for $I \neq 0$ the limit does not

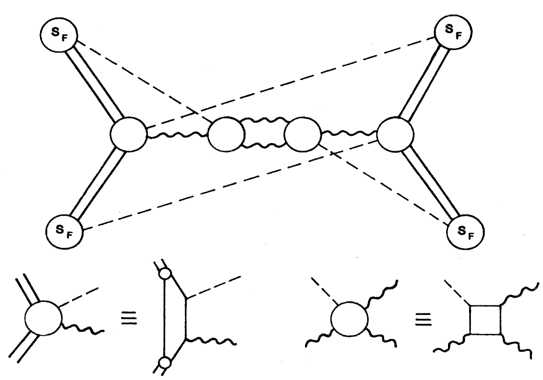

FIG. 16. An elastic-scattering diagram with four box-anomaly couplings.

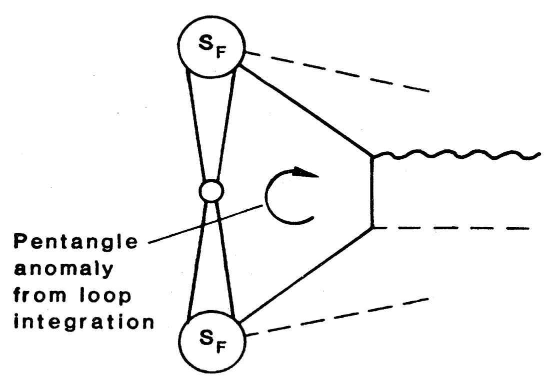

FIG. 18. The pentangle anomaly as a candidate baryon coupling.

exist, that is

$$A^I_{NN'} \underset{M^2\to 0}{\longrightarrow} -\infty, \quad I\neq 0 \tag{4.2}$$

while for the color-zero kernels

$$A^0_{NN'}(M^2,g_0,\vec{k}^{\,1}_\perp,\ldots,\vec{k}^{\,N'+N}_\perp) \underset{M^2\to 0}{\longrightarrow} \widetilde{A}^{\,0}_{NN'}(\vec{k}^{\,1}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp), \tag{4.3}$$

where $\widetilde{A}^{\,0}_{NN'}$ is a scale-invariant (generalized) function satisfying

$$\widetilde{A}^{\,0}_{NN'}(\mu k^1_\perp,\ldots,\mu\vec{k}^{\,N+N'}_\perp)=\widetilde{A}^{\,0}_{NN'}(\vec{k}^{\,1}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp). \tag{4.4}$$

When the kernels $A^I_{NN'}$ are iterated the infinity (4.2) is exponentiated for a basic reason. It is well known that in a non-Abelian Higgs theory the gluons Reggeize.[14,27] Regge behavior is an exponentiation with respect to rapidity. In particular, for $I=1$ the two-gluon kernel gives the leading-logarithm behavior

$$\sum_n y^n\left[\int A^1_{22}\right]^n=e^{y\alpha(q^2)}, \tag{4.5}$$

where $\alpha(q^2)$ is the leading-order Regge trajectory. [Equation (4.5) is, of course, a formal representation of an infinite set of convolution integrals of the kernel A^1_{22}.] When $M^2\to 0$ we have

$$\alpha(q^2) \underset{M^2\to 0}{\sim} \ln M^2 \tag{4.6}$$

and for the special case of A^1_{22} this infinity is that of (4.2). When we go to higher-order gluon kernels there is further exponentiation associated with Regge cut behavior.[2,27]

Since the infinity (4.2) is due to the noncancellation of infrared divergences for a color nonzero system, we clearly expect it to persist as we go to higher-order calculations. Since unitarity ensures[28] that Regge pole and cut behavior also persists to all orders, we feel it is safe to assume a general result which we can write formally as

$$\sum y^n\left[\int A^I\right]^n \underset{M^2\to 0}{\sim} e^{-\infty}=0, \quad I\neq 0. \tag{4.7}$$

A closely related property of the A^0 kernels is that if any subset of the arguments of $\widetilde{A}^{\,0}_{NN}$ is zero, then the scale-invariance property (4.4) is violated and an infinity appears,

$$\widetilde{A}^{\,0}_{NN'}(\vec{k}^{\,1}_\perp,\cdots,\vec{k}^{\,N+N'}_\perp) \underset{\vec{k}^{\,1}_\perp\cdots\vec{k}^{\,i}_\perp\to 0,\; i<N}{\longrightarrow} -\infty. \tag{4.8}$$

This infinity is again exponentiated due to Regge behavior.

The scale invariance of $\widetilde{A}^{\,0}_{NN'}$ causes two effects. In the ultraviolet transverse-momentum region the $A^0_{NN'}$ behave as nonintegrable kernels

$$\int\prod_i \frac{d^2\vec{k}_i}{\vec{k}_i^{\,2}+M^2}|A^0|^2=\infty \tag{4.9}$$

even when $M^2\neq 0$. This produces the well-known fixed cut in the angular momentum plane,[27] violating the Froissart bound when the lowest-order perturbative values are taken for the $\widetilde{A}^{\,0}_{NN'}$. In the massless limit we can hope to use asymptotic freedom to replace g_0 by the running coupling constant $g(\vec{k},\Lambda)$ and this will be just sufficient to eliminate the infinity of (4.9). However, a thorough understanding of the infrared divergences we are discussing is necessary before this can be done in practice.

The exponentiation effects of (4.7) and (4.8) suppress all infrared regions of transverse-momentum diagrams *apart from the region where all k_i are uniformly zero and the total color is zero.* The central point of this section is that *if the scale-invariance property (4.4) holds, then an infrared divergence from such a region propagates unchanged through any number of gluon interactions.* This can be seen immediately by first noting that

$$I^0_{NN'}(\vec{k}^{\,N+1}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp)=\int\frac{d^2\vec{k}^{\,1}_\perp d^2\vec{k}^{\,2}_\perp\cdots d^2\vec{k}^{\,N}_\perp}{(\vec{k}^{\,1}_\perp)^2(\vec{k}^{\,2}_\perp)^2\cdots(\vec{k}^{\,N}_\perp)^2}\times\widetilde{A}^{\,0}_{NN'}(\vec{k}^{\,1}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp) \tag{4.10}$$

$$=I^0_{NN'}(\mu\vec{k}^{\,N+1}_\perp,\ldots,\mu\vec{k}^{\,N+N'}_\perp). \tag{4.11}$$

That is, the scale-invariance property propagates through the interaction integrals. Therefore an overall logarithmic divergence produced as

$$\vec{k}^{\,1}_\perp\sim\vec{k}^{\,2}_\perp\sim\cdots\vec{k}^{\,N}_\perp\to 0 \tag{4.12}$$

similarly persists. That is,

$$\begin{aligned}\int\frac{d^2\vec{k}^{\,1}_\perp}{(\vec{k}^{\,1}_\perp)^2}\cdots\frac{d^2\vec{k}^{\,N}_\perp}{(\vec{k}^{\,N}_\perp)^2}\frac{d^2\vec{k}^{\,N+1}_\perp}{(\vec{k}^{\,N+1}_\perp)^2}\cdots\frac{d^2\vec{k}^{\,N+N'}_\perp}{(\vec{k}^{\,N+N'}_\perp)^2}A^0_{NN'}(\vec{k}^{\,0}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp)\\ \underset{M^2\to 0}{\sim}\int\frac{d^2\vec{k}^{\,1}_\perp\cdots d^2\vec{k}^{\,N}_\perp\cdots d^2\vec{k}^{\,N+N'}_\perp}{(\vec{k}^{\,1}_\perp)^2\cdots(\vec{k}^{\,N+N'}_\perp)^2}\frac{d^2\vec{k}^{\,N+N'+1}_\perp\cdots d^2\vec{k}^{\,N+N'+N''}_\perp}{(\vec{k}_\perp^{\,N+N'+1})^2\cdots(\vec{k}^{\,N+N'+N''}_\perp)^2}\\ \times A^0_{NN'}(\vec{k}^{\,1}_\perp\cdots\vec{k}^{\,N+N'}_\perp)A^0_{N'N''}(\vec{k}^{\,N+1}_\perp,\ldots,\vec{k}^{\,N+N'}_\perp,\ldots,\vec{k}^{\,N+N'+N''}_\perp)\\ \underset{M^2\to 0}{\sim}\int\cdots(A^0)^n \underset{M^2\to 0}{\sim}\ln M^2\end{aligned} \tag{4.13}$$

(provided we eliminate divergences from the regions of $\vec{k}_\perp$ space which are ultimately suppressed by exponentiation of infinities). In a multiple integral there will be a contribution to the coefficient of $\ln M^2$ from each gluon propagation interval.

The divergence (4.13) is going to play a vital role in the setting up of a high-energy theory based on the diagrams and states of the last two sections. Suppose now that the kernels $\tilde{A}^0$ are scale invariant and consider first the $M^2 \to 0$ limit of the diagram of Fig. 14. When we add the effects of interactions of the gluons in the K_- triplets, as illustrated in Fig. 19, we will find that all infrared divergences are eliminated apart from an overall divergence of the form (4.13) associated with each triplet. The divergence will couple directly to the K_-^∞ part of the states $|S_F\rangle$. Note that the $\delta^2(\vec{k}_\perp^1+\vec{k}_\perp^2+\vec{k}_\perp^3)$ factor in (2.24), if present in the coupling, will enhance the logarithmic divergence to a power. However, it is possible that we should take a smoother transverse-momentum distribution for the gluons in $|S_F\rangle$ contributing to K_-^∞. This would modify the nature of the divergence in a way we shall not attempt to discuss.

Whatever its nature, the divergence results from a set of gluons with the quantum numbers of K_-^∞ each acquiring identically zero transverse momentum. Denoting this set of gluons by dotted lines as in Fig. 20, the gluon interactions on either side of the divergent set can be absorbed into a redefinition of the state and vertex as illustrated. Note that an arbitrary number of gluons may be involved in the divergence (except that they have the quantum numbers of K_-). This is why it was irrelevant how many gluons we used initially to define $|S_F\rangle$. The question now arises as to what is the significance of the infrared divergence and what we should do with it.

The diagrams contributing to Fig. 19 do not actually give the maximum degree of divergence, diagrams of the form of Figs. 16 and 17 do. Although we shall not attempt a proof we believe that if the kernels $\tilde{A}^0$ are scale invariant, then in general the most divergent diagrams are those having the structure of Fig. 21. Such diagrams have the form that sets of gluons having the quantum numbers of K_- emerge from each external state $|S_F\rangle$ and split into subsets (indicated by a single dotted line) each of which also has the quantum numbers of K_- and which terminates on some anomalous internal interaction. The

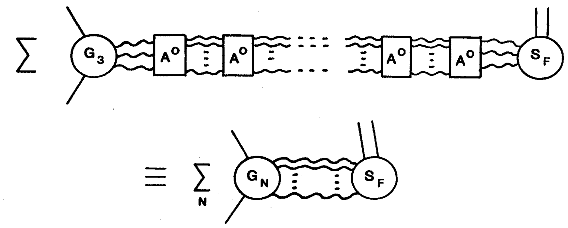

FIG. 20. Redefinition of the couplings to an infrared-divergent set of K_- gluons.

gluons in each dotted line can interact as in Fig. 19 and the dotted lines can also interact. The net effect is that there is an overall divergence (logarithmic or otherwise) of the form (4.13) for the gluons originating from each external state. Such a divergence can clearly be canceled by a normalization factor for the external states.

We conclude that if we consider the scattering of Fock space states $|S_F\rangle$ containing K_-^∞ gluons, that we will obtain an infinite S matrix. The infinity factorizes and can be interpreted as a separate infinity for each external state $|S_F\rangle$. However, from (2.29) and (2.30) we concluded that our states $|S_F\rangle$ should be multiplied by a zero normalization factor because of transverse gauge invariance. Clearly we believe the infinity and zero should cancel.

Our argument is then that a simple Fock-space state, with or without K_- gluons, should at infinite momentum ($P_+=0$) be normalized by a factor $P_+^{\,2-\delta/2}$ to have an overlap with a physical gauge-invariant state, as discussed in Sec. II. To have a finite S matrix, there must then be a matching infrared divergence for each Fock state in the naively calculated scattering amplitudes. [If the logarithmic divergence of (4.13) were the correct divergence, then we would have to take $\delta \to 4$ in (2.30). As we have emphasized both in Sec. II and in the above, we shall not attempt to determine the degree of this divergence. It is related to exactly how the transverse-momentum distribution giving $K_-^\infty \neq 0$ shrinks as we go to infinite momentum.] The matching of divergences implies *the Fock-space state must contain* K_-^∞ *gluons.* We have not discussed explicitly the kernels for quark propagation in rapidity. However, if the quarks in our state $|S_F\rangle$ do not have net color zero, then there will be a kernel for the interaction

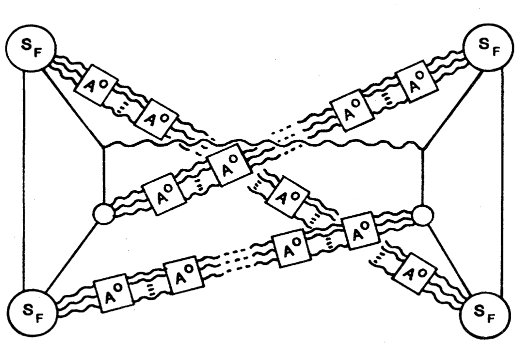

FIG. 19. Gluon interactions added to the elastic-scattering diagram of Fig. 14.

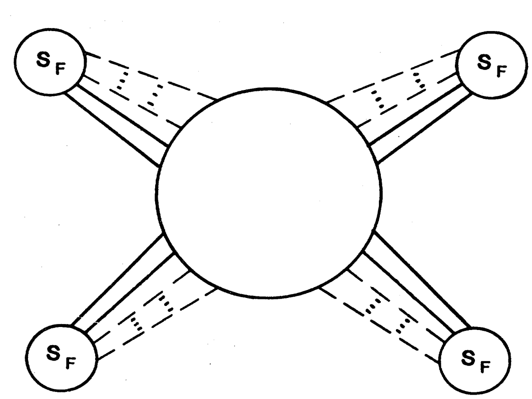

FIG. 21. The general form of maximally divergent diagrams.

of the quarks with the gluons as illustrated in Fig. 22, which will be infinite. Therefore the infrared divergence of the K_- gluons will be rapidly destroyed by infinite exponentiation as the quarks propagate a little in rapidity. So *the quarks must carry net color in the states* $|S_F\rangle$, *that is only the confinement S matrix is finite.*

The infrared divergences we have isolated originate from the scale-invariant kernel (4.3) which has avoided the infinities and consequent exponentiation (4.2) and (4.8). There are further possibilities for exponentiation and subsequent removal of the divergences. A simple possibility would be for the K_- gluons to interact with the additional exchanged transverse-momentum gluon in Fig. 19 as illustrated in Fig. 23. If the single gluon does not carry color zero there could be such an infinite interaction and consequent exponentiation. Therefore the single gluon should lie outside of the SU(2) gauge group to which the K_- gluons belong. This will be vital as we build up our complete SU(3) diffraction theory in the next section. Note that the anomaly-current structure of the K_- gluons prevents a repeated interaction of the form of Fig. 23 due to anomalous fermion loops. There has to be an odd number of anomaly currents for such an interaction and in addition we noted in the last section that there is no coupling of anomalous loops by propagating K_- gluons. Every set of K_- gluons originates from an external state. [Note that the role of the anomaly current in our analysis could not be played by an arbitrary color-zero set of gluons. In general, such a set would have a perturbative interaction (and not an anomalous interaction) with the additional exchanged gluon(s) which would result in the exponentiation of all infrared divergences, including the special class we have discussed.]

Finally, we come to the fact that the initial scale invariance of the $\tilde{A}^0_{NN'}$ results from the fixed value of the dimensionless coupling constant $g=g_0$ which appears in leading-order calculations. As we go to higher orders, the renormalization scale enters g and it becomes momentum dependent.[14] Consequently (4.3) is replaced by

$$A^0_{NN'}(M^2, g(\vec{k}_i,\Lambda), \vec{k}^{\,i}_{\perp}) \underset{M^2\to 0}{\longrightarrow} \tilde{A}^0_{NN'}(g(\vec{k}_i,\Lambda), \vec{k}^{\,i}_{\perp}) , \quad (4.14)$$

and now the presence of Λ in $\tilde{A}^0$ implies there is no need for $\tilde{A}^0$ to be scale invariant in the infrared transverse-momentum regions. In fact, we expect just the opposite. We expect the confinement scale (bag radius, etc.) to be a consequence of the growth of g in the infrared region—the scale being some function of Λ. In general, then, we expect the presence of a confinement scale to destroy the infrared scale invariance of $\tilde{A}^0$ and consequently to destroy the propagation, over large rapidity ranges, of the infrared divergence (4.13).

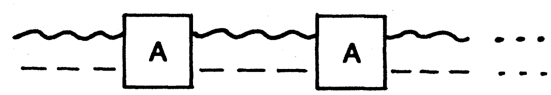

FIG. 23. Repeated interactions of the anomaly current.

This brings us back to the central point of the paper. Unless we can (in a first approximation) ignore the confinement scale, or bag radius, at infinite momentum, we do not expect to succeed with a light-cone *(parton-model)* quantization based on the perturbative vacuum. We now see the general discussion centered around a very particular technical point which we wish to emphasize. *If the color-zero kernels* $\tilde{A}^0_{NN'}$ *are infrared scale invariant, the existence of an anomaly-current component in the wave function will produce an effect which will propagate over large rapidity intervals* (and potentially participate in wee-parton interactions). If the kernels are not infrared scale invariant we expect no effect.

We can recover the infrared scale invariance of $\tilde{A}^0_{NN}$ as a general property only under very special conditions. The gauge coupling g must not grow in the infrared region. This will be a general property of a gauge theory only if the β function defined with all quarks massless has an infrared fixed point, as illustrated in Fig. 24. In this case the gauge coupling evolves only to a critical value $g=g_c$. The kernels $\tilde{A}^0$ will not be those of leading-order perturbation theory, but they will have analogous scaling properties. We expect anomalous dimensions to appear in that the logarithms of perturbation theory, e.g.,

$$\tilde{A}^0(\vec{k}_1,\vec{k}_2,\ldots)\sim \ln(k_{1i}/k_{2i})f(\vec{k}_1,\ldots,\vec{k}_2)+\cdots , \quad (4.15)$$

will be replaced by

$$\tilde{A}^0(\vec{k}_1,\vec{k}_2,\ldots)\sim (k_{1i}/k_{2i})^{\gamma}\tilde{f}(\vec{k}_1,\ldots)+\cdots , \quad (4.16)$$

where γ is some anomalous dimension. We should emphasize that the $\tilde{A}^0_{NN'}$, although infrared finite when used as kernels in the integrals discussed above, cannot be written entirely as simple functions as (4.15) may be thought to imply. They are distributions. Therefore their scaling properties will not be straightforward. Nevertheless the scale-invariance property (4.4) will hold and the infrared divergence (4.13) will propagate over large rapidities if

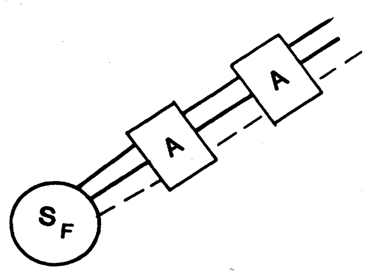

FIG. 22. Interactions of quarks and gluons via an infrared-divergent kernel.

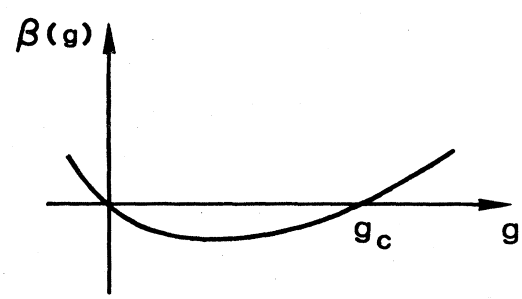

FIG. 24. An infrared fixed point of the β function.

there is an infrared, fixed point. We conclude that an infrared fixed point is vital for the success of our infinite-momentum quantization and the *(infinite-momentum)* derivation of confinement we have proposed.

V. HIGH-ENERGY SCATTERING IN QCD

In this section we will briefly outline how we build a high-energy formalism for a (very particular) theory with SU(2) gauge invariance and then how we use this to determine the conditions under which critical Pomeron high-energy behavior occurs in QCD. We also want to discuss the interrelation with confinement and chiral-symmetry breaking. The previous sections have been intended to demonstrate the existence of, rather than to provide comprehensive description of, the building blocks for our formalism.

All of the previous analysis has been concerned with SU(2) gauge invariance, although it has become apparent that gluons outside of the SU(2) group are needed. We also analyzed the SU(2) infrared divergences with a single mass scale. In effect we are implementing the principle of complementarity[17] referred to in the Introduction. This says that if fundamental-representation Higgs scalars are used for the Higgs mechanism we will not encounter any confinement phase transition as we decouple the scalars to restore the gauge symmetry. (We should point out that this decoupling is a nontrivial limit in which vector masses are taken to zero while scalar masses are simultaneously taken to infinity.) By making this decoupling one representation at a time the gauge symmetry is built up through the sequence (with vector mass scales shown)

$$\mathrm{SU}(1) \underset{M_1^2\to 0}{\longrightarrow} \mathrm{SU}(2) \underset{M_2^2\to 0}{\longrightarrow} \mathrm{SU}(3) \rightarrow \cdots \; . \tag{5.1}$$

This construction is vital for us. We build the high-energy behavior of SU(3) QCD by first constructing the theory in which the gauge symmetry is SU(2) and the remaining symmetry is broken by one SU(3) triplet of Higgs scalars. In turn the SU(2) theory is built up using the results of the previous sections in which the first limit of (5.1) is utilized.

We wish the SU(2) high-energy theory we construct to be complete, well defined and have all the properties discussed in the previous sections. We require asymptotic freedom for both the gauge coupling g and the Higgs coupling Λ. This is required to avoid the ultraviolet problem (4.9). It is a very strong requirement satisfied only if the SU(3) asymptotic freedom constraint on the number of fermions is saturated. If there are five flavors of triplet quarks, as experiment indicates, then there are only three possibilities for the quark content of our theory: (A) 16 color triplets of quarks, (B) 6 color triplets and 2 color sextets of quarks, and (C) 5 color triplets, 1 color sextet, and 1 color octet of quarks. [We have already indicated our strong preference for (B) in the Introduction.] Having added this many quarks there are several indications[18] that there will also be an infrared fixed point for both the gauge coupling and the Higgs-boson coupling. The infrared fixed point is, of course, strictly a property of the theory with all quarks (and scalars) massless. However, the high-energy behavior we are concerned with holds for

$$S \gg m_{q_i}{}^2 \gg t \tag{5.2}$$

for all quark masses m_{q_i}. Therefore the asymptotic behavior should be (qualitatively) independent of the m_{q_i} and we can take m_{q_i} as close to zero as is necessary to ensure the behavior of coupling constants is (qualitatively) that of the massless theory. (The width of the diffraction peak will, however, depend on m_{q_i} as is discussed in Ref. 29).

The states of our theory are, in first approximation, the infinite-momentum states $|S_F\rangle$ of Sec. II. For simplicity we consider only scalar mesons containing a quark-antiquark pair and a set of K_-^∞ gluons. Note that while K_-^∞ is an axial-vector component, the quark-antiquark pair may be either in a vector or an axial-vector state and combine with K_-^∞ to produce either a scalar or a pseudoscalar meson. Consequently, our SU(2) states will be parity doubled (although not necessarily degenerate in mass). The "Born" diagram for high-energy scattering will be essentially that of Fig. 16. However, this diagram can really be regarded as representative of a vast class of diagrams, as we shall describe.

We first give the group structure of the gluons involved and introduce some diagrammatic notation to describe them. Writing the SU(3) gluons in 3×3 matrix form we have

$$\left[\begin{array}{ccc} & & \times \\ & \mathrm{SU}(2) & \times \\ \times & \times & \times \end{array}\right] \begin{array}{l} \left.\right\} \mathrm{SU}(2)\ \text{doublet} \\ \left.\right\} \mathrm{SU}(2)\ \text{singlet} \end{array} . \tag{5.3}$$

There are three massless gluons forming an SU(2) triplet which individually we denote by a dotted line as in Fig. 25, while a set of K_-^∞ gluons will be denoted by a dashed line as in previous sections. There are two SU(2) doublets with mass $(\sqrt{3}/2)M_2$ which we denote by alternate dashes and dots as in Fig. 25 and finally a singlet with mass M_2 which we denote by a wavy line. Quarks are denoted by a solid line as before.

As we compute higher-orders of perturbation theory the singlet gluon Reggeizes due to its coupling to the doublets. That is, it lies on a Regge trajectory given, in first approximation, by a transverse-momentum bubble diagram

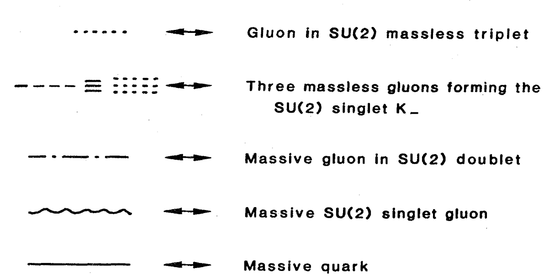

FIG. 25. Notation for gluons and quarks when the SU(3) gauge symmetry of QCD is broken to SU(2).

$$\alpha(q^2)=1+\frac{g^2(q^2-M_2{}^2)}{c\pi^2}\times\int\frac{d^2\vec{k}}{(\vec{k}^2+\frac{3}{4}M_2{}^2)[(q-\vec{k})^2+\frac{3}{4}M_2{}^2]}\,. \quad (5.4)$$

We denote the Reggeization due to a sum of transverse-momentum diagrams by a small circle as illustrated in Fig. 26. The doublets can be given infrared finite trajectories with $\alpha(q^2)\rightarrow-\infty$ as $q^2\rightarrow 0$ and so consequently their exchange over large rapidity intervals can be neglected.

With the notation of Figs. 25 and 26 our Born diagram for high-energy scattering, of the form of Fig. 16, is now as illustrated in Fig. 27. Each gluon in each K_- set now carries strictly zero transverse momentum and so each such set couples only to the anomaly in the fermion loop to which it is attached. Since each K_- set is to produce a transverse-momentum infrared divergence to be factorized onto an external state, it must travel over a large rapidity interval. This determines that the central quark and gluon structure (which can only contribute over a finite rapidity interval) must be located (in rapidity) well away from either end of the rapidity interval.

All the (large) rapidity dependence of the diagram of Fig. 27 will therefore come from the Reggeized singlet propagators and will be of the form

$$e^{y_1\alpha(q^2)}e^{y_2\alpha(q^2)} \quad (5.5)$$

$$\underset{y_1+y_2\sim y\rightarrow\infty}{\sim} e^{y\alpha(q^2)}\equiv S^{\alpha(q^2)}\,, \quad (5.6)$$

where $\alpha(q^2)$ is given by (5.4). Since the diagram corresponds to the exchange of an even number of vectors, the Regge-pole behavior (5.6) will appear in the even-signature amplitude. Clearly Fig. 27 represents an extraordinary collaboration of all the elements of the quark-gluon structure in QCD to produce a genuine *Pomeron Regge pole* as the leading high-energy behavior of a theory.

The separation of the full rapidity interval into two subintervals in Fig. 27 is not unique and in fact it could be split into an arbitrary finite number of intervals. The general class of diagrams which gives (5.6) is therefore illustrated in Fig. 28, where all anomaly interactions are represented by circles. Figure 27 is then, as we indicated earlier, only the simplest of a vast class of sums of perturbative diagrams providing the initial Regge pole approximation for the Pomeron. We use a wide wavy line as indicated in Fig. 28 to represent our first Pomeron-Regge pole approximation.

The next question is whether there is a triple-Pomeron coupling. This coupling has a physical significance in the hadron S matrix since dispersion theory relates it to

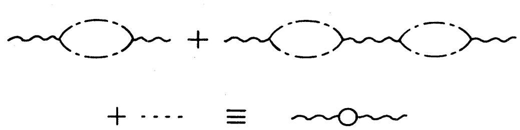

FIG. 26. Transverse-momentum diagrams giving the Reggeization of the SU(2)-singlet gluon.

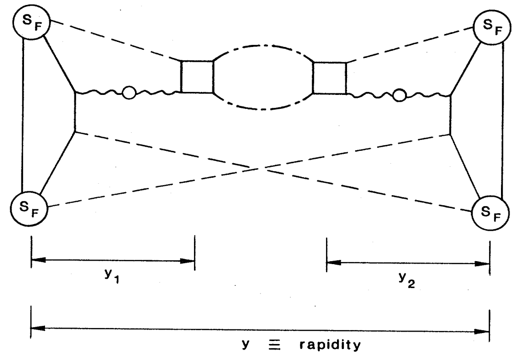

FIG. 27. The simplest elastic-scattering diagram for the Pomeron.

physical hadron states. It is not surprising therefore that the coupling is not generated by a simple anomaly, but rather exists by virtue of the Pomeron's coupling to the pion (and other hadrons). That is, a multiple-quark loop which can be interpreted as a pion loop (or loops) will generate such a coupling as in Fig. 29—although probably a nonplanar diagram is necessary.

The triangle anomaly coupling appearing in Fig. 17 effectively gives vacuum production of Pomerons, as illustrated in Fig. 30. Note also that it follows from (5.6) that the Pomeron Regge trajectory is identical to that on which the SU(2) singlet lies. Therefore we know that the trajectory satisfies

$$\alpha(t=M_2{}^2)=1\,, \quad (5.7)$$

as illustrated in Fig. 31. It seems then that we have all the features of the super-critical Pomeron.[30] If this is the case, then the limit $M_2{}^2\rightarrow 0$, or restoring the full gauge invariance to SU(3), will give the critical Pomeron.

While we are very optimistic that this limit does give the critical Pomeron (for at least one of the fermion possibilities above), much remains to be done to establish this. The limit is very nontrivial and must produce many important effects in a way which remains to be understood. Let us begin with some properties of the Pomeron that must be checked.

It may seem fairly obvious from (5.7) that $\alpha(t=M_2{}^2=0)=1$, but in fact we must establish that $\alpha'(t=0)$ does

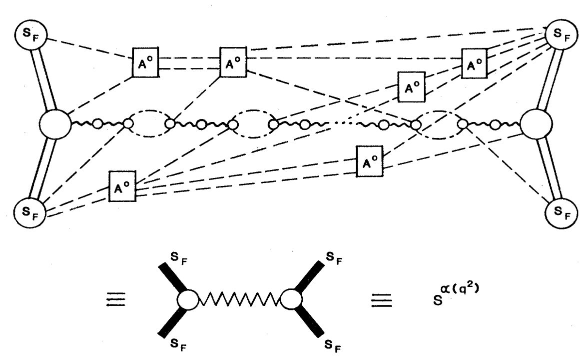

FIG. 28. A typical elastic-scattering diagram contributing to the Pomeron.

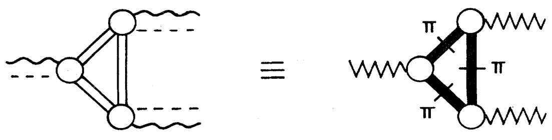

FIG. 29. A candidate triple-Pomeron coupling.

not blow up (too fast) as $M^2 \to 0$. It must also be established that supercritical-Pomeron graphs represent entirely all the gauge-theory contributions.

A further feature is that the singlet gluon trajectory must decouple from the Pomeron as $M^2 \to 0$. Similarly, all quark and gluon states that are SU(2) singlets, but are not SU(3) singlets, must decouple. There is also the question of the status of chiral-symmetry breaking during this process. We have noted above that our meson states will be parity doubled—although they need not be degenerate in mass. To begin discussion of the chiral limit, we first note that Fig. 29, and analogous diagrams, imply that the triple-Pomeron coupling r_0 will satisfy (if m_q is the relevant quark mass)

$$r_0 \underset{m_q \to 0}{\sim} \frac{1}{m_q} . \tag{5.8}$$

Thus the chiral limit will be equivalent[29] to the strong-coupling limit of our Pomeron theory—which will not exist unless $\alpha(0)=1$. Consequently, we believe the chiral limit does not exist for the SU(2) *high-energy* theory we construct. As $M_2{}^2 \to 0$, or $\alpha(0) \to 1$, Reggeon interactions of the Pomeron with hadrons (regarded as Reggeons) become increasingly important. It is the limit $\alpha(0) \to 1$ which produces wee partons and as the discussion in the Introduction indicates, we expect their presence to be necessary to allow a chiral limit with spontaneous symmetry breaking. The Reggeon interaction as $\alpha(0) \to 1$ will be responsible for the realization of this argument in our formalism.

To see first that our mesons (pions) can act as the relevant Goldstone bosons we need to consider their Regge trajectories as discussed in Ref. 2. A pion is (approximately) a bound state of two fermions and the gluons of K_-, whose trajectory $\alpha_\pi(t)$ should, near $t=4m_q{}^2$, pass through a "nonsense" state of two fermions (and any number of gluons). That is,

$$\alpha_\pi = j = n_1 + n_2 - 1 = \tfrac{1}{2} + \tfrac{1}{2} - 1 = 0 , \tag{5.9}$$

and so gives the physical particle at this point. In contrast, the nucleon trajectory should similarly pass through the nonsense point

$$\alpha_N = j = \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2} - 2 = -\tfrac{1}{2} , \tag{5.10}$$

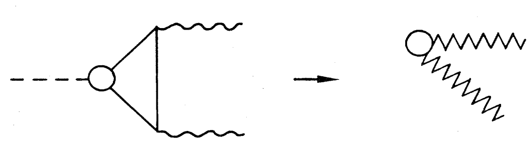

FIG. 30. Vacuum production of Pomerons due to the triangle anomaly.

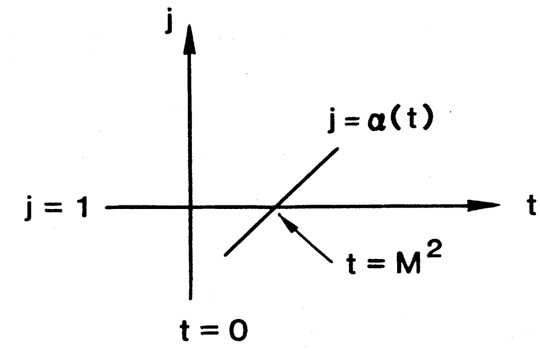

FIG. 31. The Regge trajectory $\alpha(t)$.

which does not give a physical particle. In the chiral limit of zero quark mass we therefore expect

$$\alpha_\pi(t=0)=0, \quad \alpha_N(t=0) = -\tfrac{1}{2} , \tag{5.11}$$

in which case the pion trajectory gives a zero-mass particle while the nucleon does not.

The vital question we have to answer is, however, what distinguishes the pseudoscalar pion and its scalar partner. Consider a pseudoscalar pion state $|\pi_F\rangle$ and an analogous scalar partner state $|\tilde{\pi}_F\rangle$, with vector and axial-vector valence-quark components, respectively. The relevant Reggeon interactions with the Pomeron will again be given by fermion-loop anomalies, as illustrated in Fig. 32. Since the π_F coupling involves vector couplings (apart from the K_- coupling), it will be regularized to satisfy Ward identities while the $\tilde{\pi}_F$ coupling will involve axial vectors and so the same Ward identities will be softly broken. We believe such identities will weaken the pion-Pomeron interaction at $t=0$ sufficiently to allow the pion to interact with the Pomeron as $\alpha(0) \to 1$ and not be strongly affected. Its scalar partner, on the other hand, will be strongly affected and we believe it will be driven from the physical sheet of the angular momentum plane for positive t.

We do not wish to elaborate on this argument here since it would not be appropriate and in any case we need a more detailed development of our formalism. However, we believe that the $M_2{}^2 \to 0$ limit not only produces the critical Pomeron, but after it is taken the chiral limit can be taken and the result is spontaneous chiral-symmetry breaking. The hiding of one parity hadron trajectory by the critical Pomeron is the technical realization of the wee-parton argument of Kogut and Susskind,[5] quoted in the Introduction. That the regularization of anomalies in the pion-Pomeron interaction determines the nature of the

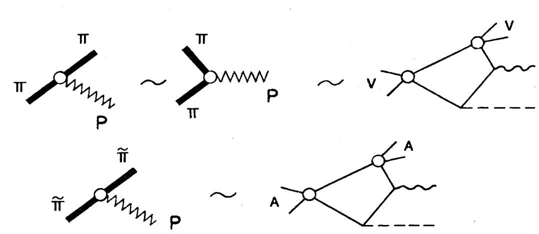

FIG. 32. Box-anomaly diagrams for the $\pi\pi P$ and $\tilde{\pi}\tilde{\pi}P$ couplings.

symmetry breaking can probably be viewed as the ability of the wee partons to carry vacuum information.

Finally, we would like to elaborate briefly on the connection between phase-transition phenomena in QCD and in Reggeon field theory to which we referred in the Introduction. $\alpha_P(0)>1$ formally defines the supercritical Pomeron which was the subject of much controversy[12,30] some years ago. If we successfully connect the above SU(2) theory with our supercritical Pomeron solution,[30] we will determine that supercritical Reggeon field theory describes the partial breaking of gauge symmetry and the loss of spontaneous chiral-symmetry breaking in QCD—by the addition of a Higgs system.

We have not discussed it in this paper, but in other papers[2] we have suggested that our analysis could be extended to show that if QCD has fewer fermions than discussed above, for example N_F ($\equiv$number of triplet flavors) <16, then the Pomeron will be subcritical $[\alpha_P(0)<1]$. If we then simply increase N_F through 16 and beyond, we will potentially reach an alternative supercritical-Pomeron theory. Since asymptotic freedom is lost such a theory is undefined at short distances but may make sense as a deconfined theory at large distances. We believe that *with a transverse-momentum cutoff* this situation corresponds to the expanding-disk solution of supercritical-Pomeron theory.[12] This solution would therefore represent a situation in which the complete SU(3) gauge symmetry is effectively preserved but all gluons are deconfined. [The relation of the confinement and chiral-symmetry phase transitions to N_F in a cutoff (lattice) theory is discussed by Banks and Zaks in Ref. 18.]

We suggest therefore that the phase-transition analysis of Reggeon field theory was inherently difficult (and controversial) since it was actually anticipating the full complexity of the confinement and chiral-symmetry-breaking phase transitions in QCD.

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