Couplings of low-lying glueballs to light quarks, gluons, and hadrons

John M. Cornwall and A. Soni
Department of Physics, University of California, Los Angeles, California 90024
(Received 20 October 1983)

We derive a set of QCD sum rules for the operator $\int dx\, G_\mu^\nu G_\nu^\mu$, which is in a sense a generalization of the trace anomaly. Combining these with plausible phenomenology, we arrive at a semiquantitative picture of the couplings between the two lowest-lying glueballs (called S for $J^{PC}=0^{++}$ and P for 0^{-+}) and light quarks, gluons, and hadrons. These couplings suggest the existence of an approximate chiral/dilatation symmetry carried by the glueball interactions, with large and calculable symmetry-breaking terms arising primarily from the glueball masses. The couplings are of typical hadronic size, even though suppressed in the large-N limit. The QCD sum rules mentioned above are closely related to the one-loop effective action, although derived quite differently; we believe the sum rules partially justify use of the effective action for small field strengths. A somewhat speculative generalization of sum rules to axial-anomaly densities gives an approximate evaluation of the Witten sum rule for the θ dependence of the vacuum energy. The S-P chiral symmetry is related to the quark $U_A(1)$ symmetry through the Peccei-Quinn mechanism; this symmetry is badly broken by the P mass.

I. INTRODUCTION

One would like to know much more about the glueballs of QCD than is currently known. At the moment, there are two experimental candidates for light glueballs with masses <2 GeV: The $\iota(1440)$ with $J^{PC}=0^{-+}$ (Ref. 1) and the $\theta(1670)$ which is 2^{++} (Ref. 2). There are numerous calculations of the masses of (quarkless) glueballs,[3–6] with good agreement between Monte Carlo calculations[3] and calculations of the authors[5] based on using the Schrödinger equation with massive ($m=500$ MeV) gluons. Both these methods yield glueballs with the quantum numbers and masses of ι and θ, but for the 0^{++} glueball the situation is worse: no convincing experimental candidate, and a substantial spread in theoretical mass values, from 700 to 1200 MeV.

Let us ignore this potential embarrassment for QCD and proceed to the next logical step: calculating (more realistically, estimating) the couplings of glueballs to quarks and hadrons. In this paper we consider only the lightest glueballs, the S (0^{++}) and P (0^{-+}), using these symbols to denote states unmixed with quarks. These play special roles because (as discussed by other authors[7]) they have the quantum numbers of the trace of the energy-moment tensor and of the axial anomaly. Therefore, they should be important in describing the breaking of dilatation symmetry and $U_A(1)$ symmetry, where $U_A(1)$ is the axial-vector baryon current. Since the masses of S and P are not small, these symmetries will be badly broken; in particular, the η' will pick up a mass of $\sim M_S, M_P$. The machinery for finding the η' mass in the chiral limit has been set up by Witten,[8] who shows how it is governed by the zero-momentum propagator of the axial anomaly.[9]

The above-mentioned authors[7] and others[10] have gone some way in establishing the phenomenological couplings that S and P must have in order to be consistent with known Ward identities and the like, but to our knowledge no one has tried to calculate the S and P couplings directly from QCD. If one can calculate the S and P masses by some dynamical scheme, one also ought to be able to calculate couplings (i.e., wave functions). Thus, our Schrödinger-equation approach,[5] based on the dynamical generation of a gauge-invariant effective gluon mass,[11] yields in principle the S and P couplings to gluons, from which all other couplings follow. As we discuss in the Appendix, this does not work at all even though the masses are reasonably well predicted. This is a familiar story: approximate dynamical schemes do better on energy levels than they do on wave functions.

We therefore abandon the direct dynamical approach and try instead a combination of QCD sum rules and plausible, if somewhat speculative, phenomenology. We shall see that the sum rules, although closely related to the perturbative one-loop effective action,[12] demand a nonperturbative interpretation for consistency. They can be considered as a generalization of the relation of the trace anomaly[13] to the vacuum energy.

The first set of sum rules, infinite in number, refers to zero-momentum matrix elements of the gluon field density,

$$g^2\sum G^a_{\mu\nu}G_a^{\mu\nu}(x)=g^2G\cdot G \tag{1.1}$$

(i.e., the trace anomaly), which has the same quantum numbers as the S glueball; later we will identify the right-hand side (RHS) of (1.1) with a specific polynomial in S. The sum rules are derived under the condition that the β function of (quarkless, N_c colors) QCD is well approximated by its first term:

$$\beta(g)=-bg^3+\cdots\ ,\quad b=\frac{11N_c}{48\pi^2}\ . \tag{1.2}$$

This limitation, of course, is not a bar to the appearance of nonperturbative effects, which will involve the factor $\exp(-1/bg^2)$. It is possible, as we mention in Sec. II, that the sum rules' derivation can be extended to remove condition (1.2) without a qualitative change in the nature of the sum rules. The problem in this extension is knowing precisely how to implement nonperturbatively the normal product of two fields such as occurs in (1.1). We recognize the uncertainty ensuing in trading g for a mass scale when only the first term in $\beta(g)$ is used, and believe that this leads to errors of order $\pm 30\%$ in quantitative applications.

The sum rules are not derived from an effective action, but they imply one. It turns out that the effective action required by the sum rules is the one-loop action,[12] without an imaginary part (whose presence signifies instability of the vacuum with a superposed constant color field[14]). Use of the one-loop action for small field strengths is properly subject to criticism on the grounds that its derivation cannot be justified, since the Landau ghost pole in the running coupling constant is reached. Our derivation makes it clear that the Landau ghost singularity arises from the sum rules for an indefinitely large number of $G\cdot G$ operators. Saving only the sum rules with a finite number of operators can never lead to a singularity in the effective action. On the other hand, the finite sum rules always lead to an effective action corresponding to an effective potential with a nonperturbative minimum (i.e., $\langle G\cdot G\rangle > 0$). We use this fact to argue for a nonvanishing expectation value of S, and find the value

$$\langle S\rangle^2 = \frac{bg^2}{2M_S{}^2}\langle G\cdot G\rangle, \quad \langle S\rangle \simeq 130 \text{ MeV} . \tag{1.3}$$

The numerical value follows from the phenomenological estimate[15] $g^2\langle G\cdot G\rangle \simeq 0.47$ GeV^4, and the predicted[3-6] value of the S mass $M_S \simeq 1$ GeV.

The sum rules invoked so far help to construct an effective action for S. To complete the picture, we need sum rules for the axial-anomaly density

$$G\cdot\widetilde{G} \equiv \sum G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a(x), \quad \widetilde{G}^{\mu\nu}_a = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta a} , \tag{1.4}$$

since this density has the quantum numbers of the P glueball. Witten[8] has expressed the two-point function of (1.4) in terms of $\partial^2 E_{\text{vac}}/\partial\Theta^2$, but the latter quantity is unknown. We cannot use the same techniques here as used for the $G\cdot G$ sum rules, but based on the close relation of the latter to the one-loop action, we consider the one-loop action as extended to depend both on $G\cdot G$ and $G\cdot\widetilde{G}$ for constant fields.[16] This action is expressed as a proper-time integral whose integrand has a series expansion in powers of $G\cdot G$ and $G\cdot\widetilde{G}$. However, the proper-time integrals have infrared (large-time) divergences for each term of the series, divergences which are reflected in the overall integral as the Landau ghost. We speculate that the nonperturbative induction of short-range gauge-field correlations for the individual terms in the power-series expansion of the integrand leads to infrared convergence of the corresponding integrals, so that it makes sense to compare the coefficients of $(G\cdot G)^2$ and $(G\cdot\widetilde{G})^2$ in this expansion.[17] These coefficients are readily calculated, and are essentially equal. Combined with previous information this leads to a specific value for the Witten sum rule.[8,10,18] Knowing this value allows us to generate an infinite set of sum rules for Green's functions with two $G\cdot\widetilde{G}$ fields and any number of $G\cdot G$ fields, all at zero momentum.

Taken together, these sum rules yield an effective action for $G\cdot G$ and $G\cdot\widetilde{G}$, in which we save only up to quadratic terms. The next step is to translate this into an effective action for (zero-momentum) S, P fields, which we promote to finite, small momentum by adding kinetic terms. This requires the specific operator connection between $G\cdot G$, $G\cdot\widetilde{G}$, and S,P. Unfortunately, this connection is necessarily ambiguous, because S has the quantum numbers of the vacuum.

The near equality of the coefficients of $(G\cdot G)^2$ and $(G\cdot\widetilde{G})^2$ strongly suggests a chiral symmetry relating $G\cdot G$ and $G\cdot\widetilde{G}$, which we can in fact identify with $U_A(1)$ with the help of the Peccei-Quinn mechanism.[19] There is a particularly elegant solution to the operator-ambiguity problem, which expresses the chiral symmetry both in the $(G\cdot G, G\cdot\widetilde{G})$ basis and the (S,P) basis in the sense that the (S,P) kinetic terms and quartic couplings are exactly symmetric, while the mass terms (*required* by our sum rules) necessarily violate both a dilatation symmetry and the chiral symmetry. The dilatation symmetry is, of course, broken by $\langle S\rangle$, and if the mass terms were chirally symmetric, the P would have to be a massless Goldstone boson. This is not what happens; instead, our resolution of the operator-ambiguity problem leads to the relation $M_S = M_P$. Now this is not likely to be the case in nature; other calculations suggest that $M_S \simeq 0.7 M_P$ is closer to the mark.[3-6] The point is not that we are off by 30% or so in relating M_S to M_P, but rather that both these masses are ~ 1 GeV—in other words, that P is not a Goldstone boson. If it were, it would raise a host of problems with $U_A(1)$-symmetry breaking which are very well known.

These remarks concerning the P are speculative, because nothing that we can *derive* forces us to the chiral interpretation that we end up using. Yet as far as we can see, it is consistent with known facts. It turns out that many predictions of the theory are insensitive to the operator ambiguity, because this ambiguity is probed largely by one-loop corrections to the tree-level effective Lagrangian for S and P. We show in Sec. II that loop corrections to the tree-level process are governed by a reasonably small parameter.

Next we turn to the couplings of S and P to quarks, gluons, and hadrons. The couplings to gluons are, of course, read off from the operator relations between S,P and $G\cdot G$, $G\cdot\widetilde{G}$; knowing these, we could calculate the coupling to quarks and hadrons directly from QCD (at least in principle). However, if we are only interested in zero-momentum coupling to light quarks this is not necessary, since the S interpolating field is (up to a constant related to $\langle S\rangle$) the trace of the energy-momentum tensor;[13] that is, S acts like a dilaton:

$$T^\mu_\mu = +\frac{\beta(g)}{2g}G\cdot G \simeq -\frac{bg^2}{2}G\cdot G . \tag{1.5}$$

(Terms involving $M_Q\overline{\psi}\psi$ for light quarks are numerically unimportant.) At zero momentum, T^{μ}_{μ} has universal matrix elements related to the mass of the particle involved. We also do not need to calculate explicitly the zero-momentum coupling of P to light quarks, because this coupling is chirally related to the S coupling: S and P appear in the combination $S+i\xi\gamma_5 P$, with $\xi\simeq 1$. The number ξ is the square root of the earlier-mentioned ratio of coefficients of $(G\cdot G)^2$ and $(G\cdot\widetilde{G})^2$ in the effective action. The overall strength of the coupling is such that half the constituent mass of a quark is generated through its coupling to $\langle S\rangle$, the rest coming from other QCD processes.

We have said that we do not need to calculate these zero-momentum couplings explicitly; nevertheless, in Sec. III we offer some simple examples of one-dressed-loop graphs. The point here is that such graphs give very nearly the answer we know they must give, by invoking the trace anomaly.

One may now ask whether the couplings so calculated have much to do with predicting scalar and pseudoscalar glueball widths, decay modes, branching ratios, and the like. Unfortunately, several problems arise: (1) glueballs may be readily produced in quarkonium decay, and the coupling S and P to *heavy* quarks, even at zero momentum transfer, requires substantial modification of the simple dilaton picture given here for light quarks; (2) mass-shell couplings may deviate substantially from the zero-momentum couplings we calculate; (3) S and P are unmixed with quarks (by definition), unlike the physical particles studied by experimentalists. A separate paper will be devoted to these issues.

The S coupling to hadrons at zero momentum is such as to generate a large part (perhaps all, within our errors) of any hadron's mass. It is natural, therefore, to ask whether this mass generation might not be recognized, as the phenomenological level, as part of a Higgs mechanism. The phenomenological Lagrangian is then renormalizable. (Actually, this is a dubious virtue since such Lagrangians are strongly coupled in the energy regime where renormalizability makes a difference.) It has been known for a long time[20,21] how to do this, and it turns out that one is always left with at least a massive scalar and a pseudoscalar flavor-singlet particle, as well as perhaps other flavor-bearing massive scalars and pseudoscalars. However, it is frustrating that, while the scalar is 0^{++}, the pseudoscalar is 0^{--} and not 0^{-+}. From the viewpoint of QCD the 0^{--} cannot be formed as a $\bar{q}q$ state, but only as a $\bar{q}q\bar{q}q$ or three-gluon state. If it is a glueball it should be considerably heavier than the 0^{++}, and has no recognizable approximate symmetry connecting it to this particle.

In connection with phenomenological hadron Lagrangians, we point out that such a Lagrangian describing massive vectors (or pseudovectors) has soliton states which resemble glueballs, and might in fact be the appropriate realization of these at the level of observable hadrons. These solitons are easily found[22] from a phenomenological gauge-invariant description of massive vectors without Higgs particles, and are essentially strongly coupled versions of the 't Hooft—Polyakov monopole (with, however, no long-range fields, since the gauge symmetry is completely broken). Because the coupling is strong, it is not clear that the semiclassical attack of Ref. 22 is even approximately valid, but for what it is worth we report in Sec. III that such an approach yields a 0^{++} mass of around 1 GeV.

II. ZERO-MOMENTUM SUM RULES AND THE EFFECTIVE ACTION

In this section we derive a set of sum rules for vacuum expectation values of the zero-momentum field operators $\int dx\, G\cdot G$, $\int dx\, G\cdot\widetilde{G}$. The sum rules as derived are approximate, because we approximate β by its lowest-order value $-bg^3$ ($b=11N_c/48\pi^2$ in quarkless N_c-color QCD). The infinite set of sum rules involving only $G\cdot G$ gives essentially the one-loop effective action as calculated in perturbation theory, but the derivation does not invoke perturbation theory, nor does it involve special choices of zero-momentum field configurations. We consider the sum rules as partial justification for using the one-loop action outside the regime of large $G\cdot G$ where perturbation theory is justified. The Landau ghost of perturbation theory shows itself as a singularity in the effective action $\Omega(G\cdot G)$ at the origin; this singularity is not present in the effective actions based on any finite number of sum rules, since these actions are polynomials. Moreover, all these actions show nonperturbative extrema. We use the simplest of them to construct an effective action for S.

We cannot derive sum rules for $G\cdot\widetilde{G}$ (and P) the same way, so we turn instead to the one-loop action as a function of $G\cdot G$ and $G\cdot\widetilde{G}$.[16] We speculate that nonperturbative effects modify certain proper-time integrals so that they converge at large proper times (i.e., in the infrared regime), which allows us to compare the contributions of $(G\cdot G)^2$ and $(G\cdot\widetilde{G})^2$ to the action. From this comparison plus a specific operator relation between the set (S,P) and the set $(G\cdot G, G\cdot\widetilde{G})$, we find an action for S and P.

A. Scalar sum rules

It is convenient to scale out the coupling constant by defining the new fields

$$\bar{A}^a_\mu = gA^a_\mu\ ,$$
$$\bar{G}^a_{\mu\nu}=\partial_\mu\bar{A}^a_\nu-\partial_\nu\bar{A}^a_\mu+\epsilon^{abc}\bar{A}^b_\mu\bar{A}^c_\nu=gG^a_{\mu\nu}\ . \tag{2.1}$$

Then the generating functional at zero external source is

$$Z=e^{iW}=\int(d\bar{A})\exp\left[-\frac{i}{4g^2}\int dx\,\bar{G}\cdot\bar{G}\right]. \tag{2.2}$$

The numerical significance of W is that it measures the vacuum energy:

$$-i\ln Z=W=-\int dx\,E_{\rm vac}=-\int dx\,\tfrac{1}{4}\langle T^\mu_\mu\rangle\ , \tag{2.3}$$

where $\langle T^\nu_\mu\rangle$ is the stress-energy tensor of the vacuum.

The trace anomaly in the vacuum sector is derived as follows. Apply $g^3\partial/\partial g$ to $\ln Z$ and get

$$\langle\bar{G}\cdot\bar{G}\rangle=-2g^3\frac{\partial}{\partial g}E_{\rm vac}\ . \tag{2.4}$$

Now $E_{\rm vac}$ is a renormalization-group (RG) invariant of dimension 4, so

$$0=\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}\right]E_{\rm vac}=\left[4+\beta\frac{\partial}{\partial g}\right]E_{\rm vac}\ , \qquad (2.5)$$

where μ is the renormalization point. Combining (2.1)–(2.5) yields

$$E_{\rm vac}=\frac{\beta}{8g}\langle G\cdot G\rangle\ . \qquad (2.6)$$

With (2.3), this yields one consequence of the trace anomaly.[13] It is the $N=0$ element of our sum rules. With the approximation (1.2), the form we use is

$$E_{\rm vac}=-\frac{bg^2}{8}\langle G\cdot G\rangle\ . \qquad (2.7)$$

The rest of the sum rules are derived by repeatedly applying $g^3\partial/\partial g$ to $\ln Z$ and using (2.5) with $\beta\simeq-bg^3$. To display the sum rules compactly, introduce the notation

$$\theta(x)=\frac{bg^2}{8}G\cdot G\ . \qquad (2.8)$$

For $N\geq 1$ the sum rules are

$$i^N\int dx_1\cdots dx_N\langle T\theta(x_1)\cdots\theta(x_N)\theta(0)\rangle_{\rm conn}=\langle\theta\rangle\ , \qquad (2.9)$$

where the subscript conn means that only connected contributions are saved. Note that the left-hand side (LHS) of the $N=1$ sum rule is positive definite, and therefore so must be the RHS. The requirement that the sum rules be well defined[23] immediately leads to $\langle\theta\rangle>0$, thus a nonperturbative realization of QCD.

The derivation we give only works in the approximation $\beta\simeq-bg^3$. It can be "improved" by invoking the renormalization group and the trace anomaly, but we will not concern ourselves with that here. There is little of phenomenological import to be gained by such improvement.

Let us construct the effective action which generates these sum rules. First define $W(J)$ by

$$e^{iW(J)}=\int d(\bar{A})\exp\left[-\frac{i}{4g^2}\int dx\,\bar{G}\cdot\bar{G}+\frac{ibJ}{8}\int dx\,\bar{G}\cdot\bar{G}\right]\ . \qquad (2.10)$$

The source J is independent of x since we are interested in zero-momentum sum rules. The effective zero-momentum field θ is defined by

$$\int dx\,\theta\equiv\frac{\partial W}{\partial J} \qquad (2.11)$$

and the effective action is, as usual, the Legendre transform of W:

$$\Gamma(\theta)=W(J)-\int dx\,J\theta,\quad \frac{\partial\Gamma}{\partial\theta}=-\int dx\,J\ . \qquad (2.12)$$

Evidently,

$$\left[\frac{\partial}{\partial J}\right]^{N+1}W\bigg|_{J=0}=i^N\int dx\,dx_1\cdots dx_N\langle T\theta(x_1)\cdots\theta(x_N)\theta(0)\rangle_{\rm conn}=\int dx\langle\theta\rangle\ , \qquad (2.13)$$

where the last line follows from (2.9). In what follows, one should consider $\langle\theta\rangle$ to be a fixed number, actually independent of θ or J; its true significance will be reinstated at the end.

It follows from (2.13) that

$$W=\int dx\langle\theta\rangle e^J \qquad (2.14)$$

and from (2.11) that

$$\theta=\langle\theta\rangle e^J,\quad J=\ln(\theta\langle\theta\rangle^{-1})\ . \qquad (2.15)$$

The differential equation (2.12) for Γ leads to

$$\Gamma(\theta)=-\int dx\,\theta\ln\left[\frac{\theta}{e\langle\theta\rangle}\right]\ , \qquad (2.16)$$

where we impose the condition that $\Gamma(0)=0$, as in perturbation theory. This $\Gamma(\theta)$ is just the one-loop result, except for imaginary contributions which are artifacts of perturbation theory. It has a maximum at $\theta=\langle\theta\rangle$ ($J=0$) of value $\int dx\langle\theta\rangle$; that is, the vacuum energy is $-\langle\theta\rangle$, as required by the trace anomaly (2.7).

This maximum (minimum of the effective potential) is not reliable if derived from perturbation theory, but there is no reason to doubt its significance based on the sum rules (2.9), which are *not* tied to perturbation theory. Perturbation theory is valid when $\theta\gg\langle\theta\rangle$, or by (2.15) when J is large. The large-N derivatives of $W(J)$ are sensitive to the large-J behavior, so the large-N sum rules probe the perturbative regime. What happens if we keep only a small number of sum rules and construct an effective action based only on these? It is easy to see that using only the first N of the sum rules (2.9) is equivalent to keeping only $N+1$ terms in the power-series expansion of Γ in (2.16) in the argument, $\theta\langle\theta\rangle^{-1}-1$. For each N, the resulting $\Gamma[N]$ has a maximum at $\theta=\langle\theta\rangle$, and the value of the maximum is $\int dx\langle\theta\rangle$, just as in (2.16). Next we will construct an effective action for S based on saving only a small number of sum rules.

B. Effective action for S

We construct an effective zero-momentum action for S which is renormalizable, that is, a polynomial of degree 4 at most. Begin by expanding the action (2.16) around its maximum, saving only up to quadratic terms:

$$\Gamma(\theta)\simeq\Gamma_2(\theta)\equiv\int dx\left[\tfrac{1}{2}\langle\theta\rangle+\theta-\tfrac{1}{2}\frac{\theta^2}{\langle\theta\rangle}\right]\ . \qquad (2.17)$$

This action correctly yields the $N=0$ (trace-anomaly) sum rule (2.6) as well as the $N=1$ sum rule (2.9). It has a maximum at $\theta=\langle\theta\rangle$, of value $\int dx\langle\theta\rangle$, just as the full expression (2.16) does. However, it does not vanish at the origin, as (2.16) does.

Compare (2.17) to the standard scalar action for constant fields

$$\Gamma_S=\Gamma_S(S=0)+\int dx\left[-\frac{{M_S}^2}{8\langle S\rangle^2}S^4+\frac{{M_S}^2}{4}S^2\right] \quad (2.18)$$

with the wrong sign for the mass term, so $\langle S\rangle\neq 0$. By comparison with (2.17), an operator relation which makes the two actions equal is

$$S^2(x)=\frac{4\theta(x)}{{M_S}^2}=\frac{bg^2}{2{M_S}^2}G\cdot G\ . \quad (2.19)$$

This relation need only apply at zero momentum transfer, but we will suppose that it holds for a range of small momenta. The tree-level vacuum expectation value of (2.19) has already been given in (1.3), yielding $\langle S\rangle\simeq 130$ MeV. The term $\Gamma_S(S=0)$ is equal to $\int dx\ \frac{1}{2}\langle\theta\rangle$, and represents contributions to the vacuum energy other than from the S glueball.

To complete the action we add to (2.18) the usual kinetic term

$$\int dx\ \tfrac{1}{2}\partial_\mu S\partial^\mu S\ . \quad (2.20)$$

We extract the physical propagating field with the definition

$$S'=S-\langle S\rangle\ . \quad (2.21)$$

For constructing the S matrix, an appropriate interpolating field for S' comes from expanding (2.19) and dropping the S'^2 term:

$$\begin{aligned}S'(x)&=\frac{2[\theta(x)-\langle\theta\rangle]}{\langle S\rangle {M_S}^2}+\cdots\\&={M_S}^{-1}\langle\theta\rangle^{-1/2}(\theta-\langle\theta\rangle)+\cdots\ .\end{aligned} \quad (2.22)$$

It is S' which appears [through (2.22)] in the connected sum rules (2.9). The $N=1$ sum rule involves the zero-momentum S' propagator, and is by construction exactly satisfied, just as the vacuum expectation value (1.3) exactly yields (2.6) when it is used in the scalar action (2.18).

An effective action, to be useful, must be reasonably accurate at the tree level. One finds, by calculating some one-loop graphs, that these multiply corresponding tree-level graphs by a factor (up to logarithms and numbers of order unity)

$$\epsilon=\frac{{M_S}^2}{16\pi^2\langle S\rangle^2}\simeq 0.4\ . \quad (2.23)$$

The numerical value follows from $M_S\simeq 1$ GeV, $\langle S\rangle\simeq 130$ MeV which we adopt as nominal. We could, of course, wish for a smaller expansion parameter, but that is not under our control (except in the large-N_c limit, where $\langle S\rangle\sim N_c$, $M_S\sim 1$). Observe that to the extent that the $O(S'^2)$ corrections to the operator relation (2.22) constitute loop effects, they are of $O(\epsilon)$.

There are other alternatives for forming an effective action for S which is renormalizable. Saving the first four terms in the expansion (2.16) (i.e., the first four sum rules) yields

$$\begin{aligned}\Gamma(\theta)&\simeq\Gamma_4(\theta)\\&\equiv\langle\theta\rangle\int dx\left(\tfrac{1}{4}+\tfrac{11}{6}y-\tfrac{3}{2}y^2+\tfrac{1}{2}y^3-\tfrac{1}{12}y^4\right)\ ,\end{aligned} \quad (2.24)$$

where $y=\langle\theta\rangle^{-1}\theta$. Clearly this effective action requires a linear relation between S and θ, which is identified by requiring $\partial^2\Gamma/\partial S^2=-{M_S}^2\int dx$ at the extremum; this leads to

$$S=\frac{\theta}{M_S\langle\theta\rangle^{1/2}}\ . \quad (2.25)$$

The S-matrix interpolating field S' is the same as given in (2.22), but $\langle S\rangle$ is smaller by a factor of 2. Likewise, the contribution to the action from states other than S is $\frac{1}{2}$ that in (2.17). As a final example, one might require that S saturate the action [i.e., $\Gamma_S(S=0)=0$] instead of saturating the sum rules. An appropriate quadratic action is

$$\Gamma\simeq\bar{\Gamma}\equiv\int dx(2\theta-\theta^2\langle\theta\rangle^{-1})\ , \quad (2.26)$$

which has, as the other actions do, an extremum at $\theta=\langle\theta\rangle$ of value $\int dx\langle\theta\rangle$. Using this one finds by reference to (2.18)

$$S^2=\frac{2\theta}{{M_S}^2}\ , \quad (2.27)$$

which is $\frac{1}{2}$ the value in (2.19). The interpolating field S' is $\sqrt{2}$ times that given in (2.22), with the consequence that the $N=1$ sum rule (2.9) is only 50% saturated by the S' contribution.

These examples of alternative actions to the simplest choice (2.17) lead us to expect ambiguities of order $\sqrt{2}$ in defining S' in terms of θ, about of the same order as the error in using a tree-level action in the first place [see (2.23)].[24] For the rest of this paper we stick to (2.17) because it can be usefully extended to incorporate terms in $G\cdot\tilde{G}$ in a chirally symmetric way.

C. Extension to pseudoscalars

No useful technique similar to applying $g^3\partial/\partial g$ to $\ln Z$ is available for the pseudoscalar density, defined by

$$\tilde{\theta}(x)\equiv\frac{b}{8}g^2G\cdot\tilde{G}(x)\ . \quad (2.28)$$

Drawing on the close relation between the scalar sum rules (2.9) and the one-loop effective action (2.16), we exploit the one-loop action for constant fields as a function of $G\cdot G$ and $G\cdot\tilde{G}$ (Refs. 16 and 25) to derive sum rules for two insertions of $\tilde{\theta}$ and any number of insertions of θ, all at zero momentum transfer.

For covariantly constant fields, all components of the field-strength tensor commute in color space. We can therefore diagonalize all $\bar{G}_{\mu}^{a\nu}$ in Lorentz indices simultaneously. The eigenvalues are of the form $\pm i\bar{B}^a$, $\bar{E}^a$ where $\bar{B}^a$ (magnetic eigenvalues) and $\bar{E}^a$ (electric eigenvalues) are real color vectors. We introduce the matrices

$$\bar{E}=\sum T_a E^a,\ \ \bar{B}=\sum T_b E^b\ , \tag{2.29}$$

where the T_a are group generators in the adjoint representation [normalized so that $\mathrm{Tr}T_aT_b=N_c\delta_{ab}$ for the color group $SU(N_c)$]. The effective action is[16]

$$\Gamma=\int dx\left[-\frac{1}{4g_0{}^2}\bar{G}\cdot\bar{G}-\frac{1}{16\pi^2}\mathrm{Tr}\int_\delta^\infty\frac{ds}{s^3}\left[\frac{\bar{B}s}{\sin\bar{B}s}\frac{\bar{E}s}{\sinh\bar{E}s}[(2\sinh^2\bar{E}s-\sin^2\bar{B}s)+1]-1\right]\right]\ . \tag{2.30}$$

The trace is over the color matrices, and the cutoff δ at small proper times is absorbed in the bare coupling constant g_0. In general, Γ depends on other invariants besides $G\cdot G$ and $G\cdot\tilde{G}$ (Ref. 25); we suppress such dependence by taking $\bar{E}^a$ parallel to $\bar{B}^a$ in color space. Except for imaginary parts, the usual one-loop result (2.16) is found by setting $\bar{E}=0$, $2\,\mathrm{Tr}\bar{B}^2=N_c\bar{G}\cdot\bar{G}$.

The action (2.30) is not expandable about the origin as a power series in $\bar{E}$, $\bar{B}$; instead it has a logarithmic singularity characteristic of perturbation theory. For any fixed s, the integrand is expandable, and the singularity arises from a large-s divergence when one attempts to integrate the expansion term by term. This divergence is the usual infrared divergence associated with massless gluons, that is, associated with field-strength correlations which are long-range. In fact, there is every reason to believe that field-strength correlations are short-range, which amounts to saying that an essential nonperturbative modification of (2.30) is a cutoff of the large-s part of the integrand. Such a cutoff is furnished, for example, by a gluon mass,[11] which multiplies the integrand by e^{im^2s}. In that case, the modified one-loop action can be expanded in a power series, a necessary step if we are to compare the present results with the truncated sum-rule actions such as (2.17) or (2.24).

Therefore, let us speculate that it makes sense to modify (2.30) with a large-s cutoff, and expand the integrand. For $N_c=3$ the term quadratic in $G\cdot G$ and $G\cdot\tilde{G}$, which we modify with a crude cutoff in proper time, is

$$\Gamma=-\frac{1}{16\pi^2b^2}\frac{127}{10}\int dx\int_0^{s_c}ds\,s(\theta^2+\xi^2\tilde{\theta}^2)+\cdots\ , \tag{2.31}$$

where, in analogy with (2.8), we define

$$\tilde{\theta}=\frac{b}{8}g^2G\cdot\tilde{G} \tag{2.32}$$

and the crucial parameter ξ^2 is given by

$$\xi^2=\tfrac{121}{127}\ . \tag{2.33}$$

We construct an effective action by identifying the θ^2 term in (2.31) with the θ^2 term in the quadratic action (2.17), thus requiring the identification (for $N_c=3$)

$$\frac{1}{2\langle\theta\rangle}=\frac{127}{10}\frac{1}{16\pi^2b^2}\int_0^{s_c}s\,ds\ . \tag{2.34}$$

Numerically, this is satisfied if $s_c\sim(0.5\ \mathrm{GeV})^{-2}$, a reasonable value. Finally, our speculative modification of the action (2.17) to include the lowest-order $\tilde{\theta}$ term is

$$\Gamma(\theta,\tilde{\theta})\sim\int dx\left[\tfrac{1}{2}\langle\theta\rangle+\theta-\frac{1}{2\langle\theta\rangle}(\theta^2+\xi^2\tilde{\theta}^2)\right]\ . \tag{2.35}$$

The corresponding source functional for a scalar source J_S coupled to θ, and pseudoscalar source coupled to $\tilde{\theta}$, is found to be

$$W(J_S,J_P)=\int dx\langle\theta\rangle\left[1+J_S+\tfrac{1}{2}J_S{}^2+\frac{1}{2\xi^2}J_P{}^2\right] \tag{2.36}$$

in which the J_S terms will be recognized as the expansion of (2.14) with $J\rightarrow J_S$.

We are now in a position to estimate the Witten sum rule, by taking $\partial^2W/\partial J_P{}^2$ at $J_S=J_P=0$. It is easy to find that

$$i\int dx\langle T\tilde{\theta}(x)\tilde{\theta}(0)\rangle=\xi^{-2}\langle\theta\rangle\ . \tag{2.37}$$

By expressing (2.37) in the form

$$Z^{-1}\int(d\bar{A})\exp\left[-\frac{i}{4g^2}\int dy\,\bar{G}\cdot\bar{G}\right]i\int dx\,\tilde{\theta}(x)\tilde{\theta}(0)=\xi^{-2}\langle\theta\rangle\ , \tag{2.38}$$

and repeatedly applying $g^3\partial/\partial g$ to both sides one finds the set of sum rules

$$i^{N+1}\int dx\,dx_1\cdots dx_N T\langle\theta(x_1)\cdots\theta(x_N)\tilde{\theta}(x)\tilde{\theta}(0)\rangle_{\mathrm{conn}}=\xi^{-2}\langle\theta\rangle\ , \tag{2.39}$$

analogous to the sum rules (2.9) involving θ only.

It would not be difficult to guess the general form of (2.37); the point is that we have an (admittedly speculative) approach to the parameter ξ, which suggests that this is nearly unity. Let us consider earlier approaches[8,10] to finding the RHS of (2.37). Arnowitt and Nath[10] give the following value, based on canonical commutators:[8]

$$i\int dx\langle T\tilde{\theta}(x)\tilde{\theta}(0)\rangle=\tfrac{1}{2}bg^2\langle\theta\rangle\ . \tag{2.40}$$

This cannot be quite correct, since the RHS is not renormalization-group invariant, which happens because the derivation of (2.40) uses canonical commutation relations for unrenormalized fields. More detailed study shows that g^2 should be replaced by b^{-1} times a number of order unity, as in (2.37). It is phenomenologically reasonable that ξ in this equation should be nearly unity, since the Witten sum rule[8] for the η' mass yields $(700-850)\xi^{-1}$ MeV for this mass in the chiral limit (the lower value is for two flavors, the higher value for three flavors).

The next step, given the action (2.35), is to identify $\tilde{\theta}$ with a suitable function of S, and P, and to write down a renormalizable effective action for the glueballs. In doing so we will take the opportunity to fit the results into a chirally invariant scheme, although other options are possible.

D. Effective action for S and P: Speculations on approximate chiral invariance

We are tempted to make these speculations because ξ is so nearly unity, according to (2.33). If ξ were exactly one, there is a chiral symmetry with a simple realization both on the fundamental gluon fields $G^a_{\mu\nu}$ and on the composite fields S and P, and this chiral symmetry is related to $U_A(1)$ acting on quarks through the Peccei-Quinn mechanism.[19] It is explicitly broken by a mass term for the P, and this breaking appears in a natural way.

Consider first the quadratic term in the action (2.35), in the limit $\xi=1$; this term is proportional to $\theta^2+\tilde{\theta}^2$. In Minkowski space, the latter is invariant under the duality transformation

$$G^a_{\mu\nu}\to G^a_{\mu\nu}\cos\beta+\tilde{G}^a_{\mu\nu}\sin\beta \tag{2.41}$$

(recall that in Minkowski space $\tilde{G}\cdot\tilde{G}=-G\cdot G$). The invariant densities transform as

$$\begin{aligned} G\cdot G&\to G\cdot G\cos 2\beta+G\cdot\tilde{G}\sin 2\beta\ ,\\ G\cdot\tilde{G}&\to G\cdot\tilde{G}\cos 2\beta-G\cdot G\sin 2\beta\ . \end{aligned} \tag{2.42}$$

Of course, the term linear in θ in the action (2.35) is not invariant under the above.

The following operator identification allows us to implement the same chiral rotations on S and P; it generalizes (2.19):

$$\begin{aligned} S^2(x)-P^2(x)&=\frac{4\theta(x)}{M_S{}^2}\ ,\\ 2S(x)P(x)&=\frac{4\xi\tilde{\theta}(x)}{M_S{}^2}\ . \end{aligned} \tag{2.43}$$

(We write ξ explicitly, although for the moment it has the value unity.) Then the transformations (2.42) are equivalent to

$$\begin{aligned} S&\to S\cos\beta+P\sin\beta\ ,\\ P&\to P\cos\beta-S\sin\beta\ , \end{aligned} \tag{2.44}$$

and the *full* effective action, including all terms of (2.35) plus kinetic terms for S and P, is

$$\Gamma=\Gamma(S=P=0)+\int dx\left[\tfrac{1}{2}\partial_\mu S\partial^\mu S+\tfrac{1}{2}\partial_\mu P\partial^\mu P-\frac{M_S{}^2}{8\langle S\rangle^2}(S^2+P^2)^2+\frac{M_S{}^2}{4}(S^2-P^2)\right]\ . \tag{2.45}$$

All but the last term are invariant under (2.44), while the S^2-P^2 term, which is just the transcription of the θ term in (2.35), is not. This is so even if $\xi\neq 1$. In writing (2.45) we have used the tree-level formula $\langle S\rangle^2=4M_S{}^{-2}\langle\theta\rangle$; the correction from the P^2 term in (2.43) is suppressed by the smallness parameter ϵ of (2.23).

It is easy to see from (2.45) the prediction $M_S=M_P$, while if the last term were S^2+P^2 instead of S^2-P^2, the P would be a *massless* Goldstone boson. Now it is certainly not the case in nature that $M_S=M_P$ exactly; there is probably a 40% discrepancy, and in any event no reason to expect the existence of some symmetry which enforces $M_S=M_P$. Indeed there is no such symmetry; we have simply enforced by fiat the requirement of simple transformation properties for S and P under (2.41). It is, of course, possible to modify the connection (2.43) between S,P and $\theta,\tilde{\theta}$ so that the correct phenomenological relation between M_S and M_P is obtained, and then chiral symmetry is violated in the quartic couplings as well as in the mass term. But the modest levels of accuracy we can achieve in any case make it reasonable for us to persist with the invented chiral-transformation laws (2.41)–(2.44). The point really is that this chiral symmetry makes it natural that M_P is $\sim$GeV, and not very much smaller.

It is evident that the tree-level relation

$$\tilde{\theta}\simeq\frac{M_S{}^2\langle S\rangle}{2\xi}P \tag{2.46}$$

plus $M_S=M_P$ leads to exact saturation of the $\tilde{\theta}$ sum rule (2.37). Let us consider further consequences of this relation which involves massless quarks. Peccei and Quinn[19] have shown that the chiral quark transformation

$$\psi\to\exp\left[\frac{i}{2}\alpha\gamma_5\right]\psi \tag{2.47}$$

leads to a change in the QCD action for an infinitesimal transformation $\delta\alpha$:

$$\delta\Gamma=-\delta\alpha\int dx\frac{g^2N_F}{32\pi^2}G\cdot\tilde{G}\ , \tag{2.48}$$

where N_F is the number of massless quarks for which (2.47) applies. On the other hand, an infinitesimal change of S and P as in (2.44), measured by an angle $\delta\beta$, leads to a change in the effective action (2.45) of

$$\delta\Gamma=\int dx\,M_S{}^2\langle S\rangle P\delta\beta=\int dx\frac{\xi bg^2}{4}G\cdot\tilde{G}\delta\beta\ , \tag{2.49}$$

where in the second equality we used (2.43). By comparison to (2.48),

$$-\delta\beta=\frac{N_F\delta\alpha}{8\pi^2 b\xi}\ , \tag{2.50}$$

which indicates that the chiral transformation (2.44) is an element of the usual $U_A(1)$ associated with quarks. The chiral charge associated with S,P is not simply related to the chiral charge of quarks, for general N_F.

It will certainly not be easy to check experimentally the glueball couplings of the action (2.45), so at this point we seem to be left only with the crude relation $M_S=M_P$ as a measurable consequence of our speculations. In Sec. III, we will see that the parameter ξ occurs in the couplings to quarks and hadrons, where there are much better chances for experimental verification.

III. COUPLINGS OF GLUEBALLS TO GLUONS, LIGHT QUARKS, AND HADRONS

In this section we consider the couplings of S and P to other strongly interacting states, when all momenta involved are small and only light quarks are involved. Most of the couplings are trivially deduced from what we have already done, with the exception of P couplings to quarks and hadrons. We illustrate some general principles by explicit calculation at the one-dressed-loop level. It has already been remarked that zero-momentum couplings are not necessarily directly relevant to experiments.

A. Couplings of S and P to gluons

We will write an effective Lagrangian for the purpose of generating Feynman rules; this Lagrangian is not to be confused with the effective Lagrangians discussed in Sec. II. It follows directly from the operator relations (2.43) that the coupling of the propagating field $S'=S-\langle S\rangle$ and of P to two gluons is described by (in this approximation $M_S=M_P$)

$$\mathscr{L}_{\rm eff}=\frac{bg^2}{4\langle S\rangle}S'G\cdot G+\frac{bg^2\xi}{4\langle S\rangle}PG\cdot\widetilde{G}\ . \tag{3.1}$$

In view of the trace anomaly (1.5), the first term of this is the same as $-(2\langle S\rangle)^{-1}S'T^\mu_\mu$ (in the absence of massive quarks).

B. Couplings to light quarks and hadrons

We wish to calculate the coefficients in the effective Lagrangian,

$$\mathscr{L}_{\rm eff}=-\bar{\psi}(G_S S+i\gamma_5 G_P P)\psi+\cdots\ , \tag{3.2}$$

for quarks with zero mass in the Lagrangian. If it is accepted that S is an operator proportional to T^μ_μ, that is, S acts like a dilaton,[26] then in principle one need not calculate anything to find the zero-momentum coupling of S to on-shell quarks, since the matrix elements of T^μ_μ are completely determined in this configuration. The concept of an on-shell quark with a specific mass is at best heuristically useful, but we will pursue it.

If $\langle S\rangle$ is not zero, the quarks pick up an effective mass of order $G_S\langle S\rangle$, plus contributions from other processes not mediated by S. Let us call this constituent mass M_Q. The matrix element of T^μ_μ at zero momentum transfer

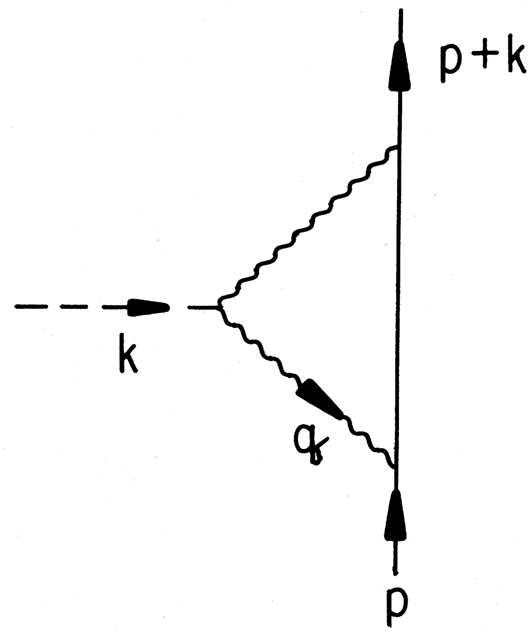

FIG. 1. Coupling of S or P to quarks via two gluons.

with on-shell quarks is just M_Q, so from the operator relation between S' and T^μ_μ [see (1.5) and (2.43)] we immediately deduce

$$G_S=\frac{M_Q}{2\langle S\rangle}\ . \tag{3.3}$$

Numerically $G_S\simeq 1$, for usually quoted values of M_Q and $\langle S\rangle$ as given in (1.3).

It is worthwhile looking for some simple approximation in which a relation like (3.3) can be directly calculated, if only to build confidence in the necessary approximations. Consider the graph of Fig. 1, showing the coupling of S' to two quarks. We will calculate this graph at $k=0$ using free vertices and a free massive quark propagator, but with massive gluon "propagators"[27] of the type advocated in Ref. 11; this modification of the propagator removes an infrared divergence associated with the limit $M_Q=0$, which we will presently pursue. The modified propagator is

$$d_{\mu\nu}(q)=(-g_{\mu\nu}+\text{ gauge terms})d(q^2)(bg^2)^{-1}\ , \tag{3.4}$$

$$d^{-1}(q^2)=(q^2-M_G{}^2)\ln\left[\frac{-q^2+4M_G{}^2}{\Lambda^2}\right]\ , \tag{3.5}$$

where Λ is the renormalization-group mass. This propagator is a good approximation to the solution of a certain nonlinear equation[11] in which $\beta(g)$ has been set equal to $-bg^3$.

Now consider the limit $M_Q \ll M_G$, to simplify the presentation. M_Q can be neglected everywhere except in the numerator of the quark propagator, and likewise P can be put to zero everywhere except in that numerator, in order to go to the quark mass shell. A straightforward calculation then gives

$$G_S = \frac{3iC_F M_Q}{b\langle S\rangle(2\pi)^4}\int d^4q\, d^2(q) . \tag{3.6}$$

Here C_F is the quark Casimir eigenvalue. As needed, G_S is indeed proportional to $M_Q\langle S\rangle^{-1}$, and consistency with (3.3) requires the condition

$$1 = \frac{6iC_F}{(2\pi)^4 b}\int d^4q\, d^2(q) . \tag{3.7}$$

Remarkably, this contains no reference to the quarks, except through C_F. The RHS of (3.7) is about $\frac{1}{2}$, which we consider an adequate check of consistency given the approximations made. An infinity of higher-order graphs also contribute, an example of which is Fig. 2. It is important to note that a consistency condition of the same general type as (3.7) also occurs[11] in studying the nonlinear "propagator" equation whose solution is (approximately) (3.4) and (3.5); the condition determines the gluon mass M_G. Generating a gluon mass is the same as giving S a nonzero expectation value, and now we see that generating a quark mass is likewise the same as $\langle S\rangle \neq 0$ and involves the same sort of consistency condition.

Next consider the P coupling, using the approximation of Fig. 1. The numerator of this graph needs some comment; it is

$$4\epsilon^{\mu\nu\alpha\beta}q_\mu(q+k)_\nu\gamma_\alpha(\not p-\not q+M_Q)\gamma_\beta . \tag{3.8}$$

Evidently this is linear in the quark momentum transfer k, which must not be set to zero to begin with. However, we can set $k=p=0$ in other factors of the graph and replace the terms quadratic in q, i.e., $-q_\mu \not q$, by $-\frac{1}{4}q^2\gamma_\mu$. We will show that the M_Q term in (3.8) is of higher order in k and p when sandwiched between on-shell spinors, so ignore it for now. Then (3.8) becomes $-6iq^2\gamma_5\not k$, and sandwiched between spinors yields

$$-\bar u(p+k)6iq^2\gamma_5\not k u(p) = 12iq^2M_Q\bar u\gamma_5 u . \tag{3.9}$$

This is to be compared to the numerator for the S glueball, which is $12q^2M_Q\bar u u$. The term linear in M_Q in (3.8) is proportional to $M_Q q_\nu k_\mu\gamma_5\sigma^{\mu\nu}$, whose matrix elements between spinors are easily shown to be $O(M_Q{}^2k, M_Qk^2)$.

It is now clear that the P graph is the same as the S graph, except that $\bar u u$ is replaced by $i\xi\bar u\gamma_5 u$, and so

$$G_P = \xi G_S \tag{3.10}$$

for light quarks and zero momentum. For $\xi=1$, the combination $S+i\xi\gamma_5 P$ transforms under (2.44) with a phase $e^{-i\gamma_5\beta}$; if β were equal to the quark angle α in (2.46), the effective Lagrangian (3.2) would be chirally invariant. Combining (3.10) with (3.3) leads to a sort of Goldberger-Treiman relation

$$G_P = \frac{\xi M_Q}{2\langle S\rangle} , \tag{3.11}$$

but this is not fully justified, since we derived (3.10) only under the approximation of Fig. 1 which does not quite yield (3.3).

Let us briefly discuss the couplings of S,P to hadrons made of light quarks. For S, these follow directly from the proportionality of the glueball operator to T^μ_μ, and at zero momentum generalize (3.3) to

$$G_{S\bar BB} = \frac{M_B}{2\langle S\rangle} , \tag{3.12}$$

$$G_{Smm} = \frac{M_m{}^2}{2\langle S\rangle} \tag{3.13}$$

for baryons B and mesons m of masses M_B, M_m which are made of light quarks. If taken literally these have interesting phenomenological consequences (e.g., $S\to\pi\pi$ very weakly), but mixing of S with $\bar q q$ states and extrapolation to the S mass shell may lead to large corrections. Within the approximation of Fig. 1, one also has $G_{P\bar BB} = \xi G_{S\bar BB}$, and appropriate chiral relations between S and P couplings for mesons. Such chiral relations may be badly broken.

Because $\langle S\rangle$ is not large, all these couplings are rather strong, which is not the naive expectation from the large-N_c limit.[28,29] In this limit, couplings of glueballs to hadrons are $O(N_c{}^{-1})$, and indeed that is the case, since from (1.3) and $\langle G\cdot G\rangle \sim N_c{}^2$ it follows that $\langle S\rangle \sim N_c$. However, the constant of proportionality is crucial and our work shows that $N_c=3$ is not large enough for substantial suppression, just as $M^2(\eta')$ is large even though of order $N_c{}^{-1}$. In general, then, one does not expect unusually small widths for glueballs, and to depend on this for an experimental signature may be misleading.

We have already noted in the Introduction that these couplings of S and P to hadrons cannot be the remnants of a Higgs coupling, because the P has the wrong charge-conjugation eigenvalue. This is easy to see, by invoking the old arguments[30] that requiring high-energy tree unitarity on the S matrix automatically leads to Higgs couplings which generate masses for spin-1 particles. Consider, for example, a chiral $SU(N_f)\times SU(N_f)$ hadron symmetry (N_f=number of flavors) which has massless vector

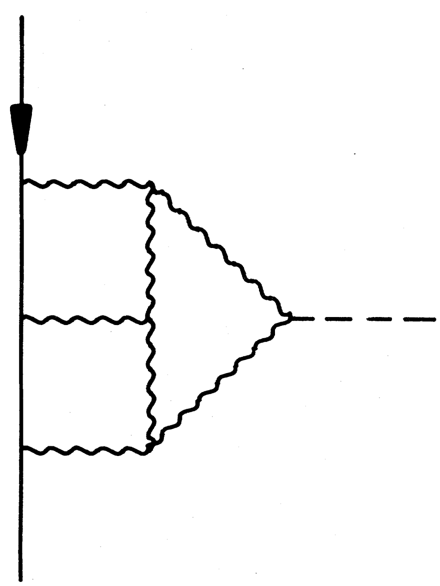

FIG. 2. An example of other coupling processes of glueballs to quarks.

mesons V_μ and axial-vector mesons A_μ, both with $CP=1$. The Higgs couplings which make these massive are also necessary to make the tree-level S matrix well behaved at high energies. In particular, for the process $VV\to AA$ a pseudoscalar flavor singlet is necessary, which couples to $V_\mu A^\mu$ and thus has $C=-1$. So we cannot use unitarity to argue in favor of extra flavor singlets like S and P in dealing with hadronic phenomenology.

However, it may be that particles of these quantum numbers appear as solitons of phenomenological hadron Lagrangians of the usual type, with vector-meson mass terms not associated with the Higgs mechanism. We can illustrate this for an S-type soliton using a gauge-invariant mass term for ρ mesons, constructed with the aid of auxiliary pure-gauge fields. It has been shown[22] that this leads to a soliton of the 't Hooft—Polyakov variety, except that there are no long-range (monopole) fields because all isospin components of the ρ meson are massive. The mass of the soliton is, in the semiclassical approximation,[22,31]

$$M_{\rm sol}=3.05m_\rho\left[\frac{4\pi}{g_\rho{}^2}\right]. \qquad (3.14)$$

With $m_\rho\simeq 750$ MeV, $g_\rho{}^2/4\pi\simeq 2$, we find $M_{\rm sol}\simeq 1200$ MeV. Of course, the semiclassical approximation is not to be trusted for such a large coupling constant, nor is it clear what relation this phenomenological soliton bears (if it exists) to states constructed directly from the underlying QCD Lagrangian.

Note added: After this work was finished, Claude Bernard brought our attention to the work of Bhanot, Rabinovici, Seiberg, and Woit,[32] who have evaluated the dependence of the vacuum energy on the vacuum angle Θ for the color group SU(2), using lattice methods. These authors, working at a lattice β of 2.1 (or lattice spacing $a^{-1}=530$ MeV), find

$$\left.\frac{\partial^2 E_{\rm vac}(\Theta)}{\partial\Theta^2}\right|_{\Theta=0}=(6.6\pm0.4)\times10^{-3}a^{-4}$$
$$\simeq 5.2\times10^{-4}\ {\rm GeV}^4\,.$$

We can evaluate the dependence of $E_{\rm vac}$ on Θ by using $\Gamma=-\int d^4x\,E_{\rm vac}$, and inducing a finite Θ via the transformation (2.42) or (2.44), with $\beta=\Theta(8\pi^2 b\xi)^{-1}$ according to (2.50). In a parity-conserving theory, Γ is to be evaluated at $\theta=\langle\theta\rangle$ and $\tilde\theta=0$. Let us, as in the main text, set $\xi=1$. Then (2.35) and (2.42) yield

$$E_{\rm vac}(\Theta)=-\frac{bg^2}{8}\langle G\cdot G\rangle\cos\left[\frac{\Theta}{4\pi^2 b}\right].$$

In accordance with general large-N_c arguments,[8] this is of the form $N_c{}^2(\Theta/N_c)$. This implies that the coefficient of Θ^2 is independent of N_c (for large N_c, at least), and suggests the possibility of comparing the SU(2)-lattice value with our phenomenological SU(3) value. We find, for $N_c=3$,

$$\left.\frac{\partial^2 E_{\rm vac}}{\partial\Theta^2}\right|_{\Theta=0}=\frac{1}{2b}\left[\frac{g}{8\pi^2}\right]^2\langle G\cdot G\rangle$$
$$\simeq 5.4\times10^{-4}\ {\rm GeV}^4\,.$$

The close agreement is fortuitous, and perhaps should not be taken too seriously because of the difference in N_c. However, we hope that future lattice work will be done on the Θ dependence of $E_{\rm vac}$ for $N_c=3$.

Note added in proof. Shifman has informed us that he and his collaborators have also derived the scalar sum rules in our Eq. (2.9) and discussed some of the applications which we have made in a way similar, but not identical, to ours.[33-35] These authors have not discussed the extension to pseudoscalars and the ensuing broken chiral symmetry.

ACKNOWLEDGMENTS

We thank K. Bardakci and M. Halpern for reminding us of the existence of Ref. 20. This work was supported in part by the National Science Foundation and the work of A. Soni in part by DOE Grant No. DE-AM03-76SF0034/PA-AE-AT03-81ER40024/Task J (Outstanding Junior Investigator Program).

APPENDIX

We give the relations between coupling constants and wave-function integrals for the Schrödinger-equation approach to S and P glueballs.

Let the effective couplings between glueballs and gluons be described by the Lagrangian

$$\mathcal{L}=\gamma_S SG\cdot G+\gamma_P PG\cdot\tilde G\,. \qquad ({\rm A1})$$

After factoring out appropriate projection operators and spherical harmonics, the nonrelativistic partial-wave amplitude in the 0^{++} two-gluon channel is

$$t(0^{++})=e^{i\delta}\sin\delta$$
$$=\tfrac{3}{2}(N_c{}^2-1)\left[\frac{k}{8\pi M_S}\right]\frac{16\gamma_S{}^2M_G{}^4}{s-M_S{}^2-iM_S\Gamma_S}\,, \qquad ({\rm A2})$$

where k is the c.m. momentum and s is the Mandelstam energy variable. In the 0^{-+} channel,

$$t(0^{-+})=(N_c{}^2-1)\left[\frac{k^3}{8\pi M_P}\right]\frac{128\gamma_P{}^2M_G{}^2}{s-M_P{}^2-iM_P\Gamma_P}\,. \qquad ({\rm A3})$$

For a nonrelativistic S-wave bound state, the pole term in t can be written in terms of integrals over the wave function:

$$t=-\frac{kM_G}{4\pi}\frac{I^2}{E_B-E}\,, \qquad ({\rm A4})$$

where $s=(2M_G+E)^2$, $s_B=(2M_G+E_B)^2$ in terms of the binding energy E_B ($s_B=M_S{}^2$ or $M_P{}^2$), and the integral I is

$$I=\left.\int d^3r\,e^{i\vec k\cdot\vec r}V(r)\psi_B(r)\right|_{\vec k^2=M_GE_B}\,. \qquad ({\rm A5})$$

Here $V(r)$ is the binding potential and $\psi_B(r)$ the wave function, normalized so $\int d^3r\,|\psi_B|^2=1$. For a bound state of the usual sort, $E_B<0$ and k is imaginary, but this is not the case for glueballs which are more massive than their constituents.

According to (3.1),

$$\gamma_S = \frac{bg^2}{4\langle S \rangle} \simeq 1 \text{ GeV}^{-1}, \quad \gamma_P = \frac{bg^2 \xi}{4\langle S \rangle} \simeq 1 \text{ GeV}^{-1} . \qquad (A6)$$

Here we simply take $bg^2 \simeq \frac{1}{2}$, to get numerical values. This corresponds to $\alpha_s \sim 0.5$ for the energy scales of interest (~ 1 GeV).

Combining (A2) and (A4) yields ($N_c = 3$)

$$\gamma_S{}^2 = \frac{I^2 M_S}{24 M_G{}^2} , \qquad (A7)$$

which is to be compared to the value in (A6). For the nonrelativistic wave functions of Ref. 5, we have calculated I and find values of γ_S roughly twice or more as large as that given in (A6), and similarly for γ_P. There are major uncertainties in using (A7) having to do with using a nonrelativistic approximation, and so instead of proceeding further along these lines, which would be correct in principle for a nonrelativistic bound state, we have adopted another approach in the main body of this paper. This leads to the quoted values in (A6).

[1]For a recent review, see E. Bloom, in *Proceedings of 21st International Conference on High Energy Physics, Paris, 1982,* edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. 43, C3-407 (1982)]; C. Edwards *et al.,* Phys. Rev. Lett. 49, 259 (1982).

[2]C. Edwards *et al.,* Phys. Rev. Lett. 48, 458 (1982).

[3]G. Bhanot and C. Rebbi, Nucl. Phys. B189, 469 (1981); K. Ishikawa, M. Teper, and G. Schierholz, Phys. Lett. 110B, 399 (1982); Z. Phys. C 19, 327 (1983); B. Berg and A. Billoire, Nucl. Phys. B221, 109 (1983); B226, 405 (1983).

[4]R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976); J. F. Donoghue, K. Johnson, and B. A. Li, Phys. Lett. 99B, 416 (1981); C. E. Carlson, T. H. Hansson, and C. Peterson, Phys. Rev. D 27, 1556 (1983); 28, 2895(E) (1983).

[5]J. M. Cornwall and A. Soni, Phys. Lett. 120B, 431 (1983).

[6]T. Barnes, Z. Phys. C 10, 275 (1981).

[7]See, e.g., C. Rosenzweig, J. Schechter, and G. Trahern, Phys. Rev. D 21, 3389 (1980); J. Schechter, *ibid.* 21, 3393 (1980); P. De Vecchia and G. Veneziano, Nucl. Phys. B171, 253 (1980).

[8]E. Witten, Nucl. Phys. B156, 269 (1979).

[9]As further elaborated by G. Veneziano, Nucl. Phys. B159, 213 (1979).

[10]R. Arnowitt and P. Nath, *Unification of the Fundamental Particle Interactions,* proceedings of the Europhysics Study Conference, Erice, Italy, 1980, edited by S. Ferrara, J. Ellis, and P. van Nieuwenhuizen (Plenum, New York, 1980), p. 411.

[11]J. M. Cornwall, Phys. Rev. D 26, 1453 (1982). The concept of a gauge-invariant gluon mass term for QCD was first given in Ref. 22.

[12]S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134, 539 (1978); G. K. Savvidy, Phys. Lett. 71B, 133 (1977); M. J. Duff and R. Ramón-Medrano, Phys. Rev. D 12, 3357 (1976); H. Pagels and E. Tomboulis, Nucl. Phys. B143, 485 (1978). For uses of the effective potential closer in spirit to our development, see R. Fukuda and Y. Kazama, Phys. Rev. Lett. 45, 1142 (1980); S. Adler, Phys. Rev. D 23, 2905 (1981).

[13]J. C. Collins, A. Duncan, and S. D. Joglekar, Phys. Rev. D 16, 438 (1977).

[14]N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).

[15]M. A. Shifman, A. J. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979). See also B. Guberina, R. Meckbach, R. D. Peccei, and R. Rückl, *ibid.* B184, 476 (1981) who give a numerical value for $\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$ very similar to that of Shifman *et al.*

[16]A. Yildiz and P. H. Cox, Phys. Rev. D 21, 1095 (1980). For the original work on this subject, see J. Schwinger, Phys. Rev. 82, 664 (1951); also see Ref. 14.

[17]If a gluon mass of $\simeq$500 MeV (see Refs. 5 and 11) is used as the cutoff, the coefficient of $(G \cdot G)^2$ is consistent with the phenomenological value found in Ref. 15. See Sec. II.

[18]The values given in Refs. 8 and 10 are not consistent with the renormalization group.

[19]R. Peccei and H. Quinn, Phys. Rev. D 16, 1791 (1977).

[20]Such Lagrangians were written down long ago, e.g., K. Bardakci and M. B. Halpern, Phys. Rev. D 6, 696 (1972); see also I. Bars, M. B. Halpern, and Y. Yoshimura, Phys. Rev. Lett. 29, 969 (1972); Phys. Rev. D 7, 1233 (1973).

[21]J. M. Cornwall, Phys. Rev. D 22, 1452 (1980).

[22]J. M. Cornwall, in *Deeper Pathways in High-Energy Physics,* proceedings of Orbis Scientiae, 1977, Coral Gables, Florida, edited by A. Perlmutter and L. F. Scott (Plenum, New York, 1977), p. 683.

[23]We leave to the reader the problems of interpretation arising in an Abelian gauge theory, which are intimately connected with defining a renormalized, finite $\langle \theta \rangle$ for the present non-Abelian case.

[24]If the quadratic action (2.16) is used with the *exact* relation (2.18), the LHS of the sum rules (2.8) vanishes for $N \geq 2$, as it should. If the approximate relation (2.21) is used, the LHS of the $N \geq 2$ sum rules is in error by similar amounts. For example, for $N = 3$ one finds $\frac{3}{2}\langle \theta \rangle$ for the LHS, instead of the exact value $\langle \theta \rangle$.

[25]There are, of course, many other gauge-invariant densities without field-strength derivatives which appear in the general effective action [see R. Roskies, Phys. Rev. D 15, 1722 (1977)]. But none of the others is only bilinear, and we assume that couplings of S and P to two-gluon fields are dominant.

[26]See, e.g., P. Carruthers, Phys. Rev. D 2, 2265 (1970), and references therein.

[27]The "propagators" of Ref. 11 are not the canonical ones, which are nontrivially gauge-dependent, but rather objects constructed from pieces of vertices, etc., as well as canonical propagators. The modified propagators have gauge-invariant proper self-energies.

[28]E. Witten, Nucl. Phys. B160, 57 (1979).

[29]D. Robson, Nucl. Phys. B130, 328 (1977); C. E. Carlson *et al.,* Phys. Lett. 98B, 110 (1981); 99B, 353 (1981).

[30]J. M. Cornwall, D. Levin, and G. Tiktopoulos, Phys. Rev. D 10, 1145 (1974); Phys. Rev. Lett. 30, 1268 (1973); C. H. Llewellyn Smith, Phys. Lett. 46B, 233 (1973).

[31]The numerical coefficient given in Ref. 22 is in error by some 40%; it was corrected later by K. Olynyk, whom we thank.

[32]G. Bhanot, E. Rabinovici, N. Seiberg, and P. Woit, report, 1983 (unpublished).

[33]V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B191, 301 (1951).

[34]M. A. Shifman, Z. Phys. C 9, 347 (1981).

[35]A. A. Migdal and M. A. Shifman, Phys. Lett. 144B, 445 (1982).