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Quark models and the structure of the $\Delta(1232)$ resonance

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We investigate the structure of the $\Delta(1232)$ resonance within a model which contains pions (π) interacting with nucleons (N) and Δ's. With a πNN (or $\pi N\Delta$) vertex form factor obtained from quark models, we demonstrate explicitly that the perturbation series of π-N scattering converges rapidly. Thus a crossing-symmetric solution to the π-N scattering problem can be obtained reliably by perturbation expansion. We expand the π-N scattering series to fourth order in renormalized coupling constants, and then the (3,3) phase shift is calculated using both T and K matrices. The Δ contribution is found to be dominant in generating the observed resonance. Our calculation and results are compared with those of the cloudy bag model.

I. INTRODUCTION

The MIT bag model[1] and the constituent quark model (CQM)[2] are both quite successful in describing hadronic static properties. A major extension of the original bag model has been the introduction of the pion (π), as a fundamental field, into the bag-model Lagrangian.[3-8] The inclusion of the pion field is required to restore chiral symmetry, and so the resulting model is often called the chiral bag model. Recently pionic effects have also been incorporated into the CQM.[9] Here the inclusion of the pion field is not guided by any symmetry principle, but is simply necessitated by the fact that pions interact with baryons. In both cases, the three-quark core of a baryon is considered as a static, extended source of pions. It is assumed that pointlike pions are coupled directly to u and d quarks in the core; from this one can then deduce the effective pion-baryon interaction Hamiltonian. Now if the pion field is second quantized, the result is an effective field theory of π-baryon interactions. This model differs from the old static pion-nucleon (N) model in two respects. Firstly, the degree of freedom in the baryon sector has been extended to include excited states of the nucleon (Δ and N^*) explicitly. Secondly, the πNN vertex form factor is no longer arbitrary; it can be obtained from the quark wave function which is in turn determined by the static properties of the nucleon.

The notion of coupling pions directly to quarks is taken more seriously by some physicists, who tend to regard the π-quark coupling as fundamental. Then, to be consistent, one has to sum over all possible virtual quark states in a π loop such as the one shown in Fig. 1. This, however, leads to divergent baryon self-energies in the second-quantized version of the chiral bag model—the cloudy bag model (CBM).[10] We shall not address ourselves to this problem here, but shall instead regard the introduction of direct π-quark coupling as merely a convenient means of deriving an effective theory of π-baryon interactions.

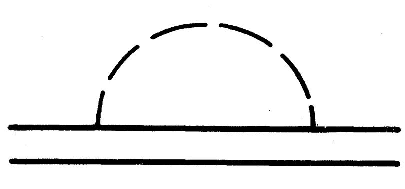

FIG. 1. Microscopic view of pionic contribution to baryon self-energy. The solid lines represent quarks and the dashed line represents a pion.

We distinguish three different descriptions of the Δ. A bare Δ or a three-quark Δ is defined as a purely three-quark bound state. A dressed Δ consists of a three-quark core surrounded by a pion cloud. Finally, a physical Δ or $\Delta(1232)$ resonance is what one experimentally observes in laboratories. The purpose of this paper is to investigate the structure of the $\Delta(1232)$ resonance within the model sketched above. On the level of an effective field theory, the underlying hadronic model (CBM or CQM) turns out to be irrelevant for our purpose. The important thing is that there exists a three-quark Δ state, which is predicted by all quark models. Moreover, as will be shown in Sec. II, form factors determined by hadronic static properties in the CQM and CBM turn out to be similar.

The formation of the $\Delta(1232)$ resonance is the most prominent feature in intermediate-energy π-N scattering. The $\Delta(1232)$ also plays an important role in many other nuclear reactions at intermediate energies. These include π-nucleus scattering, (γ,π), (π,p), and (γ,p) reactions, etc. In all these reactions the $\Delta(1232)$ formation takes place within a nucleus. In a nuclear medium the properties of the $\Delta(1232)$ are expected to be modified; this medium effect has been the subject of many discussions in recent years.[11] It is clear that a knowledge of the different components of the $\Delta(1232)$, apart from interesting in its own right, is also necessary for a deeper understanding of nuclear medium effects on its free-space properties. Furthermore, knowing the structure of the $\Delta(1232)$ is also important in $N\Delta$ coupled-channel calculations. An attempt is being made to incorporate the bare Δ in a πNN three-body calculation.[12]

The π-baryon (N and Δ) interaction Hamiltonian is given and discussed in Sec. II. The π-N scattering amplitude is calculated perturbatively in Sec. III, where comparison is also made with previous works on describing the $\Delta(1232)$ in the quark model. Results are presented and discussed in Sec. IV. Finally, summary and conclusions are given in Sec. V.

II. INTERACTION HAMILTONIAN

In the subsequent calculation, we shall include only N and Δ intermediate states. Contributions from higher excited states are expected to be small due to larger energy denominators. Also, the πNN^* coupling is suppressed for low-momentum pions because the quark wave functions of N and N^* are orthogonal.

In quark models, coupling pions directly to quarks results in the following effective π-(N,Δ) interaction Hamiltonian:[6–9]

$$H_I=\int \frac{d^3k}{(2\pi)^{3/2}}[\mathscr{H}_{\pi NN}(\vec{k})+\mathscr{H}_{\pi N\Delta}(\vec{k})+\mathscr{H}_{\pi\Delta\Delta}(\vec{k})] \tag{1}$$

with

$$\mathscr{H}_{\pi NN}(k)=\sum_\alpha i\left[\frac{4\pi}{2\omega_k}\right]^{1/2}\frac{f^{(0)}_{\pi NN}}{m_\pi}u(k)N^\dagger\tau_\alpha\vec{\sigma}\cdot\vec{k}Na_\alpha(\vec{k})+\text{H.c.}\,, \tag{2}$$

$$\mathscr{H}_{\pi N\Delta}(k)=\sum_\alpha i\left[\frac{4\pi}{2\omega_k}\right]^{1/2}\frac{f^{(0)}_{\pi N\Delta}}{m_\pi}u(k)\Delta^\dagger T_\alpha\vec{S}\cdot\vec{k}Na_\alpha(\vec{k})+\text{H.c.}\,, \tag{3}$$

$$\mathscr{H}_{\pi\Delta\Delta}(k)=\sum_\alpha i\left[\frac{4\pi}{2\omega_k}\right]^{1/2}\frac{f^{(0)}_{\pi\Delta\Delta}}{m_\pi}u(k)\Delta^\dagger\mathscr{T}_\alpha\vec{\mathscr{S}}\cdot\vec{k}\Delta a_\alpha(\vec{k})+\text{H.c.}\,, \tag{4}$$

where $a_\alpha(\vec{k})$ is the annihilation operator for a pion of charge α and momentum $\vec{k}$. $\vec{S}(\vec{T})$ is the $\frac{1}{2}$-to-$\frac{3}{2}$ spin (isospin) transition operator, and $\vec{\mathscr{S}}(\vec{\mathscr{T}})$ is the $\frac{3}{2}$ spin (isospin) operator. $f^{(0)}$'s are the bare coupling constants. If we adopt the usual convention that[13]

$$\langle \tfrac{3}{2}||S||\tfrac{1}{2}\rangle=\langle \tfrac{3}{2}||T||\tfrac{1}{2}\rangle=2$$

and (5)

$$\langle \tfrac{3}{2}||\mathscr{S}||\tfrac{3}{2}\rangle=\langle \tfrac{3}{2}||\mathscr{T}||\tfrac{3}{2}\rangle=\sqrt{15}\,,$$

then

$$f^{(0)}_{\pi N\Delta}=(\tfrac{72}{25})^{1/2}f^{(0)}_{\pi NN}\,,\quad f^{(0)}_{\pi\Delta\Delta}=\tfrac{4}{5}f^{(0)}_{\pi NN} \tag{6}$$

from the zeroth-order quark spin-isospin wave functions of the baryon cores. It is often assumed that the renormalized coupling constants (f's) also obey the same relations (6).[6,9] However, when one splits the N and Δ masses, relations (6) may no longer hold. Furthermore, coupling a pion to a bound system of quarks would inevitably involve the notion of off-shell bound states (or off-shell bags), of which we do not have any knowledge. The issue is therefore more complicated than one would naively think. In view of the above remarks, we shall regard the renormalized coupling constants $f_{\pi N\Delta}$ and $f_{\pi\Delta\Delta}$ as free parameters to be fixed by experimental data. The renormalized πNN coupling $f_{\pi NN}$ is fixed by experiment at

$$(f_{\pi NN})^2=0.08\,. \tag{7}$$

In Eqs. (2)—(4), $u(k)$ is the vertex form factor. We have assumed $u(k)$ to be identical for the πNN, $\pi N\Delta$, and $\pi\Delta\Delta$ vertices. In the CQM with a harmonic-oscillator interquark potential $u(k)$ is given by

$$u(k)=e^{-k^2/6\alpha^2}\,. \tag{8}$$

With pointlike pions, the cutoff parameter α is related to the root mean square radius of the baryon core by

$$\alpha^{-1}=r_{\text{rms}}\,. \tag{9}$$

A consistent fit to baryon masses, magnetic moments, and nucleon charge radii gives[9,14]

$$\alpha\simeq 1.2\text{ fm}^{-1}\,. \tag{10}$$

In the CBM,[6,7]

$$u(k)=3j_1(kR)/kR\,, \tag{11}$$

where R is the bag radius. Equation (11) can be well approximated by the Gaussian form of Eq. (8) with[15]

$$\alpha^{-2}=0.636R^2\,. \tag{12}$$

In the CBM calculation R was varied, together with $f_{\pi NN}$ and the dressed Δ mass ω_Δ $(=M_\Delta-M_N)$, in order to obtain a fit to the total cross section in the (3,3) channel. It turned out that $R=0.82$ fm and $\omega_\Delta=280$ MeV. However, this solution is not unique. As shown in Ref. 6, another possible solution is $R=0.22$ fm and $\omega_\Delta=\infty$ (equivalent to $f_{\pi N\Delta}=0$). We feel that R should not be considered as a free parameter in a scattering calculation. In order to have a stable bag, and avoid multiplicity of solutions, R should be determined separately by minimizing the total bag energy, including the pion-cloud contribution, with respect to the bag radius:

$$\left.\frac{\partial E(r)}{\partial r}\right|_{r=R}=0\,. \tag{13}$$

In a semiclassical approach,[16] the baryon mass spectrum and magnetic moments have been studied with pionic effects included in the bag model. The mass study also took into account the center-of-mass correction and the gluon magnetic interaction energy. The N and Δ bag radii were determined to be around 1 fm. Nucleon magnetic moments and charge radii have also been studied in the CBM.[17,18] The best fit to experimental values corresponds also to $R\approx 1$ fm. Now substituting $R=1$ fm into Eq. (12)

gives $\alpha = 1.25\ \text{fm}^{-1}$, which is very close to that obtained in the CQM [Eq. (10)]. So there seems to be an approximate agreement on the range of the pion vertex form factor in the CQM and the bag model. We shall use Eq. (8) with α given by Eq. (10) in our calculation. We will not argue about the exact value of α. As long as $\alpha \simeq 1.2\ \text{fm}^{-1}$ or, equivalently, $R \simeq 1$ fm, our calculation and conclusions will not be affected in any essential way.

III. THE Δ(1232) RESONANCE

For a long time, since the 1950's, mainly due to the work of Chew and Low,[19] the Δ(1232) resonance has been considered as a resonant state of a nucleon and pions. A more modern view is that it possesses, apart from a π-N part, a three-quark Δ part as well.[7,13] This three-quark Δ state appears naturally in the quark model of hadrons. In this work we take the latter point of view and include the Δ explicitly in our calculation. With the π vertex form factor given in Sec. II, one can show that the perturbation series of the π-N scattering amplitude converges very fast. Consequently, a crossing-symmetric solution to the (3,3)-channel π-N scattering problem can be obtained by perturbation expansion to finite order.

We first expand the π-N scattering T matrix to fourth order in renormalized coupling constants; it is diagrammatically shown in Fig. 2. To be consistent, second-order corrections to propagators and vertices should also be included in evaluating the second-order scattering diagrams. These corrections are shown in Fig. 3. Figure 3(a) renormalizes baryon masses, and Fig. 3(b) renormalizes coupling constants. The renormalization procedure can be carried out in a standard way, and is shown explicitly in Appendix A. The evaluation of diagrams shown in Fig. 2 involves straightforward manipulation of spin and isospin operators. The resulting expressions are given in Appendix B. The on-energy-shell T matrix is related to the phase shift δ_{33} by the unitarity condition, which gives[19]

$$T_{33}(k',k) = -\frac{4\pi}{2\omega_k}\frac{3}{k^3}e^{i\delta_{33}}\sin\delta_{33}\mathscr{P}_{33}(k',k)\ , \tag{14}$$

where $\mathscr{P}_{33}$ is the (3,3)-channel projection operator defined by

$$\mathscr{P}_{33}(k',k) = P_{3/2}(k',k)Q_{3/2}(\vec{k}',\vec{k}) \tag{15}$$

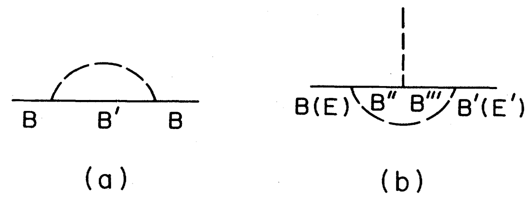

FIG. 3. (a) Pionic contribution to baryon self-energy. (b) π-baryon vertex correction. The B's stand for N or Δ and E's for energies.

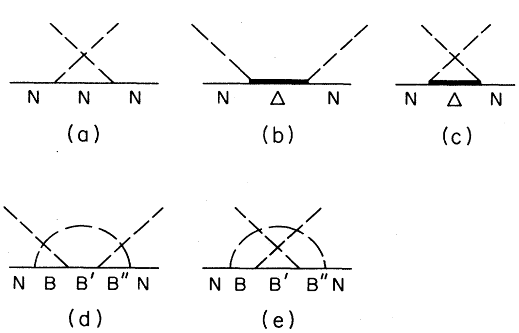

FIG. 2. Pion-nucleon scattering amplitude to fourth order. The B's stand for N or Δ.

with $P_{3/2}$ and $Q_{3/2}$ given in Appendix B. A T-matrix solution to the π-N scattering amplitude has also been obtained in the CBM.[6,7] In that calculation the Low equation was solved by including only one-pion—one-nucleon intermediate states, and neglecting crossing symmetry. This solution is equivalent to iterating Figs. 2(a) and 2(b) combined to infinite order. A fit to the experimental (3,3)-channel total cross section was then obtained by varying $f_{\pi NN}$, ω_Δ, and R. Our calculation differs from it in several respects. Firstly, we fix $f_{\pi NN}$ at the experimental value [Eq. (7)]. In Ref. 7, the value $(f_{\pi NN})^2 = 0.064$ was used. This is not satisfactory since Eq. (7) is well established. Secondly, we use a vertex form factor which is fixed by the static properties of the nucleon in the quark model (see the discussion on this point in Sec. II). Thirdly, our solution is crossing symmetric and is obtained through a consistent perturbation expansion of the scattering series; specifically, we include all diagrams up to fourth order. In this approach, the πNN and $\pi N\Delta$ vertex corrections can be taken into account in a straightforward way. Vertex corrections have not been included correctly in Ref. 7; e.g., Eq. (3.6) of that paper holds *only* for the π-N crossed Born term [Fig. 2(a)].

A different but convenient approach is to calculate the reactance matrix (K or R), for which one replaces the Green's function

$$G(E) = \frac{1}{E - H_0 + i\epsilon} \tag{16}$$

that appears in the T-matrix approach by its real part, i.e.,

$$G(E) \rightarrow G_R(E) = \frac{\text{P}}{E - H_0}\ , \tag{17}$$

where P indicates that the Cauchy principal value is taken whenever the energy denominator vanishes. And again, by unitarity, the on-energy-shell K matrix can be related to the phase shift:

$$K_{33}(k',k) = -\frac{4\pi}{2\omega_k}\frac{3}{k^3}\tan\delta_{33}\mathscr{P}_{33}(k',k)\ . \tag{18}$$

The K-matrix approach has been used in many past calculations of π-N scattering. For example, in the 1950's Blair and Chew[20] used it to calculate π-N phase shifts by expanding the scattering amplitude to fourth order in $f_{\pi NN}$ in the old static π-N model. More recently, several groups[13,21,22] have also used the K-matrix approach in calculating π-N phase shifts. They have, however, included only the lowest-order diagrams. Moreover, convergence of the perturbation series has never been established explicitly.

In an exact calculation, the T-matrix and K-matrix methods should yield identical results. This, however, is not guaranteed in a perturbative calculation truncated at finite order. Therefore, a comparison of the results from both methods actually provides a measure of the deviation of our solution from the exact one.

Finally, we point out a kinematic ambiguity which is inherent to all static models. In a strictly static sense, the total energy is given by

$$E=\omega_k=(m_\pi{}^2+k^2)^{1/2} . \tag{19}$$

However, in order to compare calculated results with experimental data, one has to decide whether k is a laboratory or center-of-mass quantity. Both choices have been made in the literature.[23] However, it is obvious that neither choice is satisfactory, because at the resonance energy E_{lab} and $E_{\mathrm{c.m.}}$ differ by as much as 35 MeV due to nucleon recoil. To remedy this embarrassing situation, we identify k with the center-of-mass pion momentum; but instead of Eq. (19) we use the exact relation

$$E=\omega_k+(k^2+M_N{}^2)^{1/2}-M_N . \tag{20}$$

This prescription amounts to including the kinetic energies of real nucleons in our calculation.

IV. RESULTS AND DISCUSSION

The main results of this calculation are shown in Figs. 4 and 5. Figure 4 corresponds to the T-matrix approach, and Fig. 5 to the K-matrix approach. While both methods are able to reproduce the data quite well, the K-matrix method provides a better overall fit. In obtaining the fits, we have varied only two parameters, $f_{\pi N\Delta}$ and ω_Δ. As for $f_{\pi\Delta\Delta}$, we used the quark-model relation

$$f_{\pi\Delta\Delta}=\tfrac{4}{5}f_{\pi NN} . \tag{21}$$

The results are not very sensitive to small changes of $f_{\pi\Delta\Delta}$ around this value. In the T-matrix approach, the best-fit

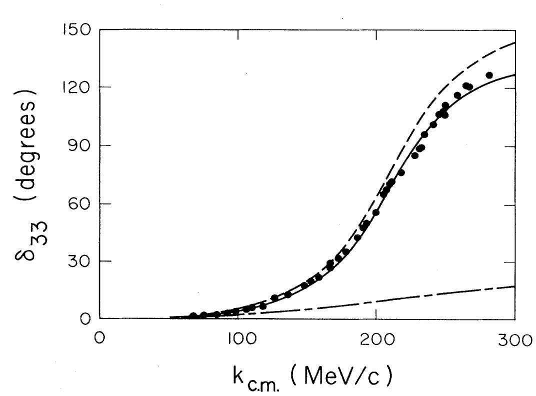

FIG. 4. Results from the T-matrix approach. The solid curve represents the best fit to the data. The dashed curve corresponds to the Δ contribution and the dash-dotted curve the Chew-Low contribution. The experimental data are from Ref. 24.

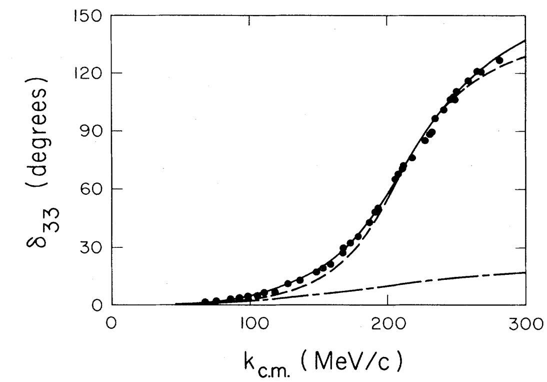

FIG. 5. Results from the K-matrix approach. The meanings of the curves are the same as in Fig. 4.

values of the parameters are $f_{\pi N\Delta}=0.624$ and $\omega_\Delta=301$ MeV, while in the K-matrix approach, $f_{\pi N\Delta}=0.523$ and $\omega_\Delta=293$ MeV. The fact that these two sets of parameters are close in magnitude indicates that higher-order corrections are not very important. For comparison, the CBM (Ref. 7) parameters are $f_{\pi N\Delta}\simeq 0.4$ and $\omega_\Delta=280$ MeV.

In order to examine the importance of different contributions, we have also calculated separately the Chew-Low contribution and the Δ contribution. The former is obtained by setting $f_{\pi N\Delta}=0$, and the latter by keeping only diagrams which contain intermediate Δ's. The results are also shown in Figs. 4 and 5, which indicate that the Δ contribution is dominant. The Chew-Low contribution is small but nevertheless not negligible. A similar conclusion was also reached in the CBM calculation.[6] From Fig. 5, we note that in the K-matrix approach, the resonance energy E_Δ is unaffected by the Chew-Low contribution. This point will become clear later on. In the T-matrix approach, if we set $f_{\pi NN}=0$, we find that the resonance position would shift upward by about 8 MeV. However, in Ref. 6, setting $f_{\pi NN}=0$ would cause the resonance peak to shift upward by about 50 MeV. This substantial difference is partly due to the smaller bag radius (0.72 fm) used in Ref. 6, and partly due to the fact that nucleon recoil energies have been neglected there.[25] It is clear that if Eq. (20) were used in Ref. 6, the Δ contribution alone would peak around $E=\omega_\Delta$, which is 294 MeV, *very close* to the physical resonance energy of 293 MeV. The small shift found in our calculation can be easily understood as follows: For the complete amplitude, δ_{33} goes through $\pi/2$ at $E=293$ MeV. When we set $f_{\pi NN}=0$, the dominance of the contribution from Fig. 2(b) implies that now $\delta_{33}=\pi/2$ when $E\simeq\omega_\Delta=301$ MeV. Therefore, a small upward shift of about 8 MeV is expected. Furthermore, a small shift is consistent with the K-matrix result of no shift at all.

In the K-matrix approach, the observed resonance energy, $E_\Delta=293$ MeV, is exactly equal to the dressed (renormalized) Δ mass ω_Δ. This result is not accidental. The reason is that, from Eqs. (A3) and (B6), the K matrix for Fig. 2(b) diverges at $E=\omega_\Delta$, while all other contributions are finite. Therefore, from Eq. (18), $\delta_{33}(E=\omega_\Delta)=\pi/2$. This result is independent of the magnitude of $f_{\pi NN}$ or

$f_{\pi N\Delta}$ ($\neq 0$). That explains why the Chew-Low contribution does not affect E_Δ. Furthermore, the result $E_\Delta = \omega_\Delta$ is also independent of the order of the perturbation expansion, because, by definition, Eq. (A3) is always true. In this sense, the result $E_\Delta = \omega_\Delta$ is exact.

As mentioned earlier, the fact that the T-matrix and the K-matrix results are similar already implies that truncation of the scattering series at fourth order is a good approximation. Nevertheless, it is worthwhile to check explicitly the convergence of the perturbation series. Figure 2(a) plus iterations with itself is usually called the "Chew-Low series." It has been argued that all terms in the series are of the same order of magnitude.[26,15] However, with the πNN form factor we use, the ratio of the first three terms of the series is actually $\lesssim 1{:}0.14{:}0.02$ (real part), indicating fast convergence.

V. SUMMARY AND CONCLUSIONS

In quark models, a physical nucleon consists of a three-quark core surrounded by a pion cloud. The π-N interaction form factor depends on the size of the core. It is interesting to note that two very different quark models (bag model and CQM) give similar form factors. Using the quark-model form factor, we have shown that the perturbation series of π-N scattering converges very fast. Therefore, a crossing-symmetric π-N scattering amplitude can be obtained in a straightforward manner by perturbation expansion. The (3,3) phase shift is calculated by perturbation expansion to fourth order in renormalized coupling constants. We find that the three-quark Δ state is very important in producing the physically observed Δ(1232) resonance. In fact the general features of the observed resonance (mass and width) are determined mainly by the Δ contribution. The contribution from the conventional Chew-Low series is small but not negligible, i.e., its inclusion is necessary to fit the data. It is pointed out that the inclusion of the nucleon recoil energy is important in obtaining a reliable fit to the data. The general feature of our solution is similar to that of the CBM.[6,7] However, the values of the parameters obtained in this work are quite different from those of Ref. 7. The differences can be attributed to the fact that in Ref. 7 R is treated as a free parameter, instead of fixing it by the baryon static properties (see Sec. II); also nucleon-recoil energies have been neglected. In addition, the πNN and $\pi N\Delta$ vertex corrections have not been included properly in Ref. 7 (see Sec. III), and the CBM solution is not crossing symmetric. Finally, it is shown that, in the K-matrix approach, the observed resonance energy E_Δ is equal to the dressed Δ mass ω_Δ; this result is independent of the order of the perturbation expansion.

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APPENDIX A

The renormalization procedure for a static theory has been given before in other works.[17,27] We include it here for completeness and clarity. In the following, $i\epsilon$ and P (Cauchy principal value) will be understood wherever needed.

1. Mass renormalization

If we denote the self-energy correction, shown in Fig. 3(a), by $\Sigma(E)$, then the inverse of the modified baryon propagator is given by

$$G'(E)^{-1} = E - M^{(0)} - \Sigma(E) , \tag{A1}$$

where $M^{(0)}$ is the bare mass. Next we expand $\Sigma(E)$ in a Taylor series around M, the renormalized mass:

$$\Sigma(E) = \mathrm{Re}\Sigma(M) + \mathrm{Re}\Sigma'(M)(E-M) + \Sigma_r(E) , \tag{A2}$$

where Re denotes the real part, and $\Sigma_r(E)$ is the remainder. Note that in the K-matrix formulation, $\Sigma(E)$ is real, and thus from Eq. (A2)

$$\Sigma_r(M) = \Sigma_r'(M) = 0 . \tag{A3}$$

Substituting Eq. (A2) into Eq. (A1) and rearranging terms, we get

$$G'(E) = Z_2 G(E) , \tag{A4}$$

where the renormalization constant Z_2 is defined by

$$Z_2 = [1 - \mathrm{Re}\Sigma'(M)]^{-1} . \tag{A5}$$

$G(E)$ is the renormalized propagator,

$$G(E)^{-1} = E - M - Z_2\Sigma_r(E) , \tag{A6}$$

and we have made the identification

$$M = M^{(0)} + \mathrm{Re}\Sigma(M) . \tag{A7}$$

2. Vertex renormalization

Let the vertex correction, shown in Fig. 3(b), be $\Gamma(E,E')$. The modified vertex function is then given by

$$V(E,E') = (Z_2 Z_2')^{1/2} f^{(0)} u(k)[1+\Gamma(E,E')] , \tag{A8}$$

where the factor $(Z_2 Z_2')^{1/2}$ comes from the propagator renormalization of the external baryon lines. Rearranging terms, we get

$$V(E,E') = f u(k) v(E,E') , \tag{A9}$$

where we have identified the renormalized coupling constant f as

$$f=\frac{(Z_2Z_2')^{1/2}}{Z_1}f^{(0)}\;; \tag{A10}$$

the renormalization constant Z_1 is defined by

$$Z_1=[1+\Gamma(0,0)]^{-1}\,, \tag{A11}$$

and

$$v(E,E')=Z_1[1+\Gamma(E,E')]\,. \tag{A12}$$

Lowest-order expressions for $\Sigma(E)$ and $\Gamma(E,E')$ can be found in Refs. 7 and 17.

APPENDIX B

In this appendix, we write down explicit expressions for the scattering diagrams shown in Fig. 2. First, we define some notations to be used later:

$$\begin{aligned}
&P_{3/2}(k',k)=\delta_{kk'}-\tfrac{1}{3}\tau_{k'}^{\dagger}\tau_k\,,\\
&P_{1/2}(k',k)=\tfrac{1}{3}\tau_{k'}^{\dagger}\tau_k\,,\\
&Q_{3/2}(\vec{k}',\vec{k})=\vec{k}'\cdot\vec{k}-\tfrac{1}{3}\vec{\sigma}\cdot\vec{k}'\vec{\sigma}\cdot\vec{k}\,,\\
&Q_{1/2}(\vec{k}',\vec{k})=\tfrac{1}{3}\vec{\sigma}\cdot\vec{k}'\vec{\sigma}\cdot\vec{k}\,,
\end{aligned} \tag{B1}$$

where the P's (Q's) are the usual isospin (angular momentum) projection operators. (The arguments will be omitted in later use.) Also,

$$\rho(a,b)=(aP_{3/2}+bP_{1/2})(aQ_{3/2}+bQ_{1/2})\,, \tag{B2}$$

$$\begin{aligned}
&F_2(k)=\frac{4\pi}{m_\pi{}^2}u^2(k)\frac{1}{2\omega_k}\,,\\
&F_4(k)=\frac{4}{3m_\pi{}^4}u^2(k)\frac{1}{2\omega_k}\,,
\end{aligned} \tag{B3}$$

and

$$I(E_1,E_2,E_3)=\int dq\,q^4[\omega_q(E-\omega_q-E_1)(E-\omega_q-E_2)(E-\omega_q-E_3)]^{-1}\,. \tag{B4}$$

1. Second-order amplitudes

For Fig. 2(a), we have

$$A_2(N)=F_2(k)(f_{\pi NN})^2v^2(E_N,E_N-\omega_k)[E_N-\omega_k-Z_2^{(N)}\Sigma_r^{(N)}(E_N-\omega_k)]^{-1}\rho(2,-1)\,. \tag{B5}$$

For Fig. 2(b), we have

$$A_2(\Delta)=F_2(k)(f_{\pi N\Delta})^2v^2(E_N,E)[E-\omega_\Delta-Z_2^{(\Delta)}\Sigma_r^{(\Delta)}(E)]^{-1}\rho(1,0)\,, \tag{B6}$$

where E_N is the initial nucleon kinetic energy in the π-N center-of-mass frame, such that $E=E_N+\omega_k$. See Eq. (20) and the discussion preceding it. The crossed Δ amplitude [Fig. 2(c)] can be obtained from $A_2(\Delta)$ by crossing the external pions, i.e.,

$$k\leftrightarrow -k' \quad \text{(isospin indices)}\,,$$

and (B7)

$$(\omega_k,\vec{k})\leftrightarrow(-\omega_k,-\vec{k}')\,.$$

Note that ω_k's in F_2 and F_4 are from pion wave-function normalizations, and therefore should not be affected by Eqs. (B7).

2. Fourth-order amplitudes

For Fig. 2(d), we have

$$A_4(N,N,N)\equiv A_4(B=N,B'=N,B''=N)=F_4(k)(f_{\pi NN})^4I(\omega_k,0,\omega_k)\rho(4,1)\,, \tag{B8}$$

$$A_4(\Delta,N,N)=A(N,N,\Delta)=F_4(k)(f_{\pi NN})^2(f_{\pi N\Delta})^2I(\omega_k+\omega_\Delta,0,\omega_k)\rho(\tfrac{2}{3},-\tfrac{4}{3})\,, \tag{B9}$$

$$A_4(N,\Delta,N)=F_4(k)(f_{\pi NN})^2(f_{\pi N\Delta})^2I(\omega_k,\omega_\Delta,\omega_k)\rho(\tfrac{5}{3},\tfrac{8}{3})\,, \tag{B10}$$

$$A_4(\Delta,\Delta,N)=A(N,\Delta,\Delta)=F_4(k)f_{\pi NN}(f_{\pi N\Delta})^2f_{\pi\Delta\Delta}I(\omega_k+\omega_\Delta,\omega_\Delta,\omega_k)\rho(-\tfrac{5}{3},\tfrac{10}{9})\,, \tag{B11}$$

$$A_4(\Delta,N,\Delta)=F_4(k)(f_{\pi N\Delta})^4I(\omega_k+\omega_\Delta,0,\omega_k+\omega_\Delta)\rho(\tfrac{1}{9},\tfrac{16}{9})\,, \tag{B12}$$

$$A_4(\Delta,\Delta,\Delta)=F_4(k)(f_{\pi N\Delta})^2(f_{\pi\Delta\Delta})^2I(\omega_k+\omega_\Delta,\omega_\Delta,\omega_k+\omega_\Delta)\rho(\tfrac{5}{3},\tfrac{25}{18})\,. \tag{B13}$$

The crossed fourth-order amplitudes [Fig. 2(e)] can again be obtained easily from the corresponding A_4's by crossing symmetry [see Eq. (B7)].

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