Bethe-Salpeter few-quark dynamics and the pion degree of freedom

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The pionic interaction of baryon (qqq) states is formulated in a Bethe-Salpeter (BS) model of harmonic confinement, characterized by a universal spring constant ($\tilde{\omega}=0.15$ GeV) and the quark mass ($m_q=0.28$ GeV) concerned, on lines similar to those developed recently for pionic couplings of meson ($q\bar{q}$) states, wherein the pionic degree of freedom is governed by the same BS dynamics as for other $q\bar{q}$ states, and *not* introduced by hand as an independent entity. The algebraic structures of the matrix elements for $NN\pi$ and $\Delta N\pi$ couplings are strongly reminiscent of the quark-pair-creation model, as found recently for meson couplings ($\rho\pi\pi$, $\omega\rho\pi$). A Lorentz-invariant adaptation of the resulting pion-baryon form factors, on lines identical to those employed successfully to the meson couplings, leads to the parameter-free predictions $(4\pi)^{-1}(G_{NN\pi})^2=13.02$ and $\Gamma(\Delta\to N\pi)=104.6$ MeV, both within 10% of experiment. It is stressed that these pionic results are consistent with the one-pion-exchange-potential structure.

The Nambu-Goldstone nature of the pion makes it readily describable by the PCAC (partial conservation of axial-vector current) language, as if it were an elementary particle, but only at the cost of its more natural entitlement to a composite ($q\bar{q}$) description requiring a more general form of dynamics applicable to all $q\bar{q}$ states. Unfortunately dynamical models, while predicting reasonable masses for most $q\bar{q}$ states,[1] do not seem to be easily amenable to the pion's properties, so much so that there exists a strong tendency in the literature to make an exception[2,3] for the pion to the $q\bar{q}$ rule. Such a point of view, which at best represents an effective description, is perfectly legitimate for the phenomenological exploration of pionic degrees of freedom in nuclei within the general philosophy of the bag picture,[4] but is less defensible for a basic theoretical understanding of hadron structures themselves. This is how one would be inclined to view certain attempts to put pionic degrees of freedom by hand within a bag dynamics[5] which tends to obscure the predictive value of the basic model, since a reasonably adequate theory of confinement should, in the first instance, aim at reconciling such apparently conflicting properties (PCAC vs composite) of pseudoscalar mesons within an integrated framework, instead of starting by making exceptions for certain (special) particles.

The purpose of this paper is to draw attention to a different model of confinement,[6] which not only explains the basic properties of the pion in company with those of other hadrons within its dynamical framework,[7,8] but also provides the correct strengths for pion couplings to the electromagnetic (EM) field,[8,9] as well as to other mesons.[9] First, the model which is characterized by two basic constants,[6–9] a universal spring constant ($\tilde{\omega}=0.15$ GeV) and the concerned (nonstrange) quark mass ($m_q=0.28$ GeV), (i) accounts simultaneously for the small pion mass (m_π) and the much larger ρ mass (m_ρ) through a $\vec{J}\cdot\vec{S}$ term[7,8] and (ii) predicts an almost correct PCAC strength f_π through the simple formula[8]

$$f_\pi=\left[\frac{3}{m_\pi}\right]^{1/2}(\beta_\pi^2/\pi)^{3/4}(1+\tfrac{3}{2}\beta_\pi^2 m_q^{-2})^{-1/2}$$
$$=89.1\ (93)\ \text{MeV}\ , \qquad (1)$$

where, here as elsewhere, the experimental value is given in parentheses. Second, its EM as well as mesonic couplings are manifested through the following parameter-free predictions:

$$\langle r_\pi^2\rangle^{1/2}=0.77\ (0.66\pm0.03\ (\text{Ref. }10);\quad 0.71\pm0.03\ (\text{Ref. }11))\ \text{fm}\ ,$$
$$\lambda_e^+(K\to\pi l\bar{l})=0.026\ (0.029\pm0.004\ (\text{Ref. }12))\ ,$$
$$\Gamma(\omega\to\pi^0\gamma)=0.888\ (0.89\pm0.01\ \ (\text{Ref. }12))\ \text{MeV}\ ,$$
$$\Gamma(\rho\to\pi\pi)=142.7\ (158\pm8\ (\text{Ref. }12))\ \text{MeV}\ .$$

In this paper we have extended the above results involving only meson states[8,9] to the baryon level and thus attempted to predict $NN\pi$ and $\Delta N\pi$ couplings within the same framework. This requires a straightforward extension of the Bethe-Salpeter (BS) technique[9] developed recently for pure meson couplings which had led[9] to structures strongly reminiscent of the quark-pair-creation model (QPCM)[13] for hadronic transitions such as $\rho\to\pi\pi$ (real) and $\omega\to\rho\pi$ (virtual). Not only are QPCM-like structures reproduced once again but the following new parameter-free results emerge:

$$(G_{NN\pi})^2/4\pi=13.02\ (14.6)\ , \qquad (2)$$

$$\Gamma(\Delta\to N\pi)=104.6\ (110\text{–}120\ (\text{Ref. }12))\ \text{MeV}\ , \qquad (3)$$

without giving any special status to the pion beyond its natural entitlement from BS dynamics as a $q\bar{q}$ state.

To indicate the essential steps, the first task is one of correctly reading out the vertex operator (at each BS vertex) from the full four-dimensional BS amplitude, so as to pave the way for application of the Feynman rules for relevant matrix elements, as outlined in Refs. 8 and 9 for the meson case. To do this for the baryon case, Fig. 1(a) represents the $NN\pi$ coupling structure (in the limit $q_\mu{}^2=0$), and Fig. 1(b) the $\Delta\to N\pi$ transition (for $q_\mu{}^2=-m_\pi{}^2$), each corresponding to the breakup (12;3) in which quark 1 "emits" the $(q\bar{q})$ pion, while quark 2 interacts with the former before and after the emission processes, and quark 3 remains a spectator throughout. (An identical diagram arises from the interchange $2\rightleftarrows 3$, but any diagram corresponding to the emission of the pion by a spectator is considered spurious.) Figure 1(a) makes use of the Ψ_3 term of the full BS amplitude Ψ $(=\Psi_1+\Psi_2+\Psi_3)$, which has the structure[14]

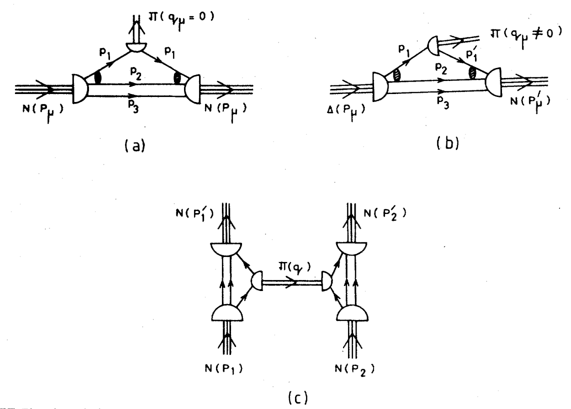

FIG. 1. (a) $NN\pi$ coupling, contribution due to (12;3) configuration; (b) (12;3) configuration for $\Delta\to N\pi$ decay matrix elements; (c) OPEP matrix element. The open semicircles denote BS vertices. The shaded ovals indicate the last interaction in the pair. For details see text.

$$\Psi_3(p_1p_2p_3)=\prod_1^3 S_F(p_i)V_3W(P)\,, \tag{4}$$

$$V_3=N_{B,B^*}(m_q{}^2+p_3{}^2)D_{12}\phi_0\delta(p_3^0-\tfrac{1}{6}P_0-\tfrac{1}{6}P_0')/2\pi i\sqrt{\delta(0)}\,, \tag{5}$$

$$W(P)=\frac{1}{\sqrt{2}}(\chi'\phi'+\chi''\phi'')\text{ for }N,\ \ \chi^s\phi^s\text{ for }\Delta\,, \tag{6}$$

so that the quantity V_3W is identifiable as the requisite vertex function at each baryon vertex. The relevant symbols are explained in a separate paper[14] dealing with EM couplings of baryons, from which we shall draw freely, including notations. The spinor $W(P)$ in Eq. (6) is defined for each baryon (four-momentum P_μ) in the composite qqq space in terms of spin-isospin functions (χ,ϕ) of standard S_3 symmetry[15] but the spin functions (χ) are expressed in a relativistic (covariant) form.[14] N_{B,B^*} is the normalization constant for the baryon (N,Δ)[14]; D_{12} is the three-dimensional energy denominator[7,14] associated with the (12) pair and ϕ_0 is the corresponding (instantaneous) wave function.[7,14] The $NN\pi$ coupling (normalized to $\bar{p}\pi^0p$) is defined through

$$G_{NN\pi}\bar{u}(P')\gamma_5U(P)\equiv\langle N\,|\,\pi\,|\,N\rangle$$
$$=\frac{2\times3}{\sqrt{3}}(2\pi)^8\int\int d^4q_{12}d\eta_3\bar{V}_3'V_3\langle W(P')\,|\,\Psi_\pi(p_1'p_1)S_F(p_2)S_F(p_3)\,|\,W(P)\rangle\,, \tag{7}$$

where Ψ_π is the four-dimensional BS amplitude for the pion.[8,9] A very similar expression holds for the $\Delta\to N\pi$ amplitude, Fig. 1(b). The factors of 2 and 3 in (7) represent, respectively, (i) the inclusion of an identical contribution arising from an interchange in the indices 2 and 3 and (ii) the effect of pion emission by all the three quarks in turn. However the color factor, instead of being unity for the EM coupling of a baryon, is now

$$\frac{1}{\sqrt{6}}\epsilon_{ijk}(B)\frac{1}{\sqrt{6}}\epsilon_{i'jk}(B')\frac{1}{\sqrt{3}}\delta_{ii'}(\pi)=\frac{1}{\sqrt{3}}\,. \tag{8}$$

The method of simplification of the right-hand side of (7) in respect of spin-isospin matrix elements and integration over four-momenta is identical to that of Ref. 14 (which in turn is patterned after Ref. 9). In particular, after the integration over dq_{12}^0 and $d\eta_3^0$, and the requisite translation over the η variable so as to bring the Gaussian factors $\phi_0\phi_0'\phi_\pi$ to a standard quadratic form free from linear ξ,η terms, Eq. (7) reduces to

$$\langle N\,|\,\pi\,|\,N\rangle=(2\pi)^6i^2 2N_B{}^2N\pi\left[\frac{\sqrt{3}}{2}\right]^4\int\int d\vec{\xi}\,d\vec{\eta}(\phi_\pi\phi_0\phi_0')2p_{2+}D_\pi(m_q{}^2+p_3{}^2)(m_q+\tfrac{1}{3}M)^2[\langle S'\rangle-\tfrac{1}{3}\langle S''\rangle]\,, \tag{9}$$

where the symbols $(\phi_\pi,2p_{2+},D_\pi)$ are defined as in Refs. 8 and 9, $\phi_0\phi_0'$ as in Ref. 14, and

$$\begin{bmatrix}\langle S'\rangle\\ \langle S''\rangle\end{bmatrix}=\begin{bmatrix}(m_q+\tfrac{1}{3}M)^2+\tfrac{4}{3}M(m_q+\tfrac{1}{3}M)+\vec{\eta}^{\,2}\\ -(m_q+\tfrac{1}{3}M)^2+\tfrac{4}{9}M(m_q+\tfrac{1}{3}M)-\tfrac{1}{3}\vec{\eta}^{\,2}\end{bmatrix}\,. \tag{10}$$

The QPCM structure of Eq. (9) is manifested through the appearance of the product of the wave functions of the three participating hadrons, together with certain (semikinematic) factors in the *numerator* only, in close analogy to the case of pure meson couplings,[9] except for the effect of the spectator requiring the additional factor $(m_q{}^2+p_3{}^2)$, arising from the normal rules of field theory. Omitting the "translated" expression for $\phi_\pi\phi_0\phi_0'$ in (9) for brevity, the final result is

$$\langle N\,|\,\pi\,|\,N\rangle=(2\pi)^6 i^2 N_B{}^2 N\pi(4m_\pi)\tfrac{1}{3}(P_+ + P'_+)(m_q+\tfrac{1}{3}M)^2(\beta_0\,|\,\beta_N)^3(\pi\beta_\pi{}^2)^{-3/4}$$
$$\times\bar{u}(P')\gamma_5 u(P)\{\exp[-F(NN)]\}[\tfrac{4}{3}(m_q+\tfrac{1}{3}M)^2+\tfrac{32}{27}M(m_q+\tfrac{1}{3}M)+\tfrac{5}{3}\beta_0{}^2]$$
$$\times(m_q{}^2-\tfrac{1}{4}M_\pi{}^2+\tfrac{3}{2}\beta_0{}^2)(m_q{}^2-\tfrac{1}{9}M^2+\tfrac{1}{6}\beta_0{}^2+\tfrac{9}{8}\beta_N{}^2)\,, \tag{11}$$

where

$$\beta_0{}^{-2}=\beta_N{}^{-2}+\tfrac{1}{2}\beta_\pi{}^{-2}\,, \tag{12}$$

$$F(NN)=\tfrac{1}{36}\beta_0{}^2\beta_N{}^{-2}\beta_\pi{}^{-2}(\vec{P}^2+\vec{P}'^2-\tfrac{1}{2}\vec{q}^2)+\tfrac{1}{9}\beta_N{}^{-2}\vec{q}^2\,. \tag{13}$$

For the exponent F of the Gaussian form factor we adopt the same philosophy as employed earlier for purely hadronic transitions,[9] viz., a four-dimensional Lorentz adaptation[16] of the three-dimensional form (13) ($\vec{P}^2\to P_\mu{}^2=-M^2$, etc.) together with a renormalization factor arising from a "reference" $\rho\rho\rho$ coupling at the symmetrical point $(-m_\rho{}^2,-m_\rho{}^2,-m_\rho{}^2)$ for each $P_\rho{}^2$, just as adopted for pure meson couplings.[9] Thus in the limit $q_\mu{}^2\to 0$, we have the replacement

$$\exp[-F(NN)]\to\exp(\tfrac{1}{18}M^2\beta_0{}^2\beta_\pi{}^{-2}\beta_N{}^{-2}-\tfrac{3}{16}m_\rho{}^2\beta_\rho{}^{-2})\,. \tag{14}$$

We note in passing that this prescription is not needed for the EM form factors of baryons,[14] just as discussed for the corresponding problem of meson couplings.[9] Insertion of result (14) in Eq. (11), together with the defining equation (7) for $G_{NN\pi}$ leads to the result (2) after numerical substitutions for the various quantities according to the BS model ($\tilde{\omega}=0.15$ GeV, $m_q=0.28$ GeV).[6-9]

For $\Delta\to N\pi$ coupling, the procedure is identical, except for $M_N\neq M_\Delta$ and $q_\mu{}^2=-m_\pi{}^2$. We merely write down the analogous matrix element to Eq. (11), after making a similar Lorentz-invariant adaptation of the corresponding three-dimensional form factor and using the same renormalization point[9] for $\rho\rho\rho$ coupling:

$$\langle p\,|\,\pi^+\,|\,\Delta^{++}\rangle=(2\pi)^6 i^2 N_{B^*}N_B N\pi\frac{P_+ + P'_+}{2\sqrt{2}}(\beta_1{}^2\beta_0{}^2\beta_N{}^{-2}\beta_\pi{}^{-2})^{3/2}(\pi\beta_\pi{}^2)^{-3/4}$$
$$\times\left[\frac{i}{M'}\bar{u}(P')q_\nu u_\nu(P)\right]B_m A_m\{\exp[-F(N\Delta)]\}2m_\pi(m_q{}^2+\tfrac{3}{2}\beta_0{}^2+\beta^2\vec{q}^2-\tfrac{1}{4}m_\pi{}^2)$$
$$\times(m_q{}^2-\tfrac{1}{9}P_0P_0'+\tfrac{1}{36}q_\mu{}^2+\tfrac{3}{8}\beta_0^2+\tfrac{9}{8}\beta_1{}^2+\tfrac{1}{4}\beta^2\vec{q}^2)\,, \tag{15}$$

where A_m and B_m are, within 0.1%, the same quantities as defined in connection with $\Delta\to N\gamma$ decay[14] and the unexplained symbols are as follows:

$$\exp[-F(N\Delta)]\to\exp[\tfrac{1}{36}\beta_0{}^2\beta_\pi{}^{-2}\beta_1{}^{-2}(M_\Delta{}^2+M_N{}^2-\tfrac{1}{2}m_\pi{}^2)+\tfrac{1}{6}\beta\beta_\pi{}^{-2}(M_\Delta{}^2-M_N{}^2)+\tfrac{1}{9}m_\pi{}^2\beta_1{}^{-2}-\beta^2\beta_0{}^{-2}m_\pi{}^2]\,, \tag{16}$$

$$2\beta_1{}^{-2}=\beta_N{}^{-2}+\beta_\Delta{}^{-2};\;\;2\beta_0{}^{-2}=\beta_\pi{}^{-2}+\beta_N{}^{-2}+\beta_\Delta{}^{-2};\;\;6\beta=\beta_0{}^2(\beta_N{}^{-2}-\beta_\Delta{}^{-2})\,. \tag{17}$$

Finally the connection

$$\Gamma(\Delta\to N\pi)=\tfrac{1}{4}\sum_{pol}|\langle p\,|\,\pi^+\,|\,\Delta^{++}\rangle|^2\frac{M_N}{M_\Delta}\frac{|\vec{q}|}{2\pi} \tag{18}$$

leads after the straightforward substitutions directly to the value (3) for the $\Delta\to N\pi$ width.

If these results for pion-baryon couplings are considered in association with the other pionic results[8,9] quoted in the beginning of this paper, as well as with those on the complete mass spectra of $q\bar{q}$ (Refs. 7 and 8) and qqq (Refs. 7 and 17) hadrons, together with still another dimension of agreements on the EM properties of baryons[14] (charge radii, magnetic moments, and $\Delta\to N\gamma$ amplitudes), they should warrant the following assessment: These results collectively constitute a strong and wide-ranging experimental base for an integrated, yet highly economical, field theoretic approach to a practicable model of harmonic confinement which not only works for both types of hadrons but also does not necessitate any special treatment[2-5] for pions (or other P mesons).

A major by-product of these pionic successes is that the BS model quantitatively reproduces the one-pion-exchange-potential (OPEP) features in the following sense. The matrix element of the N-N potential via pion exchange in this model is inferred from Fig. 1(c) as

$$V_{12}(q)=(G_{NN\pi})^2\bar{u}(P_1')\gamma_5\tau_\alpha u(P_1)\bar{u}(P_2')\gamma_5\tau_\alpha u(P_2)$$
$$\times(q^2+m_\pi{}^2)^{-1}\,, \tag{19}$$

where the spinor factors give rise to the standard OPEP structure $\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}\, \vec{\tau}_1 \cdot \vec{\tau}_2$ in the usual way,[18] and the strength and range are determined, respectively, by the two quantities $(G_{NN\pi})^2$ and $m_\pi{}^2$, as *predicted* by this very model. In this respect the value of $(G_{NN\pi})^2$, Eq. (2), seems to agree with the data to within 10%, while the consistency of the predicted pion mass with experiment (which is a major problem for QCD-oriented models) is already evidenced from the near equality of the $F(M)$ values[7,8] for ρ and π. Since no other parametric ingredient is involved in Eq. (19), it is thus seen that the BS model provides a reasonable degree of understanding of the OPEP structure, which has been a bone of contention for most models of confinement, necessitating a special status for the pion.[2–5]

To summarize, it has been shown that the BS model predicts the major pionic couplings to baryons to within 10% accuracy, using the same assumption (and basic constants $\tilde{\omega}, m_q$) as employed for an understanding of several other crucial pionic properties.[6–9] The OPEP structure then follows as a natural corollary of the model in which the pion is just another $q\bar{q}$ state in company with other mesons. A detailed report containing a fuller list of meson-baryon couplings, together with their implications on short-range N-N potentials, is in preparation.

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