Bethe-Salpeter qqq dynamics: Electromagnetic properties of baryons

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The Bethe-Salpeter model for $q\bar{q}$ and qqq systems under harmonic confinement, previously found to fit the $q\bar{q}$ and qqq mass spectra of light (u,d,s) quarks rather well with a universal spring constant ($\tilde{\omega}=0.15$ GeV) and the concerned quark mass ($m_{ud}=0.28$ GeV, $m_s=0.35$ GeV) is employed to predict some crucial electromagnetic properties of baryons as a generalization of a similar method recently developed for the electromagnetic interaction of $q\bar{q}$ mesons. The major successes, in which the relativistic features of the model have played a crucial role, are in respect of magnetic moments of baryons and the charge radius of the proton, all in very good agreement with experiment with no free parameters. In particular, $\mu_p=2.796$ (2.793) nuclear magnetons and $\langle r_p{}^2\rangle^{1/2}=0.86$ (0.87) fm. The helicity amplitudes $A_{3/2}$ and $A_{1/2}$ for $\Delta\to N\gamma$ also reveal an improved fit to data over previous quark-model calculations. The significance of these results is discussed in relation to contemporary confinement models.

I. INTRODUCTION

Electromagnetic (EM) properties of hadrons constitute one of the most basic tests of any dynamical model for quark confinement claiming to provide an integrated understanding of hadronic phenomena. In this respect, the $q\bar{q}$ system is theoretically simpler to handle, but has the disadvantage of too few EM data for extensive comparison with theoretical predictions. Baryons (qqq) on the other hand, offer a much richer variety of accurate EM data for comparison with a theoretical model, but the latter is usually a much harder exercise for qqq systems than for their $q\bar{q}$ counterparts when relativistic effects with all relevant degrees of freedom (spin, color, flavor) are taken into account. And as long as the situation with QCD—the candidate theory of strong interactions—remains fluid with respect to the long-range part (despite the advances in lattice gauge theories), dynamical models of confinement[1,2] should deserve consideration on individual merits. In the present paper we have been motivated by such considerations to study some crucial properties of baryons within the framework of a rather comprehensive Bethe-Salpeter BS model of harmonic confinement which is characterized by just *two* basic parameters, a universal spring constant ($\tilde{\omega}$) and the quark mass (m_q) concerned.[3] This model not only provides a very good description of $q\bar{q}$ (Refs. 4 and 6) and qqq (Refs. 4 and 5) mass spectra, but has also been amenable to a simple enough formulation of electroweak[6] and pionic couplings[7] of pure $q\bar{q}$ systems, with several impressive agreements with relevant data.[6,7] In this paper we propose a corresponding formulation of the EM couplings of a qqq system as an extension of the $q\bar{q}$ case,[6] together with some basic applications, viz., (i) the charge form factor of the proton, (ii) magnetic moments of baryons, and (iii) the helicity amplitudes for $\Delta\to N\gamma$. The method used is very similar to that already employed for the $q\bar{q}$ case,[6] but there are now two new features: (a) the effect of the spectator quark on the construction of the full three-body BS amplitude, and (b) the appearance of different types of permutation symmetries according to S_3 classifications in each degree of freedom (spin, flavor, momentum). Such S_3 symmetries, though familiar in the nonrelativistic (NR) quark model,[8] need a full-fledged relativistic adaptation, especially for the spin functions, before they can be used within the BS framework. In this respect, we have generalized a representation suggested long ago by Blankenbecler *et al.*,[9] so as to incorporate the full features of S_3 symmetry. This is done in Sec. II in the context of construction of the full BS amplitude for qqq systems of different S_3 symmetries, as well as the corresponding normalization constants. The method adopted is otherwise very similar to that developed in Ref. 6 for $q\bar{q}$ systems. The only additional feature concerns the four-momentum dependence of the spectator quark which is suitably incorporated in the BS amplitude commensurate with the dynamics of the model.

In Sec. III which outlines the construction of EM form factors, the spin and momentum dependence of the EM matrix elements are explicitly shown and integration over timelike momenta carried out on identical lines to those explained in Refs. 6 and 7. A simple formula is also given for the proton's charge radius whose numerical value (0.86 fm) is in surprisingly good accord with experiment (0.87 fm). Section IV is devoted to the calculation of magnetic moments of $\underline{8}$ and $\underline{10}$ (B^*) baryons, using the results of Sec. III for the general case of unequal-mass kinematics. The calculated values compare very favorably with experiment as well as with those of the vector-meson-dominance—oriented Schwinger model based on partial symmetry.[10,11] Section V sketches the application of this model to the evaluation of the $\Delta^+\to p\gamma$ helicity amplitudes, the results for which show a distinct improvement over earlier quark-model results.[1] Finally, in Sec. VI we discuss the significance of the predictions of this model vis-á-vis the corresponding results obtained in some contemporary confinement models of comparable predictive power. Applications to pionic couplings under BS dynamics are relegated to a separate paper.[12]

II. STRUCTURE OF THE THREE-BODY BS AMPLITUDE

Following closely the reasoning of Mitra and Kulshreshtha[6] (MK) for the $q\bar{q}$ case, we first write the full four-dimensional BS amplitude in the form

$$\Psi(p_1,p_2,p_3)=N_B\prod_{i=1}^{3}(m_i-i\gamma^{(i)}\cdot p_i)\Phi(p_1p_2p_3)W(P) , \quad (2.1)$$

where we expect Φ to be a scalar function of its arguments, while any additional spin dependence of Ψ is (it is hoped) contained in the factor $W(P)$ containing only the external (hadronic) four-momentum P_μ. As in the $q\bar{q}$ case,[6] an *a fortiori* justification of this construction lies in the following. After substitution of (2.1) in the full three-body BS equation,[3] the successive steps of a Gordon-reduction on the right-hand-side (RHS) kernel[3] and the instantaneous approximation to the resulting equation[4] lead exactly to the same equation as Eq. (4.5) of Ref. 4 with the instantaneous counterpart (ϕ) of the Φ function playing the role of the three-dimensional wave function. Next, since (for harmonic confinement) the spin structure of the latter equation is diagonal (not only for equal-mass kinematics[4] but also for unequal-mass quarks[5]), any additional spin dependence of ϕ cannot involve the individual quark momenta (p_i), and must necessarily be a *constant* factor depending only on P_μ. This last is indicated by the overall factor $W(P)$ in (2.1) whose structure can additionally be made to conform to the S_3 classification [without the dynamical complications that any quark-momentum (p_i) dependence of this quantity would otherwise have caused].

For the structure of $W(P)$, it is best to consider the spin and isospin functions together, so as to bring out the overall S_3 symmetry. Thus for the *ground* state of a $\underline{56}$ baryon, $W(P)$ has the symmetric forms

$$W(P)=\frac{1}{\sqrt{2}}(\chi'\phi'+\chi''\phi'') \text{ or } \chi^s\phi^s , \quad (2.2)$$

where (χ,ϕ) are spin and flavor functions of appropriate S_3 symmetries. The mixed functions (χ',χ'') of spin $\frac{1}{2}$ may be taken in a (23) basis with χ' as a (23) singlet and χ'' a (23) triplet,[8] and their relativistic structures consistent with S_3 symmetry may be expressed jointly as

$$|\chi'\rangle;|\chi''\rangle=\left[\frac{M-i\gamma\cdot P}{2M}\left[i\gamma_5;\frac{1}{\sqrt{3}}i\hat{\gamma}_\mu(P)\right]\frac{1}{\sqrt{2}}C\right]_{\beta\gamma}$$

$$\otimes[(1;\gamma_5\hat{\gamma}_\mu(P))u(P)]_\alpha , \quad (2.3)$$

where the first factor is a 4×4 matrix ($\beta\gamma$ element) in a 23 spin space[9] and the second factor a 4×1 spinor (α element) in the 1 spin space; C is a charge-conjugation matrix with the properties

$$-\tilde{\gamma}_\mu=C^{-1}\gamma_\mu C , \quad \tilde{\gamma}_5=C^{-1}\gamma_5 C ;$$

$\hat{\gamma}_\mu(P)$ is the component of γ_μ orthogonal to P_μ. Likewise the spin-$\frac{3}{2}$ function χ^s has the relativistic representation

$$|\chi^s\rangle=\left[\frac{M-i\gamma\cdot P}{2M}[-i\hat{\gamma}_\mu(P)]\frac{1}{\sqrt{2}}C\right]_{\beta\gamma}\otimes[u_\mu(P)]_\alpha . \quad (2.4)$$

The representations of (ϕ',ϕ'',ϕ^s) are of course adequate in their respective nonrelativistic forms,[8] so that the following relations hold:

$$\begin{aligned}&\langle\phi''|1;\vec{\tau}^{(1)}|\phi''\rangle=\langle\phi'|1;-\tfrac{1}{3}\vec{\tau}^{(1)}|\phi'\rangle ,\\ &\langle\phi''|1;\vec{\tau}^{(1)}|\phi^s\rangle=\langle\phi''|0;-2\sqrt{2}\vec{\tau}^{(1)}|\phi''\rangle .\end{aligned} \quad (2.5)$$

Next the structure of the scalar function Φ, must be obtained by *inversion* of the instantaneous relation[3,13] [hoping that no confusion will arise with the same notation ϕ as in Eq. (2.5) for the isospin functions]

$$\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=\int dq_{23}^0 dp_1^0\Phi(p_1p_2p_3) . \quad (2.6)$$

The inversion process was unique in the $q\bar{q}$ case[6] since the effect of the instantaneous kernel[6] which we denote by $K(12)$ for simplicity, could be at once reexpressed in terms of the three-dimensional energy denominator D_{12} through an equation of the form

$$D_{12}\phi(\vec{q})=\int d\vec{k}_{12}K(12)\phi(\vec{k}) . \quad (2.7)$$

In the present qqq case, the corresponding instantaneous equation is of the form

$$\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=\sum_{123}D_{23}{}^{-1}\int d\vec{k}_{23}K(23)\phi(\vec{p}_1\vec{p}\,'_2\vec{p}\,'_3) , \quad (2.8)$$

so that the effect of a particular pairwise kernel K_{ij} cannot be automatically expressed in terms of the corresponding energy denominator D_{ij} because of the effect of the couplings with the other pairs. However, the four-dimensional amplitude Φ, by virtue of Eq. (2.6), has an extra δ function in the energy variable of the *spectator* within our model which specifies[3] $p_3^0=m_3P^0/m_0$ (when the p_3 quark is a spectator) to complete the overall dynamical requirements.[14] As a result, the pairwise breakup of Φ corresponding to the $q\bar{q}$ relation[6]

$$2\pi i\Delta_1\Delta_2\Phi(p_1p_2)=\int d\vec{k}\,K(12)\phi(\vec{k})$$

becomes

$$2\pi i\sqrt{\delta(0)}\Phi(p_1p_2p_3)=\sum_{123}\frac{\delta(p_3^0-m_3P^0/m_0)}{\Delta_1\Delta_2}\times\int d\vec{k}_{12}K(12)\phi(\vec{p}_1\vec{p}\,'_2\vec{p}\,'_3) , \quad (2.9)$$

where $\Delta_i=p_i{}^2+m_i{}^2$ and the factor $\sqrt{\delta(0)}$ represents a normalization effect due to the finite time (T) available (for free propagation) to the spectator ($\sqrt{\delta(0)}\Longrightarrow\sqrt{T/2\pi}$). Due to the presence of the energy δ function with each pairwise kernel K_{ij} on the RHS of (2.9), it is permissible to replace the latter by the corresponding energy denominator D_{ij} in accordance with Eq. (2.8) so that Eq. (2.9) simplifies to

$$\Phi(p_1p_2p_3)=\sum_{123}\frac{D_{12}\phi(\vec{p}_1\vec{p}_2\vec{p}_3)\delta(p_3^0-m_3P_0/m_0)}{2\pi i\sqrt{\delta(0)}\Delta_1\Delta_2} , \quad (2.10)$$

where, for equal masses,[4]

$$D_{12}=\frac{4M}{3}(\vec{q}_{12}^{\,2}+m_q^2-\tfrac{1}{9}M^2+\tfrac{1}{4}\vec{p}_3^{\,2}) . \tag{2.11}$$

As a final check, the substitution of (2.10) in (2.1) leads to the same instantaneous equation for ϕ as Eq. (4.5) of Ref. 4. For the ground state, we have as in Ref. 4,

$$\phi=\phi_0(\vec{\xi},\vec{\eta})=(\pi\beta_N^2)^{-3/2}\exp[-\tfrac{1}{2}(\vec{\xi}^{\,2}+\vec{\eta}^{\,2})\beta_N^{-2}] . \tag{2.12}$$

Thus the structure of the full BS amplitude Ψ from which the vertex functions V_i can be immediately identified (for writing down the Feynman-type matrix elements) is

$$\begin{aligned}\Psi&=S_F(p_1)S_F(p_2)S_F(p_3)W(P)(V_1+V_2+V_3)\\&\equiv\Psi_1+\Psi_2+\Psi_3 ,\end{aligned} \tag{2.13}$$

where

$$S_F^{-1}(p)=i(m_q+i\gamma\cdot p) \tag{2.14}$$

and

$$V_3=N_B(m_q^2+p_3^2)\frac{D_{12}\phi_0}{2\pi i}\frac{\delta(p_3^0-P^0/3)}{\sqrt{\delta(0)}} \tag{2.15}$$

is the vertex associated with the breakup [Fig. 1(a)] in which the last interaction was in the (p_1p_2) pair with p_3 as the spectator. [Note that the vertex V_3 has the (expected) compensating factor $m_q^2+p_3^2$ against a (spurious) singularity which would otherwise arise from the propagator $S_F(p_3)$ associated with the spectator p_3.] The constant N_B is determined by the full normalization condition analogous to the $q\bar{q}$ case,[5] based on the conservation of baryonic charge,[15]

$$\begin{aligned}(2J+1)(2\pi)^{-4}P_\mu/M\\ =\sum_{\rm pol}\int\prod_{i=1}^{3}d^4p_i\,\delta^4(P-p_1-p_2-p_3)\\ \times\bar{\Psi}(p_1p_2p_3)\frac{\partial}{\partial P_\mu}\prod_{j=1}^{3}S_F^{-1}(p_j)\Psi(p_1p_2p_3) .\end{aligned} \tag{2.16}$$

Evaluation of this integral will be discussed in Sec. III in connection with the EM form factor of the baryon [Fig. 1(b)], which involves a very similar integral with the replacements $p_i\to p_i'=p_i-q$ ($q=$ photon momentum) in the final state. The final result for equal-mass kinematics is

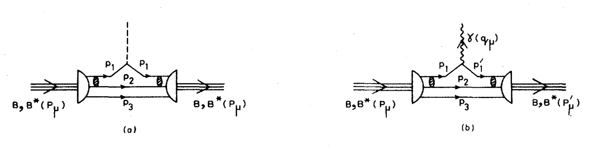

FIG. 1. (a) BS normalization contribution due to (12;3) configuration, vide Eq. (2.1) of text. (b) (12;3) configuration for baryonic EM matrix elements. The open semicircle represents the BS vertex; the shaded oval represents the last interaction of the (12) pair. For other details see text.

$$\begin{aligned}N_{B,B^*}^{-2}&=\tfrac{1}{2}\sqrt{3}(2\pi)^3M(m_q+\tfrac{1}{3}M)^2\\&\quad\times\{[(m_q+\tfrac{1}{3}M)^2+\tfrac{3}{2}\beta_{N,\Delta}^2]\\&\quad\times(m_q^2-\tfrac{1}{9}M^2+\tfrac{3}{2}\beta_{N,\Delta}^2)+\tfrac{3}{8}\beta_{N,\Delta}^4\} ,\end{aligned} \tag{2.17}$$

covering *both* N and Δ cases.

A different kind of baryon normalization which is based on the parton sum rule (for squared charges) has an absolute character and corresponds to its breakup into *free* quarks (partons) as a result of deep-inelastic electron scattering, calculated in the $P_z=\infty$ frame. The derivation, which is sketched elsewhere,[16] leads to the formula

$$N_\infty^{-2}=\frac{27}{8\sqrt{2}}(m_q+M/3)^6\left[\frac{6m_q^2}{M^2}\right]^2\left[\frac{2\pi}{3}+4\ln\tfrac{3}{2}\right] , \tag{2.18}$$

which has little formal resemblance to (2.17). The quantity (2.18) is logically more correct than (2.17) for use in connection with the rate of, e.g., proton decay,[16] where the nature of the final products $(e^+,q,\bar{q})$ makes the latter process topologically similar to the dissociation $p\to qqq$.

Before ending this section, we note the possibility of a negative N_B^2 for sufficiently large M, as implied by Eq. (2.17). While such "unpleasant" features are not unknown in BS dynamics,[17] we are unable to offer any deep insight into this question in the present context, except for suggesting that a negative N_B^2 implies some sort of instability against possible breakup into smaller masses (e.g., $\Delta\to N\pi$).

III. EM FORM FACTOR OF THE BARYONS

The EM form factor is determined by the matrix element for Fig. 1(b), where the EM interaction acts on quark 1, whose immediately preceding and following interactions are with quark 2, while quark 3 remains a spectator throughout. Another equivalent configuration (not shown) is one with the roles of quark 2 and 3 interchanged. Two more pairs of such diagrams, where the roles of quark 1 are played in turn by quark 2 and quark 3, respectively, must also be included for a total listing of matrix elements. For nonstrange baryons (with equal-mass quarks), all the three pairs of diagrams give rise to an overall factor of 3; but for strange baryons these terms must be separately evaluated because of unequal-mass kinematics. Now each of these configurations will give rise to a 3×3 matrix structure arising out of the three terms $\bar{\Psi}_i$ of $\bar{\Psi}$ and likewise Ψ_j of Ψ, as shown in the breakup, Eq. (2.13). However, because of the energy δ function in (2.15), only the diagonal terms ($i=j$) will effectively contribute, while the nondiagonal terms will be negligible, because of smearing out of the δ functions. Thus we have in an obvious notation, the following configurations:

$$(12;3),\ (13;2);\ \ (21;3),\ (23;1);\ \ (32;1),\ (31;2) , \tag{3.1}$$

where the first index refers to the quark having EM interaction, while the second index stands for the associated interacting quark before and after the EM interaction.

The last index stands for the spectator. In this counting the configurations corresponding to the EM interaction of the spectator have been ignored as dynamically spurious.

We now have the entire machinery to write down the matrix elements for the SU(6) EM interaction operator

$$\Gamma_\mu=\sum_1^3 \gamma_\mu^{(i)} \tfrac{1}{2} e\left[\lambda_s^{(i)}+\frac{1}{\sqrt{3}}\lambda_8^{(i)}\right] \tag{3.2}$$

between $|N\rangle$ and/or $|\Delta\rangle$-type states, from Fig. 1(b), in accordance with standard Feynman rules by virtue of the BS structure (2.13) which directly identifies the vertex parts and propagators in the figure. For equal-mass kinematics which we consider first, it is enough to write down the results for only the first two sets of indices in (3.1) by virtue of S_3 symmetry whose full content is illustrated by the following typical cases[18]:

$$\langle N|\Gamma_\mu|N\rangle=\frac{e}{4}\langle 1+3\tau_z\rangle\langle X'|\gamma_\mu^{(1)}|X'\rangle + \frac{e}{4}\langle 1-\tau_z\rangle\langle X''|\gamma_\mu^{(1)}|X''\rangle\ , \tag{3.3}$$

$$\langle \Delta|\Gamma_\mu|\Delta\rangle=\frac{e}{2}\langle 1+2T_z\rangle\langle X^s|\gamma_\mu^{(1)}|X^s\rangle\ , \tag{3.4}$$

$$\langle p|\Gamma_\mu|\Delta^+\rangle=2e\langle X''|\gamma_\mu^{(1)}|X^s\rangle\ , \tag{3.5}$$

which separate out the trivial isospin factors from the nontrivial spin-cum-momentum dependence of the matrix elements shown in terms of the symbols $|X'\rangle$, $|X''\rangle$ and $|X^s\rangle$, which are proportional to $|\chi'$, $|\chi''\rangle$, $|\chi^s\rangle$, Eqs. (2.2) and (2.3), as well as to some other momentum-dependent factors that arise in accordance with the Feynman rules for the entire matrix element. For example, the complete (12;3) matrix element, Fig. 1(b), is given by

$$\langle X'|i\gamma_\mu^{(1)}|X'\rangle=(2\pi)^8\int d^4q_{12}d^4\eta_3\bar{V}'_3V_3\langle\chi'(P')|S_F(p_1')i\gamma_\mu^{(1)}S_F(p_1)S_F(p_2)S_F(p_3)|\chi'(P)\rangle\ , \tag{3.6}$$

where[3]

$$m_{12}q_{12}=m_2p_1-m_1p_2,\quad m_0\eta_3=-m_{12}p_3+m_1p_2+m_2p_1\ .$$

The other $|X\rangle$ matrix elements listed in Eq. (3.3)—(3.5) can be written down by mere inspection in terms of the corresponding $|\chi\rangle$ matrix elements. The spin factor in Eq. (3.6), by virtue of the special representation (2.3), gives to the following structure:

$$\begin{aligned}\langle A\rangle &\equiv\langle\chi'|\cdots|\chi'\rangle\Delta_1\Delta_1'\Delta_2\Delta_3\\ &=\mathrm{Tr}\left[\frac{1}{\sqrt{2}}C^{-1}\gamma_5\frac{M'-i\gamma\cdot P'}{2M'}(m_2-i\gamma\cdot p_2)\frac{M-i\gamma\cdot P}{2M}\gamma_5(m_3+i\gamma\cdot p_3)\frac{1}{\sqrt{2}}C\right]\\ &\quad\otimes\bar{u}(P')(m_1-i\gamma\cdot p_1')i\gamma_\mu(m_i-i\gamma\cdot p_1)u(P)\ .\end{aligned} \tag{3.7}$$

In a similar way, we have

$$\begin{aligned}\langle B\rangle &\equiv\langle\chi''|\cdots|\chi''\rangle\Delta_1\Delta_1'\Delta_2\Delta_3\\ &=\mathrm{Tr}\left[\frac{1}{\sqrt{6}}C^{-1}i\hat{\gamma}_\nu(P')\frac{M'-i\gamma\cdot P'}{2M'}(m_2-i\gamma\cdot p_2)\frac{M-i\gamma\cdot P}{2M}i\gamma_\nu(P)(m_3+i\gamma\cdot p_3)\frac{1}{\sqrt{6}}C\right]\\ &\quad\otimes\bar{u}(P')i\hat{\gamma}_\nu(P')\gamma_5(m_1-i\gamma\cdot p_1')i\gamma_\mu(m_1-i\gamma\cdot p_1)\gamma_5 i\hat{\gamma}_\nu(P)u(P)\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}\langle\chi''|\cdots|\chi^s\rangle\Delta_1\Delta_1'\Delta_2\Delta_3&=\mathrm{Tr}\left[\frac{1}{\sqrt{6}}C^{-1}i\hat{\gamma}_\nu(P')\frac{M'-i\gamma\cdot P'}{2M'}(m_2-i\gamma\cdot p_2)\frac{M-i\gamma\cdot P}{2M}i\hat{\gamma}_\nu(P)(m_3+i\gamma\cdot p_3)\frac{1}{\sqrt{2}}C\right]\\ &\quad\otimes\bar{u}(P')\hat{\gamma}_\nu(P')\gamma_5(m_1-i\gamma\cdot p_1')i\gamma_\mu(m_1-i\gamma\cdot p_1)u_\nu(P)\ .\end{aligned} \tag{3.9}$$

In these expressions the final baryon mass (M') has been taken to be different in general from the initial baryon mass (M), in order to make these formulas applicable to more general situations such as transition form factors.

For further evaluation, the following relations among the momenta in Fig. 1(b) are useful:

$$P=P'+q\ ,\quad p_1'=p_1-q\ , \tag{3.10}$$

$$p_2=p_2'\ ,\quad p_3=p_3'\ , \tag{3.11}$$

$$(m_0/m_1)^{1.2}\xi_1=p_3-p_2=\xi_1'(m_0/m_1)^{1/2}\ , \tag{3.12}$$

$$m_0\eta_1=-2m_2p_1+m_1(p_2+p_3)\ ,$$

$$\eta_1'=\eta_1+2m_2/m_0q\ . \tag{3.13}$$

The integration variables in Eq. (3.6) may be reexpressed as

$$d^4q_{12}d^4\eta_3=dq^0_{12}d\eta^0_3d^3\xi_1d^3\eta_1[\tfrac{1}{2}(m_0/m_1)^{1/2}]^3 . \quad (3.14)$$

The evaluation of the matrix elements of the type Eq. (3.6) is conveniently carried out in the following sequence: (i) Integration over dq^0_{12} in the null-plane language gives

$$\frac{1}{2\pi i}\int dq^0_{12}\frac{\Delta_3 D_{12}D'_{12}}{\Delta_1\Delta'_1\Delta_2}=2p_{2+}\Delta_3 \quad (3.15)$$

as a direct application of the method outlined in Ref. 7, with $p_{2+}=p^0_2+p_{2z}$, etc. (ii) Integration of $d\eta^0_3$ gives merely

$$\int d\eta^0_3[\delta(p^0_3-m_3Mm_0{}^{-1})/\sqrt{\delta(0)}]^2=1 . \quad (3.16)$$

(iii) Translation over the $\vec{\xi}_1$, and $\vec{\eta}_1$ variables, viz.,

$$\vec{\eta}_1\rightarrow\vec{\eta}-m_2\vec{q}/m_0 , \quad \vec{\eta}'_1\Longrightarrow\vec{\eta}+m_2\vec{q}/m_0; \quad \vec{\xi}_1=\vec{\xi}'_1=\vec{\xi} \quad (3.17)$$

reduces the Gaussian factors for the equal-mass case to

$$\phi_0\phi'_0\Longrightarrow(\pi\beta_N{}^2)^{-3/2}\exp[-(\vec{\xi}^2+\vec{\eta}^2)\beta_N{}^{-2}-\tfrac{1}{9}q^2\beta_N{}^{-2}] . \quad (3.18)$$

After these steps, the spin-matrix elements (3.7) to (3.9) can be simplified by dropping linear terms in $\vec{\xi},\vec{\eta}$. The final results for the equal-mass case obtained through a straightforward reduction of γ matrices (for final-baryon mass same as initial-baryon mass and dropping some very small terms $\sim q^2\xi^2$ and $\sim q^2\eta^2$) are

$$\langle A\rangle=(1+\tfrac{1}{4}q^2M^{-2})\times\bar{u}(P')[a_i\bar{P}_\mu M^{-1}+a_s i\sigma_{\mu\nu}q_\nu/2M]u(P) , \quad (3.19)$$

$$\langle B\rangle=(1+\tfrac{1}{4}q^2M^{-2})\times\bar{u}(P')[b_c\bar{P}_\mu M^{-1}+b_s i\sigma_{\mu\nu}q_\nu/2M]u(P) , \quad (3.20)$$

where a Gordon reduction[3] has led to

$$i\gamma_\mu\Longrightarrow\bar{P}_\mu M^{-1}+i\sigma_{\mu\nu}q_\nu/2M , \quad 2\bar{P}_\mu=P_\mu+P'_\mu . \quad (3.21)$$

The above coefficients satisfy the following relations:

$$a_c=b_c+O(q^2) , \quad b_s=-\tfrac{1}{3}a_s . \quad (3.22)$$

The corresponding quantities $a^*_{c,s}$ arising from $B^*\rightarrow B^*$-type matrix elements (3.4) do not require a fresh calculation, since

$$a^*_{c,s}=a_{c,s} . \quad (3.23)$$

The values of $a_{c,s}$ for the general case of unequal-mass kinematics corresponding to all the configurations $(ij;k)$ of Eq. (3.1) are given in the Appendix. In particular, the full value of $a_c(q^2)$ for equal-mass kinematics, including terms of $O(q^2)$, is

$$a_c(q^2)=(m_q+\tfrac{1}{3}M)^2[(m_q+\tfrac{1}{3}M)^2+\vec{\eta}^2-\tfrac{1}{9}q^2] . \quad (3.24)$$

Note that terms of order (q^2) are relevant for the charge radius of the baryon, but not for its magnetic moment which is of $O(q)$ only. Further, the normalization integral (2.16) involves these very matrix elements in the limit $q\rightarrow0$ when only $a_c(=b_c)$ survives. The final step consists in integrating the quantities a,b weighted by other (ξ,η)-dependent factors $2p_2+\Delta_3\phi_o\phi'_0$, vide Eq. (3.15) and (3.18), viz.,

$$\langle a_c(12;3)\rangle=\int\int d\vec{\xi}\,d\vec{\eta}(\phi_0\phi'_0 2p_{2+}\Delta_3)a_c(12;3) , \quad (3.25)$$

which we denote as $\langle a_c(q^2)\rangle$ for equal-mass case (similarly for the other terms). In particular, Eq. (3.25), for the equal-mass case leads to the following connection with $N_B{}^2$ given by Eq. (2.17):

$$N_B{}^{-2}=2(2\pi)^3(\sqrt{3}/2)^3\langle a_c(q^2=0)\rangle . \quad (3.26)$$

Charge radius of the proton. For the charge form factor F of the proton which depends only on the convective part $\langle a_c\rangle$, substitution in Eq. (3.3) with $\tau_z=1$ and use of (3.18) yields

$$F(q^2)=e(1+\tfrac{1}{4}q^2M^{-2})\times[\langle a_c(q^2)\rangle/\langle a_c(0)\rangle]\exp(-\tfrac{1}{9}q^2\beta_N^{-2}) , \quad (3.27)$$

which explicitly brings out the normalization $F(0)=e$. From (3.27) and (3.24), the charge radius r_p of the proton is deduced as

$$\langle r_p{}^2\rangle=\tfrac{6}{9}\beta_N{}^{-2}-\tfrac{3}{2}M^{-2}-6\frac{\partial}{\partial q^2}\ln\langle a_c(q^2)\rangle\bigg|_{q^2=0} , \quad (3.28)$$

the last two terms representing a characteristically BS effect. Taking $\beta_N{}^2=0.029$, according to the dynamical prediction of this model,[4] the breakup between the first and last two terms of (3.28) yields

$$\langle r_p{}^2\rangle=23.00-4.38=18.62\ (18.9)\mathrm{GeV}^{-2} , \quad (3.29)$$

in excellent accord with experiment (experimental result shown in parentheses), in which the BS effect is clearly seen to have played a crucial role, in almost exact analogy to the EM form factors of mesons.[6]

g_A/g_V for the nucleon. We end this section with the prediction of g_A/g_V in the BS model. The calculation is formally identical to that of the EM matrix element outlined in the foregoing, except for the replacement

$$i\gamma_\mu\rightarrow i\gamma_5\gamma_\mu$$

in (3.2) and all subsequent equations (3.3)—(3.9). The axial-vector current is expressible as

$$\langle N|j^A_\mu|N\rangle=\bar{u}(P')i\gamma_5\gamma_\mu u(P)\langle a_{5s}\rangle/\langle a_c(0)\rangle , \quad (3.30)$$

where

$$a_{5s}=[m_1{}^2(1+Mm_0{}^{-1})^2-\tfrac{1}{3}\vec{\eta}^2]m_2{}^2(1+Mm_0{}^{-1})^2$$

and the angular brackets are meant in the sense of (3.25). Substitution of numerical values yields

$$g_A/g_V=\langle a_{5s}\rangle/\langle a_c(0)\rangle\approx0.96\ (1.22) , \quad (3.31)$$

showing a discrepancy of $\sim20\%$ for this parameter.

IV. MAGNETIC MOMENTS OF BARYONS

Magnetic moments of baryons which represent one of the most striking successes of the quark model *in principle* have had a long history. In particular, since the quark model brings out rather convincingly the effect of *compositeness* on the anomalous aspects of these moments, the nature of the dynamics employed for the purpose acquires a nontrivial significance well beyond the initial successes of nonrelativistic SU(6) theory. Now the prediction of magnetic moments, by its very nature, requires a mass (length) dimension for which the obvious candidates are (i) the elementary quark masses (m_q) as in the older nonrelativistic models and (ii) the composite baryon masses (M) which have been conjectured[19] or perhaps certain V-meson masses which can be motivated by the vector-meson-dominance (VMD) principle.[20] The precise choice comes out of the overall dynamical framework employed for the detailed calculation. Thus while the V-meson masses (m_V) enter through the VMD form of dynamics,[20,10] the baryon masses (M) seem to enter in a very natural way in the present BS model through the structure of the spinor $W(P)$, Eqs. (2.2)—(2.4), representing the external kinematics of the baryon as a whole.

The magnetic moments in this model can be extracted from the $\sigma_{\mu\nu}q_\nu$ terms in the baryon form factors evaluated in Sec. III, neglecting terms of $O(q^2)$. In particular, the nucleon magnetic moment (equal-mass case) is predicted as

$$\mu_N = \frac{e}{2M}\left[\frac{1+3\tau_z}{4}\frac{\langle a_s\rangle}{\langle a_c\rangle}+\frac{1-\tau_z}{4}\frac{\langle b_s\rangle}{\langle a_c\rangle}\right], \tag{4.1}$$

leading to the values (in nuclear magnetons)

$$\mu_p = 2.796\ (2.793),\quad \mu_n = -1.864\ (-1.912)\ , \tag{4.2}$$

in rather impressive accord with data (shown in parentheses).

Evaluation of the magnetic moments of strange baryons (Λ,Σ,Ξ) involving unequal-mass kinematics $(m_1 \neq m_2 = m_3)$ is similar in principle but requires calculation of the matrix elements for all the configurations listed in (3.1), not merely (12;3). In the Appendix, we list only the quantities $a_{c,s}$ for the different configurations, in view of the connections (3.22) and (3.23) for the b coefficients. The resulting "weighted average" quantities, in the sense of Eq. (3.25), are now

$$\langle\langle A_{c,s}\rangle\rangle = 2[\langle a_{c,s}(12;3)\rangle + \langle a_{c,s}(21;3)\rangle + \langle a_{c,s}(23;1)\rangle]\ , \tag{4.3}$$

due to relations (A5)—(A7) of the Appendix. Also,

$$\langle\langle B_s\rangle\rangle = -\tfrac{1}{3}\langle\langle A_s\rangle\rangle\ ,\quad \langle\langle B_c\rangle\rangle = \langle\langle A_c\rangle\rangle\ . \tag{4.4}$$

Finally, the magnetic moments for the different cases, taking account of the SU(3) matrix elements as in Eq. (4.1), are given by the general formula ($M=$ mass of baryon concerned)

$$\mu_{B,B^*} = \frac{e}{2M} g_{B,B^*}\langle\langle A_s\rangle\rangle/\langle\langle A_c\rangle\rangle\ , \tag{4.5}$$

$$g_B = -\tfrac{1}{3}\ (\Lambda), +1\ (\Sigma^+), -\tfrac{1}{3}\ (\Sigma^-),\quad -\tfrac{2}{3}(\Xi^0), -\tfrac{1}{3}\ (\Xi^-)\ , \tag{4.6}$$

$$g_{B^*} = 2\ (\Delta^{++}),\ \pm 1\ (\Sigma^{*\pm}),\ 0\ (\Xi^{*0}),\ -1\ (\Xi^{*-})\ . \tag{4.7}$$

The predicted and observed values, all normalized to nuclear magneton units, are shown in Table I. Also shown in the table for completeness are the predictions of the Schwinger model[10] based on partial symmetry and EM substitution (VMD).[11] The algebraic formulas, for the baryon magnetic moments (including those for charmed baryons), which are rather simple looking in this model, are given elsewhere.[11]

Table I shows, first of all, that the BS-model predictions as a whole are in surprisingly good accord with data.[21] The nucleon magnetic moments especially leave little to be desired, except for some fine-structure effects (a small symmetry-breaking term?) indicated for the neutron case. Even for the "strange" cases the discrepancy is rather small, considering the total absence of any free parameters. In this respect, the VMD-oriented Schwinger model[10,11] seems to give a somewhat complementary picture: While the agreement is not so good for the nucleon, the overall fits are slightly better for the strange baryons. Unfortunately, a general comparison between the BS and Schwinger models at the theoretical level has not been possible, but one concrete evidence of a basic harmony between these two models has been found at the pure meson-coupling level. Namely, a recent calculation of the

TABLE I. Baryon magnetic moments (in nuclear magnetons).

Baryon	BS model	Schwinger (Ref. 10)	Expt. (Refs. 21 and 22)
p	2.796	2.42	2.793
n	-1.864	-1.62	-1.913
Λ	-0.57	-0.614	-0.614 ± 0.005
Σ^+	2.626	2.355	2.33 ± 0.13
Σ^-	-0.876	-0.871	-0.89 ± 0.14
Ξ^0	-1.518	-1.356	-1.236 ± 0.01
Ξ^-	-0.751	-0.55	0.75 ± 0.07
Δ^{++}	4.438	4.839	
Σ^{*+}	2.196	1.886	
Σ^{*-}	-2.196	-1.624	
Ξ^{*-}	-2.03	-2.04	
Ξ^{*0}	0	0.365	

$\omega \to \pi^0\gamma$ amplitude was made in two different ways.[7] (i) by direct-photon coupling of the quarks in the BS model and (ii) by VMD substitution ($\rho^0 \to \gamma$), *after* calculating the $\omega \to \rho^0\pi^0$ amplitude again in the BS model. Both gave results within 1% of each other ($\Gamma \approx 888$ keV) and were in excellent accord with experimental data.[22] The present results for the baryon magnetic moments do not seem to be inconsistent with such an expectation, though an explicit VMD-oriented calculation is much more laborious in this baryonic case. Table I also records the predictions for the decimet baryons' magnetic moments for future (?) comparison with data. Magnetic moments have been calculated in certain bag models[23] as well (see Sec. VI for a discussion).

V. $\Delta \to N\gamma$ DECAY IN THE BS MODEL

Our last application in this paper concerns the EM coupling between Δ and N type states, leading to the prediction of the $\Delta^+ \to p\gamma$ helicity amplitudes ($A_{3/2,1/2}$) in the BS model. The algebra is straightforward but somewhat heavier, so we outline only the essential steps here. The matrix element in this case involves the spin part (3.9) with $M \neq M'$ as a factor of the corresponding X-matrix element (3.5). Now the appearance of unequal baryon masses in the (transition) matrix element requires a generalization in the structure of the energy δ function arising from the spectator (say 3). Since p_3^0 cannot be simultaneously equal to both $\frac{1}{3}P_0$ and $\frac{1}{3}P'_0$, this fact necessitates an overall assessment of the spectator's energy status in the context of the entire matrix element involving *both* initial and final states instead of fixing its energy arbitrarily on the basis of only one of the states. The simplest solution lies in the replacement

$$\delta(p_3^0 - \tfrac{1}{3}M) \Longrightarrow \delta(p_3^0 - \tfrac{1}{6}P_0 - \tfrac{1}{6}P'_0) \tag{5.1}$$

in an expression of the type (2.10). Strictly speaking, such a mixed quantity does *not* make sense for a single state (initial or final) such as (2.10), but only in the context of the overall matrix element in which both states are involved. As a further consistency check, the RHS of (5.1) works out as $\delta(\eta_3^0)$, which gives the expected constraint on the internal variable η_3^0, Eq. (3.14).

The calculations proceed exactly in the sequence of steps indicated in Sec. III for the EM form factors, viz., (i) integration over dq_{12}^0, $d\eta_3^0$ as in (3.14)–(3.16) and (ii) the requisite translation (3.17) in the η_1 variable (to make the Gaussian factors $\phi_0\phi'_0$ free from linear ξ,η terms) prior to the simplification of the spin factor (3.9). Thus the $\Delta \to N\gamma$ counterpart of the RHS of Eq. (3.15) is now given by the replacement

$$2p_{2+}\Delta_3 \Longrightarrow \tfrac{1}{3}(P_+ + P'_+)(m_q{}^2 - \tfrac{1}{9}P_0P'_0 + \tfrac{1}{4}\vec{\eta}^{\,2} + \tfrac{3}{8}\vec{\xi}^{\,2} + \tfrac{1}{36}q_\mu{}^2)\ , \tag{5.2}$$

where $q_\mu{}^2 = 0$ in this case, and

$$P_+ = P_0 + P_z,\ \ P'_+ = P'_0 + P'_z\ . \tag{5.3}$$

The spin-isospin factor (3.9) works out as

$$2e\langle X'' \,|\, \gamma_\mu^{(1)} \,|\, X^s\rangle = 2eA_m[B_m\bar{u}(P')\gamma_5\gamma_\mu q_\nu M^{-1}u_\nu(P) - B'_m\bar{u}(P')2i\gamma_5 u_\mu(P)]\ , \tag{5.4}$$

where

$$A_m \approx (m_q + \tfrac{1}{3}M')(m_q + \tfrac{1}{3}M) + (M+M')[\alpha(m_q + \tfrac{1}{3}M') + \alpha'(m_q + \tfrac{1}{3}M)]\ , \tag{5.5}$$

$$B_m \approx (m_q + \tfrac{1}{3}M')(m_q + \tfrac{1}{3}M) - \tfrac{1}{36}(M-M')^2 - \tfrac{1}{2}\beta(M^2 - M'^2)\left[\tfrac{1}{3} + \tfrac{1}{2}m_q\frac{M+M'}{MM'}\right]\ , \tag{5.6}$$

$$B'_m \approx \tfrac{1}{4}(m_q + \tfrac{1}{3}M')(m_q + \tfrac{1}{3}M)(M+M')^2M^{-1}M'^{-1} + \tfrac{1}{36}(M-M')^2 - \tfrac{1}{4}O(\beta)\times O(M^2 - M'^2)\ , \tag{5.7}$$

$$\alpha = \tfrac{2}{3}\beta_\Delta{}^2(\beta_N{}^2 + \beta_\Delta{}^2)^{-1}\ ,\ \alpha' = \tfrac{2}{3} - \alpha\ ,\ \ 2\beta = \alpha - \alpha'\ . \tag{5.8}$$

As A_m, B_m, and B'_m are practically constant, the complete amplitude is obtained by integrating out only over the RHS of (5.2) with respect to the distribution arising from the $\phi_0\phi'_0$ functions:

$$\int\int d\vec{\xi}\,d\vec{\eta}\,(\pi^2\beta_N{}^2\beta_\Delta{}^2)^{-3/2}\exp[-(\vec{\xi}^{\,2} + \vec{\eta}^{\,2})\beta_1{}^{-2}]\ ,$$

where

$$2\beta_1{}^{-2} = \beta_N{}^{-2} + \beta_\Delta{}^{-2}\ . \tag{5.9}$$

The final result is

$$A(\Delta \to N\gamma) = 4\pi^3\sqrt{3}eN_BN_{B^*}(P_+ + P'_+)\langle X'' \,|\, \gamma_\mu^{(1)} \,|\, X^s\rangle \times (M'/2q_0E_q)^{1/2}(\beta_1{}^4\beta_N{}^{-2}\beta_\Delta{}^{-2})^{3/2} \times [m_q{}^2 - \tfrac{1}{9}P_0P'_0 + \tfrac{3}{2}\beta_1{}^2]\ . \tag{5.10}$$

The helicity amplitudes can be read from (5.4) to be in the ratio

$$A_{1/2}{:}A_{3/2} = (E_q/M' + 1)B_m - B'_m{:}\sqrt{3}B'_m \tag{5.11}$$

and the absolute quantities work out in $\mathrm{GeV}^{-1/2}$ units as

$$\begin{aligned} A_{3/2} &= -0.205(-0.258 \pm 0.01)\ ,\\ A_{1/2} &= -0.118\ (-0.138 \pm 0.1)\ . \end{aligned} \tag{5.12}$$

These values, though somewhat lower than the corresponding experimental numbers[22] (in parentheses), nevertheless show a significant improvement over the earlier results[1] based on orthodox quark models of the seventies (though modified versions do give better fits[24]).

VI. SUMMARY AND CONCLUSIONS

In order to compare the present calculations with contemporary approaches (and many exist in the literature), it is meaningful to choose only those models whose theoretical scope (and predictive power) are comparable. For if it were for the problem of hadron couplings alone, it could have been handled with relatively few theoretical principles without a full-fledged dynamical framework. Indeed the principle of partial symmetry,[10–,18] together with the quark-pair-creation model[25] (QPCM) already provide a powerful yet highly economical (parametric) framework for a very large class of strong interactions,[26] while EM couplings merely require the additional ingredient of the gauge principle[1] *or* the VMD principle of EM substitution.[10] On the other hand, such principles are not enough for, e.g., the prediction of mass spectra which require a more explicit dynamical model. The first nontrivial attempt of this dimension was that of Feynman, Kislinger, and Ravndal[1] who gave a *relativistic* shape (together with certain rules for evaluation of current matrix elements) to the traditional NR oscillator model. However, apart from certain serious theoretical problems born out of an inadequate conceptual framework it was soon overtaken by the development of QCD, on the one hand, and the emergence of the bag philosophy on the other.[2] These gave rise to certain QCD-oriented models which still required "confinement" to be put in by hand (through "potentials", "bag boundaries," etc.) *albeit* at the cost of certain free parameters. This is certainly not the most desirable state of affairs, but must be faced until such time as nonperturbative QCD makes some genuine advances (numerical results on lattice gauge theories seem to lack stability[27]).

The present model, which belongs to this last category, stands to compare only with these, using the *twin* criteria of (i) theoretical consistency and (ii) quality and quantity of fits to diverse data ranging from mass spectra to strong and EM couplings of hadrons, all within a *single* dynamical framework. Judged by these criteria, the present (BS) model with its universal spring constant ($\tilde{\omega}$) and the quark mass (m_q) not only accounts for the hadron ($q\bar{q}$,qqq) masses[4,5] of (u,d,s) flavors in an extremely convincing manner, but has been found to provide very accurate parameter-free fits to the electroweak[6] and pionic[7] couplings of the $q\bar{q}$ systems. These should perhaps be judged together with the present results on the EM couplings of baryons as well as certain crucial *predictions* (derived separately[12,7]) on the pionic couplings, viz.,

$$(G_{NN\pi})^2/4\pi = 13.02\ (14.6)\ ,$$

$$\Gamma(\Delta\to N\pi) = 104.6\ (110-120)\ \text{MeV}\ ,$$

$$\Gamma(\rho\to 2\pi) = 143\ (158\pm 8)\ \text{MeV},^{7}$$

for an overall assessment of the status of the BS model. It is important to note that the BS model *does not* require the pionic (or kaon, etc.) degree of freedom to be put in by hand. Not only is the pion's mass "understood" within the theoretical framework of this model[4] without extra assumptions, but even its couplings to leptons,[6] to the EM field[6,7] and to other hadrons,[7,12] come out equally naturally as part of the pion's BS dynamics as a $q\bar{q}$ system.

In contrast, the bag model,[2,28] though primarily relying on a single basic parameter (R), frequently requires additional assumptions (on bag shapes, etc.)[28] to widen and/or improve the range of its mass predictions.[29] Its range of applications is often limited to static phenomena.[30] More importantly, the insertion by hand of pionic degrees of freedom[31] within the bag framework must be regarded as a serious compromise of the theoretical status of the model, and any experimental success of the model *after* such a modification must reckon with this major weakness. It is from this angle that we are inclined to view the successes of the so-called cloudy bag model on the EM, etc., properties[23] of the nucleon without reference to the dynamics played by the input pionic degrees of freedom together with the crucial parameters (m_π,f_π), while their effects on hadron mass spectra are yet unknown.[23]

Our reservations,[3,4] despite the impressive successes, on QCD-oriented NR oscillator models,[32] are of a more conceptual nature in so far as it is difficult to see *a priori* how a basically NR dynamics can be justified for intrinsically light quarks where genuine relativistic effects are bound to arise, despite efforts to subsume them through suitably designed parameters.[32] Similar remarks apply to the predictions of such NR models to $qq\bar{q}\bar{q}$ systems,[33] for which a relativistic Bethe-Salpeter dynamics produces an entirely different scenario.[34]

To summarize, we have extended the BS model of $q\bar{q}$ to qqq systems characterized by two basic constants ($\tilde{\omega}$,m_q), from the study of mass spectra and electroweak and pionic couplings of ($q\bar{q}$) mesons to the investigation of EM properties of baryons without further assumptions. Particular care has been taken to preserve the role of the "spectator" in the definition of transition matrix elements which are directly amenable to the language of Feynman diagrams. The results on the proton's charge radius as well as the baryon magnetic moments are excellent, while the amplitudes for $\Delta\to N\gamma$ decay represent a considerable improvement over several earlier quark-model calculations. These results stand in sharp constrast to those of certain modified bag models wherein the pionic degrees of freedom (with concomitant parameters m_π and f_π) are inserted by hand, thus obscuring the possibility of a direct comparison with such models on the basis of their (comparable) experimental successes alone. On the other hand, the pionic couplings of baryons come out correctly (within 10% accuracy) in this model without giving any special status for the pion. These and related details are given in a separate paper.[12] Prediction of proton decay in this model is given elsewhere.[16]

APPENDIX

For evaluation of the magnetic moments of baryons, the relevant terms are of $O(q)$. We list the distinct quantities arising from the six configurations, Eq. (3.1) of text, under the assumption $m_1\neq m_2=m_3$,

$$a_c(12;3)=a_c(13;2)=m_2{}^2(1+M/m_0)^2[m_1{}^2(1+M/m_0)^2+\vec{\eta}^{\,2}]\ , \tag{A1}$$

$$a_c(21;3)=a_c(23;1)=a_c(31;2)=a_c(32;1)$$
$$=m_1m_2(1+M/m_0)^2[m_2{}^2(1+M/m_0)^2+\tfrac{1}{4}\vec{\eta}^{\,2}+m_0/4m_1)\xi^2]\ , \tag{A2}$$

$$a_s(12;3)=a_s(13;2)=m_2{}^2(1+M/m_0)^2[m_1{}^2(1+M/m_0)^2+\tfrac{1}{3}\vec{\eta}^{\,2}+4m_1m_2M/m_0(1+M/m_0)]\,, \tag{A3}$$

$$a_s(21;3)=a_s(23;1)=a_s(31;2)=a_s(32;1)$$
$$=m_1m_2(1+M/m_0)^2[m_2{}^2(1+M/m_0)^2+\tfrac{1}{12}\vec{\eta}^{\,2}+(m_0/12m_1)\vec{\xi}^{\,2}+2m_{12}m_2M/m_o(1+M/m_0)]\,. \tag{A4}$$

The corresponding weighted quantities which may be written down, according to definition (3.25) of the text, satisfy the relations

$$\langle a_{c,s}(12;3)\rangle=\langle a_{c,s}(13;2)\rangle\,, \tag{A5}$$

$$\langle a_{c,s}(21;3)\rangle=\langle a_{c,s}(31;2)\rangle\,, \tag{A6}$$

$$\langle a_{c,s}(23;1)\rangle=\langle a_{c,s}(32;1)\rangle\,. \tag{A7}$$

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