Quark theory of charmed-D-meson two-body nonleptonic weak decays

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Quark spectator and tadpole graphs as predicted by the Cabibbo—Glashow-Iliopoulos-Maiani weak current and Hamiltonian density combine to give a complete description of charmed $D^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$, and $D^+ \to \bar{K}^0\pi^+$ Cabibbo-angle-enhanced decays and $D^+ \to \bar{K}^0 K^+$, $\pi^0\pi^+$, and $D^0 \to K^-K^+$, $\pi^-\pi^+$ Cabibbo-angle-suppressed decays.

I. INTRODUCTION

Now that the experimental data on charmed-D-meson decays appear to follow some systematic patterns,[1] it is time for theorists to reexamine the two-body nonleptonic decays and determine to what extent these decays are driven by the underlying Cabibbo[2]—Glashow-Iliopoulos-Maiani[3] (GIM) weak SU(4) quark current

$$j_\mu^W = \bar{u}\gamma_\mu^L(d\cos\theta_C + s\sin\theta_C) + \bar{c}\gamma_\mu^L(-d\sin\theta_C + s\cos\theta_C)\ . \tag{1}$$

In a recent paper,[4] it was demonstrated that the long-standing problem of the origin of the nonleptonic $K_{2\pi}$ $\Delta I = \frac{1}{2}$ rule is explained by quark tadpole graphs built up from the fundamental current (1) dominating the large $K^0_{2\pi}$ decays. This tadpole enhancement is complemented by the small $\Delta I = \frac{3}{2}$ $K^+_{2\pi}$ decay being properly scaled in magnitude and in sign[4] by vacuum saturation of the conventional Cabibbo weak Hamiltonian density[2]

$$H_{JJ} = \frac{G_W}{2\sqrt{2}}(j_\mu^W j_W^{\mu\dagger} + j_\mu^{W\dagger} j_W^\mu)\ , \tag{2}$$

which corresponds directly to the quark spectator graph.

In this work we extend these ideas to the nonleptonic $\Delta I = 0,1$ $D_{K\pi}$, and $\Delta I = \frac{1}{2}, \frac{3}{2}$ $D_{K\bar{K}}$ and $D_{\pi\pi}$ weak decays. The Cabibbo-angle-enhanced $D_{K\pi}$ decays cannot receive contributions from quark tadpoles (with $\Delta I = \frac{1}{2}$); instead they are controlled by quark spectator and to a lesser extent color-suppressed spectator graphs. In Sec. II, we show that the equivalent vacuum saturation of the $D_{K\pi}$ matrix elements of (2) given (1) are reasonably consistent with experiment, with only $D^0 \to \bar{K}^0\pi^0$ on the borderline of being incompatible.

On the other hand, the Cabibbo-angle-suppressed $D^+_{K\bar{K}}$ and $D^+_{\pi\pi}$ decays $D^+ \to \bar{K}^0K^+$ and $D^+ \to \pi^0\pi^+$ can receive contributions from spectator but not color-suppressed spectator graphs. However, the $\Delta I = \frac{1}{2}$ quark tadpole can drive the $\Delta I = \frac{1}{2}$ $D^+ \to \bar{K}^0K^+$ decay—but not the $\Delta I = \frac{3}{2}$ $D^+ \to \pi^0\pi^+$ decay. We show in Sec. III that these conclusions are not inconsistent with data.

Finally, in Sec. IV, we demonstrate that the Cabibbo-angle-suppressed decays $D^0_{K\bar{K}}$, $D^0_{\pi\pi}$, (i.e., $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$) are driven by *both* quark spectator and tadpole diagrams. That is, the $D^0_{K\bar{K}}$ decay rate is correctly predicted by vacuum-saturating the spectator graph and applying current algebra and PCAC (partial conservation of axial-vector current) to the (slightly smaller) tadpole graph. While such graphs *add* for $D^0_{K\bar{K}}$, they *subtract* for $D^0_{\pi\pi}$ decay, and this result also reproduces the observed[1] branching ratio $B(D^0 \to K^+K^-/\pi^+\pi^-) \sim 4$. In Sec. V, we summarize these patterns and suggest that, taken *together*, quark spectator and tadpole graphs do indeed verify that the Cabibbo-GIM quark current (1) and Hamiltonian density (2) drive both K- and D-meson nonleptonic decays.

II. CABIBBO-ANGLE-ENHANCED $D_{K\pi}$ DECAYS

These decay amplitudes excite only $\Delta I = 1$, 0 components of H_W and from (1) and (2) are proportional to $\cos^2\theta_C$.

A. $K^+_{\pi^+\pi^0}$

By way of review, we first consider $K^+ \to \pi^+\pi^0$ decay since H_W then transforms like $\Delta I = \frac{3}{2}$ and therefore the $\Delta I = \frac{1}{2}$ s-d quark tadpole cannot play a role.[4] Instead, $K^+_{\pi^+\pi^0}$ is driven by the quark spectator and color-suppressed spectator graphs of Fig. 1, or equivalently by the corresponding vector-dominance-model (VDM) K^*-pole graphs of Fig. 2. Both Figs. 1(b) and 2(b) are suppressed by a factor of 3 due to color and we ignore them when computing Figs. 1(a) or 2(a) by vacuum saturation of (2) given (1):

$$a_{3/2} \equiv -i\langle \pi^+\pi^0 | H_{JJ} | K^+ \rangle \tag{3a}$$

$$= \frac{iG_W}{2\sqrt{2}}\sin\theta_C\cos\theta_C \langle \pi^0 | J_\mu^{4-i5} | K^+ \rangle \times \langle \pi^+ | J^{1+i2,\mu} | 0 \rangle \tag{3b}$$

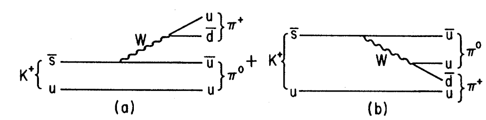

FIG. 1. Quark spectator (a) and color-suppressed spectator (b) diagrams for $K^+ \to \pi^+\pi^0$ decay.

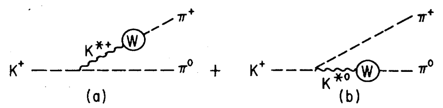

FIG. 2. Vector-dominance pole graphs for $K^+ \to \pi^+\pi^0$ decay.

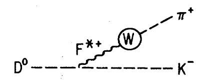

FIG. 4. Vector-dominance pole graph for $D^0 \to K^-\pi^+$ decay.

$$= \frac{G_W}{2\sqrt{2}} \sin\theta_C \cos\theta_C f_\pi (m_K{}^2 - m_\pi{}^2)$$

$$\approx 1.92 \times 10^{-8} \text{ GeV} \tag{3c}$$

with $f_\pi \approx 93$ MeV and $\theta_C \approx 13.2°$ taken as positive. Since the observed $\Delta I = \frac{3}{2}$ amplitude is[4] $a_{3/2} = (1.83 \pm 0.01) \times 10^{-8}$ GeV, where the sign is chosen in agreement with the tadpole dominance of $a_{1/2}$, we regard vacuum saturation, i.e., the equivalent quark spectator graph Fig. 1(a), as approximately describing $K^+_{\pi^+\pi^0}$ decay.[5]

B. $D^0_{K^-\pi^+}$

Since here $\Delta I = 1$, 0, the $\Delta I = \frac{1}{2}$ tadpoles cannot contribute. Instead we can approach $D^0 \to K^-\pi^+$ decay in a manner similar to $K^+_{\pi\pi}$ decay since it can be described by the quark spectator graph of Fig. 3 or the VDM F^*-pole graph of Fig. 4. Vacuum saturation of $D^0_{K^-\pi^+}$ then leads to the amplitude

$$M_{-+} \equiv i \langle K^-\pi^+ | H_W | D^0 \rangle \tag{4a}$$

$$= \frac{iG_W}{2\sqrt{2}} \cos^2\theta_C \langle K^- | J_\mu^{13+i14} | D^0 \rangle$$

$$\times \langle \pi^+ | J^{1+i2,\mu} | 0 \rangle \tag{4b}$$

$$= \frac{G_W}{2} \cos^2\theta_C f_\pi (m_D{}^2 - m_K{}^2)$$

$$\approx 1.67 \times 10^{-6} \text{ GeV} . \tag{4c}$$

This compares well with the observed amplitude[1]

$$| M(D^0_{K^-\pi^+}) |_{\text{expt}} = (8\pi m_D{}^2 \Gamma_{-+} q_{\text{c.m.}})^{1/2}$$

$$= (1.83 \pm 0.41) \times 10^{-6} \text{ GeV.} \tag{5}$$

C. $D^0_{\bar{K}^0\pi^0}$

This amplitude is color suppressed in either the quark-model version of Fig. 5 or the equivalent VDM D^*-pole graph of Fig. 6. Unfortunately, these graphs cannot be vacuum saturated. However, M_{-+} and $\sqrt{2}M_{00}$ have the same quark structure except for the sharing of color in Fig. 5 relative to Fig. 3. Thus the quark model predicts[6]

$$| M_{-+} / \sqrt{2} M_{00} | = 3 . \tag{6}$$

Combining (6) with (4c) then suggests the scale

$$| M_{00} | \approx 0.40 \times 10^{-6} \text{ GeV} , \tag{7a}$$

which is only moderately within the range of experiment,[1]

$$| M(D^0_{\bar{K}^0\pi^0}) |_{\text{expt}} = (8\pi m_D{}^2 \Gamma_{00} / q_{\text{c.m.}})^{1/2}$$

$$= (1.67 \pm 0.55) \times 10^{-6} \text{ GeV} . \tag{7b}$$

The smallness of M_{00} relative to M_{+-} as predicted by (3) is not inconsistent with analogous branching-ratio observations[1] on nonleptonic decays having final-state resonances:

$$B(D^0 \to \bar{K}^0\rho^0 / K^-\rho^+) = 0.01^{+0.17}_{-0.01} , \tag{8a}$$

$$B(D^0 \to \bar{K}^{*0}\pi^0 / K^{*-}\pi^+) = 0.4^{+1.9}_{-0.4} . \tag{8b}$$

D. $D^+_{\bar{K}^0\pi^+}$

Now there exists both quark spectator and color-suppressed spectator $D^+ \to \bar{K}^0\pi^+$ diagrams, as depicted in Fig. 7, with analog VDM F^* and D^* pole graphs which we do not bother to display. Since these graphs are just the sum of the spectator graphs of Figs. 3–4, and the color-suppressed spectator graphs of Figs. 5–6, we may infer the well known $D_{K\pi}$ amplitude sum rule

$$M_{-+} + \sqrt{2} M_{00} = M_{0+} . \tag{9}$$

Alternatively, combining the final-state K and π isospins into $I = \frac{1}{2}$ and $\frac{3}{2}$ combinations, we may write the $D_{K\pi}$ amplitudes as (including possible final-state interactions)

$$M_{-+} = -\tfrac{2}{3} M_{1/2} e^{i\delta_{1/2}} + \tfrac{1}{3} M_{3/2} e^{i\delta_{3/2}} , \tag{10a}$$

$$\sqrt{2} M_{00} = \tfrac{2}{3} M_{1/2} e^{i\delta_{1/2}} + \tfrac{2}{3} M_{3/2} e^{i\delta_{3/2}} , \tag{10b}$$

$$M_{0+} = M_{3/2} e^{i\delta_{3/2}} , \tag{10c}$$

from which the sum rule (9) also follows.

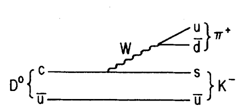

FIG. 3. Quark spectator diagram for $D^0 \to K^-\pi^+$ decay.

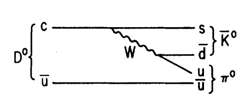

FIG. 5. Color-suppressed quark spectator diagram for $D^0 \to \bar{K}^0\pi^0$ decay.

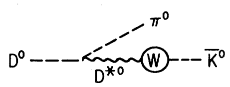

FIG. 6. Vector-dominance pole graph for $D^0 \to \bar{K}^0\pi^0$ decay.

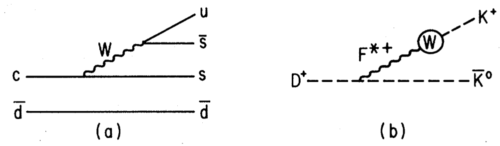

FIG. 8. Quark spectator and VDM pole graphs for $D^+ \to \bar{K}^0 K^+$ decay.

In order to take advantage of (9), however, we must determine the relative sign of M_{-+} and M_{00} in (6). Fierz reshuffling of the fermion fields in (2) suggests[6] $M_{-+}/M_{00} > 0$. On the other hand, Eq. (10) requires the "large" $M_{1/2}$ [i.e., the 6 SU(3) or 20 SU(4) component of H_W] to cancel between (10a) and (10b) in order to obtain the "small" $M_{3/2}$ [i.e., the $\underline{15}^*$ SU(3) or $\underline{84}$ SU(4) component of H_w] in (10c). The same situation occurs for the K^+_{+0} graphs of Figs. 1—2 so that the large $\Delta I = \frac{1}{2}$ component of H_W cancels between Figs. 1(a) and 1(b) or between Figs. 2(a) and 2(b), leaving only the small $\Delta I = \frac{3}{2}$ component of H_W to drive $K^+_{\pi^+\pi^0}$ decay. Such cancellation patterns suggest a relative minus sign[7] between M_{-+} and M_{00} on the left-hand side of (9), then predicting from the vacuum-saturation scale of 4(c) [and therefore also from (7a)]

$$M_{0+} = (1.67 - 0.56) \times 10^{-6}\ \text{GeV}$$
$$= 1.11 \times 10^{-6}\ \text{GeV}\ . \tag{11a}$$

This value agrees well with the magnitude of the measured amplitude[1]

$$|M(D^+_{\bar{K}^0\pi^+})|_{\text{expt}} = (8\pi m_D{}^2 \Gamma_{0+}/q_{\text{c.m.}})^{1/2}$$
$$= (1.15 \pm 0.20) \times 10^{-6}\ \text{GeV}\ . \tag{11b}$$

While (11b) does seem to suggest a relative minus sign in (11a), the experimental error on the D^0 lifetime is too large to be certain of (11a) or (11b); we must wait for the new Mark III results to be sure.

In a similar manner we should account for the color-suppressed $K^+_{\pi^+\pi^0}$ graph, as noted in Ref. 7, now $1 - \frac{1}{3} = \frac{2}{3}$ of the magnitude of (3c). We suggest that this 30% mismatch between theory and experiment is a final-state interaction effect.

III. CABIBBO-ANGLE-SUPPRESSED $D^+_{K\bar{K}}, D^+_{\pi\pi}$ DECAYS

These amplitudes excite $\Delta I = \frac{1}{2}$, $\frac{3}{2}$ components of H_W and are suppressed, being proportional to $\sin\theta_C \cos\theta_C$. While the data are sparse for these decay modes, they are nonetheless significant because they expose the $\Delta I = \frac{1}{2}$ c-u quark tadpole which then combines with the quark spectator or VDM graphs.

A. $D^+_{\bar{K}^0 K^+}$

This pure $\Delta I = \frac{1}{2}$ transition is driven by both the spectator or VDM graphs of Fig. 8 and also by the c-u quark tadpole of Fig. 9(b). Looking first at the spectator graph of Fig. 8, vacuum saturation (VS) of (2) gives for $f_K \approx 1.2 f_\pi \approx 112$ MeV,

$$M_{\text{VS}}(D^+_{\bar{K}^0K^+}) = \frac{iG_W}{2\sqrt{2}} \sin\theta_C \cos\theta_C$$
$$\times \langle \bar{K}^0 | J^{13+i14}_\mu | D^+ \rangle \langle K^+ | J^{4+i5,\mu} | 0 \rangle$$
$$= \frac{G_w}{2} \sin\theta_C \cos\theta_C f_K (m_D{}^2 - m_K{}^2) \tag{12a}$$
$$\approx 4.7 \times 10^{-7}\ \text{GeV}. \tag{12b}$$

Concerning the tadpole contribution, recall that the $K^0_{2\pi}$ amplitudes are controlled[4,8,9] by the $\Delta I = \frac{1}{2}$ s-d quark "self-energy" tadpole of Fig. 9(a). We believe that questions concerning the elimination of these kinetic tadpoles by SU(4) rotations or quantum-chromodynamic and quantum-flavor-dynamic transformations have been adequately answered in Ref. 4. These self-energy quark tadpoles are at the core of the respective one-particle transitions $\langle 0 | H_{\text{tad}} | \bar{K}^0 \rangle$ and $\langle 0 | H_{\text{tad}} | D^0 \rangle$ depicted in Figs. 10(a) and 10(b), which in turn generate the rapidly varying $K_{2\pi}$ and $D_{K\bar{K}}$ pole graphs of Figs. 11(a) and 11(b).

At this point one invokes current algebra and PCAC to relate Figs. 10 and 11 as[4,9]

$$-\langle \pi^0\pi^0 | H_{\text{tad}} | \bar{K}^0 \rangle \approx \frac{1}{2f_\pi{}^2} \langle 0 | H_{\text{tad}} | \bar{K}^0 \rangle\ , \tag{13a}$$

$$-\langle \bar{K}^0 K^+ | H_{\text{tad}} | D^+ \rangle \approx \frac{1}{f_K{}^2} \langle\ | H_{\text{tad}} | D^0 \rangle\ . \tag{13b}$$

To estimate the magnitude of (13b), we note that the structure of Fig. 10(a), as shown in Ref. 4, is

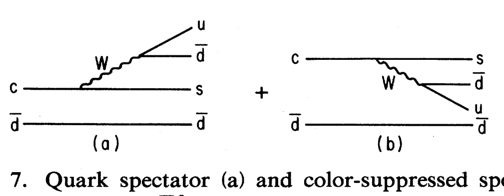

FIG. 7. Quark spectator (a) and color-suppressed spectator (b) diagrams for $D^+ \to \bar{K}^0\pi^+$ decay.

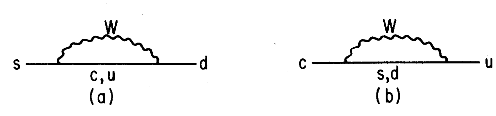

FIG. 9. $\Delta I = \frac{1}{2}$ s-d quark tadpole (a) and c-u quark tadpole (b).

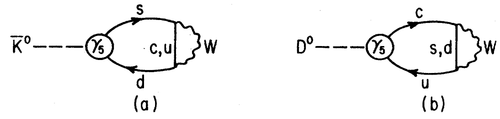

FIG. 10. $\bar{K}^0$ to vacuum (a) and D^0 to vacuum (b) tadpole graphs.

$$\langle 0 | H_{\text{tad}} | \bar{K}^0 \rangle \propto (m_s - m_d)(m_c^2 - m_u^2) ,$$

where the factor of $m_s - m_d$ derives from the s-d quark loop in Fig. 9(c) and the factor $m_c^2 - m_u^2$ is universally generated[10] by the GIM current (1) in Fig. 9(a). Switching $d \leftrightarrow u$ and $s \leftrightarrow c$ then converts Fig. 9(a) to 9(b) and

$$\langle 0 | H_{\text{tad}} | D^0 \rangle \approx \frac{(m_c - m_u)(m_s^2 - m_d^2)}{(m_s - m_d)(m_c^2 - m_u^2)} \langle 0 | H_{\text{tad}} | \bar{K}^0 \rangle \tag{14a}$$

$$\approx i\, 2.3 \times 10 \text{ GeV}^3 , \tag{14b}$$

where $m_c \approx 1550$ MeV, $m_s \approx 510$ MeV, $m_u \approx m_d \approx 340$ MeV are the constituent quark masses and the magnitude of $\langle 0 | H_{\text{tad}} | \bar{K}^0 \rangle$ and sign are taken from theory[4] or from experiment.[1] Substituting (14b) back into (13b) then predicts the tadpole amplitude

$$M_{\text{tad}}(D^+_{\bar{K}^0 K^+}) = i \langle \bar{K}^0 K^+ | H_{\text{tad}} | D^+ \rangle$$
$$\approx 1.8 \times 10^{-7} \text{ GeV} . \tag{15a}$$

Adding together (12b) and (15a), we obtain the total $D^+_{\bar{K}^0 K^+}$ amplitude

$$M(D^+_{\bar{K}^0 K^+}) = 6.5 \times 10^{-7} \text{ GeV} , \tag{15b}$$

which is quite consistent with observation,[1]

$$| M(D^+_{\bar{K}^0 K^+}) |_{\text{expt}} = (8\pi m_d^2 \Gamma_{D^+ \bar{K}K} / q_{c.m.})^{1/2}$$
$$= (6.0 \pm 2.1) \times 10^{-7} \text{ GeV} . \tag{15c}$$

Apart from the excellent agreement between (15b) and (15c), the point is that the spectator and tadpole amplitudes are theoretically predicted to add; if they instead interfered destructively, the results would be reduced to 2.9×10^{-7} GeV, which is not in as good agreement with (15c) as is (15b).

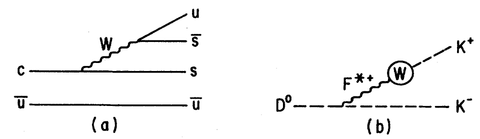

FIG. 12. Quark spectator (a) and equivalent vector-dominance pole graph (b) for $D^0 \to K^- K^+$ decay.

B. $D^+_{\pi^0 \pi^+}$

This decay excites only $\Delta I = \frac{3}{2}$ components of H_W, so the c-u tadpole is ruled out. As for the spectator or VDM graph, vacuum saturation of (2) for $D^+ \to \pi^0 \pi^+$ leads to

$$M(D_{\pi^- \pi^+})^+ = \frac{i G_W}{2\sqrt{2}} \sin\theta_C \cos\theta_C \times \langle \pi^0 | J_\mu^{11+i12} | D^+ \rangle \langle \pi^+ | J^{1+i2,\mu} | 0 \rangle \tag{16a}$$

$$\approx -\frac{G_W}{2\sqrt{2}} \sin\theta_C \cos\theta_C f_\pi (m_D^2 - m_\pi^2)$$
$$\approx -3.0 \times 10^{-7} \text{ GeV} , \tag{16b}$$

which also is not inconsistent with experiment,[1]

$$| M(D^+_{\pi^0 \pi^+}) |_{\text{expt}} \langle (6.3 \pm 1.5) \times 10^{-7} \text{ GeV} . \tag{16c}$$

IV. CABIBBO-ANGLE-SUPPRESSED D^0_{KK}, $D^0_{\pi\pi}$ DECAYS

Like the $K^0_{2\pi}$ decays, these D^0 amplitudes are proportional to $\sin\theta_C \cos\theta_C$ and excite $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ components of H_W such that *both* quark spectator and tadpole graphs occur together for a given process.

A. $D^0_{K^- K^+}$

The equivalent quark spectator and VDM pole graphs for $D^0 \to K^- K^+$ are displayed in Fig. 12. Vacuum saturation of Fig. 12 then corresponds to

$$M_{\text{VS}}(D^0_{K^- K^+}) = \frac{i G_W}{2\sqrt{2}} \sin\theta_C \cos\theta_C \times \langle K^- | J_\mu^{13+i14} | D^0 \rangle \langle K^+ | J^{4+i5,\mu} | 0 \rangle \tag{17a}$$

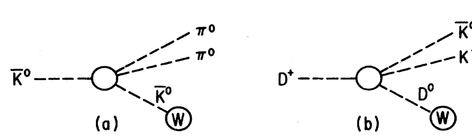

FIG. 11. Rapidly varying $\bar{K}^0 \to \pi^0 \pi^0$ pole diagram (a) and $D^+ \to \bar{K}^0 K^+$ pole diagram (b).

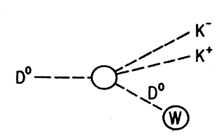

FIG. 13. Rapidly varying pole graph for $D^0 \to K^- K^+$ decay.

$$\approx \frac{G_W}{2}\sin\theta_C\cos\theta_C f_K(m_D{}^2-m_K{}^2)$$

$$\approx 4.7\times10^{-7}\ \text{GeV}\ . \tag{17b}$$

This amplitude must be added to the rapidly varying tadpole diagram of Fig. 13. Using (13) and (14), the latter graph gives

$$M_{\text{tad}}(D^0_{K^-K^+})\equiv i\langle K^-K^+\,|\,H_{\text{tad}}\,|\,D^0\rangle \tag{18a}$$

$$=-\frac{i}{f_K{}^2}\langle 0\,|\,H_{\text{tad}}\,|\,D^0\rangle$$

$$\approx 1.8\times10^{-7}\ \text{GeV}\ . \tag{18b}$$

The total $D^0_{K^-K^+}$ amplitude is then the sum of (17) and (18):

$$M(D^0_{K^-K^+})\approx 6.5\times10^{-7}\ \text{GeV}\ , \tag{18c}$$

which is quite close to observation,[1]

$$|\,M(D^0_{K^-K^+})\,|_{\text{expt}}=(6.4\pm0.8)\times10^{-7}\ \text{GeV}\ . \tag{18d}$$

B. $D^0_{\pi^-\pi^+}$

The corresponding quark spectator and VDM pole graphs are depicted in Fig. 14. Vacuum saturation analogous to (17) gives

$$M_{\text{VS}}(D^0_{\pi^-\pi^+})=\frac{iG_W}{2\sqrt{2}}(-\sin\theta_C)\cos\theta_C$$

$$\times\langle\pi^-\,|\,J^{11+i12}_\mu\,|\,D^0\rangle\langle\pi^+\,|\,J^{1+i2,\mu}\,|\,0\rangle \tag{19a}$$

$$=-\frac{G_W}{2}\sin\theta_C\cos\theta_C f_\pi(m_D{}^2-m_\pi{}^2)$$

$$\approx -4.2\times10^{-7}\ \text{GeV}\ , \tag{19b}$$

where the minus sign in (19) is due to the explicit structure of the GIM quark current (1). On the other hand, the rapidly varying tadpole diagram of Fig. 15 has the same sign relative to Fig. 13 and Eqs. (13b) and (18b):

$$M_{\text{tad}}(D^0_{\pi^-\pi^+})\equiv i\langle\pi^-\pi^+\,|\,H_{\text{tad}}\,|\,D^0\rangle \tag{20a}$$

$$\approx -\frac{i}{2f_\pi{}^2}\langle 0\,|\,H_{\text{tad}}\,|\,D^0\rangle$$

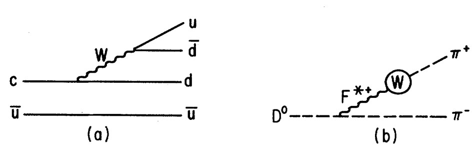

FIG. 14. Quark spectator (a) and equivalent vector-dominance pole graph (b) for $D^0\to\pi^-\pi^+$ decay.

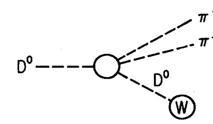

FIG. 15. Rapidly varying pole graph for $D^0\to\pi^-\pi^+$ decay.

$$\approx 1.3\times10^{-7}\ \text{GeV}\ . \tag{20b}$$

Combining (19) and (20), the net $D^0_{\pi\pi}$ amplitude is

$$M(D^0_{\pi^-\pi^+})\approx -2.9\times10^{-7}\ \text{GeV}\ . \tag{21}$$

The ratio of the $D^0_{K\bar K}$ to $D^0_{\pi\pi}$ amplitudes (18c) and (21) then leads to the predicted branching ratio

$$B(D^0\to K^-K^+/\pi^-\pi^+)$$

$$=(q^K_{\text{c.m.}}/q^\pi_{\text{c.m.}})\,|\,M(D^0_{K^-K^+})/M(D^0_{\pi^-\pi^+})\,|^2$$

$$\approx 4.3\ , \tag{22a}$$

which is consistent with the measured branching ratio[1]

$$B(D^0\to K^-K^+/\pi^-\pi^+)_{\text{expt}}=3.4\pm1.9\ . \tag{22b}$$

We note the importance of the *sign* of the tadpole contributions (18) and (20) relative to the spectator amplitudes (17) and (19), determined by the sign of $\langle 0\,|\,H_{\text{tad}}\,|\,\bar K^0\rangle$ in (14). The latter sign is fixed by the experimental $K_{2\pi}$ amplitude ratio $a_{3/2}/a_{1/2}\approx 0.05>0$ and independently by the $K_{2\pi}$ $\Delta I=\frac{3}{2}$ quark spectator to $\Delta I=\frac{1}{2}$ quark tadpole ratio.[4] If this sign were opposite, then the branching ratio (22a) would be $\frac{1}{4}$ rather than 4. We regard this as a very significant test of the spectator plus tadpole theory—combined with the GIM current (1) and Cabibbo Hamiltonian (2).

To assure the reader that mixing quark spectator and tadpole graphs is a legitimate enterprise, we stress that *both* contributions stem from the current-algebra—PCAC premise. While this is obvious for the tadpole amplitudes from the structure of (13), it was Sakurai[11] who stressed the connection between the VDM and current-algebra and PCAC. With reference to the $D_{K\pi}$ decays, it was shown in Ref. 7 that one must add the results of taking the K and π soft, leading to, for $f_K=f_\pi$,

$$-f_\pi M_{-+}=\langle K^-\,|\,[Q^{\pi^-},H_{JJ}]\,|\,D^0\rangle$$

$$+\langle\pi^+\,|\,[Q^{K^+},H_{JJ}]\,|\,D^0\rangle$$

$$=\frac{1}{\sqrt{2}}\langle\pi^+\,|\,H_{JJ}\,|\,F^+\rangle\ , \tag{23a}$$

$$-f_\pi M_{00}=\langle\bar K^0\,|\,[Q^{\pi^0},H_{JJ}]\,|\,D^0\rangle+\langle\pi^0\,|\,[Q^{K^0},H_{JJ}]\,|\,D^+\rangle$$

$$=\tfrac{1}{2}\langle\bar K^0\,|\,H_{JJ}\,|\,D^0\rangle\ , \tag{23b}$$

$$-f_\pi M_{+0}=\langle\bar K^0\,|\,[Q^{\pi^-},H_{JJ}]\,|\,D^+\rangle$$

$$+\langle\pi^+\,|\,[Q^{K^0},H_{JJ}]\,|\,D^+\rangle$$

$$=\frac{1}{\sqrt{2}}[\langle\pi^+\,|\,H_{JJ}\,|\,F^+\rangle+\langle\bar K^0\,|\,H_{JJ}\,|\,D^0\rangle]\ . \tag{23c}$$

The final right-hand sides of (23a)–(23c) simulate both the quark spectator plus color-suppressed spector graphs of Figs. 3, 5, and 7 and at the same time follow the VDM graphs of Figs. 4, 6, and 8, while satisfying the sum rule (9).

V. SUMMARY

We have shown that Cabibbo-angle-enhanced $D_{K\pi}$ decays are controlled by quark spectator or equivalently VDM pole graphs; that Cabibbo-angle-suppressed $D^+_{K\bar{K}}$ decay is driven by the quark spectator and the $\Delta I=\frac{1}{2}$ c-u quark tadpole graphs; that $D^+_{\pi\pi}$ decay cannot receive contributions from tadpoles, but $D^0_{K\bar{K}}$ and $D^0_{\pi\pi}$ are driven by both quark spectator and tadpole diagrams. In our opinion, all of this points very convincingly to the underlying Cabibbo-GIM quark current as the true origin of nonleptonic weak decays—not only for K mesons[4] and D mesons, but also for hyperon decays as well,[9,12] where $\Delta I=\frac{1}{2}$ tadpoles cannot occur.

In passing, it should be obvious to the reader that we have deemphasized the QCD modification of the weak Hamiltonian density (2) in terms of the QCD factors[13,6] $f_\pm$. Perhaps they are necessary for inclusive rates, but it is not clear what calculation they improve for two-body nonleptonic decays. The $D_{K\pi}$ branching ratio $B(D^0\to K^-\pi^+/\bar{K}^0\pi^0)$ goes from 18 in (6) to ~ 40 with the $f_\pm$ factors included,[6] even more removed from present experiment. Moreover, such $f_\pm$ factors appear not to be necessary for $K_{2\pi}$ or $D_{K\bar{K}}$, $D_{\pi\pi}$ or hyperon decays; they would partially spoil the good agreement that already exists between theory and experiment in these cases. A theoretical justification for ignoring the QCD factors $f_\pm$ for quark spectator decays is that gluons exchanged between quarks within the same hadron have been accounted for, e.g., in the $\langle 0|A_\mu|\pi^+\rangle$ and $\langle \bar{K}^0|V_\mu|D^+\rangle$ current matrix elements and gluons exchanged between quarks within different hadrons can be ignored because of gluon confinement.

Note added in proof. The key relative minus sign between the color-enhanced and color-suppressed spectator graphs of Fig.7 for $D^+\to\bar{K}^0\pi^+$ decay suggested on phenomenological grounds below Fig. 6 in fact has a theoretical basis. The factor-of-3 color suppression in Fig. 7(b) at the weak vertex follows from the Fierz reshuffling of the quark fields in H_{JJ} in (2). But then the strong vertices $\langle \bar{K}^0|V_\mu^{F^{*+}}|D^+\rangle$ and $\langle \pi^+|V_\mu^{D^{*0}}|D^+\rangle$ are of opposite sign because $\bar{K}^0$, F^{*+}, π^+, D^{*0} have a net relative Condon-Shortley negative phase when converting the $q\bar{q}$ quark states to Cartesian language. This mismatch between $q\bar{q}$ quark and Cartesian states is more easily seen[14] for $K^+\to\pi^+\pi^0$ decay, also leading to a $1-\frac{1}{3}=\frac{2}{3}$ suppression of the color-enhanced amplitude (3).

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