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$K \rightarrow 2\pi$ decays and K-π matrix elements of $\Delta S = 1$ transition operators

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It is pointed out that the K-π matrix element given by Shifman, Vainshtein, and Zakharov using the vacuum-insertion method is inconsistent with the chiral-symmetry constraint. The matrix elements of the left-handed current $\times$ current operators obtained by this method are however consistent with chiral symmetry and give a qualitative understanding of the $\Delta I = \frac{3}{2}$ $K \rightarrow 2\pi$ decay amplitude. The $\Delta I = \frac{1}{2}$ amplitude seems to be dominated by a pure $\Delta I = \frac{1}{2}$ operator as indicated by the calculated $K_L \rightarrow 2\gamma$ decay rate; the nature of this interaction is discussed.

With the advent of quantum chromodynamics (QCD) as a possible theory of strong interactions, some progress has been made during the past ten years towards an understanding of the octet dominance and the $\Delta I = \frac{1}{2}$ rule for nonleptonic hyperon and kaon decays within the framework of the standard Weinberg-Salam theory of weak interactions. In the works of Gaillard and Lee (GL) and others,[1] because of short-distance QCD effects, the Wilson coefficients[2] of the (1,8) and (1,27) pieces of the $\Delta S = 1$ four-quark operator in the effective Lagrangian are, respectively, enhanced and suppressed by a factor of 2.5 relative to the free-field value, producing qualitatively an octet enhancement. There are indications that this is not sufficient to account for the large observed octet $\Delta I = \frac{1}{2}$ amplitude, which is larger than the $\Delta I = \frac{3}{2}$ part by a factor of 20. In fact, from PCAC (partial conservation of axial-vector current) and chiral-symmetry constraints one can relate the $K \rightarrow 2\pi$ amplitude to K-π transition matrix elements. From the quark-counting rule alone, we know that the K-π matrix element for the (1,8) piece is smaller than that for the $\Delta I = \frac{3}{2}$, (1,27) piece by a factor of 2, leaving only a small enhancement of the $\Delta I = \frac{1}{2}$ amplitude, far below the data. It appears that the standard four-quark operator of the current$\times$current type cannot be the dominant contribution to nonleptonic decays and that some new operators must be involved. These new operators must transform as a (1,8) representation under SU(3)$\times$SU(3) since the current-algebra Callan-Treiman-type relations between $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decays are satisfied to a good accuracy. The usual penguin-type $\Delta S = 1$ interactions[3] advocated by Shifman, Vainshtein, and Zakharov (SVZ) are possible new contributions, although their relative importance is uncertain since the SVZ estimate for this contribution is questionable for *the simple reason that the chiral-symmetry property and PCAC constraints are not satisfied by the SVZ expression,* as will be clear in the following discussion. One can, however, carry out a phenomenological analysis to determine the fraction of the penguin part relative to the total $\Delta I = \frac{1}{2}$ amplitude. For this purpose one needs to evaluate the K-π matrix element from the four-quark operators of the effective Lagrangian. This matrix element is also related to the K^0-$\bar{K}^0$ transition matrix element of the effective $\Delta S = 2$ operator which is built from the same $(V - A)$ currents and behaves as (1,27) under SU(3)$\times$SU(3). A knowledge of the matrix element of the four-quark operator is thus essential to the analysis of nonleptonic K decays and K_S-K_L mass difference. In this paper we shall present an analysis of nonleptonic $K \rightarrow 2\pi$ decay based on the valence-quark approximation (i.e., only the $q\bar{q}$ component in the meson wave function) for the K-π matrix element (which shows that in the free-quark model, the saturation by the vacuum intermediate state is exact). The advantage of using this approximation is that it may be justified to some extent and that the K-π matrix element obtained is explicitly quadratic in momentum, consistent with chiral-symmetry and PCAC constraints. This is not the case with the MIT-bag-model calculation as the bag-model description of the pseudoscalar-meson octet considered as Goldstone bosons of the chiral SU(3)$\times$SU(3) symmetry is rather questionable and the matrix element obtained does not have this momentum dependence explicitly displayed. For this reason we shall not use the bag model in the following. The main results of our analysis can be summarized as follows.

(i) The $\Delta I = \frac{3}{2}$ amplitude is qualitatively understood using the K-π matrix elements in the vacuum-insertion approximation.

(ii) The SVZ estimate of penguin matrix elements is inconsistent with chiral-symmetry and PCAC constraints which require that the K-π matrix element of (1,8) operators must be momentum-dependent. This invalidates their claim that penguin interactions can account for most of the $\Delta I = \frac{1}{2}$ amplitude.

(iii) Assuming a K_L-π^0 transition dominated by purely $\Delta I = \frac{1}{2}$, (1,8) operators, then the $K_L \rightarrow 2\gamma$ amplitude induced by the mixing of K_L with π^0, η, η' produces a decay rate in agreement with experiment to within 30%.

To proceed we present here some important properties of the K-π transition and $K \rightarrow 2\pi$ decay amplitudes derived from SU(3)$\times$SU(3) symmetry and PCAC constraints for use in the following analysis.

An elegant way of implementing chiral-symmetry and PCAC constraints on kaon nonleptonic decay amplitudes is to construct a phenomenological effective Lagrangian involving only the pseudoscalar-meson fields which transform nonlinearly[4–6] under SU(3)×SU(3) (the usual flavor symmetry). The decay amplitudes are obtained from tree diagrams and automatically satisfy current-algebra Callan-Treiman-type relations in the soft-pion limit, provided that the effective Lagrangian contains only terms transforming as (1,8) or (1,27) representations of SU(3)×SU(3). This is true for the nonleptonic-weak-interaction effective Lagrangian in the standard Weinberg-Salam theory with left-handed $V-A$ currents. Since chirality is not affected by short-distance QCD effects, the four-quark operators in the GL effective Lagrangian contain only (1,8) and (1,27) pieces built from left-handed quark fields of the current×current type. We have[3,7]

$$\mathscr{L}_w(\Delta I=\tfrac{1}{2})=-\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C C_1 O_1 \ , \quad (1)$$

$$\mathscr{L}_w(\Delta I=-\tfrac{3}{2})=-\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C C_4 O_4 \quad (2)$$

for the main contributions to $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes. The operators O_1 and O_4 are normal ordered and are products of two color-singlet $V-A$ currents given by

$$\begin{aligned} O_1&=\mathscr{H}_A-\mathscr{H}_B \ , \\ O_4&=\mathscr{H}_A+\mathscr{H}_B-\mathscr{H}_C \end{aligned} \quad (3)$$

with

$$\begin{aligned} \mathscr{H}_A&=\bar{d}\gamma_\mu(1+\gamma_5)u\bar{u}\gamma_\mu(1+\gamma_5)s \ , \\ \mathscr{H}_B&=\bar{u}\gamma_\mu(1+\gamma_5)u\bar{d}\gamma_\mu(1+\gamma_5)s \ , \\ \mathscr{H}_C&=\bar{d}\gamma_\mu(1+\gamma_5)d\bar{d}\gamma_\mu(1+\gamma_5)s \ . \end{aligned} \quad (4)$$

C_1 and C_4 are Wilson's coefficients due to QCD short-distance effects calculated by Gaillard and Lee and SVZ and known numerically to be 2.5 and 0.4, respectively. These values seem not to depend strongly on the choice of the QCD mass scale μ. Qualitatively we thus see that QCD short-distance effects enhance the $\Delta I=\frac{1}{2}$ piece and suppress the $\Delta I=\frac{3}{2}$ piece by a large factor.

The operator responsible for the $\Delta I=\frac{3}{2}$ transition is given by Eq. (2) and is the only contribution. However, other operators like the penguin interactions found by SVZ can also be responsible for the $\Delta I=\frac{1}{2}$ transition. They can be approximated by a local four-quark operator of the form[3,7]

$$\begin{aligned} \mathscr{L}_w(\text{penguin})&=-\frac{G_F}{2\sqrt{2}}C_5\sin\theta_C\cos\theta_C O_5 \ , \\ O_5&=\sum_a J_\mu^a\bar{d}\gamma_\mu(1+\gamma_5)\lambda^a s \ , \end{aligned} \quad (5)$$

where C_5 is first order in the QCD coupling constant $\alpha_s(\mu^2)$ and is known to be quite small ($C_5\sim 0.05$—0.02). J_μ^a are the color-octet gauge vector currents transforming as a singlet (1,1) representation under flavor SU(3)×SU(3). Since the color-octet $\Delta S=1$ left-handed currents

$$\bar{d}\gamma_\mu(1+\gamma_5)\frac{\lambda^a s}{2}$$

transform as (1,8), the penguin interactions given by Eq. (1) contain only (1,8) terms and give rise to a purely $\Delta I=\frac{1}{2}$ amplitude. Having shown that the $\Delta S=1$ nonleptonic-decay effective Lagrangian contains only (1,8) and (1,27) terms, we can now write down a phenomenological Lagrangian for the K-π transition and $K\to 2\pi$ decay. Following Cronin we have for the (1,8) piece[4]

$$\mathscr{L}_w(\text{octet})=-\frac{cG_F}{4\sqrt{2}f^4}\text{Tr}(\lambda_6\partial_\mu M\partial_\mu M^\dagger) \ , \quad (6)$$

where M is a 3×3 pseudoscalar-meson coupling matrix transforming as a $(3,\bar{3})$ representation of SU(3)×SU(3) and satisfying the unitary condition $MM^\dagger=1$. M can be expanded in terms of the pseudoscalar-meson fields Φ_i ($i=1,\ldots,8$) as

$$M(f\Phi)=1+2if\Phi+2(if\Phi)^2+\cdots \ , \quad (7)$$

where

$$\Phi=\sum_{i=1,8}\Phi_i\frac{\lambda_i}{\sqrt{2}}$$

and higher-order terms depending on the form of M are irrelevant to the K-π transition and $K\to 2\pi$ decays and have been dropped. f is the inverse of the pion decay constant ($f^{-1}=f_\pi=m_\pi$).

Using (7) we get

$$\mathscr{L}_w(\text{octet})=-c\frac{G_F}{\sqrt{2}f^2}\times\text{Tr}(\lambda_6\partial_\mu\Phi\partial_\mu\Phi+\text{i}f\lambda_6\{\partial_\mu\Phi,[\Phi,\partial_\mu\Phi]\}) \ , \quad (8)$$

which is precisely of the form $(V-A)\times(V-A)$. It is important to note that only derivative couplings are allowed for the nonleptonic-decay effective Lagrangian. This is *the main difference between nonleptonic K decays and electromagnetic interactions involving pseudoscalar-meson electromagnetic mass differences and $\eta\to 3\pi$ decay.*[2,5] The one-photon contribution which can have terms behaving like (8,8) and the tadpole u_3 term belonging to $(3,\bar{3})$ can give rise to nonderivative couplings. These important characteristics follow from a beautiful theorem due to Coleman, Wess, and Zumino on the nonlinear realization of chiral symmetry.[6] Thus, the K-π transition and $K\to 2\pi$ decay amplitudes are *quadratic in momenta.*

The $\Delta I=\frac{1}{2}$ amplitudes for $K_S\to\pi^+\pi^-$ and the K-π transition are then given by

$$A(K_S(p)\to\pi^+(k)\pi^-(q))=i\frac{cG_F}{2}f_\pi(2p^2-k^2-q^2) \ , \quad (9a)$$

$$A(K^0\to\pi^0)=-\frac{A(K^-\to\pi^-)}{\sqrt{2}}=+c\frac{G_F}{2}f_\pi{}^2 q(\pi)\cdot q(K) \ . \quad (9b)$$

The $\Delta I=\frac{3}{2}$ amplitude is obtained from $\mathscr{L}_w$(27-plet) built from the product $(V-A)\times(V-A)$. It is given by O_4 with the quark fields replaced by the pseudoscalar-meson fields. Thus,

$$\mathscr{L}_w(\text{27-plet}) = -c'\frac{G_F}{\sqrt{2}}[(V-A)\times(V-A)]_{I=3/2}\ , \qquad (10)$$

from which the $\Delta I=\frac{3}{2}$ part of the $K\to 2\pi$ decay amplitude and K-π transition are given by

$$A(K^-(p)\to\pi^-(k)\pi^0(q)) = ic'\frac{G_F}{2}f_\pi(3p^2-2k^2-q^2)\ , \qquad (11a)$$

$$A(K^0\to\pi^0)=\sqrt{2}A(K^-\to\pi^-) = -c'G_F f_\pi{}^2 q(\pi)\cdot q(K)\ . \qquad (11b)$$

In obtaining (9) and (10) we have dropped terms of the form $\partial_\mu K_i V_{\mu j}$ ($j=1,2,3$) which, owing to the conservation of the isovector vector currents ($\partial_\mu V_{\mu j}=0$), can be expressed as a four-divergence and can therefore be discarded without affecting the physics. Thus, no terms antisymmetric in pion momenta [e.g., terms of the form $p\cdot(k-q)$ are present in (9a) and (11a)].

By letting all the meson momenta on the mass shell we get the physical decay amplitudes which are now proportional to $(m_K{}^2-m_\pi{}^2)$ and are first order in SU(3)-symmetry-breaking effects. This is in agreement with the theorem of Gell-Mann which tells us that $K\to 2\pi$ decay amplitudes must vanish in the SU(3)-symmetry limit.[8] Note also the quadratic momentum dependence of the K-π matrix element as mentioned above and any model which does not produce a K-π transition with this property is not acceptable. Similarly the $K\to 3\pi$ decay amplitudes can be obtained from (6) and (10). In particular, the $\Delta I=\frac{1}{2}$ amplitudes obtained with the parameter c determined from the $K\to 2\pi$ decay rate reproduce quite well the measured decay rates and the slope parameters.[4] This confirms the validity of the phenomenological Lagrangian approach to nonleptonic decays of K mesons as well as the SU(3)×SU(3) symmetry properties for the nonleptonic weak interactions.

Our theoretical understanding of kaon nonleptonic decay is thus reduced to computing c and c' which are obtained from the K-π matrix elements of the $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ operators, respectively. These matrix elements can be computed using the vacuum-insertion method as usually done in the literature. The same matrix elements can also be obtained with the relativistic $q\bar{q}$ wave functions. To illustrate the chiral-symmetry properties for the matrix elements, we shall use the latter method. Let us first consider the K-π matrix element for the $\Delta I=\frac{3}{2}$ O_4 operator. It is exactly the bound-state matrix element of the $\Delta S=1$ $q\bar{q}\to q\bar{q}$ transitions due to the W-exchange process in the limit $m_W{}^2\to\infty$ and is given according to the usual prescription by

$$A(K^-\to\pi^-) = \int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\bar{\psi}(k,k-p)T_w(k,k-p;k',k'-p)\psi(k',k'-p)\ , \qquad (12)$$

where the trace over the quark loop is to be taken whenever appropriate and $T_w(k,k-p;k',k'-p)$ (or simply T_w) is the virtual $\Delta S=1$ $q\bar{q}\to q\bar{q}$ transition matrix element. Since the short-distance QCD effects have already been factorized out and absorbed into C_4, the K-π matrix elements do not depend on the $q\bar{q}$ wave function at short distance and T_w can be taken to be independent of the virtual-quark momenta and is given by the free-quark matrix elements of O_4. The integrations over virtual-quark momenta k,k' can be carried out independently and the matrix elements thus obtained depend only on the long-distance matrix elements

$$\langle 0|:\bar{q}\gamma_5 q:|\pi\rangle \text{ and } \langle 0|:\bar{q}\gamma_\mu\gamma_5 q:|\pi\rangle\ ,$$

which appear in the Wilson short-distance expansion of the meson wave function.[9] The s-channel-annihilation part of T_w for the K^--π^- transition is identified with $\mathscr{H}_A$ and is given by

$$\langle K^-|\mathscr{H}_A|\pi^-\rangle = \mathrm{Tr}[\gamma_5(a+b\not{p})\gamma_\mu(1+\gamma_5)] \times\mathrm{Tr}[\gamma_\mu(1+\gamma_5)\gamma_5(a-b\not{p})]\ , \qquad (13)$$

where

$$\int\frac{d^4k}{(2\pi)^4}\psi(k,k-p) = \gamma_5(a+b\not{p})\ . \qquad (14)$$

a and b are scalar functions of p^2 (the meson momentum) and are considered to be independent of p^2 in the limit of chiral symmetry where the pseudoscalar-meson octet is considered as massless Goldstone bosons. More precisely we have

$$4a = \langle 0|:\bar{q}\gamma_5 q:|\pi\rangle\ ,$$
$$-4bp_\mu = \frac{ip_\mu}{f} = \langle 0|:\bar{q}\gamma_\mu\gamma_5 q:|\pi\rangle\ . \qquad (15)$$

In the phenomenological Lagrangian for K decays we have neglected all SU(3)-violation effects other than those given by the mass terms. The K-π matrix elements in Eqs. (10) and (11) should therefore be computed only in the SU(3) limit given by

$$f_K=f_\pi=f^{-1}\ .$$

Equation (13) then gives

$$\langle K^-|\mathscr{H}_A|\pi^-\rangle = f_\pi{}^2p^2\ , \qquad (15')$$

which has been obtained previously using the approximation by the vacuum intermediate state.[10] This is not surprising since we know from (12) that only quark field operators which annihilate π^-, K^- are present in $\mathscr{H}_A$.

The t-channel-exchange part of T_w is given by

$$\langle K^-|\mathscr{H}_B|\pi^-\rangle = \tfrac{1}{3}\ \mathrm{Tr}[\gamma_5(a+b\not{p})\gamma_\mu(1+\gamma_5) \times\gamma_5(a-b\not{p})\gamma_\mu(1+\gamma_5)]\ , \qquad (16)$$

where the factor $\frac{1}{3}$ comes from color counting. In terms of f_π we thus have

$$\langle K^-|\mathscr{H}_B|\pi^-\rangle = \tfrac{1}{3}f_\pi{}^2p^2\ . \qquad (16')$$

It is important to note that *because of the* $(V-A)\times(V-A)$ *structure of* $\mathcal{H}_B$, *only the b term contributes to (16)* and the matrix element thus obtained satisfies PCAC constraints. The same result can also be obtained from a Fierz transformation of $\mathcal{H}_B$ into an annihilation term as usually done in the literature.[3,7] Note also that once a and b can be expressed in terms of measurable quantities as given by (15), *no detailed knowledge of the* $q\bar{q}$ *wave function is needed* for K-π matrix elements.

For the $\mathcal{H}_C$ term, we have similarly

$$\langle K^- \mid \mathcal{H}_C \mid \pi^- \rangle = 0 \; .$$

Hence,

$$\langle K^- \mid O_4 \mid \pi^- \rangle = \tfrac{4}{3} f_\pi{}^2 p^2 \; , \tag{17a}$$

$$\langle K^0 \mid O_4 \mid \pi^0 \rangle = \sqrt{2} \langle K^- \mid O_4 \mid \pi^- \rangle = \tfrac{4}{3}\sqrt{2} f_\pi{}^2 p^2 \; , \tag{17b}$$

which obey the $\Delta I = \frac{3}{2}$ rule given in Eq. (11b). For $\Delta I = \frac{1}{2}$ matrix elements, we get similarly

$$\langle K^- \mid O_1 \mid \pi^- \rangle = \tfrac{2}{3} f_\pi{}^2 p^2 \; , \tag{18a}$$

$$\langle K^0 \mid O_1 \mid \pi^0 \rangle = -\frac{1}{\sqrt{2}} \langle K^- \mid O_1 \mid \pi^- \rangle = -\frac{\sqrt{2}}{3} f_\pi{}^2 p^2 \; , \tag{18b}$$

which clearly obey the $\Delta I = \frac{1}{2}$ rule. The above expressions for K-π matrix elements are quadratic in momentum and thus satisfy chiral-symmetry and PCAC constraints.

By comparing (17) and (18) we have

$$\langle K^- \mid O_1 \mid \pi^- \rangle = \tfrac{1}{2} \langle K^- \mid O_4 \mid \pi^- \rangle \; . \tag{19}$$

From (9) and (10) we thus have

$$\frac{c'}{c} = \frac{2C_4}{C_1} = \frac{1}{3} \tag{20}$$

[using the SVZ calculated values for C_1 and C_4 ($C_1 = 2.5$, $C_4 = 0.5$)], showing a small enhancement of the $\Delta I = \frac{1}{2}$ amplitude far below the measured values determined from $K_S \to \pi\pi$ and $K^+ \to \pi^+\pi^0$ decay rates. Experimentally[11] we find

$$\begin{aligned} &|\, c \,|_{\text{expt}} = 1.04 \pm 0.1, \quad |\, c' \,|_{\text{expt}} = 0.0327 \; , \\ &\left| \frac{c'}{c} \right|_{\text{expt}} = 0.0314 \; . \end{aligned} \tag{21}$$

Thus, Eq. (20) already tells us that the $\Delta I = \frac{1}{2}$ effective Lagrangian of Gaillard and Lee is insufficient to explain the large $\Delta I = \frac{1}{2}$ amplitude. In fact, using (17) and (18) we find

$$\begin{aligned} &c' = \tfrac{2}{3} C_4 \sin\theta_C \cos\theta_C \; , \\ &c = +\tfrac{1}{3} C_1 \sin\theta_C \cos\theta_C \; . \end{aligned} \tag{22}$$

With $C_1 = 2.5$, $C_4 = 0.4$ we get

$$c'(\text{theory}) = 0.053, \quad c(\text{theory}) = +0.17 \; . \tag{22$'$}$$

The magnitude of the calculated value of c is smaller than experiment by a factor of 6. Thus, almost all the $\Delta I = \frac{1}{2}$ amplitudes must be due to additional $\Delta S = 1$ interactions which are purely $\Delta I = \frac{1}{2}$ and may be of the SVZ penguin type. Depending on the relative sign of this purely $\Delta I = \frac{1}{2}$ amplitude and the four-quark $\Delta I = \frac{1}{2}$ amplitudes [given by (22)], we infer that

$$\begin{aligned} &f = 0.8, \quad \text{same sign} \; , \\ &f = 1.2, \quad \text{opposite sign} \; , \end{aligned} \tag{23}$$

where f is the fraction of the penguin contribution to the total amplitude.

For the $\Delta I = \frac{3}{2}$ amplitude, the calculated value of c' given in (22′) is of the right order of magnitude and is only 60% larger than the measured value. A slight reduction of C_4 and inclusion of chiral-symmetry-breaking effects as well as $I = 2$ $\pi\pi$ final-state interactions[12] can bring the discrepancy down to the level of 20—30 %. Also the contributions to the K-π transition from the low-lying-hadron intermediate state (π, ω, A_1, etc.) in the T product of two currents evaluated in the manner prescribed[2] by Wilson can account for part of the $\Delta I = \frac{3}{2}$ amplitude as found recently by Pham and Sutherland.[13] Allowing for the uncertainties in the calculation, we may say that the vacuum-insertion method gives a qualitative understanding of the $\Delta I = \frac{3}{2}$ amplitude and that this technique gives the right order of magnitude for the four-quark K-π matrix elements, although we do not know how to justify this method on a theoretical basis.

We have thus showed that the $\Delta I = \frac{1}{2}$ amplitude seems to be dominated by a pure $\Delta I = \frac{1}{2}$ interaction. SVZ have given an estimate for this contribution using the penguin operator given by (5) and the valence-quark approximation. Using (12) we can express the penguin K-π matrix element as

$$\begin{aligned} \langle K^- \mid O_5 \mid \pi^- \rangle = \tfrac{16}{3}\, \mathrm{Tr}[&\gamma_5(a + b\not{p})\gamma_\mu(1+\gamma_5) \\ &\times \gamma_5(a - b\not{p})\gamma_\mu] \; . \end{aligned} \tag{24}$$

As pointed out by SVZ, because of the pure vector nature of the gauge vector currents, both a and b contribute to the K-π transition. We get

$$\langle K^- \mid O_5 \mid \pi^- \rangle = \tfrac{16}{3}(16a^2 - 8b^2 p^2) \; . \tag{25}$$

The second term in (25) is quadratic in momentum, consistent with PCAC constraints, and is given by $f_\pi{}^2 p^2$ as for the normal O_1, O_4 contribution. The first term which is obtained from the matrix element $\langle 0 \mid :\bar{q}\gamma_5 q: \mid \pi \rangle$ [Eq. (15)] is independent of the meson momenta and therefore violates chiral-symmetry and PCAC constraints. This term seems to be quite large and has been estimated by SVZ using the standard value for current quark mass. In this way they claimed that the K-π matrix element given by (25) is sufficiently large to account for the fraction of the penguin contribution. *This is incorrect since any terms independent of momenta must be discarded as they violate the SU(3)×SU(3) property of the penguin interactions* which, as mentioned above, must belong to the (1,8) representation.

It is not surprising to realize that the valence-quark approximation cannot be used to evaluate the penguin K-π

matrix element. Nonperturbative effects due to spontaneous breakdown of chiral symmetry must be taken into account to restore the quadratic momentum dependence of the K-π matrix element.

For convenience we present here a more pedestrian way to see the quadratic momentum dependence of the K-π matrix element for the penguin operator. Using standard current-algebra techniques we get

$$\langle K^- \mid O_5 \mid \pi^- \rangle = \frac{p^2 - m_\pi^2}{f_\pi m_\pi^2} \left[i p_\mu \int d^4x \exp(ip\cdot x) \langle K^- \mid T[A_\mu^{1+i2}(x) O_5(0)] \mid 0 \rangle + \langle K^- \mid [Q_5^{1+i2}, O_5] \mid 0 \rangle \right] . \tag{26}$$

The commutator piece belongs to the (1,8) representation of SU(3)$\times$SU(3) (isospin-rotated form of O_5) and in terms of the phenomenological Lagrangian [see Eq. (8)] it is at least bilinear in the pseudoscalar-meson field and hence the second term in (26) vanishes. The T-product part can be separated into terms regular as $p_\mu \to 0$ and the pion-intermediate-state contribution given by

$$\frac{p^2}{m_\pi^2} \langle K^- \mid O_5 \mid \pi^- \rangle .$$

Thus, the matrix element $\langle K^- \mid O_5 \mid \pi^- \rangle$ is given entirely by the regular term and is therefore quadratic in momentum. It may then be possible to calculate the K-π matrix element using (26) with PCAC constraints satisfied by the vanishing of the matrix element of the commutator term. We leave this question open to further analysis as we do not know how to evaluate the T-product term at the quark or hadronic level. When this is done, then the large momentum-independent term in the SVZ estimate makes no contribution and we are left with only the momentum-dependent part (p^2 term). Although we are unable to say much about the magnitude of the p^2 term, we feel it is rather unlikely that this term, being of the same order as the four-quark-operator matrix elements, could produce a large $\Delta I = \frac{1}{2}$ amplitude since C_5 is quite small and there are no other large mass scales besides m_c^2 in the factor $\ln m_c^2/\mu^2$. If this qualitative argument is correct, then some other mechanism for the $\Delta I = \frac{1}{2}$ rule must be sought. Such an additional contribution to the $\Delta I = \frac{1}{2}$ amplitude would probably come from long-distance nonperturbative contributions which must be evaluated at the hadronic level according to the prescription[2] of Wilson. These contributions come from the Born term and other low-lying resonances which can be evaluated in the same manner as in the calculation of the π^+-π^0 mass difference. The only difference are the contributions to K-π matrix elements from *charmed-hadron intermediate states (D, D*, etc.), which are purely* $\Delta I = \frac{1}{2}$. Assuming vector-meson dominance for the form factors involved (e.g., ρ, F^* dominance, etc.), then the light hadrons contribute to K-π transition terms of the order $O(G_F p^2 m_\rho^2)$, while the D,D^* contributions are of the order $O(G_F p^2 m_D^2)$, which is larger than the π,ω terms by a factor 5—10, quite sufficient to produce a $\Delta I = \frac{1}{2}$ amplitude of the right magnitude and to enhance the $\Delta I = \frac{1}{2}$ part relative to the $\Delta I = \frac{3}{2}$ part which receives no contribution from D, D^*, etc. For details of the calculation we refer the reader to the work[13] of Pham and Sutherland.

We shall devote the rest of this paper to a phenomenological calculation of the $K_L \to 2\gamma$ decay rate to see the importance of these new purely $\Delta I = \frac{1}{2}$ interactions. For this purpose, we shall assume that these additional $\Delta I = \frac{1}{2}$ interactions come from an effective operator which transforms as (1,8) under SU(3)$\times$SU(3) and behaves like the $s \to d$ off-shell transition with respect to ordinary SU(3). We shall denote the sum of the SVZ penguin and the nonperturbative contributions as simply the *pure* $\Delta I = \frac{1}{2}$ *contribution.* Using this property we shall now proceed to the calculation of the $K_L \to 2\gamma$ decay rate which can give information on the $\Delta I = \frac{1}{2}$, K-π transition. The long-distance part of this amplitude is induced by the mixing of K_L to low-lying mesons (π^0, η, η', etc.), and the short-distance part is given approximately by the $\bar{s}d \to \gamma\gamma$ transition caused by W exchange. The short-distance part has been calculated[10] by Gaillard and Lee and found to be quite small in the chiral-symmetry limit and can be ignored. The long-distance part (dispersive contribution) is given simply by the π^0,η,η' pole (pole-dominance approximation) contribution. We have

$$\langle K_L \mid \mathscr{L}_w \mid 2\gamma \rangle = \sum_{i=\pi^0,\eta,\eta'} \frac{a_{K_L i} \langle i \mid \mathscr{L}_{\rm EM} \mid 2\gamma \rangle}{(m_K^2 - m_i^2)} , \tag{27}$$

where $a_{K_L i}$ is the K_L-i transition induced by nonleptonic interactions defined as in (9a) or (11a) and $\langle i \mid \mathscr{L}_{\rm EM} \mid 2\gamma \rangle$ is the electromagnetic (EM) decay amplitude for pseudoscalar mesons into two photons. In terms of $q\bar{q}$ states, the pseudoscalar-meson nonet is defined as follows:

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) ,$$
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) , \tag{28}$$
$$\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) .$$

We shall assume in the following that the K_L-i transition is given by the $\Delta I = \frac{1}{2}$ (1,8) piece. In the SU(3) limit we have

$$a_{K_L \eta} = \frac{1}{\sqrt{3}} a_{K_L \pi^0} . \tag{29}$$

Because of the large cancellation between the π^0 and η poles in Eq. (27) in the exact-SU(3) limit, the pseudoscalar-meson pole contributions are quite sensitive to η-η' mixing as well as deviations from the exact-SU(3) relation between $a_{K_L\pi^0}$ and $a_{K_L\eta}$ [Eq. (29)]. These effects must now be taken into account. Since $\langle K_L \mid \mathscr{L}_w \mid q\bar{q} \rangle$ are related to $\langle 0 \mid \bar{q}\gamma_\mu\gamma_5 q \mid q\bar{q} \rangle$, SU(3)-violation effects in

$a_{K_L i}$ are given by the ratio f_K/f_π to a good approximation. This can be most easily seen by noting that the effective Lagrangian responsible for the K_L-$(q\bar{q})_{O^-}$ transition is of the $A_\mu A_\mu$ type so that relative to $a_{K_L\pi^0}$, $\mathcal{L}_w$ can be written as

$$\mathcal{L}_w = -c\frac{G_F}{\sqrt{2}} f_\pi f_{q\bar{q}} \partial_\mu K_L \partial_\mu \varphi_{q\bar{q}} , \tag{30}$$

where $\varphi_{q\bar{q}}$ is the pseudoscalar-meson field operator associated with $(q\bar{q})_{O^-}$ state. Equation (30) gives rise to SU(3)-violation effects of $a_{K_L\eta}$ and $a_{K_L\eta'}$ relative to $a_{K_L\pi^0}$. Assuming

$$\begin{aligned} &f_\pi = f_{d\bar{d}} , \\ &f_K = f_{d\bar{s}} = f_{d\bar{d}}(1+\epsilon) , \\ &f_{s\bar{s}} = f_{d\bar{s}}(1+\epsilon) , \end{aligned} \tag{31}$$

we have

$$\frac{f_K}{f_\pi} = 1+\epsilon \tag{32}$$

so that for the pure $\Delta I = \frac{1}{2}$ part we have

$$\frac{a_{K_L s\bar{s}}}{a_{K_L d\bar{d}}} = \frac{f_{s\bar{s}}}{f_{d\bar{d}}} = 1+2\epsilon+O(\epsilon^2) . \tag{33}$$

The nonpenguin amplitude induced by O_1 ($s\bar{d} \to u\bar{u}$ transition) is simply proportional to $f_{u\bar{u}}$ which is just f_π. Note that to first order in the SU(3)-symmetry-breaking parameter ϵ, (31) reproduces the Gell-Mann—Okubo—type relation[14] for the pseudoscalar-meson decay constants:

$$4f_K - f_\pi = 3f_{\eta_8} + O(\epsilon^2) . \tag{34}$$

From the $\eta' \to 2\gamma$ decay rate,[15] we know that the system π^0-η-η' can be described by the pseudoscalar-meson nonet with a η-η' mixing angle $\theta_P = 10.5°$ and a possible negligibly small glueball component in the η'. The observed $\eta,\eta' \to 2\gamma$ decay amplitudes can also be parametrized in terms of the $\pi^0 \to 2\gamma$ amplitude and θ_P using this nonet structure. The physical η,η' can thus be defined as

$$\begin{aligned} &\eta = \eta_8 \cos\theta_P + \eta_0 \sin\theta_P , \\ &\eta' = -\eta_8 \sin\theta_P + \eta_0 \cos\theta_P . \end{aligned} \tag{35}$$

Including also the nonpenguin contribution, we thus get

$$\begin{aligned} &a_{K_L\eta} = \frac{a_{K_L\pi^0}}{\sqrt{3}}(1+\delta) , \\ &a_{K_L\eta'} = -\frac{2\sqrt{2}}{\sqrt{3}} a_{K_L\pi^0}(1+\delta') \end{aligned} \tag{36}$$

with

$$\begin{aligned} &\delta = 4\epsilon f - 2\sqrt{2}\sin\theta_P(\tfrac{3}{2}f - \tfrac{1}{2} + \epsilon f) , \\ &\delta' = f(1+\epsilon) - \tfrac{1}{2}(1-f) + \frac{\sin\theta_P}{2\sqrt{2}}(1+4\epsilon f) - 1 , \end{aligned} \tag{37}$$

f being the fraction of the pure $\Delta I = \frac{1}{2}$ term in the K_L-π^0 matrix element given in (23).

From (37) we see that for $\sin\theta_P > 0$, effects of SU(3) violation and η-η' mixing tend to cancel out largely, however, with $\epsilon = 0.28$ the overall effect is still large and increases the pole contributions. For a large f, the η' contribution becomes considerable and tends to cancel out the η contribution since $A(\eta \to 2\gamma)$ and $A(\eta' \to 2\gamma)$ are of the same sign. Consequently, important cancellation between η and η' contributions may occur depending on the value of f.

In terms of the $\pi^0 \to 2\gamma$ amplitude, we thus obtain

$$A(K_L \to 2\gamma) = -c\frac{G_F m_p^2}{\sqrt{2}} \left[\frac{f_\pi^2}{m_p^2}\right] \times (2.05 - 1.15f) A(\pi^0 \to 2\gamma) . \tag{38}$$

For the purely penguin interaction, $f = 1$ and the branching ratio of $\Gamma(K_L \to 2\gamma)$ is calculated to be

$$B(K_L \to 2\gamma)_{\text{theory}} = 6.57 \times 10^{-4} , \tag{39}$$

to be compared with the measured value[11]

$$B(K_L \to 2\gamma)_{\text{expt}} = (4.9 \pm 0.5) \times 10^{-4} . \tag{40}$$

Thus, theory agrees with experiment to within 30%. This must be considered as a successful prediction for the decay rate $\Gamma(K_L \to 2\gamma)$ considering the approximation involved. Probably a better agreement can be obtained with $f = 1.1$—1.2 and we may conclude that the pure $\Delta I = \frac{1}{2}$ part is likely to be of the opposite sign relative to the normal amplitude given by the O_1 contribution in (22).

In the preceding analysis we have seen that the $\Delta I = \frac{3}{2}$ $K^+ \to \pi^+\pi^0$ amplitude can be qualitatively understood using QCD-suppressed effective current$\times$current interactions and the valence-quark approximation for the K-π matrix element (vacuum-insertion method). This allows us to estimate the free-quark box diagram for the K_S-K_L mass difference using this approximation as usually done in the literature.[10] We give here some comments concerning this matrix element.

In the standard model with left-handed currents, the K_S-K_L mass difference is given by the K^0-$\bar{K}^0$ transition induced by the $\Delta S = 2$ operator O_{JJ} of the $(V-A)\times(V-A)$ type,[10,16]

$$O_{JJ} = :\bar{s}\gamma_\mu(1+\gamma_5)d\bar{s}\gamma_\mu(1+\gamma_5)d: ,$$

which is of the same form as O_1 and O_4 but transforms as a (1,27) representation of SU(3)$\times$SU(3). Using (13) and (16) we then get the usual expression obtained by the vacuum-intermediate-state approximation,

$$\langle K^0 | O_{JJ} | \bar{K}^0 \rangle = \tfrac{8}{3} f_K^2 p^2 , \tag{41}$$

which is quadratic in momentum in agreement with PCAC constraints. Incidentally we note that the one-pion intermediate-state contribution[16] given by Shrock and Treiman is not actually needed. Also their expression has a part which is independent of momentum and therefore violates chiral-symmetry constraints. In fact, as pointed out in the preceding analysis, in the free-quark model, the vacuum intermediate state is the only contribution to the K^0-$\bar{K}^0$ matrix element for the simple reason that the Wick-ordered operator $:\bar{s}\gamma_\mu(1+\gamma_5)d:$ must annihilate K^0. To go beyond the valence-quark approximation one would have to include the $q\bar{q}$ glue component in the wave function which is expected to increase the K^0-$\bar{K}^0$ matrix element relative to the valence-quark approximation.

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