Abrupt onset of scaling violation

Julián Chela-Flores*

International Centre for Theoretical Physics, Trieste, Italy and Departamento de Física, Universidad Simón Bolívar Apartado 80659, Caracas Venezuela (Received 13 September 1982; revised manuscript received 24 October 1983)

We present a novel interpretation of deep-inelastic scattering data. In our approach the Q^2 evolution from perfect scaling starts with a discontinuity in the structure function F_2 , due to strong correlations among the sea quarks. This effect is compatible with the data gathered in the kinematic region available so far, but will bring new predictions in the kinematic region which will be reached in the near future. Precise experimental suggestions are indicated so as to distinguish between QCD incorporating strong correlations, leading to abrupt scaling violations, and current QCD without such correlations, leading to a smooth logarithmic scale breaking.

I. INTRODUCTION

A fairly complete set of parton distribution functions of nucleons, mesons, and even the photon, has been obtained since high-energy leptons were shown to have a nonzero probability to scatter off hadrons with large transfers of energy ν and momentum Q.

The kinematic region of the $v-Q^2$ plane which we have been able to probe has been high Q and high $x=Q^2/2M_pv$, and medium values of Q for very small values of x. This is well understood: the energy and angular distributions of the scattered leptons show a certain type of correlation known as scaling, which arises from elastic lepton-parton collisions (where the parton is a fermion).

An improvement in our understanding of hadron structure has been reported recently, namely, the value of the Rparameter, which may be defined as the ratio of the number of spin-0 to spin- $\frac{1}{2}$ partons. It is reported that R is very small¹ (0.06±0.012±0.025) for $x \ge 0.4$ and $Q^2=38$ GeV.² There are nonzero values of R for small x, but R is still badly known in the kinematic region of small x and large Q^2 .

From the above it follows that there still remains a large unexplored kinematic region (small x and large Q^2). This is further aggravated by the fact that the most recent experiments trying to enter the new kinematic region disagree at the 10% level outside statistical errors.¹

On the other hand, the correct ground state of quantum chromodynamics (QCD) has so far eluded a thorough understanding. A great diversity of pictures have been tried, suggesting how it might be formed. However, so far we cannot be completely certain of its structure.

We believe that there might be sufficient evidence for a type of correlation in the QCD ground state, beyond scaling, which the above-mentioned kinematic region will reveal. There appears to be enough indications for such correlations in the data gathered so far.

The remaining part of this paper has been laid out as follows: In Sec. II we have introduced the assumptions of an extended (E) quark-parton model (QPM), which will allow us to study the new type of correlations in the QCD ground state, which also occur in various forms of condensed matter. In Sec. III we have considered the significance of the new correlations implied by the EQPM. In Sec. IV we have justified the use of the generalized Hartree-Fock approximation. We continue, in Sec. V, to consider the available data (cf. Sec. VA). We indicate, in Sec. V B, how the data was interpreted in order to learn about the gap evolution for values of the x variable which range within the new kinematic region considered in our paper. Three figures and one table supplement this section. Finally, in Sec. VI, we discuss the new predictions of the EQPM which go beyond QCD without correlations, not only in the sharp break of the structure function, but also in the expected behavior of the R parameter.

II. A SECOND TYPE OF CORRELATION

Quantum chromodynamics has succeeded in incorporating the QPM into a gauge theory predicting a *smooth* logarithmic scale breaking. This is well adjusted to the data for low momentum transfer ($Q \leq 100 \text{ GeV}^2$).¹ In view of the presence of uncertainties in QCD due to uncalculated power-law terms from higher-twist operators,² it seems worthwhile to consider further nonperturbative effects supplementing the basic ideas of the QPM, the nature of which lie just beyond QCD. We consider below the well known pairing phenomenon of quantum liquids, with a highly correlated ground state yielding an energy gap Δ in the spectrum, related to the coupling constant V [cf. Eq. (8) below] by means of an essential singularity, basically arising from a nonperturbative mechanism.³⁻¹¹

In order to understand how the energy gap will show up in scattering cross sections, we adopt the following assumptions:

(A) The nucleon may be viewed as a multiple-quark structure, accompanied by a sea of virtual quarkantiquark pairs. The internal quantum numbers of the nucleon are carried by the valence quarks, and the sea carries the vacuum quantum numbers. The quark q_k^{\dagger} and antiquark \bar{q}_k^{\dagger} of the sea are assumed to interact through some short-range force, so that the sea is unstable to the formation of pairs [cf. assumption (D) below]. With a fermion transformation we may introduce operators of creation and annihilation of quasiparticle excitations Q_k^{\dagger} , \bar{Q}_k^{\dagger} , respectively (cf. Sec. IV below).

(B) The masses of the quasiparticles are shown to be such that the use of the Hartree-Fock nonrelativistic methods becomes plausible (cf. Sec. IV below).

(C) We assume a probability distribution for the sea quarks which contains a divergent number of partons of very low momentum; this is precisely the reason that induces us to believe that many-body methods should be applied to hadronic problems.¹² In order to make a definite start we assume a probability distribution of the type¹³

$$dP_s(x) = dx(x^2 + \mu^2/P^2)^{-1/2}, \qquad (1)$$

where we denoted the parton fractional momentum by xand the quark mass by μ ; finally, \vec{P} denotes the nucleon total three-momentum. A more comprehensive probability distribution has been given by Ranft,¹⁴ but our conclusions do not change significantly; we only require a divergent number of partons of very low momenta. Equation (1) seems to be superseded by recent QCD calculations of hadron structure,¹⁵ which lead to a simple baglike picture of hadrons.^{16,17} Yet, we must emphasize that the two aspects of hadron structure which we are assuming in Eq. (1), namely, a set of a few valence quarks plus the sea, are backed by overwhelming experimental evidence.¹

(D) Our fourth assumption is that the hadron has a third important class of constituents, namely, a sea of virtual gluons, the quanta of the color-force field, which are exchanged between the quarks and between the gluons themselves.

(E) The role of the gluons becomes of secondary importance in our Hartree-Fock approach, since its effect is averaged by the interaction term $V_{kk'}$ [cf. Eq. (6) below]. This is in exact analogy with the effect of the phonon field in the theory of superconductivity, where the phonon-phonon interactions are neglected to first order.

(F) Our microscopic quantum-liquid model of hadron structure will undoubtedly require us to take into account the valence quarks, which in general may be considered as broken Cooper pairs. However, for the reasons mentioned in the Introduction, we restrict our attention to partons of very low momenta. Our final assumption is that the sea quarks dominate in this region. There is ample evidence supporting this assumption.¹ Since in Secs. V and VI we will look at lepton-hadron deep-inelastic scattering, which probes essentially the sea, we shall not develop in detail the theory for the valence quarks.

III. Q^2 EVOLUTION OF THE STRUCTURE FACTORS

The standard approach to the Q^2 evolution of the structure functions splits the quark-antiquark pairs arising in the sea; these components are called flavor-singlet contributions. It should be emphasized that no correlations in this many-fermion system have been envisaged so far. Following a general result by Cooper,¹⁸ given an attractive force among such fermions, condensation occurs through the well known mechanism of pair correlations (simply referred to as Cooper pairs).

Besides the flavor-singlet contributions, there are flavor-nonsinglet distributions, which arise from unpaired-quark contributions. Such unpaired quarks are usually taken as valence quarks, but given the correlations we study in this paper, these flavor-nonsinglet contributions could arise from "broken pairs" of the proton system, viewed as a quantum liquid. The concept of a broken pair is well understood in superconductors, as well as in superfluid ³He.¹⁹

As Q^2 increases from low values, the following situation occurs: At initial values of a few GeV the distance scale being probed is long, and quantum processes, such as emission and reabsorption of quark-antiquark virtual pairs, are obscured. For larger values of Q^2 , two options are presented.

(i) If there is no strong correlation among sea quarks, the distance scale being probed is much smaller, and the quark may be resolved into a quark and a gluon. Altarelli and Parisi have described this situation.20 The approximate solution to the QCD equations of Buras and Gaemers^{21,22} in fact produced smooth functions for $xF^{NS}(x,Q^2)$, $xF^{S}(x,Q^2)$, and $xG(x,Q^2)$, the nonsinglet, singlet-structure factors, and gluon distribution function, respectively, times the x variable. This solution produces a fit to the CERN-Dortmund-Heidelberg-Saclay (CDHS) early data²³⁻²⁵ for a kinematic range 5 < v < 200 GeV, 0 < x < 0.7, $1 < Q^2 < 160$ GeV². This data clearly shows the QCD-like behavior predicted. The more recent CDHS data with an extended kinematic range reaching 5 < v < 280 GeV, 0 < x < 0.7, $1 < Q^2 < 200$ GeV² also supports the QCD-like behavior.¹

(ii) If there is a strong Cooper-pair correlation among the sea quarks, as we increase Q^2 , the smaller distance scale being resolved would produce, for small x (≤ 0.2), a smooth QCD type of growth in the structure-function evolution $F_2(x,Q^2)$. However, no smooth fit to the data of the usual type $(1-x)^{\gamma}$, $x^{\alpha}(1-x)^{\beta}$, would be able to reproduce what happens as $Q \rightarrow \Delta$, since for such a value of Q, there would arise a discontinuity in the structure function. The reason is that the virtual photon has been incapable, up to that value, of breaking the pair and, for $Q = \Delta$, such a possibility begins to occur, yielding an increased collision probability; hence, an increment in F_2 would follow. While we are suggesting to supply QCD with the missing correlations of the sea quarks, in this option (ii), our predictions will differ clearly from QCD in the as-yet-untested new kinematical region. In Secs. V and VI we will show that in this region there are some differences between QCD without correlations and QCD with correlations.

IV. THE VALATIN-BOGOLIUBOV TRANSFORMATION

Let the creation operator of a d quark of the QPM be denoted by d_{κ}^{\dagger} , and let the corresponding antiparticle creation operator be denoted by $\bar{d}_{\kappa}^{\dagger}$; here, the Greek letter κ denotes spin σ and momentum \vec{k} . From our experience with the familiar fermion pairing in the phenomenon of superconductivity, we learn that the fermion representation most naturally linked with experiment is not necessarily the most convenient fermion representation in second quantization. Therefore, Valatin²⁶ and Bogoliubov²⁷ prefer new quasiparticle excitations (more energetic than the free particles) to the free conduction electrons. Likewise, we prefer to change to a more convenient fermion representation of "quasiquarks," whose creation operator is denoted by Q_{κ}^{\dagger} . In the present case, we prefer to consider the quasiquark D_{κ}^{\dagger} with its antiparticle $\overline{D}_{\kappa}^{\dagger}$. The quarks and quasiquarks are related by

$$D_{\kappa} = f_{\kappa} d_{\kappa} - g_{\kappa} d_{-\kappa}^{\dagger} , \qquad (2)$$

$$D_{\kappa}^{\dagger} = f_{\kappa} d_{\kappa}^{\dagger} - g_{\kappa} d_{-\kappa} , \qquad (3)$$

$$D_{-\kappa} = f_{\kappa} d_{-\kappa} + g_{\kappa} d_{\kappa}^{\dagger} , \qquad (4)$$

$$D_{-\kappa}^{\dagger} = f_{\kappa} d_{-\kappa}^{\dagger} + g_{\kappa} d_{\kappa} , \qquad (5)$$

where f_{κ} and g_{κ} are the usual free parameters to be determined by minimizing the Hamiltonian. There is a corresponding Valatin-Bogoliubov (VB) transformation linking antiquarks and antiquasiquarks.

In terms of the D operators, we obtain a contribution towards the total Hamiltonian of the type

$$H^{(D)} = \sum_{\kappa} \epsilon_{(D)\kappa} (D^{\dagger}_{\kappa} D_{\kappa} + \overline{D}^{\dagger}_{\kappa} \overline{D}_{\kappa}) + \sum_{\kappa'} V_{kk'} (D^{\dagger}_{\kappa} \overline{D}^{\dagger}_{-\kappa} \overline{D}_{-\kappa'} D_{\kappa'}) , \qquad (6)$$

where $\epsilon_{(D)\kappa}$ denotes the independent fermion energy.

We may complete the total Hamiltonian with contributions $H^{(i)}$, where *i* runs through all the flavors. It is to be noted that, just as phonons do not appear explicitly in mediating pairing in superconductivity, here gluons are understood to contribute towards the average Hartree-Fock potential $V_{\kappa\kappa'}$. Consequently, the total Hamiltonian is

$$H_{\rm tot} = \sum_{i} H^{(i)} , \qquad (7)$$

where the index *i* runs through U, \overline{U} , D, \overline{D} , S, \overline{S} , etc. We take the single-particle energies and interaction $V_{\kappa\kappa'}$ to be equal in the above contributions to the total Hamiltonian. We may then minimize the expectation value of H with respect to the Bardeen-Cooper-Schrieffer (BCS) ground-state wave function²⁸ (which is the vacuum of $Q^{\dagger}\overline{Q}^{\dagger}$). This leads, following the same steps as BCS, to a solution for the gap Δ , with the simple average interaction

$$V_{kk'} = \begin{cases} -V, \text{ for } |\vec{\mathbf{k}}|, |\vec{\mathbf{k}}'| \le \delta, \\ 0, \text{ otherwise,} \end{cases}$$
(8)

where δ is a small parameter. We have further assumed, for simplicity, that the averages over $\langle Q_{\kappa}^{\dagger} \overline{Q}_{-\kappa}^{\dagger} \overline{Q}_{-\kappa'} Q_{\kappa'} \rangle$ give equal contributions to the total energy of the ground state. Similarly, we find that the *D* quasiquark has a "broken-pair" energy given by¹⁹

$$E_{(D)\kappa} = (\epsilon_{(D)\kappa}^{2} + \Delta^{2})^{1/2} .$$
(9)

For a sufficiently large energy gap

$$E_{(D)\kappa} \gg \epsilon_{(D)\kappa} . \tag{10}$$

In macroscopic superconductivity this situation does not arise, since the phonons are incapable of mediating a pairing force significant enough to set a sizable difference between the conduction electrons and the more energetic quasiparticle (broken pair).

To summarize the main points of this section we may say that given the inequality (10), the more appropriate choice for the hadronic constituents is the fermion representation, in which their energy is higher, namely, U,D,S,\ldots . Therefore, in spite of the fact that we wrote the Hamiltonian formalism in terms of this representation, it is more convenient to choose the free-fermion representation u,d,s,\ldots for writing the Hartree-Fock (nonrelativistic equations). In view of the fact that the effective mass in the Valatin-Bogoliubov representation is a function of the energy gap Δ (which will be found below to have a large value), the nonrelativistic Hartree-Fock approach is justified, unlike the relativistic QPM,¹³ since U,D,S,\ldots have essentially small (and hence relativistic) masses.

V. CALCULATION OF THE GAP VARIATION

A. Discussion of the data

The simple QPM picture, with the addition of a correlated BCS-type sea, allows us to refer to the eN, μN , and vN data,²⁹⁻³¹ in which depletion of high-momentum partons is observed, as well as an increase in the lowmomentum parton distribution, as the momentum transfer squared (Q^2) increases. In particular, the European Muon Collaboration (EMC) (Ref. 29) measured F_2 in the range $3 < Q^2 < 150 \text{ GeV}^2$, and 0.015 < x < 0.65 for carbon, as well as for iron targets. Other data by different groups does not change significantly in the kinematic region considered.³² More data has also been obtained recently and supports the earlier work of the EMC.^{33,34} The lepton projectile emits a current, whose energy in the laboratory is v, and whose mass squared is $-Q^2$. From the assumption (c) of Sec. II, we find that the number of sea quarks increases as $(x^2)^{-1/2}$, viewing the collision in the rest frame of the projectile.

B. Evolution of the gap

We see that for v sufficiently high, the parton momentum decreases, so that \vec{k} and \vec{k}' lie outside the range where we have assumed $V_{kk'}$ to be nonvanishing [cf. Eq. (8)]. This is analogous to the partially broken Meissner effect, where in the Abrikosov-vortex regime, pairs and broken pairs coexist. The variation of the energy gap for the temperature range between absolute zero and the critical temperature is a smoothly decreasing function. In the BCS weak-coupling theory such a function may be represented by Fig. 1(a).²⁸ Cooper pairing becomes less probable for partons of very low momentum, since from Eq. (8) the parton partners of the correlated pair will no longer have a large probability of having their momenta in the very narrow band around the Fermi momentum. This situation may be described in analogy with superconductivity [cf. Fig. 1(a)]: a "hot proton" is, in our picture, a



FIG. 1. (a) Expected functional behavior between the energy gap Δ and temperature T. (b) Expected $\Delta(x)$ curve for x values for the allowed range in the sea.

hydrogen nucleus is being probed by a lepton projectile, sufficiently energetic to enter the kinematic region of very-low-momentum partons. The projectile will be able to resolve a correlated sea, with an underlying small energy gap. Otherwise, we may speak of a "cold proton." In this case the probed sea will show a larger energy gap, since the Cooper partners will have a larger probability of having their energy in the required narrow band around the Fermi momentum. The general trend is expected to be a BCS-type curve shown in Figure 1(b): Δ is a smooth decreasing function with decreasing x; the precise analytic shape will be inferred phenomenologically below [cf. part (b) of Sec. VI]. Beyond $x \sim 0.25$, the probability of finding a Cooper pair with momentum fraction x inside the proton becomes negligible, since the valence quarks take over.

C. Novel feature of the EQPM

From the above considerations it follows that increments of Q^2 will eventually reach values comparable with Δ^2 , say, Q^2 will reach Q_{Δ}^2 . Before reaching the Q_{Δ}^2 value, the current will be unable to transfer sufficient energy in order to interact with the sea condensate and, thus, we would expect a flat $F_2(Q^2)$ function, since there would be no interaction (this is the case of perfect scaling). To summarize the above arguments we may say that the reason why there is scaling for $Q^2 < Q_{\Delta}^2$ or alternatively the reason why there is a pointlike structure of the sea condensate is because of the strong correlations (the BCS pairing).

VI. DISCUSSION

It should be mentioned that a structured vacuum wih particle-antiparticle Cooper-pair correlations was con-

structed previously.³⁵ However, the gap evolution was not identified, as we hope to have shown in Sec. V. We now discuss various aspects of the sharp break in the F_2 function.

(a) In Sec. V we have used well established data in order to reconstruct $\Delta(x)$, so as to infer at which values of Q^2 the sharp break ought to occur. In reconstructing $\Delta(x)$ we are guided by the general behavior of $\Delta(T)$ [or $\Delta(x)$ in the present context]; but we are uncertain, *a priori*, of the precise analytic shape. All we know is that Δ is expected to be a smooth increasing function of the x variable. Hence, Figs. 1(b) and 3 are not expected to coincide, except in the general trend already mentioned in Sec. V B.

(b) The probability for finding more particles increases beyond Q^2 , so at the critical value $Q_{\Delta} = \Delta$, a sharp increment will follow for higher values of Q^2 . In Fig. 2, we show the behavior of $F_2(x, Q^2)$ for five values of the x variable at which the EMC experiments were carried out. In order to draw Fig. 2, we concentrated our attention in a domain where our model clearly displays a departure from the QCD-neglected correlations. Therefore, the curve (x = 0.015) cannot be searched for the sharp breaks, since in this curve the statistical errors will not allow us to dis-



FIG. 2. The structure function F_2 showing the sharp break. The EMC data on hydrogen has been compared with the QCD model based on the Altarelli-Parisi equations (Ref. 20) [the fit is shown by the dashed line following Norton (Ref. 29)]. The new fit (solid line) is suggested by an understanding of the role played by the conjectured energy gap, as explained in the text. The curves correspond (from top to bottom) to the following values of the Feynman x variable: 0.015, 0.040, 0.080, 0.125, and 0.175, respectively.

cern between the smooth logarithmic departure from scaling (expected from QCD) and the sharply broken curve, signaling the onset of the breaking of the Cooper pairs. Therefore, we searched the EMC data at the further available values of Q^2 , and for values of x for which F_2 is still sensitive to sea-quark contributions. We chose x = 0.040and x = 0.080. Yet, since we know that the "cold proton" (cf. Sec. VB) will have a significant energy gap, the approximate shape of which is shown in Fig. 1(b), the curve will be as in Fig. 3, since Δ is expected to be a smooth function. Then it will pass through the above two EMC points, as well as through the origin. [In the rest frame of the projectile, there will be an infinite number of sea quarks, from Eq. (1); each parton will then carry x = 0, thus making it impossible for the gap to have any finite value.]

(c) Within the kinematic region considered $(0 < Q^2 < 100)$ GeV^2) the QCD smooth fit is satisfactory. The sharp break we have suggested for the same fit is entirely qualitative, and would require further research in order to compute Q_{Δ}^2 and the slope of the sharp break of $F_2(x,Q^2)$ directly from the theory. Further experiments would indicate this effect by interpolating more values of Q^2 . Alternatively, larger values of Q^2 (outside the present kinematic region) should display a stronger deviation of the almost linear fit for x = 0.125 and x = 0.175, for example. We feel that the relevance of our quantum-liquid model is to emphasize the presence of the strong correlations, which are missing from standard theory, and explain the overall trend of the data for $F_2(x, Q^2)$ for small x (up to about x = 0.2) and small Q^2 (up to about $Q^2 = 100$ GeV²); but nontrivial departures are forseen for larger values of Q^2 .

(d) To complete the explanation, in Fig. 3 the EMC points were selected where the approximate perfect scaling gives an indication that the curve is beginning to rise. Owing to severe kinematic constraints, the most recent experiments have not reached values of Q^2 high enough to reveal the broken F_2 curve. Yet, we may already be observing such an effect: the points denoted as "prediction" in Fig. 3 for x < 0.200 are smooth interpolations, which do explain the data in terms of a "precautious" onset of Cooper-pair breaking, due to the inefficient energy gap of the "hot proton." Since from Fig. 1 we expect the Δ to



FIG. 3. The energy gap Δ for all allowed x values of sea quarks. The QCD values coincide with the x axis (absence of sea-quark pairing).

vary smoothly as a function of x, the points denoted as "prediction" for x > 0.080 are smooth extrapolations. The perfect scaling now appearing in the data (up to x = 0.175) will eventually show a sharp break, due to the efficient energy gap of the "cold proton." To be more specific, from the corresponding values of x and Q^2 shown in Table I, we highlight the prediction that a sharp break occurs at $F_2(x = 0.20, Q^2 = 31.4 \text{ GeV}^2)$; but even higher values of Q^2 would be required to display the departure convincingly.

VII. CONCLUSION

In the new kinematic region the R parameter will differ from the expectations of QCD without our correlations. For small enough values of x, the energy gap will tend to disappear (cf. Figs. 1 and 3). Hence, for such values of Q^2 in which the sharp rise has not yet ocurred (cf. Fig. 2), the current is unable to resolve the Cooper pair into two fermions, thus giving $R \neq 0$. However, if the same experiment is continued to yet smaller values of x, the R parameter would necessarily come down, since $\Delta \rightarrow 0$, as $x \rightarrow 0$ (cf. Fig. 3), under the assumption that the underlying spin-0 partons are a manifestation of the strongly correlated Cooper pairs. The data reported by Eisele¹ regarding the R parameter is not yet in the required new kinematic region.

Since new experiments will take us to Fermilab Tevatron energies, the ultimate clear manifestation of the energy gap, as well as the more general phenomenon of superfluidity of hadronic matter, will become evident in the not-too-distant future, when the unexplored kinematic region of small x and large Q^2 becomes available to experiments and is, therefore, understood better.

ACKNOWLEDGMENTS

Discussions on the subject of this paper with Professor John Strathdee and Professor E.C.G. Sudarshan are gratefully acknowledged. Financial support from the Instituto stituto Internacional de Estudios Avanzados, IDEA (Caracas) made it possible to start this work at the International Centre for Theoretical Physics, Trieste. We would like to thank Professor Abdus Salam for hospitality at ICTP during a visit.

TABLE I. Values of the fractional momentum $x = Q^2/2M_\rho v$, and the corresponding sharp-break parameter Q_{Δ} and the energy gap Δ .

x	Q_{Δ}^2 (GeV ²)	Δ (GeV)
0.015	0.49	0.70
0.040	3.00	1.70
0.080	9.00	3.00
0.125	17.47	4.18
0.175	26.62	5.16
0.200	31.36	5.60

- *Also at Instituto Internacional de Estudios Avanzados, Caracas, Venezuela.
- ¹F. Eisele, in Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982, edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. <u>43</u>, C3-337 (1982)].
- ²D. Politzer, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-659 (1982)].
- ³Y. Nambu and G. Jona-Lasinio, Phys. Rev. <u>122</u>, 345 (1961).
- ⁴G. Eilam and Y. Gell, Phys. Rev. D <u>8</u>, 3930 (1973).
- ⁵H. Kleinert, Phys. Lett. <u>59B</u>, 163 (1975).
- ⁶G. F. Chapline, Phys. Rev. <u>D11</u>, 156 (1975).
- ⁷S. Eliezer and R. M. Weiner, Phys. Rev. D <u>13</u>, 87 (1976).
- ⁸H. Terezawa, Y. Chikasinge, and K. Akama, Phys. Rev. D <u>15</u>, 486 (1977).
- ⁹J. Chela-Flores, Phys. Rev. D <u>18</u>, 2632 (1978).
- ¹⁰J. Chela-Flores and F. De Lisa, Nuovo Cimento <u>58A</u>, 67 (1980).
- ¹¹J. Chela-Flores and V. Varela, Phys. Rev. D <u>27</u>, 1248 (1983).
- ¹²J. Kogut and L. Susskind, Phys. Rep. <u>8C</u>, 75 (1973).
- ¹³J. Kuti and V. Weisskopf, Phys. Rev. D <u>4</u>, 3418 (1971).
- ¹⁴J. Ranft, in *Hadronic Matter of Extreme Energy Density*, proceedings of the workshop, Erice, Italy, 1978, edited by N. Cabibbo and L. Sertorio (Plenum, New York, 1980), p. 246.
- ¹⁵G. Alberi and G. Goggi, Phys. Rep. <u>74C</u>, 1 (1981).
- ¹⁶C. G. Callan, R. F. Dashen, and D. J. Gross, Phys. Rev. D <u>19</u>, 1826 (1979).
- ¹⁷G. E. Brown and M. Rho, Phys. Lett. <u>82B</u>, 177 (1979).
- ¹⁸L. N. Cooper, Phys. Rev. <u>104</u>, 1189 (1956).

- ¹⁹A. Leggett, Rev. Mod. Phys. <u>47</u>, 331 (1975).
- ²⁰G. Altarelli and G. Parisi, Nucl. Phys. <u>B126</u>, 298 (1977).
- ²¹A. J. Buras and K. J. F. Gaemers, Phys. Lett. <u>71B</u>, 106 (1977).
- ²²A. J. Buras and K. J. F. Gaemers, Nucl. Phys. <u>B132</u>, 249 (1978).
- ²³J. G. H. de Groot et al., Z. Phys. C 1, 143 (1979).
- ²⁴J. G. H. de Groot et al., Phys. Lett. <u>82B</u>, 292 (1979).
- ²⁵J. G. H. de Groot et al., Phys. Lett. <u>82B</u>, 456 (1979).
- ²⁶J. G. Valatin, Nuovo Cimento <u>7</u>, 843 (1958).
- ²⁷N. N. Bogoliubov, J. Phys. (Moscow) <u>11</u>, 23 (1947).
- ²⁸J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. <u>108</u>, 1175 (1957).
- ²⁹P. R. Norton, in *High Energy Physics*—1980, proceedings of the XXth International Conference Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 777.
- ³⁰A. Bodek et al., Phys. Rev. D <u>20</u>, 1417 (1979).
- ³¹H. Weerts, in High Energy Physics-1980 (Ref. 29), p. 777.
- ³²C. Y. Prescott, Report No. SLAC-PUB-2625, 1980 (unpublished).
- ³³D. Haidt, in Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982 (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-5 (1982)].
- ³⁴The European Muon Collaboration, J. J. Aubert *et al.*, Phys. Lett. <u>123B</u>, 123 (1983).
- ³⁵K. P. Sinha, C. Sivaram, and E. C. G. Sudarshan, Found. Phys. <u>6</u>, 65 (1976).