Thermodynamics of quark jets. II. Baryon production

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A Monte Carlo analysis based on a recent thermodynamic model for jets is used to model quark fragmentation into hadrons. Quantitative estimates of the baryon-to-meson ratio N_B/N_M and kaon and hyperon production are made.

I. INTRODUCTION

In the preceding paper,[1] a method was developed for using the thermodynamic properties of a one-dimensional quark gas[2-4] as a model for jets. The quarks were in a grand canonical ensemble and the multiplicity of hadrons in jet events was calculated. In this paper we carry out Monte Carlo calculations to trace the cascade evolution of the jets including baryon production and give a quantitative estimate of the baryon-to-meson ratio.

The basic picture of the physical process is the following: the annihilation of the e^+e^- pair creates a $q\bar{q}$ pair held by a string; the successive breaking of the string generates a cascade of hadrons. The string breaking must always be such that each resulting multiquark state is a singlet. The quarks are treated as classical particles [fundamental representations of SU(3)] but their interactions via SU(3) gauge fields are treated exactly. The interaction Hamiltonian was postulated in Eqs. (1), (2), and (3) and is given by

$$H=-\frac{\alpha}{4}\sum_{i<j}\lambda_i\lambda_j\,|x_i-x_j|\ ;$$

λ_i's are matrices belonging to the adjoint representation of SU(3), α is the string tension, and x_i's are the positions of the quarks.

II. GRAND CANONICAL PRESSURE ENSEMBLE

Since we wish to deal with a system in which free creation and annihilation of quarks takes place, the natural quantity to consider is the thermal grand canonical Green's function. However, the ordinary volume-fixed grand canonical partition function proves unsuitable for Monte Carlo modeling of a many-particle system at temperatures and densities as "hot" as jets.[5] This partition function is given by

$$Z(\beta,L,\mu)=\sum_{n_q=0}^{\infty}\sum_{n_{\bar{q}}=0}^{\infty}e^{-N\mu L}Z^N Q_N(L,T)W_N\ , \qquad (2.1)$$

where $n_q,n_{\bar{q}}$ are the number of quarks and antiquarks, $N=n_q+n_{\bar{q}}$, but N as a subscript represents $(n_q,n_{\bar{q}})$, and L=volume of the system. $z=2mK_1(m/T)$ is the single-particle kinetic partition function, and depends on the flavor of the particle through the mass. We will denote by $z_f(T)$ the value of this function for a specific flavor f when necessary. W_N is the number of ways to form an SU(3) singlet. When $n_q\geq n_{\bar{q}}$,

$$W_N=\sum_{a=0}^{n_{\bar{q}}}\sum_{b=0}^{n_{\bar{q}}-a}\begin{bmatrix}n_{\bar{q}}\\a,b\end{bmatrix}\begin{bmatrix}n_q\\a,b\end{bmatrix}\left[1-\frac{(n_q+1-b)(n_q-a-b)}{(a+1)(b+1)}+\frac{(n_q-a-b)(n_q+1-2b)}{(a+1)(a+2)(b+1)}\right],$$
$$Q_N(v,T)=\int\prod_i dx_i e^{-\beta C_i|x_{i+1}-x_i|}\ , \qquad (2.2)$$

where the x_i integrations are performed in a fixed volume L for all orderings of particles. Since the Casimir C_i of the color flux between the ith and $i+1$th particle is correlated with C_{i-1} and C_{i+1}, Q_N involves the calculation of 2^N terms

$$Q_N=\sum_{\{C_i\}}\int_0^{x_1}\int_{x_1}^{x_2}\cdots\int_{x_{n-1}}^{L}e^{-\beta\alpha C_i(x_2-x_1)}\cdots e^{-\beta\alpha C_n(L-x_{n-1})}\prod_i dx_i\ . \qquad (2.3)$$

Thus to simplify calculations, we use the pressure partition function, the pressure p being the conjugate variable to the volume L:

$$\pi_N(\beta,p,\mu)=\int_0^\infty dL\,\exp\left\{-L\left[p-\frac{1}{L}\ln Q_N(\beta,L,\mu)\right]\right\}. \qquad (2.4)$$

$\pi_N(\beta,p,\mu)$ can be calculated explicitly whereas direct analytic calculation of Q_N is impossible. Thus the grand pressure partition function is

$$\sum_N e^{-\beta\mu N}\pi_N(\beta,p,\mu)=Z_p\ . \qquad (2.5)$$

This is physically reasonable, as, for systems as "hot" as jets, forces keeping a system together control pressure and densities rather than the volume, thus P and T are suitable external variables.

Pressure-fixed ensembles also show Koba-Nielsen-Olesen (KNO) scaling.[1] This implies that the fluctuations

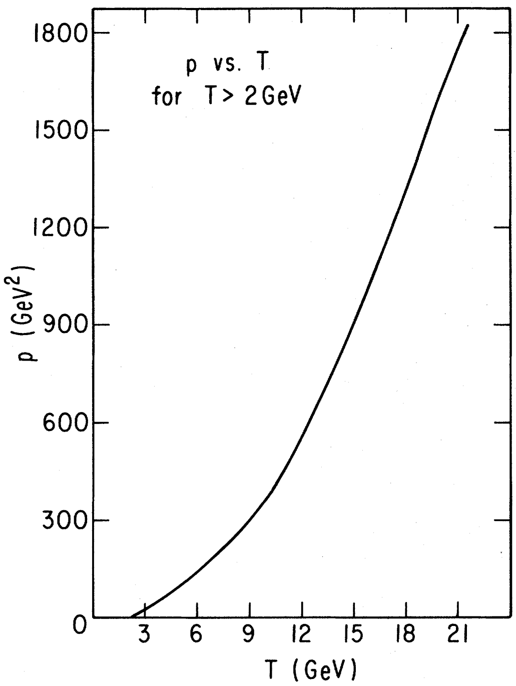

FIG. 1. Equation of state for the pressure-fixed ensemble (P vs T) for high temperatures ($T \gg \sqrt{\alpha}$).

in the number of particles are very large. This behavior is reflected in our ensemble which tends to show "critical" behavior for small fluctuations in pressure. This means that it is extremely sensitive to pressure changes and has a tendency to "shrink down" or blow up; thus the pressure has to be fine tuned to ensure equilibrium.

III. THE MONTE CARLO ANALYSIS

We start with an initial configuration of n_q quarks and $n_{\bar{q}}$ antiquarks at a high temperature T, and a pressure p. The pressure is determined numerically to ensure equilibrium. The equation of state thus obtained is plotted in Figs. 1 and 2. The analytic form for high and low T is

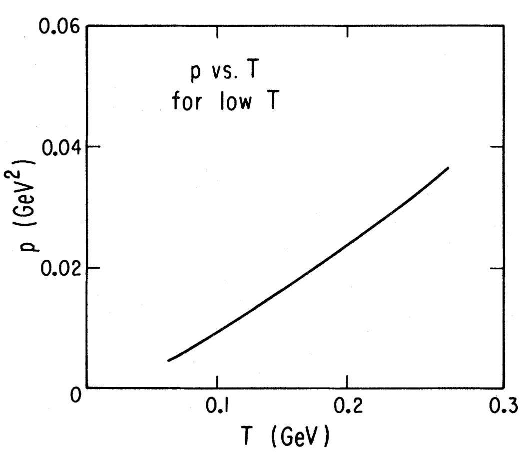

FIG. 2. Equation of state for the pressure-fixed ensemble (P vs T) for low temperatures ($T \lesssim \sqrt{\alpha}$).

determined to be

$$\begin{aligned} &p \sim 4T^2 \text{ for } T \gg \sqrt{\alpha}\ , \\ &p \sim \sqrt{\alpha}T \text{ for } T \lesssim \sqrt{\alpha}\ , \\ &\alpha = \text{string tension}\ . \end{aligned} \tag{3.1}$$

The initial color configuration of the system is represented by a sequence of irreducible representations $\mathrm{IR}(i) = (p_i, q_i)$ of SU(3). For $0 \le i \le N = n_q + n_{\bar{q}}$, IR($i$) is the representation formed by the first i particles of the system and is the color flux between particles i and $i+1$. The IR's satisfy two conditions:

$$\begin{aligned} &(p_0, q_0) = (p_N, q_N) = (0,0)\ , \\ &(p_{i+1}, q_{i+1}) = (p_i \pm 1, q_i),\ (p_i, q_i \mp 1)\ ,\text{or } (p_i \mp 1, q_i \pm 1)\ . \end{aligned} \tag{3.2}$$

If the upper signs hold in Eq. (3.2) then particle $i+1$ is a quark and IR($i+1$) is in $3 \otimes \mathrm{IR}(i)$, while, if the lower signs hold, then particle $i+1$ is an antiquark and IR($i+1$) is in $\bar{3} \otimes \mathrm{IR}(i)$. As an example, a system consisting solely of mesons would be represented by

$$IR(i) = \begin{cases} (0,0) & \text{for } i \text{ even,} \\ (1,0) \text{ or } (0,1) & \text{for } i \text{ odd}\ . \end{cases}$$

We begin each simulation with the system in the many-meson state and with flavors assigned to the quarks at random, in accordance with their masses and the initial temperature. We then iterate these three steps:

(1) Thermalize the system by the sweeping process described below.

(2) Make "observations" of the internal state of the system, with several sweeps between observations.

(3) Reduce the temperature and return to step one.

The iteration continues until one of two conditions is met—either the pressure drops to zero, or there have been no creations or annihilations in the system for five consecutive iterations of the steps (1), (2), and (3). In practice, these two conditions are met nearly simultaneously. We call a system "frozen" when it reaches this state.

The sweep process consists of applying one or more of three possible changes to each link of the system in turn. The first change applied is a color flux change. Each link has a color $\mathrm{IR}(i) = (p_i, q_i)$ denoting the flux between particles i and $i+1$. Depending on the values of IR($i-1$) and IR($i+1$), there may be up to six possible values of IR$'(i)$ which are consistent with the neighbors. [That is, which are contained in both $(3 \oplus \bar{3}) \otimes \mathrm{IR}(i-1)$ and $(3 \oplus \bar{3}) \otimes \mathrm{IR}(i+1)$.] These possible values of IR$'(i)$ are assigned probabilities proportional to $(\alpha C_i + p)^{-1}$ where C_i is the Casimir operator of IR$'(i)$, and one is then chosen at random. Note that for high temperatures, the pressure p is large and all possible IR$'(i)$ values become equally likely. Sometimes the chosen IR$'(i)$ implies that a quark and an antiquark have moved past each other. If this is the case, the flavor information is updated to reflect this. If the particles at either end of link i are both quarks or both antiquarks, then their flavors are swapped with probability $\frac{1}{2}$.

The second change considered is the insertion of a q-$\bar{q}$ pair at the current link of the system. This is only allowed at links with nonzero flux. If a pair is inserted it entails the creation of two new links with colors $\mathrm{IR}'(i+1)$ and $\mathrm{IR}'(i+2)$. $\mathrm{IR}'(i+1)$ must be contained in $\mathrm{IR}(i)\otimes(3\oplus\bar{3})$ while $\mathrm{IR}'(i+2)=\mathrm{IR}(i)$. The inserted pair may also be of any flavor. The proposed insertion is accepted or rejected according to the probability ratio

$$\frac{p(\text{insert})}{p(\text{not insert})}=\frac{T}{(\alpha C'_{i+1}+p)}\frac{T}{(\alpha C'_{i+2}+p)}\times\left[\sum_f z_f{}^2\right]\frac{W(n_q+1,n_{\bar{q}}+1)}{W(n_q,n_{\bar{q}})}\,. \tag{3.3}$$

If accepted, a flavor is then assigned according to the weights z_f.

The last change considered at each link is the deletion of a q-$\bar{q}$ pair. To be a candidate for deletion, a link i must have the same flavor of particles at each end and must satisfy $\mathrm{IR}(i)\neq 0$, $\mathrm{IR}(i-1)=\mathrm{IR}(i+1)\neq 0$. The latter guarantees that the two particles consist of one quark and one antiquark. The probability of deletion is essentially the reciprocal of Eq. (3.3). If deletion is chosen, the links i and $i+1$ are removed from the system. We delete with the probability

$$\frac{p(\text{delete})}{p(\text{not delete})}=\frac{(\alpha C_i+p)}{T}\frac{(\alpha C_{i+1}+p)}{T}z_{f_i}{}^2\frac{W(n_q-1,n_{\bar{q}}-1)}{W(n_q,n_{\bar{q}})}\,. \tag{3.4}$$

Repeating the process at each link constitutes one sweep of the system. We repeatedly sweep the system until equilibrium is established and categorize clusters of quarks that form singlets. If, for a link i, $C(i)=C(i+3)=0$, the cluster is a baryon; if $C(i+2)=C(i)=0$, the cluster is a meson.

At higher temperature, exotic states of the form $qq\bar{q}\bar{q}$, etc., also exist. We then lower the temperature of the system and repeat the process until the system freezes. When the system is frozen we find that all the singlet clusters are either baryons or mesons; all the exotic states have decayed to baryons or mesons.

A remark is in order here. The condition for insertion is more often satisfied than the condition for deletion, so the fugacity $\xi=1$. The system behaves as if it has an effective "fugacity" for pair creation.

The masses we have used in the calculation are[6] $m_u=m_d=0.300$ GeV, $m_s=0.500$ GeV, $m_c=1.6180$ GeV, $m_b=4.84$ GeV, giving single-particle partition functions z_u, z_d, z_s, z_c, and z_b.

IV. RESULTS AND COMMENTS

The output of a typical run is tabulated in Table I. We see that at high temperatures most of the quarks are free. As we lower the temperature, two-, three-, four-, five-, and six-quark states form. Further lowering of temperature causes the exotic states to decay to two- and three-quark states until at the "freezing" point only mesons and baryons are left.

Examining the system for $u\bar{u}$, $u\bar{d}$, and $d\bar{u}$ states gives the number of two-quark states that are pions. Counting $s\bar{u}$, $s\bar{d}$, $\bar{s}d$ and $\bar{s}u$ states gives the number of kaons. Pro-

TABLE I. Clustering in the stage of adiabatic expansion.

Temperature (GeV)	10	8	4	2	1	0.8	0.5	10^{-2}	10^{-3}	10^{-4}
1. Percentage of quarks forming mesons	0.56	2.88	3.23	4.81	4.28	14.65	39.47	82.53	84.51	84.00
2. Percentage of quarks forming baryons	0.00	0.00	1.21	2.81	2.81	5.37	14.09	14.05	15.79	16.00
3. Percentage of quarks forming *QQQQ* stages	0.74	0.00	1.61	7.69	5.88	8.46	17.61	1.52	0	0
4. Percentage of quarks forming higher quark stages (5 and 6 quarks)	0.46	0.00	2.42	0.00	13.24	13.77	14.73	1.90	0	0
5. Percentage of free quarks (7 or more quarks)	98.24	97.92	91.53	86.00	73.80	57.75	14.09	0.00	0	0
6. Percentage of mesons which are pions										75
7. Percentage of mesons which are kaons										25

TABLE II. c.m. energy dependence of N_B/N_M, N_K/N_M, and N_Λ/N_M.

c.m. energy $\sqrt{s}$ (GeV)	100	42.5	36	25	16
$\frac{N_B}{N_M}$	0.204±0.1	0.1831±0.13	0.139±0.017	0.099±0.025	0.074±0.01
$\frac{N_K}{N_M}$	0.406±0.1	0.39 ±0.17	0.309±0.07	0.238±0.02	0.193±0.008
$\frac{N_\Lambda}{N_M}$	0.09 ±0.06	0.07 ±0.02	0.051±0.015	0.026±0.01	0.017±0.005

tons and neutrons are not distinguished in the results of this model and are found by tabulating the three-quark states containing the u and d quarks and no s or c quarks. The number of such states is denoted by N_B.

Table II and Fig. 3 show the behavior of the ratios N_B/N_M, N_K/N_M, and N_Λ/N_M, of nucleons, kaons, and hyperons to total mesons, respectively, as a function of energy. Experimental data[7] for hadronic-species multiplicity is usually presented in terms of total multiplicity of various species. Since our model uses a one-dimensional approximation, we make predictions regarding the ratios of the particle species rather than their total multiplicity. Moreover, if the branching picture presented in paper I is correct, each transverse branch behaves as a one-dimensional quark system, in which hadronization takes place independent of the other branches. Hence, the ratios N_K/N_M, N_B/N_M, and N_Λ/N_M should remain independent of transverse-momentum effects, whereas the total multiplicity is dependent on the number of branches, and thus on transverse-momentum effects. In Fig. 4, the ratio of the charged-particle species to charged mesons calculated in the Monte Carlo simulation is compared with experiment. It should be noted that our model does not distinguish between neutral and charged particles. Since most experimental data shows charged-hadron multiplicity, in comparing our predictions with experiment we assume that charged and neutral particles appear with equal probability. The number of charged-particle states is then given by weighting the total number of particle states (charged + neutral) by the charged-to-neutral-particle ratio for that species.

There are large fluctuations in these volumes reflecting the statistical (KNO) behavior of the ensemble. It should be noted that we have used a thermodynamics ensemble to model particle production in jets. In jets however, the total number of particles is small ($N \sim 10$—20). Nevertheless, we make predictions only for the ratios of the particle species rather than the total number of particles of each species produced. Hence, the thermodynamic (N large) approximation gives good results.

In conclusion, we find that the Monte Carlo simulation of a one-dimensional quark gas in a pressure-fixed ensemble gives a good qualitative illustration of the clustering process in e^+e^- collisions. The quantitative results show good agreement with experimental data. The main feature of our program is the incorporation of baryon production in a natural way, and the fact that we can trace the step-by-step evolution of the particles at each stage of the clustering process. We have also demonstrated a practical application for pressure-fixed thermodynamic ensembles, which are particularly suitable for describing the thermodynamics of jets. Further investigations on the origin of KNO scaling in the thermodynamic context are in progress.

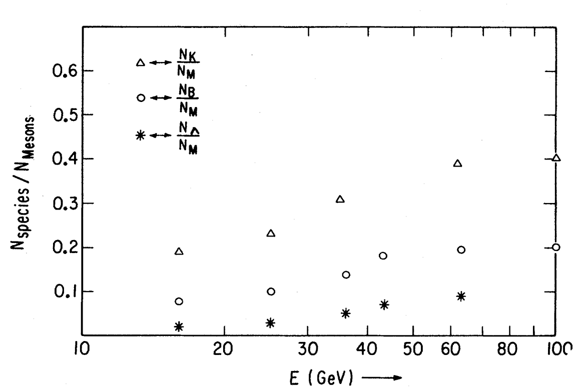

FIG. 3. Ratio of the number of particle species to total number of mesons as a function of center-of-mass energy. N_B/N_M is the ratio of nucleons to mesons, N_K/N_M is the ratio of kaons to mesons, N_Λ/N_M is the ratio of lambda hyperons to mesons.

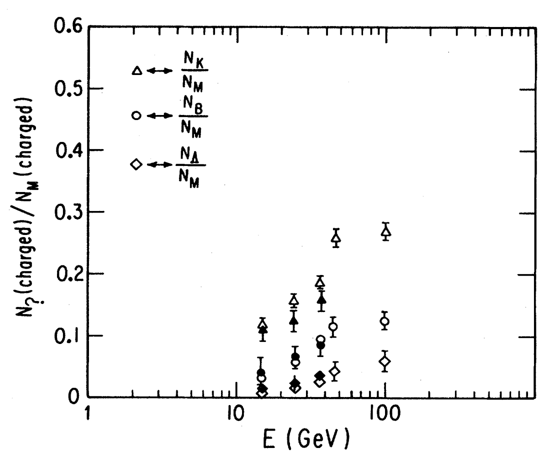

FIG. 4. Comparison of the Monte Carlo results for the charged-particle species to charged mesons with experiment. Filled symbols are experimental points; open symbols are predictions.

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[1]B. Bambah, preceding paper, Phys. Rev. D 29, 1323 (1984).

[2]Y. Nambu and B. Bambah, Phys. Rev. D 26, 2871 (1982).

[3]Y. Nambu, B. Bambah, and M. Gross, Phys. Rev. D 26, 2875 (1982).

[4]M. Gross, Phys. Rev. D 27, 432 (1982).

[5]R. Hagedorn, Z. Phys. C 17, 265 (1983).

[6]These are constitutent quark masses obtained from Particle Data Group, Phys. Lett. 111B, 1 (1982).

[7]TASSO Collaboration, DESY Report No. 82-073 (unpublished).