Tensor mesons in the unitarized quark model. Is there a glueball component in the f and f' mesons?

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We analyze the tensor-meson $q\bar{q}$ $1P$ multiplet within the unitarized quark model applied previously to the other $1P$ multiplets. The experimental mass spectrum and the total widths are fitted with essentially only four parameters almost independently of the hadronic form factor (cutoff). In order to fit the many different branching ratios related by flavor symmetry a larger cutoff in the form factor (a smaller bare-hadron radius) is needed than used in our previous analyses. The $f\to\pi\pi$, $K\bar{K}$ and $f'\to\pi\pi$, $K\bar{K}$ widths indicate that there may be a glueball mixing in the f and f' contributing as much as 15% to $f\to\pi\pi$.

I. INTRODUCTION

In a series of earlier papers,[1] we studied the 0^{++}, 1^{++}, 1^{+-}, 1^{--}, and (apart from the pion) 0^{-+} low-mass $q\bar{q}$ multiplets within a coupled-channel framework referred to as the unitarized quark model (UQM). This model can quite successfully, with few parameters, account for several quite different phenomena, such as signs and magnitudes of Okubo-Zweig-Iizuka—rule—violating angles, Q_A-Q_B mixing, and even the quite unconventional experimental properties of the 0^{++} mesons. In a related paper[2] with application to $c\bar{c}$ and $b\bar{b}$ spectroscopy our model was generalized by combining it with the quark-pair-creation (QPC) model[3] and with usual potential models.

One crucial input in the UQM is that it takes into account a very large number of flavor- and spin-related two-body continuum channels ($\pi\pi$,$K\bar{K}$,$\pi\eta$,$\pi\rho$,$K^*\bar{K}^*$, etc.). For each continuum channel we introduce couplings and vertex functions in accord with conventional phenomenology. Thereby the resonances obtain finite widths to open channels.

More importantly, however, we also use the expression for the hadronic widths to define through unitarity and analyticity a coupled-channel formalism. Thereby one also obtains, without new parameters, a contribution to the real part of the mass matrix, which comes from the hadronic loop diagrams (i.e., self-energies from, e.g., A_2-$K\bar{K}$-A_2 and f-$K\bar{K}$-f'). This gives mass shifts and mixing angles which depend sensitively on the positions of the nearest thresholds. Although one starts with a simple ideally mixed "bare" mass spectrum, after unitarization the physical mass spectrum can be quite distorted and intermixed, as was found in particular to be the case for the 0^{++} mesons.

This paper completes our previous analyses of the "old" $q\bar{q}$ $1P$ states. For details of the model we refer to our previous work and we emphasize here only the novel ingredients relevant for this application.

II. THE ABSORPTIVE PART OF THE MASS MATRIX

The crucial input in "unitarizing" a mass spectrum is to assume a good theoretical expression for the coupling to the continuum states. This requires, on the one hand, internal-symmetry relations between the effective coupling constants and, on the other hand, a functional form for the hadronic vertex functions. If mixing can be neglected, one can then easily obtain finite hadronic widths of the resonances. This introduces an imaginary part $(-i\Gamma/2)$ to the meson mass.

More generally, with several resonances which can mix one writes for the imaginary part of the squared mass matrix

$$\mathrm{Im}[M_{ij}^{A}(s)]^2=-\sum_{\substack{\text{thresholds}\\ B,b;C,c}}\gamma_{ibc}^{ABC}\gamma_{jbc}^{ABC}\frac{(k_b^{\text{c.m.}})^{2l+1}}{\sqrt{s}}\times F_{ibc}^{ABC}(s)F_{jbc}^{ABC}(s)\theta(s-s_{\text{th}})\,, \quad (1)$$

where γ^{ABC} is an effective coupling constant. The superscripts denote the multiplet, the subscripts the multiplet member, and $F_{ibc}^{ABC}(s)$ is a hadronic vertex function.

Within the QPC model E can be computed as an overlap of three $q\bar{q}$ wave functions. In Ref. 1 we made the simplifying assumption that F is independent of internal symmetry and is of the form

$$F_{ibc}^{ABC}(s)=\exp[-(k_b/k_{\text{cutoff}})^2/2]\,, \quad (2)$$

where k_{cutoff} was found to be >0.6 GeV/c and for the fits presented $k_{\text{cutoff}}=0.7$ GeV/c. Within the QPC one obtains Eq. (2) assuming a harmonic potential and equal quark masses. The cutoff corresponds then to a hadronic radius of $\sqrt{6}/k_{\text{cutoff}}=0.7$ fm. It is quite obvious that Eq. (2) is a great oversimplification of the actual situation. The cutoff can be argued to depend on flavor, such that

heavier $q\bar{q}$, being smaller, have larger cutoff. Furthermore, the analytic form of F is certainly more complicated than in Eq. (2). It depends on details of left-hand singularities, or alternatively on the form of the $q\bar{q}$ potential and the quark masses assumed. In Ref. 2, for heavy $c\bar{c}$ and $b\bar{b}$ states Eq. (2) was replaced by considerably more complicated overlap integrals calculated within the QPC model. For higher radial excitations one gets additional factors[4] (Laguerre polynomials) but in the present application, where only the first radial excitation is considered, the detailed analytic form of the vertex function should be less important.

We have also tried a form factor, which modifies the centrifugal barrier following a procedure familiar in nuclear physics (see Ref. 5):

$$k^2F^2(s)\rightarrow |kRh_1^{(l)}(kR)|^{-2}\exp[-(k/k_{\text{cutoff}})^2] \tag{3a}$$

$$=\frac{(kR)^4}{(kR)^4-3(kR)^2+9}\exp[(-k/k_{\text{cutoff}})^2]\ ,\quad \text{for } l=2\ . \tag{3b}$$

Here $h_1^{(l)}$ is the spherical Hankel function and R is a parameter for the "interaction radius" which is[5] 0.8 fm or less. In our actual fits to the data no significant improvement was found when using Eq. (3). To fit the flavor relations for branching ratios one needs $F\simeq$constant for $k\lesssim 0.6$ GeV which means that both R and $\sqrt{6}/k_{\text{cutoff}}$ should be remarkably small (<0.5 fm).

III. THE INTERNAL-SYMMETRY RELATIONS

In Eq. (1) we need flavor and spin relations between large numbers of different thresholds. We take into account all pseudoscalar-pseudoscalar (PP), pseudoscalar-vector (PV), and vector-vector (VV) thresholds as in Ref. 1. They are listed in Table I.

In the QPC model the ratios of the reduced widths of 2^{++} PP, PV, and VV decays can be predicted from spin and angular momentum overlaps (9-j symbols). They are for K^{**}

$$\tilde{\Gamma}(K^{**}-K\pi):\tilde{\Gamma}(K^{**}-K^*\pi):\tilde{\Gamma}(K^{**}-K^*\rho)=(\gamma^{TPP})^2:(\gamma^{TPV})^2:(\gamma^{TVV})^2$$
$$=1\ (D\text{ wave}):\tfrac{3}{2}\ (D\text{ wave}):\tfrac{8}{3}\ (D\text{ wave})+\tfrac{4}{3}\ (S\text{ wave})\ . \tag{4}$$

For VV loops we have both an S-wave and a D-wave part. However, rather than to calculate both in some model-dependent way we parametrize the VV thresholds as if they were only S waves and with γ^{TVV} varied as a free parameter, i.e., not fixed by Eq. (4). This is reasonable since (i) the VV loop contributions to $\text{Re}(M^2)$ are small anyway since they are relatively distant, and (ii) there are other (neglected) thresholds in the same energy region ($0^{++}0^{++}$, etc.) which are at least crudely faked by the phenomenological parameter γ^{TVV}.

As to flavor symmetry, if one imposes the Okubo-Zweig-Iizuka (OZI) rule and C invariance for the bare coupling constant the flavor relations are unique and given by [cf. Fig. 1(a)]

$$\gamma_{iab}^{TPP}=\gamma^{TAB}\text{Tr}(\Delta_i^T\Delta_a^A\Delta_b^B)_{\pm}\ , \tag{5}$$

$$+\text{sign for } AB=PP \text{ or } VV\ ,$$

$$-\text{sign for } AB=PV\ ,$$

where Δ_i^T, etc., are the usual $N\times N$ matrices ($N=$ number of flavors) and i the meson index within the multiplet. The relations (5) are also valid when one includes OZI-rule violation in the meson propagators by letting Δ_a^A depend on the η-η' mixing:

$$\Delta_\eta^P=(\cos\delta^P)\Delta_{s\bar{s}}^P+(\sin\delta^P)\Delta^P_{(u\bar{u}+d\bar{d})/\sqrt{2}}\ .$$

For the P and the V mesons we use the values

$$\delta^P=-46^\circ \text{ and } \delta^V=+3^\circ \text{ (cf. Ref. 6)}\ .$$

More generally, one could add flavor-symmetry but OZI-rule-violating vertex terms [Figs. 1(b)–1(d)] of the form

$$\gamma^b\text{Tr}(\Delta_i^T\Delta_a^A)\text{Tr}\Delta_b^B+\gamma^{b'}\text{Tr}(\Delta_i^T\Delta_b^B)\text{Tr}\Delta_a^A$$
$$+\gamma^c\text{Tr}\Delta_i^T\text{Tr}(\Delta_a^A\Delta_b^B)+\gamma^d\text{Tr}\Delta_i^T\text{Tr}\Delta_a^A\text{Tr}\Delta_b^B \tag{6}$$

for the C-symmetric couplings TPP or TVV. In fact, these are the most general C-parity- and flavor-symmetry-obeying couplings for mesons belonging to a flavor nonet, be they $\bar{q}q$, $\bar{q}q\bar{q}q$, or glueballs.

If there is a substantial contribution from glueball or four-quark states which mix with the $q\bar{q}$ states, one could argue that especially γ^b and γ^c could be substantial (the last term being multiply disconnected in the OZI sense should be small). It should also be pointed out that the terms (6) only contribute to those diagrams where a flavor

TABLE I. The two-body continuum states included in our calculation (omitting charm).

Resonance	PP thresholds	PV thresholds	VV thresholds
f,f'	$\pi\pi,K\bar{K},\eta\eta,\eta\eta',\eta'\eta'$	$\pi\rho,K\bar{K}^*+\text{c.c.}$	$\rho\rho,K^*\bar{K}^*,\omega\omega,\phi\phi,(\phi\omega)$
A_2	$\pi\eta,\pi\eta',K\bar{K}$	$\pi\rho,K\bar{K}^*+\text{c.c.}$	$\rho\rho,K^*\bar{K}^*,(\rho\phi)$
K^{**}	$\pi K,\eta K,\eta' K$	$K\rho,K\omega,K\phi,\pi K^*,\eta K^*,\eta' K^*$	$\rho K^*,\omega K^*,K^*\phi$

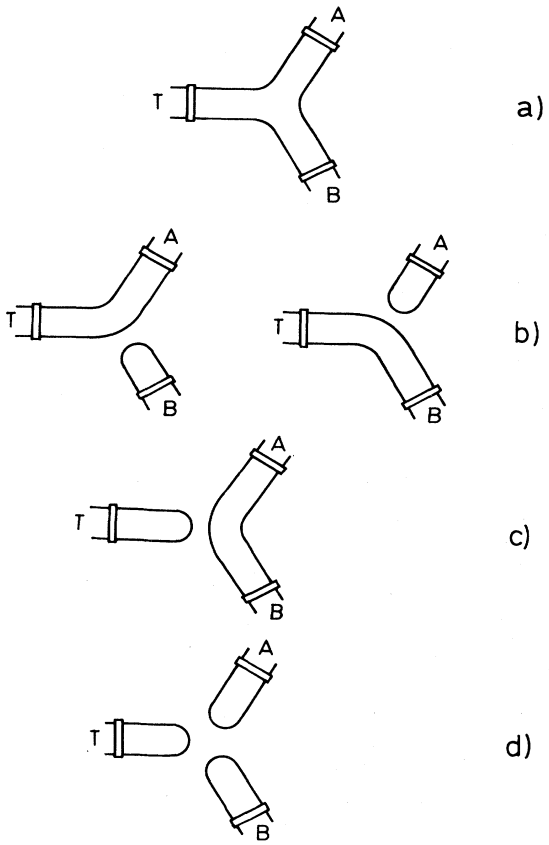

FIG. 1. (a) Quark diagram for the vertex function and Eq. (5); (b), (c), and (d) quark diagrams for the terms in Eq. (6).

singlet is involved, and therefore their overall weight in the fit can be argued to be small. However, as we discuss in Sec. IV a 15% glueball admixture in the f and f' is indicated by the data (Sec. IV B).

IV. COMPARISON WITH EXPERIMENT

In our model there are five parameters: (i) the bare $\bar{u}u$-meson mass m_0, which for a given form factor F is fixed by the f mass, (ii) the bare $s\bar{u}$-meson mass (fixed by K^{**} for given F), (iii) the coupling constant γ^{TPP}, which can be fixed by the best determined width, (iv) the coupling parameter γ^{TVV} which weighs the contribution from high-lying VV loops (and other neglected thresholds), and (v) the parameter of the form factor (k_{cutoff}). On the other hand, there is a large number (about 20) of independent data points (4 masses, 4 total widths, and many branching ratios to be predicted). In particular one can distinguish the following quantities, many of which are rather insensitive to the parameters just listed.

A. Branching ratios related by flavor symmetry

There are several branching ratios related by flavor symmetry, and which can be compared with data. In Table II we make comparisons without form factor ($F=1$, or a large value on $k_{\text{cutoff}} > 1.5$ GeV/c) and neglecting the f-f' mixing ($\delta^T=0°$) as well as glueball mixing in the f and f'. As can be seen, the overall agreement with data is quite good.

However, the agreement in $f\to K\bar{K}/\pi\pi$ may be fortuitous, since with a conventional value[6] for f-f', $\delta^T \simeq -7°$, it increases by $\sim 30\%$. Moreover, with a glueball (GB) component one can write for the reduced widths

$$\frac{\tilde{\Gamma}(f\to K\bar{K})}{\tilde{\Gamma}(f\to\pi\pi)} = \frac{\left|1-\sqrt{2}\tan\delta^T + \frac{\sqrt{6}}{\cos\delta^T}\frac{\gamma^{\text{GB}}}{\gamma^{TPP}}\right|^2}{3\left|1+\frac{\sqrt{6}}{2\cos\delta T}\frac{\gamma^{\text{GB}}}{\gamma^{TPP}}\right|^2}$$

$$\simeq \tfrac{1}{3}(1-2\sqrt{2}\delta^T+\sqrt{6}\gamma^{\text{GB}}/\gamma^{TPP}) , \qquad (7)$$

where γ^{GB} is normalized as γ^c in Eq. (6). If γ^{GB} arises through mixing with a heavy glueball state, the glueball contribution should be positive and of the same sign as the conventional $s\bar{s}$ increasing $f\to K\bar{K}/\pi\pi$. Similarly a flavor-independent cutoff [Eq. (2)] increases this ratio as pointed out by Achasov *et al.*[7] Thus, it seems likely that in order to understand the present world-average, partial-width $f\to K\bar{K}$ one needs some flavor dependence in the form factors or coupling constants which suppresses $K\bar{K}$ in comparison with $\pi\pi$. On the other hand, the issue is not yet experimentally settled; the ANL group[9] obtained an experimental value for $(f\to K\bar{K})/(f\to\pi\pi)$ of 0.047 ± 0.005, which is 33% larger than the world average, and would be easier to understand in more detailed models.

TABLE II. Branching ratios related by flavor symmetry.

Branching ratio	Experimental value (Ref. 6)	$F=1$ model $\begin{bmatrix}\text{flavor}\\ \text{factor}\end{bmatrix}\times K^5$	Experiment / ($F=1$, $\delta^T=0$)model
$f\to K\bar{K}/\pi\pi$	0.035 ± 0.0023	$\frac{1-2\sqrt{2}\delta^T}{3}\times 0.111=0.050$	0.95 ± 0.05
$A_2\to K\bar{K}/\pi\eta$	$0.33\ \pm0.04$	$0.979\times0.36=0.35$	0.95 ± 0.12
$A_2\to\eta'\pi/\eta\pi$	<0.14	$1.02\times0.044=0.045$	<2.5
$K^{**}\to K\rho/K^*\pi$	$0.36\ \pm0.05$	$1.00\times0.33=0.33$	1.11 ± 0.15
$K^{**}\to K\omega/K^*\pi$	$0.17\ \pm0.06$	$0.393\times0.25=0.10$	$1.7\ \pm0.6$

TABLE III. Ratios between flavor-related widths.

Width ratio	Experimental value	$F=1$ model $\begin{bmatrix}\text{flavor}\\ \text{factor}\end{bmatrix}\times k^5/m^2$	$\frac{\text{Experiment}}{F=1 \text{ model}}$
$\frac{f\to\pi\pi}{K^{**}\to K\pi}$	3.3 ±0.4	$2\times 1.25=2.50$	1.32±0.16
$\frac{A_2\to\eta\pi}{K^{**}\to K\pi}$	0.36±0.03	$0.695\times 0.55=0.38$	0.94±0.08
$\frac{f'\to K\bar{K}}{K^{**}\to K\pi}$	1.67±0.3	$\frac{4}{3}\times 0.60=0.80$	2.1 ±0.4

B. Other flavor relations

We also have flavor-symmetry relations between partial widths of different resonances shown in Table III. In particular we note that without glueball mixing the $(f\to\pi\pi)/(K^{**}\to K\pi)$ ratio is 30% too large experimentally. Here, since the phase spaces are nearly equal, this quantity should not depend on the form factor F. On the other hand, a glueball component modifies the $f\to\pi\pi$ width:

$$\frac{\tilde{\Gamma}(f\to\pi\pi)}{\tilde{\Gamma}(K^{**}\to K\pi)}=\frac{6\left|\cos\delta^T+\frac{2}{\sqrt{6}}\frac{\gamma^{\mathrm{GB}}}{\gamma^{TPP}}\right|^2}{3}, \tag{8}$$

where we used the same notation as in Eq. (7). With the glueball contribution positive, as expected if it arises from mixing with a heavy flavor-singlet state, the data suggest

$$\frac{2}{\sqrt{6}}\frac{\gamma^{\mathrm{GB}}}{\gamma^{TPP}}\simeq 0.15 . \tag{9}$$

Also the $f'\to K\bar{K}$ width is experimentally about twice as large (Table II) as expected without a glueball component. With a glueball one has for the reduced widths

$$\frac{\tilde{\Gamma}(f'\to K\bar{K})}{\tilde{\Gamma}(K^{**}\to K\pi)}=\frac{4}{3}\left|\cos\delta^T+\frac{1}{\sqrt{2}}\sin\delta^T+\frac{2}{\sqrt{3}}\frac{\gamma^{\mathrm{GB}}}{\gamma^{TPP}}\right|^2, \tag{10}$$

and using Eq. (9) one would have a 40% increase, improving considerably the agreement with experiment. This raises the question of whether such a large glueball contribution is consistent with the very small OZI-rule-violating decay $f'\to\pi\pi$? In fact, here the contribution of the glueball has the opposite sign than that of the f-f' mixing. One has for the $\pi\pi/K\bar{K}$ ratio

$$\frac{\Gamma(f'\to\pi\pi)}{\Gamma(f'\to K\bar{K})}=\frac{3}{2}\left|\sin\delta^T+\frac{2}{\sqrt{6}}\frac{\gamma^{\mathrm{GB}}}{\gamma^{TPP}}\right|^2\left[\frac{k_\pi}{k_K}\right]^5 . \tag{11}$$

Beusch *et al.*[8] give an experimental limit <0.0086 on this number, whereas Pawlicki *et al.*[9] quote 0.012±0.004. Using the value from the analysis of Martin and Ozmuth,[10] 0.0075±0.0025, one gets

$$\left|\sin\delta^T+\frac{2}{\sqrt{6}}\frac{\gamma^{\mathrm{GB}}}{\gamma^{TPP}}\right|=0.04\pm 0.01 , \tag{12}$$

which with $\delta^T=-7°$ could allow for even such a large value for the glueball part as quoted above in Eq. (9).

The presence of a glueball component in the f and f' has been discussed by several authors, especially Rosner,[11] Nikolić,[12] Schnitzer,[13] and Donoghue and collaborators,[14] with a similar conclusion as ours.

A natural question is whether the $\theta(1640)$, which probably is also a 2^{++} state, could explain the discrepancy just discussed. With the presently available data it no doubt could, especially if one allows the θ to be a mixture of a glueball and a $q\bar{q}$ radial excitation including both $s\bar{s}$ and $u\bar{u}+d\bar{d}$, as the analysis of Ono and Pene[4] indicates. At present we have not extended our unitary mixing analysis of the $L=1$ $q\bar{q}$ multiplets to include radial excitations, mainly because of the lack of reliable data. No doubt such an analysis will be very interesting in the near future.

Another probe of the flavor content of the neutral states is provided by the two-photon decays $f\to\gamma\gamma$, $f'\to\gamma\gamma$, and $A_2\to\gamma\gamma$. The formula (5) is also applicable for these couplings if Δ_γ is the diagonal matrix whose diagonal elements are the quark charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$. One finds, adding a glueball component z in the f or f',

$$R_f=\frac{\Gamma(f\to\gamma\gamma)}{\Gamma(A_2\to\gamma\gamma)}\left[\frac{m_{A_2}}{m_f}\right]^3 =\frac{25}{9}\left[\cos\delta_T-\frac{\sqrt{2}}{5}\sin\delta_T\right]^2(1-z^2) , \tag{13}$$

$$R_f'=\frac{\Gamma(f'\to\gamma\gamma)}{\Gamma(A_2\to\gamma\gamma')}\left[\frac{m_{A_2}}{m_{f'}}\right]^3 =\frac{2}{9}\left[\cos\delta_T+\frac{5}{\sqrt{2}}\sin\delta_T\right]^2(1-z^2) . \tag{14}$$

Equation (13) gives an upper bound $R_f\le 3$ ($R=3$ for $\delta^T=-16°$ and $z=0$). We use for R_f a value obtained from the JADE data,[15] $R_f=3.0\pm 0.7$. This is consistent with the bound especially when one takes into account the error bars. One finds that the glueball component z is small (<0.3) and $|\delta|^T$ is small $[-30°<\delta_T(m_f)<2°]$. Unfortunately, R_f does not give a good determination of δ^T.

For the ratio $R_{f'}$ we use a value 0.12±0.3 obtained from[15] the TASSO group $[\Gamma(f'\to\gamma\gamma)B(f'\to\kappa\bar{\kappa})$

TABLE IV. Ratios between PV and PP thresholds.

Branching ratio	Experimental value	$F=1$ model $\left[\begin{matrix}\text{internal-}\\ \text{symmetry}\\ \text{factor}\end{matrix}\right]\times k^5$	$\frac{\text{Experiment}}{F=1 \text{ model}}$
$K^*\to K\pi/K^*\pi$	0.55±0.06	$\frac{3}{2}\times 0.14=0.21$	2.6±0.3
$A_2\to\pi\eta/\pi\rho$	4.8 ±0.5	$5.76\times 0.32=1.82$	2.6±0.3

$=0.11\pm0.02\pm0.04$ keV] and $\Gamma(A_2\to\gamma\gamma)$ of the JADE group (0.84±0.15 keV) and a value $B(f'\to K\bar{K})=0.72$ obtained from our unitary analysis (see Sec. III D). Inserting this in Eq. (14) one finds

$$\delta^T(m_{f'})=-4.3°\pm1.5° . \tag{15}$$

This value is not sensitive to a small glueball component ($z<0.2$). Note that in spite of the rather crude determination of $R_{f'}$ one obtains a small error bar on δ^T. Thus, $R_{f'}$ of Eq. (14) gives a good determination of δ^T, contrary to R_f. The value of δ^T obtained in (15) is consistent with the naive Gell-Mann–Okubo mass-formula value $\delta^T=-7°\pm3°$ (using quadratic masses cf. Ref. 6). It is in even better agreement with our value $-4°$ found in our unitary mixing analysis using masses and strong widths as discussed in Sec. IV D.

C. Ratios between $T\to PP$ and $T\to PV$ partial widths

There are two relations between PP and PV decays which can be tested, independent of the flavor relations discussed previously. They are listed in Table IV, and compared with the model without the form factor and using the QPC-model relations (4). The agreement is not very good, which can be interpreted in two ways: either (i) the form factor Eq. (2) is crucial [with $k_{\text{cutoff}}\simeq0.7$ GeV (as we used in the fits presented in Ref. 1) the PP/PV width ratios would be suppressed by about a factor of 2, improving the agreement considerably], or (ii) the spin-angular-momentum overlaps of the QPC model are rather crude estimates of the PP/PV ratios. The fact that the $(f\to\pi\pi)/(f\to K\bar{K})$ ratio favors a large cutoff suggests the second alternative, but at present we cannot settle this issue.

TABLE V. Fit to 2^{++} meson masses and total widths to PP and PV final states with parameters of Table IV. The prediction of the f-f' mixing δ^T is also given.

Quantity	Experiment (MeV)	Unitarized model (MeV)
m_f	1273±5	1273
Γ_f^{PP+PV}	160±20	166
m_{A_2}	1318±5	1317
$\Gamma_{A_2}^{PP+PV}$	100±5	107
$m_{f'}$	1520±10	1521
$\Gamma_{f'}$	75±10	70
$m_{K^{**}}$	1434±5	1434
$\Gamma_{K^{**}}$	100±10	92
$\delta^T(m_f)$		$(-6.3+i0.71)°$
$\delta^T(m_{f'})$		$(-4.0-i1.31)°$

D. Fit to masses and total widths

The branching ratios discussed above test internal-symmetry predictions which are rather independent of the unitarization. An exception is the mixing between resonances such as f-f' mixing, which, however, is small in the present application. On the other hand, the fit to the mass splittings and to the total widths depends more sensitively on the unitarization.

We distinguish, in particular, the A_2-f mass splitting and the ideal-mixing violating mass difference $m_f+m_{f'}-2m_{K^{**}}$, which before unitarization would be zero. We find with the cutoff fixed to 0.7 GeV/c (and no glueball)

$$m_{A_2}-m_f=43.9 \text{ MeV}$$

[experiment (Ref. 6) 45±7 MeV] ,

$$2m_{K^{**}}-m_{f'}-m_f=73.2 \text{ MeV}$$

[experiment (Ref. 6) 75±25 MeV] .

The deviation from the ideal mixing angle δ^T is found to be

$$\delta^T(m_f)=(-6.3+i0.2)° ,$$

$$\delta^T(m_{f'})=(-4.0-i1.4)° .$$

It is complex and mass dependent as it must be in a unitary framework. Two of the experimental masses (m_f and $m_{K^{**}}$) fix the bare mass m_0 and quark mass splittings of our model.

In Table V we give a complete list of the fit to the masses and total widths when the cutoff is fixed to 0.7 GeV/c and in Table VI the values of the parameters are given.

TABLE VI. Values of parameters for 2^{++} mesons in the unitarized quark model using $k_{\text{cutoff}}=0.7$ GeV/c.

Parameter	Value
m_0	1819 MeV
m_s-m_n	45.7 MeV
γ^{TPP}	0.996 GeV^{-1}
$\gamma^{TPV}/\gamma^{TPP}$	$\sqrt{3}$ (fixed by QPC)
$\gamma^{TVV}/\gamma^{TPP}$	1.19

We also have tried the modified form factor of Eq. (3) and larger values of k_{cutoff} without any essential improvement in the fit to the data of Table V. Equally good fits were obtained for k_{cutoff} up to 1.5 GeV/c. The parameters (Table VI), in particular the bare masses, are of course very sensitive to the choice of cutoff. The larger the cutoff, the larger is m_0 and the smaller the quark mass difference $m_s - m_u$. As discussed in Secs. IV A–IV C, a larger cutoff would be preferred in order to fit the flavor-related branching ratios (see also Ref. 8), whereas the QPC-model predictions for the PP/PV ratios would favor a low cutoff. Our choice of 0.7 GeV/c in Tables V and IV is motivated by the fact that this cutoff was chosen in our previous fits to the other multiplets and makes the parameters comparable. Clearly, by introducing further phenomenological parameters into the model one could make a more detailed fit to the branching ratios, but it is clear from our analysis that there would be no difficulty in fitting the mass spectrum within the general framework of the unitarized quark model.

Of particular interest are the predictions of the $\eta\eta$ branching ratios of the f and f', which could be measured in future experiments. Our model predicts $B(f\to\eta\eta/K\bar{K})=0.10$ and $B(f'\to\eta\eta/KK)=0.16$. [*Note added in proof.* Recently,[16] a value $B(f\to\eta\eta)=(5.2\pm1.7)\times10^{-3}$ has been measured by a CERN-Serpukhov group; this gives $B(f\to\eta\eta/K\bar{K})=0.017\pm0.07$.]

Finally we point out that the bare-mass parameters (Table VI) obviously depend on the choice of cutoff, the number of partial waves (S and D waves), and the number of thresholds ($PP, PV, VV, \ldots$) included. In Ref. 1 we included, for obvious reasons, only S-wave PP decays for the 0^{++} states and S-wave PV and VV channels for the 1^{++} and 1^{+-} states. In order to make a meaningful comparison of all $1P$ $q\bar{q}$ bare masses, i.e., calculate how much of, say, the A_2-A_1 mass splitting is due to hadronic mass shifts, one must simultaneously include at least all S- and D-wave PP, PV, and VV continuum states. This is a calculation which is still to be done. But, it is clear from our present investigations that a substantial part of the fine structure between the 2^{++}, 1^{++}, 1^{+-}, and 0^{++} states can be accounted for by hadronic shifts.

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