Construction of effective actions in superspace

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A functional method of calculating superspace effective actions in supersymmetric theories is developed by use of superfield functional integrals and multipole-expansion techniques. The superspace effective actions for supersymmetric QED and the supersymmetric CP^{n-1} model are constructed by this method without reference to component-field calculations.

I. INTRODUCTION

The superfield formalism[1] exhibits its inherent advantages in actual calculations as well as in the formulation of supersymmetric theories.[2] Superfield perturbation theory[3,4] not only simplifies perturbative calculations but also reveals some nonrenormalization theorems[4,5] essential to the development of super grand-unification ideas.[6] Superfield techniques have also been applied to the evaluation of effective potentials.[7–10] In view of the utility of superfields, it will be important to explore various techniques for handling them.

The purpose of this paper is to present a functional method for the construction of effective actions in supersymmetric gauge theories. The key ingredient in our approach is a multipole expansion of superfields in superspace; this enables us to express the nonlocal one-loop functional directly in a series of gauge-invariant local superfield products.

Recently, Veneziano and Yankielowicz[11] have proposed an effective Lagrangian which incorporates properly the anomaly structure of supersymmetric quantum chromodynamics. Subsequently, D'Adda *et al.*[12] have elucidated the low-energy structure of the superfield effective action by an explicit calculation in the supersymmetric CP^{n-1} model; their construction is based on a component-field effective action translated into a unique superspace action under some ansatz. Our approach can directly lead to the superspace action, as shown later.

In Sec. II, we derive some functional-integral formulas to be used in the succeeding sections. In Sec. III, we illustrate our basic algorithm by deriving the one-loop effective action for supersymmetric quantum electrodynamics. The resulting action is a superfield version of Schwinger's result[13] for quantum electrodynamics. In Sec. IV, we construct the effective action for the composite supermultiplet in the supersymmetric CP^{n-1} model by a method different from that of Sec. III. Examination of the effective potential shows that supersymmetry is unbroken in this model. Section V is devoted to concluding remarks.

II. FUNCTIONAL INTEGRALS OVER SUPERFIELDS

In this section we study functional integrals over chiral superfields with emphasis on how to handle the chirality constraints involved.

Our notations are those of Ref. 14 modified so that the space-time metric is $g_{\mu\nu}=(1,-1,-1,-1)$. For example, the covariant derivatives take the form $D_\alpha=\partial/\partial\Theta^\alpha-(\not p\overline{\Theta})_\alpha$ and $\overline{D}_{\dot\alpha}=-\partial/\partial\overline{\Theta}^{\dot\alpha}+(\Theta\not p)_{\dot\alpha}$ with $(\not p)_{\alpha\dot\alpha}=p_\mu(\sigma^\mu)_{\alpha\dot\alpha}$, $p_\mu=i\partial_\mu$, and $(\sigma^\mu)_{\alpha\dot\alpha}=(1,\sigma^k)_{\alpha\dot\alpha}$. In addition, extensive use will be made of the short-hand notation $z=(x^\mu,\Theta^\alpha,\overline{\Theta}^{\dot\alpha})$, $d^8z=d^4x\,d^2\Theta\,d^2\overline{\Theta}$, $d^6z=d^4x\,d^2\Theta$, and $d^6\overline{z}=d^4x\,d^2\overline{\Theta}$; we shall call d^8z, d^6z, and $d^6\overline{z}$ the full, chiral, and antichiral measures in superspace, respectively. Of great use is matrix notation such as $\Xi(z_1,z_2)=\langle z_1|\Xi|z_2\rangle$, where $|z_i\rangle=|x_i,\Theta_i,\overline{\Theta}_i\rangle$ $(i=1,2)$ denote the eigenstates of the coordinate operator $z=(x,\Theta,\overline{\Theta})$; matrix multiplication is defined in terms of the full measure in such a way that

$$\langle z_1|\Xi\Sigma|z_2\rangle=\int d^8z_3\langle z_1|\Xi|z_3\rangle\langle z_3|\Sigma|z_2\rangle\ .$$

Let us first evaluate the Gaussian integral over the chiral superfield $\Phi(x,\Theta,\overline{\Theta})=\Phi(z)$:

$$W[\Xi]=\int[d\Phi][d\overline{\Phi}]\exp\left[i\int d^8z_1d^8z_2\overline{\Phi}(z_1)\times\Xi(z_1,z_2)\Phi(z_2)\right],\qquad(2.1)$$

where $\Phi(z)$ and its Hermitian conjugate $\overline{\Phi}(z)$ are subject to the constraint $\overline{D}_{\dot\alpha}\Phi=D_\alpha\overline{\Phi}=0$. Instead of explicitly carrying out this constrained functional integral, it is advantageous to cast it into the differential form

$$\delta W/W=i\int d^8z_1d^8z_2\delta\Xi(z_1,z_2)\langle\Phi(z_2)\overline{\Phi}(z_1)\rangle_c\ .\qquad(2.2)$$

Then W is reconstructed out of the superfield propagator $\langle\Phi\overline{\Phi}\rangle_c$ (the connected two-point Green's function), which, in the present case, is easier to derive than W itself. A simple way to obtain the propagator is to look for the minimum of the action in (2.1) with the source term $\int d^6z\,J(z)\Phi(z)+\int d^6\overline{z}\,\overline{J}(z)\overline{\Phi}(z)$ added. In matrix notation, the superfield equation

$$-\tfrac{1}{4}D^2\Xi\Phi+\overline{J}=0\qquad(2.3)$$

is solved for Φ in the form

$$\Phi=4(\overline{D}^2D^2\Xi)^{-1}\overline{D}^2\overline{J}\ .\qquad(2.4)$$

Note that $\overline{D}^2\overline{J}$ has the same chirality as Φ; correspondingly $\overline{D}^2D^2\Xi$ has been inverted in the sense

$$(\overline{D}^2D^2\Xi)_z\mathscr{G}(z,z_1)=1_{--}(z,z_1)\ ,\qquad(2.5)$$

which gives the symbolic expression for the inverse

$$\mathscr{G}=(\bar{D}^2D^2\Xi)^{-1}1_{--}\ , \qquad (2.6)$$

where the chiral δ function[3] $1_{--}(z,z_1)=\langle z\,|\,(-\frac{1}{4}\bar{D}^2)\,|\,z_1\rangle$ acts as a δ function in combination with chiral measures. The propagator is derived from (2.4) by a functional differentiation with respect to the antichiral source $\bar{J}$:

$$\langle\Phi(z_1)\bar{\Phi}(z_2)\rangle_c=i\langle z_1\,|\,(\bar{D}^2D^2\Xi)^{-1}\bar{D}^2D^2\,|\,z_2\rangle\ . \qquad (2.7)$$

Equation (2.2) is now cast into the functional form

$$\delta\ln W=-\mathrm{Tr}[(\bar{D}^2D^2\Xi)^{-1}\bar{D}^2D^2\delta\Xi]\ , \qquad (2.8)$$

which is integrated to give the desired expression for W:

$$W[\Xi]=\exp[-\mathrm{Tr}\ln(\bar{D}^2D^2\Xi)]\ , \qquad (2.9)$$

where the trace Tr is taken with the full measure $\int d^8z$. Note that $W[\Xi]$ has absolute normalization since $W[1]=1$ (i.e., the free-field case).

The second example we consider is supersymmetric quantum electrodynamics (SQED) described by the matter-field action[15]

$$\mathscr{A}=\int d^8z(\bar{\Phi}_1\Xi\Phi_1+\Phi_2\Sigma\bar{\Phi}_2)+m\int d^6z\,\Phi_1\Phi_2$$
$$+m\int d^6\bar{z}\,\bar{\Phi}_1\bar{\Phi}_2\ , \qquad (2.10)$$

where $\Phi_1(z)$ and $\Phi_2(z)$ are chiral superfields with opposite electric charges. For generality, Ξ and Σ are treated independently in what follows: SQED is obtained by the choice $\Xi=e^{2eV}$ and $\Sigma=e^{-2eV}$ in terms of the vector superfield $V(z)$.

It is convenient to write $\mathscr{A}$ in the matrix form

$$\mathscr{A}=\Psi^*\cdot\Omega\cdot\Psi\ , \qquad (2.11)$$

$$\Omega=\begin{bmatrix} m1_{--} & -\frac{1}{4}\bar{D}^2\Sigma 1_{++} \\ -\frac{1}{4}D^2\Xi 1_{--} & m1_{++}\end{bmatrix}\ , \qquad (2.12)$$

where $\Psi=(\Phi_1,\bar{\Phi}_2)^{\mathrm{tr}}$, $\Psi^*=(\Phi_2,\bar{\Phi}_1)$, $1_{--}=-\frac{1}{4}\bar{D}^2$, and $1_{++}=-\frac{1}{4}D^2$. The dot implies summation over superspace-coordinate labels using appropriate chiral or antichiral measures, as indicated in the structure of Ω. As before, we solve the equation of motion to get the propagator $\langle\Psi\Psi^*\rangle_c$, or equivalently Ω^{-1}:

$$\Omega^{-1}=\begin{bmatrix} m\Delta_-1_{--} & \frac{1}{4}\Delta_-\bar{D}^2\Sigma 1_{++} \\ \frac{1}{4}\Delta_+D^2\Xi 1_{--} & m\Delta_+1_{++}\end{bmatrix}\ , \qquad (2.13)$$

where $\Delta_-=(m^2-\frac{1}{16}\bar{D}^2\Sigma D^2\Xi)^{-1}$ and $\Delta_+=(m^2-\frac{1}{16}D^2\Xi\bar{D}^2\Sigma)^{-1}$. The Ω^{-1} is defined so that $\Omega\cdot\Omega^{-1}=\mathrm{diag}(1_{--},1_{++})$.

Let us denote by $W=W[\Xi,\Sigma]$ the functional integral over Ψ and Ψ^* with the action (2.11). Its response to the variation $\delta\Omega$ caused by $\delta\Xi$ and $\delta\Sigma$ takes the form

$$\delta W/W=-\mathscr{T}\mathrm{r}(\delta\Omega\cdot\Omega^{-1})\ , \qquad (2.14)$$

where the trace $\mathscr{T}\mathrm{r}$ is taken with appropriate chiral or antichiral measures. Using (2.13) and the operator identity $\Delta_+D^2\Xi\bar{D}^2=D^2\Xi\Delta_-\bar{D}^2$, we rewrite this as

$$\delta W/W=-\mathscr{T}\mathrm{r}(\Delta_-\delta\Delta_-{}^{-1}1_{--})\ , \qquad (2.15)$$

where $\delta\Delta_-{}^{-1}=-\frac{1}{16}(\bar{D}^2\delta\Sigma D^2\Xi+\bar{D}^2\Sigma D^2\delta\Xi)$. On integration, this yields the expression

$$\ln W=-\mathscr{T}\mathrm{r}[\ln(\Delta_-{}^{-1})1_{--}]\ . \qquad (2.16)$$

Here $\ln\Delta_-{}^{-1}$ may be defined by the parametric-integral representation:[13]

$$\ln\Delta_-{}^{-1}=-\int_0^\infty d\tau(1/\tau)\exp(-i\tau\Delta_-{}^{-1})\ , \qquad (2.17)$$

where, as usual, $\Delta_-{}^{-1}$ in the exponent is understood to possess an infrared-convergence factor $\Delta_-{}^{-1}-i0_+$ (or $m^2\to m^2-i0_+$).

Although $\ln(\Delta_-{}^{-1})1_{--}$ is chiral, $\ln\Delta_-{}^{-1}$ is not. The nonchiral part of $\exp(-i\tau\Delta_-{}^{-1})$, however, is equal to $\exp(-i\tau m^2)$, which has a vanishing contribution to $\ln W$ because $\mathscr{T}\mathrm{r}1_{--}=0$. Consequently, $\ln\Delta_-{}^{-1}$ itself can effectively be regarded as chiral in (2.16); in view of the relation $\int d^8z=\int d^6z(-\frac{1}{4}\bar{D}^2)$, this fact allows us to replace $\mathscr{T}\mathrm{r}$ combined with 1_{--} simply by the trace Tr. In this way we are led to the following equivalent representations of W:

$$\ln W=-\mathscr{T}\mathrm{r}[\ln(m^2-\tfrac{1}{16}\bar{D}^2\Sigma D^2\Xi)1_{--}] \qquad (2.18)$$

$$=-\mathscr{T}\mathrm{r}[\ln(m^2-\tfrac{1}{16}D^2\Xi\bar{D}^2\Sigma)1_{++}] \qquad (2.19)$$

$$=-\mathrm{Tr}\ln(m^2-\tfrac{1}{16}\bar{D}^2\Sigma D^2\Xi)\ , \qquad (2.20)$$

where the traces are taken with the chiral, antichiral, and full measures, respectively. Equation (2.19) is an antichiral analog of (2.18).

Equations (2.9) and (2.18)–(2.20) summarize in the form of functional formulas the improved superfield Feynman rules of Grisaru *et al.*[4]

An interesting decomposition law follows from the comparison of (2.1) with (2.10) for $m=0$, when Φ_1 and Φ_2 are decoupled:

$$\mathrm{Tr}\ln(\bar{D}^2\Sigma D^2\Xi)=\mathrm{Tr}\ln(D^2\bar{D}^2\Sigma)+\mathrm{Tr}\ln(\bar{D}^2D^2\Xi)\ . \qquad (2.21)$$

Note that $\mathrm{Tr}\ln(D^2\bar{D}^2\Sigma)$ vanishes when Σ is either a chiral or an antichiral local superfield $\Sigma(z)$, as seen from the original integral expression (2.1); it is a simple exercise to verify this directly with the functional expression.

The SQED action (2.10) is invariant under the gauge transformation $V\to V'=V+i(\Lambda-\bar{\Lambda})$, $\Phi_1\to e^{-2ie\Lambda}\Phi_1$, $\bar{\Phi}_1\to e^{2ie\bar{\Lambda}}\bar{\Phi}_1$, etc., where the phases $\Lambda(z)$ and $\bar{\Lambda}(z)$ are chiral and antichiral superfields, respectively. Gauge invariance is manifest in formulas (2.18)–(2.20), e.g., for (2.18), $I[V]=\ln(\cdots)1_{--}$ transforms as

$$I[V]=e^{2ie\Lambda}I[V']e^{-2ie\Lambda}\ . \qquad (2.22)$$

Gauge invariance is also embodied in $W[\Xi]$ defined by the functional integral (2.1), such that $W[\Xi]=W[\Xi']$ with $\Xi'=e^{-i\bar{\Lambda}}\Xi e^{i\Lambda}$. It is easy to verify this invariance property directly with the functional formula (2.9) by use of (2.21).

III. SUPERSYMMETRIC QED

In this section we present our basic formalism and construct the low-energy effective action for SQED. We shall focus on the one-loop contribution from the matter fields, $\Gamma=-i\ln W$, where $\ln W$ is given by (2.18)–(2.20), and treat V as a background superfield.

Let us consider (2.18) cast in the parametric form

$$\Gamma[V]=-i\int d^6w\int_0^\infty d\tau(1/\tau)e^{-\tau m^2}\langle w\,|\,e^{\tau L}1_{--}\,|\,w\rangle\ , \tag{3.1}$$

$$L=\tfrac{1}{16}\bar{D}^2e^{-V}D^2e^{V}\ , \tag{3.2}$$

where w designates the superspace coordinate $w=(u^\mu,\xi^\alpha,\bar{\xi}^{\dot\alpha})$. In the above, for notational convenience, we have set $2eV\to V$ since the coupling constant e can easily be recovered. In addition, assuming that $m^2-L>0$, we have deformed the τ-integration contour (effectively $\tau\to -i\tau$); for other values of L, Γ is defined by a suitable analytic continuation specified by the $i0_+$ prescription.

The $\Gamma[V]$ is a gauge-invariant nonlocal functional of V, as illustrated in Fig. 1(a). We attempt to evaluate it for a slowly varying superfield V by generalizing the concept of a multipole expansion[16] formally to superspace: Let us look at Fig. 1(a), and expand the slowly varying fields V at various superspace positions in multipoles at some fixed position, which we take to be $w=(u,\xi,\bar{\xi})$ in (3.1), as depicted in Fig. 1(b). This gives rise to a systematic expansion of $\Gamma[V]$ in a series of local superfield products. Remember, however, that the expansion in the fermionic coordinate $(\Theta,\bar\Theta)$ is only a formal procedure which is independent of how fast $V(z)$ varies in real space-time; thus, the range (or wavelength) of each superfield product is determined by its x dependence rather than by the number of covariant derivatives involved.

The expansion procedure is made manifestly supersymmetric and gauge-invariant by use of suitable transformations introduced below. Let us denote the relative superspace coordinate by $\hat{z}=(r^\mu,\theta^\alpha,\bar\theta^{\dot\alpha})=z-w$, i.e.,

$$x^\mu=r^\mu+u^\mu,\ \Theta^\alpha-\theta^\alpha+\xi^\alpha,\ \text{and}\ \bar\Theta^{\dot\alpha}=\bar\theta^{\dot\alpha}+\bar\xi^{\dot\alpha}\ . \tag{3.3}$$

We first consider the unitary operator

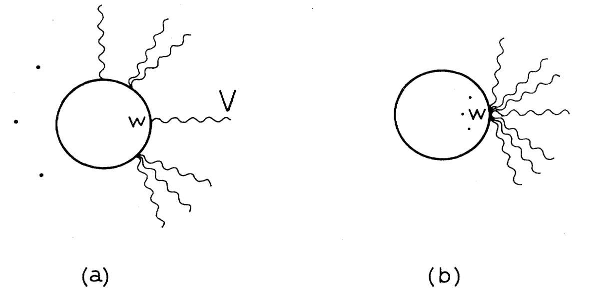

FIG. 1. (a) Schematic representation of the one-loop effective action $\Gamma[V]$. Wavy lines represent background superfields V coupled to the quantum loop of the matter superfield. (b) The superfields V at different positions are expanded in multipoles at a fixed position (here chosen to be w) in superspace.

$$U_I(\theta,\xi)=\exp(\xi\not{p}\bar\theta-\theta\not{p}\bar\xi)\ , \tag{3.4}$$

which serves to convert the covariant derivatives

$$D_\alpha=\partial/\partial\Theta^\alpha-(\not{p}\bar\Theta)_\alpha\equiv D_\alpha(\Theta,\bar\Theta)$$

and

$$\bar{D}_{\dot\alpha}=\bar{D}_{\dot\alpha}(\Theta,\bar\Theta)\ ,$$

into those defined in the $(x,\theta,\bar\theta)$ coordinate:

$$U_ID_\alpha(\Theta,\bar\Theta)U_I^{-1}=D_\alpha(\theta,\bar\theta)$$

and

$$U_I\bar{D}_{\dot\alpha}(\Theta,\bar\Theta)U_I^{-1}=\bar{D}_{\dot\alpha}(\theta,\bar\theta)\ . \tag{3.5}$$

The vector superfield $V(z)$ thereby undergoes the unitary transformation $V(x,\Theta,\bar\Theta)\to V_I(x,\theta,\bar\theta)=U_IV(x,\Theta,\Theta)U_I^{-1}$ with V_I given by

$$V_I(x,\theta,\bar\theta)=V(x+i\xi\sigma\bar\theta-i\theta\sigma\bar\xi,\ \theta+\xi,\ \bar\theta+\bar\xi)\ . \tag{3.6}$$

This $V_I(x,\theta,\bar\theta)$ may be regarded as being a result of either a supertranslation on $V(x,\theta,\bar\theta)$ or an inverse supertranslation on $V(x,\xi,\bar\xi)$. Hence, recalling the obvious reciprocal $(p_\mu\leftrightarrow -p_\mu)$ structures[14] of supercharge differential operators and covariant derivatives, we can write V_I in the form

$$V_I(x,\theta,\bar\theta)=\exp[\theta^\alpha D_\alpha(\xi,\bar\xi\dot\alpha)+\bar\theta_{\dot\alpha}\bar{D}^{\dot\alpha}(\xi,\bar\xi)]V(x,\xi,\xi)\ , \tag{3.7}$$

where the covariant derivatives act on $V(x,\xi,\bar\xi)$. Its power-series expansion in θ and $\bar\theta$ contains, as its coefficients, superfields in $(x,\xi,\bar\xi)$:

$$V_I(x,\theta,\bar\theta)=V_0(x,\theta,\bar\theta)+\Lambda_I(x,\theta,\bar\theta)+\bar\Lambda_I(x,\theta,\bar\theta)\ , \tag{3.8}$$

$$V_0(x,\theta,\bar\theta)=\theta\sigma^\mu\bar\theta v_\mu(x,\xi,\bar\xi)+\bar\theta^2\theta W(x,\xi,\bar\xi)+\theta^2\bar\theta\,\bar{W}(x,\xi,\bar\xi)+\tfrac{1}{2}\theta^2\bar\theta^2C(x,\xi,\bar\xi)\ , \tag{3.9}$$

with the coefficients given by

$$\begin{aligned}
&v_\mu(x,\xi,\bar\xi)=\tfrac14(D\sigma_\mu\bar{D}-\bar{D}\bar\sigma_\mu D)V\ ,\\
&W_\alpha(x,\xi,\bar\xi)=-\tfrac14\bar{D}^2D_\alpha V\ ,\\
&\bar{W}_{\dot\alpha}(x,\xi,\bar\xi)=-\tfrac14 D^2\bar{D}_{\dot\alpha}V\ ,\\
&C(x,\xi,\bar\xi)=\tfrac18 D^\alpha\bar{D}^2D_\alpha V=-\tfrac12 D^\alpha W_\alpha\ ,
\end{aligned} \tag{3.10}$$

where $V=V(x,\xi,\bar\xi)$. The $\Lambda_I(x,\theta,\bar\theta)$ is chiral in $(x,\theta,\bar\theta)$ [i.e., $\bar{D}_{\dot\alpha}(\theta,\bar\theta)\Lambda_I=0$], with the expansion

$$\Lambda_I(x,\theta,\bar\theta)=e^{-i\theta\partial\bar\theta}[\tfrac12 V+\theta^\alpha(D_\alpha V)-\tfrac14\theta^2(D^2V)]\ , \tag{3.11}$$

where $V=V(x,\xi,\bar\xi)$; similarly for its antichiral partner $\bar\Lambda_I(x,\theta,\bar\theta)$.

The unitary operator U_I, acting on $|\,w\rangle$, is a unit operator: $U_I\,|\,w\rangle=|\,w\rangle$. Consequently, for the integrand $\langle w\,|\cdots|\,w\rangle$ in (3.1), the effect of the present transformation is simply to replace the operators $(D,\bar{D},V)$ within it by those defined in the $(x,\theta,\bar\theta)$ coordinate. In addition, the chiral parts Λ_I and $\bar\Lambda_I$ in $V_I(x,\theta,\bar\theta)$ can be gauged away since the transformation law (2.22) is carried over to the $(x,\theta,\bar\theta)$ coordinate.

The transformation law (3.7) applies to general superfields and preserves Hermiticity or chirality of the original superfields. For a chiral superfield $\Phi(x,\Theta,\bar\Theta)$, the transformed field $\Phi_I(x,\theta,\bar\theta)$ is given by (3.11) with V replaced by $\Phi(x,\xi,\bar\xi)$. In this way, the present unitary transformation generates a multipole expansion of superfields in the fermionic coordinate.

It is important to examine how the coefficient superfields in (3.10) behave under the gauge transformation $V\to V'=V+i(\Lambda-\bar\Lambda)$ at position $(x,\xi,\bar\xi)$. Obviously, $W_\alpha(x,\xi,\bar\xi)$, $\bar W_{\dot\alpha}(x,\xi,\bar\xi)$, and $C(x,\xi,\bar\xi)$ remain invariant. The $v_\mu(x,\xi,\bar\xi)$ undergoes the change $v_\mu(x,\xi,\bar\xi)\to v'_\mu(x,\xi,\bar\xi)$ with

$$v'_\mu(x,\xi,\bar\xi)=v_\mu(x,\xi,\bar\xi)+\partial_\mu[\Lambda(x,\xi,\bar\xi)+\bar\Lambda(x,\xi,\bar\xi)]\ , \tag{3.12}$$

which is analogous to the transformation law of the ordinary vector field. From this analogy we immediately see that, by a suitable choice of $\Lambda+\bar\Lambda$, $v'_\mu(x,\xi,\bar\xi)$ can be expressed as a functional of $\mathscr{F}_{\mu\nu}=\partial_\mu v_\nu-\partial_\nu v_\mu$ defined at the fixed position $w=(u,\xi,\bar\xi)$. The necessary procedure is essentially the same as for ordinary gauge theories,[16] with the result

$$v'_\mu(x,\xi,\bar\xi)=\sum_{n=0}^{\infty}\frac{1}{n!(n+2)}(r\cdot\partial)^n r^\nu\mathscr{F}_{\nu\mu}[v(u,\xi,\bar\xi)]\ , \tag{3.13}$$

where $r^\mu=x^\mu-u^\mu$ and the derivative $r\cdot\partial=r^\mu\partial/\partial u^\mu$ acts on $\mathscr{F}_{\nu\mu}$. It is possible to express $\mathscr{F}_{\mu\nu}[v]$ in terms of W_α and $\bar W_{\dot\alpha}$:

$$\mathscr{F}_{\mu\nu}[v]=\tfrac{1}{4}(D\sigma_{\mu\nu}W-\bar D\,\bar\sigma_{\mu\nu}\bar W)\ , \tag{3.14}$$

where $(\sigma^{\mu\nu})_\alpha{}^\beta=i\frac{1}{2}(\sigma^\mu\bar\sigma^\nu-\sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta$ and $(\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}=i\frac{1}{2}(\bar\sigma^\mu\sigma^\nu-\bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta}$.

Let us denote by $H(\tau)$ the matrix element $\langle w|\cdots|w\rangle$ we have been considering. The nonlinear gauge transformation that effects the multipole expansion in r^μ, Eq. (3.13), leaves $H(\tau)$ invariant, leading to the expression

$$H(\tau)=(0|e^{\tau\mathscr{L}}1_{--}|0),\quad \mathscr{L}=\tfrac{1}{16}\bar D^2e^{-\mathscr{V}}D^2e^{\mathscr{V}}\ , \tag{3.15}$$

where $\mathscr{V}=\mathscr{V}(r,\theta,\bar\theta)$ is equal to $V_0(x,\theta,\bar\theta)$ with its coefficient $v_\mu(x,\xi,\bar\xi)$ replaced by $v'_\mu(x,\xi,\bar\xi)$ in (3.13). To indicate that the operators in (3.15) act on the relative coordinate $\hat z=z-w=(r,\theta,\bar\theta)$, we have introduced the notation $|\hat z)=|r,\theta,\bar\theta)$ for its eigenstate, i.e., $|\hat z)=|z=\hat z+w\rangle$ and $|0)\equiv|0,0,0)=|w\rangle$. Remember that $H(\tau)$ depends on the fixed coordinate w only through the coefficient superfields of $\mathscr{V}(r,\theta,\bar\theta)$, which are functionals of $W_\alpha(w)$ and $\bar W_{\dot\alpha}(w)$ alone, as seen from (3.10) and (3.14). In terms of the component fields of $V(x,\xi,\bar\xi)$,

$$V(x,\xi,\bar\xi)=\xi B(x)\bar\xi+\bar\xi^2\xi\chi(x)+\xi^2\bar\xi\bar\chi(x)+\tfrac{1}{2}\xi^2\bar\xi^2D(x)+\cdots\ , \tag{3.16}$$

$W_\alpha(x,\xi,\bar\xi)$ is written as

$$W_\alpha=e^{-i\xi\partial\bar\xi}[\chi_\alpha(x)+\xi_\alpha D(x)-i\tfrac{1}{2}f_{\mu\nu}(x)(\sigma^\mu\bar\sigma^\nu\xi)_\alpha+i\xi^2(\partial\bar\chi(x))_\alpha]\ , \tag{3.17}$$

where $f_{\mu\nu}(x)=\partial_\mu B_\nu(x)-\partial_\nu B_\mu(x)$; analogously for $\bar W_{\dot\alpha}(x,\xi,\bar\xi)$.

In order to calculate $H(\tau)$, we consider the operator $G(\tau)=e^{\tau\mathscr{L}}1_{--}$, which satisfies the differential equation

$$(d/d\tau)G(\tau)=\mathscr{L}G(\tau) \tag{3.18}$$

with $G(0)=1_{--}$. We solve this for the matrix element $G(\tau;\hat z)=(\hat z|G(\tau)|0)$; then $H(\tau)=G(\tau;0)$. Practically, it is convenient to consider the Laplace transform $\tilde G(s;\hat z)=\int_0^\infty d\tau\, e^{-s\tau}G(\tau;\hat z)$, which, being chiral in $\hat z=(r,\theta,\bar\theta)$, is expanded in the form

$$\tilde G(s;r,\theta,\bar\theta)=e^{-i\theta\partial\bar\theta}[A(s;r)+\theta^\alpha\psi_\alpha(s;r)+\theta^2F(s;r)]\ . \tag{3.19}$$

Then the component fields obey the set of equations

$$\begin{aligned}&[s-(\Pi^2+\tfrac{1}{2}C)]A+\tfrac{1}{2}W\psi=0\ ,\\ &(s-\Pi\bar\Pi)\psi+WF+\Pi\bar WA=0\ ,\\ &[s-(\Pi^2-\tfrac{1}{2}C)]F-\tfrac{1}{2}\bar W^2A+\tfrac{1}{2}\bar W\bar\Pi\psi=\delta^4(r)\ ,\end{aligned} \tag{3.20}$$

where $\Pi_\mu=p_\mu-\frac{1}{2}v'_\mu(x=r+u,\xi,\bar\xi)$, $p_\mu=i\partial/\partial r^\mu$, $\Pi=\Pi_\mu\sigma^\mu$, and $\bar\Pi=\Pi_\mu\bar\sigma^\mu$. We shall evaluate $H(\tau)$ in a slow-field approximation where $W_\alpha(x,\xi,\bar\xi)$ and $\bar W_{\dot\alpha}(x,\xi,\bar\xi)$ are taken to be independent of x^μ; correspondingly, $C(x,\xi,\bar\xi)$ is now independent of x^μ while $v'_\mu(x,\xi,\bar\xi)$ is linear in r^μ. This approximation is equivalent to regarding the component fields $\chi_\alpha(x)$, $\bar\chi_{\dot\alpha}(x)$, $D(x)$, and $f_{\mu\nu}(x)$ as constant fields in the effective action, as seen from (3.17).

Let us solve (3.20) for $A(s;r)$, which, in the present approximation, is cast in the compact form

$$A(s;r)=2e^2W^2y(y^2-a^2)^{-1}(y^2-b^2)^{-1}\delta^4(r)\ , \tag{3.21}$$

where $y=s-\Pi^2$, $a=eC(w)$, and $b^2=(\frac{1}{2}e\mathscr{F}_{\mu\nu}\sigma^{\mu\nu})^2=\frac{1}{2}e^2(\mathscr{F}^2+i\mathscr{F}\tilde{\mathscr{F}})$ with $\mathscr{F}^2\equiv\mathscr{F}_{\mu\nu}{}^2$ and $\mathscr{F}\tilde{\mathscr{F}}=\frac{1}{2}\epsilon_{\mu\nu\rho\tau}\mathscr{F}^{\mu\nu}\mathscr{F}^{\rho\tau}$. Here we have recovered the electric charge e. From (3.21) follows immediately $H(\tau)$:

$$H(\tau)=2e^2W^2[\cosh(a\tau)-\cosh(b\tau)]((0|\exp(\tau\Pi^2)|0))/(a^2-b^2)\ , \tag{3.22}$$

where $|0))$ stands for the $r^\mu=0$ eigenstate of the coordinate operator r^μ. The $|0))\to|0))$ transition amplitude is familiar from Schwinger's work:[13]

$$((0|\exp(\tau\Pi^2)|0))=(i/16\pi^2\tau^2)\det[e\mathscr{F}\tau/\sin(e\mathscr{F}\tau)]^{1/2}\ , \tag{3.23}$$

where $\mathscr{F}$ denotes a matrix $\mathscr{F}^\mu{}_\nu$ and the determinant det acts on the Lorentz index. After some algebra, we get the following expression for the one-loop effective action:

$$\Gamma[V]=(e^2/16\pi^2)\int d^6w\, W^2/(a^2-b^2)\times\int_0^\infty d\tau(1/\tau)e^{-\tau m^2}h(\tau)\ , \tag{3.24}$$

$$h(\tau)=[\cosh(a\tau)-\cosh(b\tau)]\mathrm{Im}(b^2)/\mathrm{Im}(\cosh b\tau). \tag{3.25}$$

In the present approximation, $\mathcal{F}_{\mu\nu}(x,\xi,\bar{\xi})=f_{\mu\nu}(u)$, $C(x,\xi,\xi)=D(u)$, and $e^2\int d^2\xi W^2=a^2-b^2+2e^2 i\chi\partial\bar{\chi} \approx a^2-b^2$. The imaginary part of $h(\tau)$ is equal to $-\mathrm{Im}(b^2)=-\frac{1}{2}e^2 f_{\mu\nu}\tilde{f}^{\mu\nu}$, which is a four-divergence; hence, $\Gamma[V]$ turns out to be real, as it should be.

It is straightforward to expand $\Gamma[V]$ in powers of $1/m^2$. The $O(m^0)$ term contains an ultraviolet divergence, which can be removed by the wave-function renormalization of the vector superfield V. In component fields, the $O(1/m^4)$ term is written as

$$\Gamma^{(-4)}=\tfrac{1}{12}(e^2/4\pi m^2)^2\int d^4u\,[(D^2-\tfrac{1}{2}f^2)^2+(\tfrac{1}{2}f\tilde{f})^2]\ . \tag{3.26}$$

The superfield action (3.24) correctly reproduces the component-field result to all orders in $1/m^2$. It is a simple exercise to decompose (3.24) rewritten in terms of component fields into three pieces such that

$$\Gamma[V]=\Gamma_F(e,m)+\Gamma_B(e,m^2-eD)$$
$$+\Gamma_B(-e,m^2+eD)\ ,$$

where $\Gamma_F(e,m)$ denotes the one-loop contribution from a Dirac fermion of charge e and mass m while $\Gamma_B(e,m^2)$ denotes the contribution from a scalar field of charge e and $(\mathrm{mass})^2$ m^2. One can verify this structure by a direct calculation based on the component-field Lagrangian.

We have also carried out a calculation, starting with (2.20) written in terms of the full-measure trace Tr, and confirmed[17] (3.24). The calculation is somewhat more tedious than the one presented here. It is therefore important to start with a suitable choice of functional formulas. Obviously, Eq. (2.20) is most suited for standard perturbation theory.

IV. SUPERSYMMETRIC CP^{n-1} MODEL

The supersymmetric CP^{n-1} model in its $N=2$ formulation is based on the action[12,18]

$$\mathcal{A}=\int d^2x\, d^2\Theta\, d^2\bar{\Theta}[\bar{\Phi}^a e^V\Phi^a-(n/g)V] \tag{4.1}$$

expressed in terms of $N=2$ chiral superfields Φ^a ($a=1,\dots,n$), their conjugates $\bar{\Phi}^a$, and an $N=2$ vector superfield V in two dimensions. The V, which has no kinetic term at the tree level, is an SU(n)-singlet composite superfield to be expressed in terms of Φ^a and $\bar{\Phi}^a$. In the large-n limit, it is promoted to represent a supermultiplet of composite particles. The purpose of this section is to derive the effective action for this SU(n)-singlet superfield V.

It is advantageous to regard (4.1) as a dimensionally reduced version of the same action written in terms of $N=1$ superfields in four dimensions, since the two-component spinor formalism can handle the dimensionally reduced $N=2$ case equally well. We here adopt this standpoint and inherit all the notation used in Sec. III.

We choose $x^\mu=(x^0,x^3)$ as our two-dimensional coordinate. The covariant derivatives become simpler in two dimensions: $D_1=\partial/\partial\Theta_2-p_+\bar{\Theta}_2$, $D_2=-\partial/\partial\Theta_1+p_-\bar{\Theta}_1$ and $\bar{D}_1=-\partial/\partial\bar{\Theta}_2+\Theta_2 p_+$, $\bar{D}_2=\partial/\partial\bar{\Theta}_1-\Theta_1 p_-$, where $p_\pm=p_0\pm p_3$. There are only two nonvanishing anticommutators of D_α and $\bar{D}_{\dot{\alpha}}$: $\{D_1,\bar{D}_1\}=2p_+$ and $\{D_2,\bar{D}_2\}=2p_-$, which make the $N=2$ supersymmetry manifest. In two dimensions, a four-vector B_μ is split into a Lorentz vector $B_\mu=(B_0,B_3)$ and two Lorentz scalars B_1 and B_2.

The quantum correction to the tree-level Lagrangian $-(n/g)V$ [in Eq. (4.1)] is derived from $W[\Xi]$ in (2.1) with $\Xi=e^V$. The representation (2.9) is not particularly suited for our present purpose because of the nontrivial behavior of $\bar{D}^2D^2\Xi$ under gauge transformations. As an alternative, we consider the differential form (2.2) or (2.8). In view of the gauge-transformation law of the propagator $\langle\Phi'\bar{\Phi}'\rangle=\bar{e}^{i\Lambda}\langle\Phi\bar{\Phi}\rangle e^{i\bar{\Lambda}}$, it will be evident how to apply the multipole-expansion procedure of Sec. III to the differential form.

Let us substitute $\Xi=e^V$ and $\delta\Xi=\Xi\delta V$ into (2.8) and represent $(\bar{D}^2D^2\Xi)^{-1}$ in a parametric form. Then, as before, we take the argument $w=(u^\mu,\xi^\alpha,\bar{\xi}^{\dot{\alpha}})$ of δV to be a fixed position and carry through the multipole-expansion procedure. The result is

$$\delta\Gamma=-in\int d^6w\,\mathcal{M}(V)\delta V(w)\ ,$$
$$\mathcal{M}(V)=\int_0^\infty d\tau(\partial/\partial\tau)\langle 0\,|\exp(\tau\mathcal{N}[\mathscr{V}])\,|\,0\rangle\ , \tag{4.2}$$
$$\mathcal{N}[\mathscr{V}]=\tfrac{1}{16}\bar{D}^2D^2\exp(\mathscr{V})\ ,$$

where $d^6w=d^2u\,d^2\xi\,d^2\bar{\xi}$ now denotes the full measure and $\mathscr{V}$ stands for the vector superfield $\mathscr{V}(r,\theta,\bar{\theta})$ introduced in (3.15). Among the four-vector coefficients $v'_\mu(x,\xi,\bar{\xi})$ in $\mathscr{V}(r,\theta,\bar{\theta})$, the two-vector part (v'_0,v'_3) is given by (3.13). As seen from (3.12), v'_1 and v'_2 are gauge invariant. Let us make their Lorentz-scalar nature explicit by writing $S=\frac{1}{2}(v_1-iv_2)$ and $S^*=\frac{1}{2}(v_1+iv_2)$. By virtue of (3.10), they are expressed in the form

$$S(x,\xi,\bar{\xi})=\tfrac{1}{2}D_1\bar{D}_2V(x,\xi,\bar{\xi})$$

and (4.3)

$$S^*(x,\xi,\bar{\xi})=\tfrac{1}{2}D_2\bar{D}_1 v(x,\xi,\bar{\xi})\ .$$

Using the expression (3.16) for $V(x,\xi,\bar{\xi})$, $S(x,\xi,\bar{\xi})$ is expanded in component fields,

$$S=e^I\{\phi(x)+\xi_1\bar{\chi}_2(x)-\bar{\xi}_2\chi_1(x)+\xi_1\bar{\xi}_2[D(x)+if_{03}(x)]\}\ , \tag{4.4}$$

where $I=-\xi_1\bar{\xi}_1 p_-+\xi_2\bar{\xi}_2 p_+$ and $\phi(x)=\frac{1}{2}[B_1(x)-iB_2(x)]$; analogously for S^*.

The S and S^* are the gauge-invariant coefficient superfields of lowest dimension (one in units of mass) appearing in $\mathscr{V}(r,\theta,\bar{\theta})$. The $\mathscr{V}(r,\theta,\bar{\theta})$, as a matter of fact, is a functional of S and S^* alone. To see this explicitly, let us here introduce some notation: The spinor $\xi_\alpha=(\xi_1,\xi_2)$ is converted to a new Dirac spinor $\xi'_\alpha=(\xi_1,\bar{\xi}_2)$ by interchanging $\xi_2\leftrightarrow\bar{\xi}_2$. This induces the rearrangement of covariant derivatives:

$$D_\alpha\to\mathscr{D}_\alpha=(-\bar{D}_1,D_2)\ \text{and}\ \bar{D}_{\dot{\alpha}}\to\mathscr{D}_{\dot{\alpha}}=(-D_1,\bar{D}_2)\ . \tag{4.5}$$

These new derivatives obviously satisfy the same superalgebra as D_α and $\bar{D}_{\dot{\alpha}}$. Note that S and S^* are rewritten as

$$S=-\tfrac{1}{4}\overline{\mathscr{D}}^2V(x,\xi,\overline{\xi}) \text{ and } S^*=-\tfrac{1}{4}\mathscr{D}^2V(x,\xi,\overline{\xi}), \quad (4.6)$$

where $\mathscr{D}^2=\mathscr{D}^\alpha\mathscr{D}_\alpha$, etc. Hence, $S(x,\xi,\overline{\xi})$ is chiral in the sense that $\overline{\mathscr{D}}_{\dot\alpha}S=0$ ($\mathscr{D}_\alpha S^*=0$). Remember that this chirality condition is different from that for Φ^a and $\overline{\Phi}^a$. In this new notation, the remaining coefficient superfields W_α, $\overline{W}_{\dot\alpha}$, C, and $\mathscr{F}_{03}$ defined in (3.10) and (3.13) are expressed as follows:

$$(W_1,\overline{W}_2)=-\mathscr{D}_\alpha S,\quad (\overline{W}_1,W_2)=-\overline{\mathscr{D}}_{\dot\alpha}S^*,$$
$$C+i\mathscr{F}_{03}=\tfrac{1}{2}\mathscr{D}^2S,\quad C-i\mathscr{F}_{03}=\tfrac{1}{2}\overline{\mathscr{D}}^2S^*, \quad (4.7)$$

where the argument $(x,\xi,\overline{\xi})$ has been suppressed.

To calculate $\mathscr{M}(V)$, we consider the operator

$$G(\tau)=\exp(\tau\mathscr{N}[\mathscr{V}])-1, \quad (4.8)$$

whose Laplace transform $\widetilde{G}(s)$ obeys the algebraic equation

$$s(s-\mathscr{N}[\mathscr{V}])\widetilde{G}(s)=\mathscr{N}[\mathscr{V}]. \quad (4.9)$$

Note that $G(\tau)$ and $\widetilde{G}(s)$ are chiral to the left, i.e., $\overline{D}_{\dot\alpha}\widetilde{G}(s)=0$. We solve (4.9) for the matrix element $(r,\theta,\theta\,|\,\widetilde{G}(s)\,|\,0)$ in the same manner as done in Sec. III. It then follows from the definition of the Laplace transformation that $\mathscr{M}(V)$ is equal to the $s\to 0$ limit of $s(0\,|\,\widetilde{G}(s)\,|\,0)$, with the result written as

$$\mathscr{M}(V)=-((0\,|\,[\Pi^2+\tfrac{1}{2}C-\tfrac{1}{2}W(\overline{\Pi})^{-1}\overline{W}]^{-1}\,|\,0)) \quad (4.10)$$

in the notation of (3.20) and (3.22). Here $\Pi_\mu=p_\mu-\tfrac{1}{2}v'_\mu(x,\xi,\overline{\xi})$ for $\mu=0$ and 3; $\Pi_1=-\tfrac{1}{2}v_1=-\tfrac{1}{2}(S+S^*)$ and $\Pi_2=-\tfrac{1}{2}v_2=-i\tfrac{1}{2}(S-S^*)$.

A remark here is in order. In case $V_I(x,\theta,\overline{\theta})$ is used for $\mathscr{V}$ in Eq. (4.2), $s(0\,|\,\widetilde{G}(s)\,|\,0)$ in general depends on the chiral components of V_I but is still equal to (4.10) for $s=0$. This provides direct verification of the fact that $\mathscr{M}(V)$ is gauge invariant, which is manifest in the functional-integral formula (2.2).

We shall evaluate $\mathscr{M}(V)$, a functional of S and S^*, in a slowly-varying-field approximation where S and S^* are taken to be constant (i.e., x independent) superfields; correspondingly, from now on, W_α, $\overline{W}_{\dot\alpha}$, C, and $\mathscr{F}_{03}$ are all taken to be constant superfields. The present approximation retains the component fields ϕ, ϕ^*, χ_α, $\overline{\chi}_{\dot\alpha}$, D, and f_{03} (but not their derivatives) in the effective action. We shall expand $\mathscr{M}(V)$ in terms of the number of covariant derivatives $\mathscr{D}$ and $\overline{\mathscr{D}}$ involved, $\mathscr{M}(V)=\mathscr{M}^{(0)}+\mathscr{M}^{(2)}+\mathscr{M}^{(4)}+\cdots$, and evaluate terms up to $\mathscr{M}^{(4)}$.

The functional $\mathscr{M}(V)$, when expanded in powers of $W(\cdots)\overline{W}$, is naturally split into three terms, $\mathscr{M}(V)=I_0+I_1+I_2$, owing to the fermionic nature of constant spinors W_α and $\overline{W}_{\dot\alpha}$. The first-order term I_1 is cast in the form[19]

$$I_1=\tfrac{1}{4}((0\,|\,(\Pi^2+\tfrac{1}{2}C)^{-2}[a_1(\Pi^2-i\tfrac{1}{2}\mathscr{F}_{03})^{-1}+a_2(\Pi^2+i\tfrac{1}{2}\mathscr{F}_{03})^{-1}]\,|\,0)), \quad (4.11)$$

where $a_1=-2S^*W_1\overline{W}_2=S^*(\mathscr{D}^\alpha S)(\mathscr{D}_\alpha S)$ and $a_2=-2SW_2\overline{W}_1=S(\overline{\mathscr{D}}_{\dot\alpha}S^*)(\overline{\mathscr{D}}^{\dot\alpha}S^*)$. This is represented in the parametric form

$$I_1=\int_0^\infty d\tau\, Y(\tau)B(\tau), \quad (4.12)$$

$$Y(\tau)=((0\,|\,\exp[\tau(\Pi^2+\tfrac{1}{2}C)]\,|\,0))=(i/4\pi)[\tfrac{1}{2}\mathscr{F}_{03}/\sin(\tfrac{1}{2}\mathscr{F}_{03}\tau)]\exp[-\tau(SS^*-\tfrac{1}{2}C)], \quad (4.13)$$

$$B(\tau)=-\tfrac{1}{4}\int_0^\tau d\rho(\tau-\rho)(a_1e^{-c^*\rho}+a_2e^{-c\rho}), \quad (4.14)$$

where $c^*=\tfrac{1}{2}(C+i\mathscr{F}_{03})=\tfrac{1}{4}\mathscr{D}^2S$ and $c=\tfrac{1}{4}\overline{\mathscr{D}}^2S^*$. Equation (4.13) follows from the two-dimensional version of (3.23). The zeroth-order term I_0 is given by (4.12) with $B(\tau)\to 1$.

It is easy to extract $\mathscr{M}^{(0)}$ and $\mathscr{M}^{(2)}$ from I_0+I_1:

$$\mathscr{M}^{(0)}=-(i/4\pi)\ln(SS^*/\mu^2), \quad (4.15)$$

$$\mathscr{M}^{(2)}=(i/4\pi)\tfrac{1}{8}[\mathscr{D}^2S+\overline{\mathscr{D}}^2S^*-(a_1+a_2)/(SS^*)]/(SS^*)=(i/4\pi)\tfrac{1}{8}[(1/S^*)\mathscr{D}^2\ln S+(1/S)\overline{\mathscr{D}}^2\ln S^*], \quad (4.16)$$

where μ is an ultraviolet cutoff.

The classical action of the CP^{n-1} model possesses superconformal symmetry, which is broken by quantum anomalies. This symmetry begins to show up here in $\mathscr{M}^{(2)}$, which is not afflicted by the anomalies: Under superconformal transformations, V, $(\overline{\mathscr{D}}^2\ln S^*)/S$, $(\mathscr{D}^2\ln S)/S^*$, and any function of these transform as total derivatives.[12] From this and (4.2) we learn that, in general, the nonanomalous part of $\mathscr{M}(V)$, such as $\mathscr{M}^{(2)}$, transforms as a total derivative.

To derive $\mathscr{M}^{(4)}$, it is necessary to evaluate I_2. Its relevant portion is given by $I_2\approx-\tfrac{1}{8}W^2\overline{W}^2/(p^2-SS^*)^4$, whose effect is taken care of by adding $-\tfrac{1}{48}\tau^3W^2\overline{W}^2=\tfrac{1}{48}\tau^3a_1a_2/(SS^*)$ to $B(\tau)$ in (4.14). In view of the superconformal symmetry, it is convenient to express $\mathscr{M}^{(4)}$ in terms of S and S^* using the relations $a_1=(SS^*)S(\mathscr{D}^2\ln S+S\mathscr{D}^2S^{-1})$, $C+i\mathscr{F}_{03}=S(\mathscr{D}^2\ln S+\tfrac{1}{2}S\mathscr{D}^2S^{-1})$, etc. The result is

$$\mathscr{M}^{(4)}=(i/4\pi)\tfrac{1}{192}[(S^{*-1}\mathscr{D}^2\ln S)^2+(S^{-1}\overline{\mathscr{D}}^2\ln S^*)^2$$
$$-(2/S)\overline{\mathscr{D}}^2(1/S^*)\mathscr{D}^2\ln S$$
$$-(2/S^*)\mathscr{D}^2(1/S)\overline{\mathscr{D}}^2\ln S^*]. \quad (4.17)$$

The effective action $\Gamma[V]=\Gamma^{(0)}+\Gamma^{(2)}+\Gamma^{(4)}+\cdots$ is obtained by integrating $\mathscr{M}[V]$ over V. Note that

$$\int d^6w(\ln S)\delta V = -\int d^4w'\delta S \ln S \,, \tag{4.18}$$

where $d^4w' = d^2u\, d^2\xi'$ denotes the chiral measure for $\xi'_\alpha = (\xi_1, \bar{\xi}_2)$. The extra minus sign originates from the relation $d^2\xi\, d^2\bar{\xi} = -d^2\xi' d^2\bar{\xi}'$ with $\bar{\xi}'_\alpha = (\bar{\xi}_1, \xi_2)$. It is now easy to construct $\Gamma^{(0)}$ out of $\mathscr{M}^{(0)}$:

$$\Gamma^{(0)} = (n/4\pi)\left[\int d^4w' S[\ln(S/\mu) - 1] + \int d^4\bar{w}' S^*[\ln(S^*/\mu) - 1]\right], \tag{4.19}$$

where $d^4w' = d^2u\, d^2\xi'$ and $d^4\bar{w}' = d^2u\, d^2\bar{\xi}'$. The cutoff μ is replaced by a reference momentum μ_R through the coupling-constant renormalization $g = g_R/Z$ with the choice $Z = 1 + (g_R/2\pi)\ln(\mu/\mu_R)$. Furthermore, the tree term can effectively be included in $\Gamma^{(0)}$ by the replacement $1 \to 1 - (2\pi/g_R)$ in (4.19). For $\mathscr{M}^{(2)}$, we first convert δV to δS or δS^* by partial integrations. The subsequent integrations over S or S^* yield the expression

$$\Gamma^{(2)} = (n/4\pi)\tfrac{1}{2}\int d^6w'(\ln S)\ln S^* \,, \tag{4.20}$$

where $d^6w' = d^2u\, d^2\xi' d^2\bar{\xi}'$. Likewise, $\mathscr{M}^{(4)}$ is integrated to give $\Gamma^{(4)}$:

$$\Gamma^{(4)} = -(n/4\pi)\tfrac{1}{48}\int d^6w'[\ln S(\mathscr{D}^2\ln S)/S^* + \ln S^*(\bar{\mathscr{D}}^2\ln S^*)/S] \,. \tag{4.21}$$

These expressions (4.19)—(4.21) coincide[20] with those obtained by D'Adda *et al.* in Ref. 12, where also the higher-order terms $\Gamma^{(n)}$ ($n \geq 6$) are derived.

The $\Gamma^{(0)}$, whose structure is precisely what Veneziano and Yankielowicz[11] have proposed, embodies the correct anomalous behavior under chiral, conformal, and superconformal transformations. Both $\Gamma^{(2)}$ and $\Gamma^{(4)}$ are invariant under these transformations. We have here left it unattempted to obtain a closed expression for higher-order terms $\Gamma^{(n)}$ ($n \geq 6$).

In Ref. 12, the component-field effective action calculated by the usual proper-time method is translated into the superspace effective action $\Gamma = \Gamma^{(0)} + \Gamma^{(2)} + \Gamma^{(4)} + \cdots$; the supersymmetrization procedure, to a certain extent, relies on inspection guided by superconformal symmetry. On the other hand, we have constructed the effective action by working entirely in superspace; no reference to component-field calculations has been needed. In particular, the multipole-expansion procedure has led us to S and S^* as a natural set of fundamental superfields and eventually to the unique effective action (4.19)—(4.21).

We conclude this section by studying the effective potential, which is most easily obtained in the component form. Let us retain only ϕ and D as constant fields and denote the one-loop effective potential by $P(\phi,\phi^*,D)$ $(= -\Gamma/\int d^2u)$. The superfield equation (4.2) combined with (4.10) implies that

$$(\partial/\partial D)P(\phi,\phi^*,D) = i\tfrac{1}{2}n\mathscr{M}(V)\,|_{\xi=\bar{\xi}=0} = -i\tfrac{1}{2}n((0\,|\,(p^2 - \phi^*\phi + \tfrac{1}{2}D)^{-1}\,|\,0)) \,,$$

$$(\partial/\partial\phi)P(\phi,\phi^*,0) = (\partial/\partial\phi^*)P(\phi,\phi^*,0) = 0 \,. \tag{4.22}$$

This uniquely determines $P(\phi,\phi^*,D)$,

$$P(\phi,\phi^*,D) = -in(2\pi)^{-2} \times \int d^2p \int_0^{D/2} d\alpha (p^2 - \phi^*\phi + \alpha)^{-1} \,, \tag{4.23}$$

which is combined with the tree term $\frac{1}{2}(n/g)D$ to give the complete one-loop effective potential. As verified readily, the effective potential, first extremized with respect to D, attains the minimum value equal to zero for $\phi^*\phi = \mu_R{}^2\exp(-4\pi/g_R)$ (and then $D=0$); hence, supersymmetry is unbroken in the present model. This example will clarify that our approach here based on (4.2) is a supersymmetric generalization of Weinberg's tadpole method.[10]

V. CONCLUDING REMARKS

In this paper we have shown how to calculate effective actions in superspace by combined use of superfield functional integrals and some operator techniques. Our approach has been developed for supersymmetric gauge theories; its extension to nongauge theories is straightforward.

Superspace functional formulas, in general, are represented in a number of ways by adopting different superspace measures to define a trace, as seen in Sec. II. A lesson we have learned in Sec. III is that a suitable choice of functional formulas to start with simplifies actual calculations.

Our superspace multipole-expansion procedure plays a key role in systematizing the derivation of effective actions. This feature will prove useful when one studies models with extended and/or local supersymmetry.

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[19]Note that terms linear in Π_μ $(\mu=0,3)$ are vanishing in the $|0))\rightarrow|0))$ matrix element.

[20]The correspondence between our notations and those of Ref. 12 is the following: $(\xi_1,\xi_2)\rightarrow(1/\sqrt{2})(\theta_R,\theta_L)$, $(D_1,D_2)\rightarrow\sqrt{2}(D_L,-D_R)$, $(\bar{D}_1,\bar{D}_2)\rightarrow\sqrt{2}(-\bar{D}_L,\bar{D}_R)$, $d^2\xi'\rightarrow d\theta_R d\bar{\theta}_L$, $V\rightarrow -V$, $(S,S^*)\rightarrow -(S,\bar{S})$, $-\frac{1}{4}\mathscr{D}^2\ln S\rightarrow\Delta/S$, and $-\frac{1}{4}\bar{\mathscr{D}}^2\ln S^*\rightarrow\bar{\Delta}/\bar{S}$.