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Derivation of the blackbody radiation spectrum from the equivalence principle in classical physics with classical electromagnetic zero-point radiation

Timothy H. Boyer
Department of Physics, City College of the City University of New York, New York, New York 10031
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A derivation of Planck's spectrum including zero-point radiation is given within classical physics from recent results involving the thermal effects of acceleration through classical electromagnetic zero-point radiation. A harmonic electric-dipole oscillator undergoing a uniform acceleration $\vec{a}$ through classical electromagnetic zero-point radiation responds as would the same oscillator in an inertial frame when not in zero-point radiation but in a different spectrum of random classical radiation. Since the equivalence principle tells us that the oscillator supported in a gravitational field $\vec{g}=-\vec{a}$ will respond in the same way, we see that in a gravitational field we can construct a perpetual-motion machine based on this different spectrum unless the different spectrum corresponds to that of thermal equilibrium at a finite temperature. Therefore, assuming the absence of perpetual-motion machines of the first kind in a gravitational field, we conclude that the response of an oscillator accelerating through classical zero-point radiation must be that of a thermal system. This then determines the blackbody radiation spectrum in an inertial frame which turns out to be exactly Planck's spectrum including zero-point radiation.

INTRODUCTION

In the textbooks in physics and indeed in the minds of all but a few physicists, the derivation of the Planck radiation spectrum for blackbody radiation is the exclusive domain of theories based upon quantum ideas. The element of discontinuity, of discreteness, is deemed essential for the theoretical understanding of this form. However, recently a contrary view[1-3] has been introduced in connection with the idea of classical electromagnetic zero-point radiation. The presence of this random classical radiation with a Lorentz-invariant spectrum modifies the ideas of classical electron theory so that three classical derivations[1-3] of Planck's spectrum now appear in the literature. Each of these three derivations analyzes the interaction of radiation with a different mechanical system.

In this article we present a fourth derivation of Planck's spectrum based upon the equivalence principle and the assumed absence of a perpetual-motion machine in a gravitational field in classical physics with classical electromagnetic zero-point radiation. The arguments presented are based upon very general principles and the functional form derived is inescapable. Here we have another careful and compelling argument for Planck's spectrum as the thermal radiation spectrum of classical physics.

DERIVATION OF PLANCK'S SPECTRUM

This article is written as a companion to the preceding article[4] which is in turn a natural sequel to an article[5] of 1980. As explained in the preceding article,[4] our work is carried out in the context of classical electron theory in which the homogeneous boundary conditions on Maxwell's equations have been changed to include classical random electromagnetic radiation with a Lorentz-invariant spectrum, classical electromagnetic zero-point radiation.

We imagine that some classical mechanical systems, which are charged and so have (weak) interactions with the classical electromagnetic field, are supported in a small laboratory in a gravitational field $\vec{g}$. The principle of equivalence tells us that the effects viewed in this laboratory supported in a gravitational field are the same as those seen in a small laboratory which is accelerated relative to an inertial frame with acceleration $\vec{a}=-\vec{g}$.

Now if our charged mechanical systems are accelerating through classical electromagnetic zero-point radiation, then we have exactly the situation described in the preceding article.[4] A small oscillator responds[6] with a mean-square displacement

$$\langle x^2\rangle=\tfrac{1}{2}(\hbar/m\omega)\coth(\pi c\omega/a)$$

and mean-square velocity

$$\langle v^2\rangle=\tfrac{1}{2}(\hbar\omega/m)\coth(\pi c\omega/a)\ .$$

This is the response of an oscillator in an inertial frame bathed not in zero-point radiation but rather in a spectrum of (Gaussian) random classical electromagnetic radiation with a spectrum

$$\rho_a(\omega)=(\omega^2/\pi^2c^3)[\tfrac{1}{2}\hbar\omega\coth(\pi c\omega/a)]\ , \tag{1}$$

where the energy density u_a is connected to the spectral density by

$$u_a=\int_0^\infty d\omega\,\rho_a(\omega)\ . \tag{2}$$

The preceding paper[4] proved this result only for a small harmonic dipole oscillator. However, because the radiation-reaction correction[7] associated with the uniform acceleration was a correction holding universally for every charged particle independent of the specific form of the mechanical potential, it seems natural to expect that every mechanical system will show the same reported cancellation between the additional factor in the radiation reaction and in the radiation spectrum found from the correlation functions. The cancellation of these acceleration-related factors removes any evidence of an event horizon from the local mechanical behavior of the systems.

Now the mechanical systems in the accelerating laboratory exchange energy with the random radiation as though they were in an inertial frame with the spectrum (1), and they also exchange energy among themselves. In general only if the mechanical systems behave as they do in thermal equilibrium will there be no average energy transfer from one mechanical system to another. However, if there were energy transfer from one mechanical system to the other, then the interaction with the radiation would resupply the energy to the system which had lost the energy, and we have the basis for a perpetual-motion machine with continual transfer of mechanical energy from one system to another. If the laws of thermodynamics forbidding perpetual-motion machines hold locally in gravitational fields, then, from the principle of equivalence, we expect them to hold in accelerating frames. This tells us that the transfer of energy described above for the accelerating system is forbidden, and that the systems must behave as though they are in thermal equilibrium. The spectrum (1) must correspond to a thermal radiation spectrum.

The temperature of the spectrum in (1) can be computed from the total thermal energy density u_T. The thermal radiation must be energy above the zero-point energy so that the density of thermal radiation energy is

$$\begin{aligned} u_T &= \int_0^\infty d\omega[\rho_a(\omega)-\rho_0(\omega)] \\ &= (\hbar/2\pi^2c^3)\int_0^\infty d\omega\,\omega^3[\coth(\pi c\omega/a)-1] \\ &= (\hbar/2\pi^2c^3)(a/\pi c)^4\int_0^\infty dx\,x^3(\coth x-1)\ . \end{aligned} \tag{3}$$

Evaluating the thermal radiation density in terms of Stefan's constant,

$$\begin{aligned} u_T &= \sigma T^4 \\ &= (8k^4/\pi^2c^3\hbar^3)\int_0^\infty dx\,x^3(\coth x-1)T^4\ , \end{aligned} \tag{4}$$

and comparing with (3), we see that $\rho_a(\omega)$ corresponds to a temperature

$$T=\hbar a/2\pi ck\ . \tag{5}$$

Thus the spectrum for thermal radiation at temperature T is

$$\rho_T(\omega)=(\omega^2/\pi^2c^3)[\tfrac{1}{2}\hbar\omega\coth(\hbar\omega/2kT)]\ , \tag{6}$$

which is just Planck's spectrum including zero-point radiation.

DISCUSSION

The work described here has important antecedents in other articles and also has connections with unsolved problems in classical theory with classical zero-point radiation. First we note that crucial insights for the present derivation were provided by the work on thermal effects of acceleration within the context of quantum field theory and general relativity. Many people have been intrigued by the fact that acceleration through the scalar quantum vacuum gave effects with Planck's spectrum. Sciama, Candelas, and Deutsch,[8] and Candelas and Sciama[9] have addressed the question as to why the spectrum should be thermal. They write "To explain the precisely thermal character of these excess excitations we appeal to the stability of the vacuum state with respect to the switching-on of a small self-interaction of the field, together with the staticity of the regime internal to a uniformly accelerating detector. These two properties are sufficient [see R. Haag, D. Kastler, and E. B. Trich-Pohlmeyer, Commun. Math. Phys. 38, 173 (1974)] to ensure that an ensemble of such detectors would indeed come into equilibrium with the vacuum at a nonzero temperature. A closely related argument is based on another presumed property of the vacuum state, namely its role as the state of lowest energy. No cyclic process in the accelerated detector could extract work from the quantum field."[8] "A converse argument can also be made. By demanding that the spectrum of vacuum fluctuations be such that there be no net transfer of energy or momentum between the field and the charge it may be inferred that the Minkowski vacuum appears to a uniformly accelerated observer to comprise a thermal bath."[9]

In this article we have made use of these ideas. However, we have taken the discussion out of the realm of quantum field theory and into the context of a purely classical theory. Also our analysis uses results for a vector field rather than for the scalar field appearing in almost all the quantum calculations, and hence our analysis takes account of certain cancellations in acceleration-related factors which are necessary to remove the influence of the event horizon from the mechanical behavior of the system and so preserve the equivalence principle. Finally this article emphasizes taking the ideas of equilibrium back one step from the comments of Candelas and Sciama concerning the stability of the quantum vacuum[9] over to a familar local gravitational setting by use of the equivalence principle.

The second problem we must touch involves problems within the classical theory with classical electromagnetic zero-point radiation. Several authors[10–12] have shown that charged classical mechanical systems with harmonics scatter classical electromagnetic zero-point radiation in such a way as to alter the character of the spectrum. This behavior seems unacceptable within the theory and is inappropriate for the derivation of Planck's spectrum given here. I believe that the classical electromagnetic interactions of appropriate systems with harmonics have not yet been calculated sufficiently accurately. Our derivation indicates that if classical physics with classical electromagnetic zero-point radiation has a consistent thermodynamics, then the thermal radiation spectrum is exactly Planck's spectrum including zero-point radiation.

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