Vacuum polarization of massive fields near rotating black holes

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The vacuum polarization of the massive scalar, spinor, and vector fields in Kerr spacetime is investigated. The explicit expression for the vacuum expectation value of the energy-momentum tensor is obtained and its properties are discussed.

I. INTRODUCTION

Hawking[1] has shown that a black hole formed by collapse spontaneously creates and emits particles as if it were a hot body with a temperature proportional to the surface gravity of the black hole. The action of the gravitational field of a black hole on the virtual vacuum quanta can provide them (and with some probability really provides) with the energy that is sufficient to make these quanta real. A part of these quanta reaches infinity and forms the Hawking radiation of the black hole. It is not, however, the only result of the action of the gravitational field on the vacuum. The states of those virtual vacuum quanta which do not become real are also affected by the gravitational field. This results in the rise of a nonzero vacuum expectation value of the energy-momentum tensor. It should be noted that the latter effect takes place even in the case when the gravitational field is not strong enough to create real particles. The change of the vacuum expectation value of the local observables under the action of the external field is known as vacuum polarization. Because of the well-known ambiguity of the particle definition in a strong gravitational field it is impossible in the general case to separate the contributions of real and virtual particles to the expectation value of the energy-momentum tensor.

The investigation of the energy-momentum tensor of quantum fields in a given spacetime background is the natural first step in studying the problem of the self-consistent quantum description of black-hole evaporation. In the case of massless fields in two-dimensional spacetime the vacuum energy-momentum tensor is essentially defined by the conformal anomalies and its properties have been described in detail.[2-5] In the four-dimensional case the situation is much more complicated. Vacuum polarization of the massless scalar field in the Schwarzschild metric has been investigated in Refs. 6–13. Candelas[7] succeeded in obtaining the explicit expression for $\langle \phi^2 \rangle_{\text{ren}}$ at the event horizon. He also found some of the components of $\langle T_\mu^\nu \rangle_{\text{ren}}$ at the horizon. Fawcett and Whiting[8] and Fawcett[9] have obtained $\langle \phi^2 \rangle_{\text{ren}}$ and $\langle T_{\mu\nu} \rangle_{\text{ren}}$ in the Schwarzschild spacetime by numerical calculations. The results obtained by Page,[10] who developed the approximation method for studying vacuum polarization in the conformally ultrastatic spacetimes, are in good agreement with the numerical results. Another approach to the approximate calculation of vacuum polarization near the event horizon can be found in Ref. 11. Much less is known about vacuum polarization near rotating black holes. The explicit expression for $\langle \phi^2 \rangle_{\text{ren}}$ at the pole of the event horizon was found in Ref. 12. Some results concerning the generalization of Candelas's consideration to the case of massless fields of higher spins in Kerr geometry can be found in Ref. 13.

The problem of vacuum polarization of massive fields in curved spacetime has some features which make it differ from the case of vacuum polarization of massless fields. The main difference lies in the fact that in the case of a sufficiently massive field when the Compton length $\lambda = h/mc$ is much smaller than the characteristic radius L of the spacetime curvature, the contributions of real particles and vacuum polarization to the energy-momentum tensor can be separated. In this case the quasiclassical approximation can be used to define the notion of a particle.[14] In the framework of this approach one can show that the probability of particle creation is exponentially small. As to vacuum polarization its contribution to $\langle T_{\mu\nu}(x) \rangle_{\text{ren}}$ is local and is completely determined by the spacetime geometry in the vicinity of the point x. The part of $\langle T_{\mu\nu} \rangle_{\text{ren}}$ which describes vacuum polarization can be expanded in powers of $(\lambda/L)^2$. For this purpose one can use the DeWitt series expansion[15] of the effective action in powers of m^{-2}. In Ref. 16 this approach was used for studying $\langle \phi^2 \rangle_{\text{ren}}$ in the Kerr metric. The value of $\langle T_{\mu\nu} \rangle_{\text{ren}}$ for a massive scalar field in the Schwarzschild geometry was obtained in our work.[17]

The aim of this paper is to investigate contributions of massive scalar, spinor, and vector fields to vacuum polarization in the gravitational field of a rotating black hole. In Sec. II the general expression for the first nonvanishing term of the $1/m^2$ expansion of the renormalized effective action for scalar, spinor, and vector fields is obtained in Ricci-flat ($R_{\mu\nu}=0$) spacetimes. The Newman-Penrose approach is used in Sec. III to obtain $\langle T_{\mu\nu} \rangle_{\text{ren}}$ in the vacuum type-D geometries. The properties of the explicit expression for $\langle T_{\mu\nu} \rangle_{\text{ren}}$ in Kerr spacetime obtained in Sec.

IV are discussed in Sec. V. It is shown in particular that the effect of dragging the surrounding spacetime into rotation by a rotating black hole causes not only the mass of the black hole but also its angular momentum to be shifted by vacuum polarization. The contributions of massive fields to the shifts of the mass and the angular momentum are found.

We use the sign conventions of Misner, Thorne, and Wheeler[18] and Planck's units: $\hbar=c=G=1$.

II. THE EFFECTIVE ACTION

The natural way to attack the problem of vacuum polarization by the gravitational field in the case of massive fields is to use the effective-action approach developed by DeWitt.[15] If Φ^i is the mean value of a bosonic quantum field ϕ^i, $\Phi^i=\langle\phi^i\rangle$, then the effective action $W[\Phi]$ is a functional such that the mean field Φ^i satisfies the equation

$$\frac{\delta W[\Phi]}{\delta\Phi^i}=0 . \tag{2.1}$$

This functional can be constructed as

$$W[\Phi]=\frac{\hbar}{i}\ln Z[J]-\Phi^i J_i , \tag{2.2}$$

where

$$Z[J]=\int D\phi \exp\left[\frac{i}{\hbar}(S[\phi]+\phi^i J_i)\right] \tag{2.3}$$

and the source J_i should be expressed as a function of Φ^i with the aid of Eq. (2.4):

$$\Phi^i=\frac{\hbar}{i}\frac{\delta\ln Z[J]}{\delta J_i} . \tag{2.4}$$

We use here DeWitt's condensed notation and $S[\phi]$ is a classical action. Differentiating (2.2) gives

$$\frac{\delta W[\Phi]}{\delta\Phi^i}=-J_i . \tag{2.5}$$

When $J_i=0$, Eq. (2.4) reduces to the definition of the mean field and Eq. (2.5) shows that this mean field satisfies (2.1). Using Eqs. (2.2)—(2.5) we can write the equation for W in the form

$$e^{iW[\Phi]}=\int D\phi \exp\left[i\left[S[\phi]+(\Phi^i-\phi^i)\frac{\delta W[\Phi]}{\delta\Phi^i}\right]\right] . \tag{2.6}$$

In the one-loop approximation one has

$$W[\Phi]=S[\Phi]-\frac{i}{2}\operatorname{Tr}\ln G^{ij} , \tag{2.7}$$

where G^{ij} is a Green's function for the differential operator

$$F_{ij}=\delta^2 S[\Phi]/\delta\Phi^i\delta\Phi^j .$$

If the action $S[\phi]$ describes a free quantum field ϕ^i propagating in a given spacetime background with a metric $g_{\mu\nu}$ then the mean value $\langle T^{\mu\nu}\rangle$ of the energy-momentum tensor

$$T^{\mu\nu}=2|g|^{-1/2}\delta S/\delta g_{\mu\nu}$$

can be written as

$$\langle T^{\mu\nu}\rangle=\frac{2}{|g|^{1/2}}\frac{\delta W}{\delta g_{\mu\nu}} . \tag{2.8}$$

Because of the presence of the second term in Eq. (2.7), which is proportional to $\hbar$ and which describes the quantum corrections, the quantity $\langle T^{\mu\nu}\rangle$ does not vanish even in the case when $\Phi^i=0$.

It should be emphasized that the Green's function G^{ij} is unambiguously defined only in the space with the positive-definite metric. It means that the effective action has a well-defined meaning in the Euclidean section of the complex spacetime metric. By using the Schwinger-DeWitt representation one can write the Green's function in the space with the Riemannian metric as

$$G^{ij}(x,x')=\frac{\Delta^{1/2}}{16\pi^2}\int_0^\infty\frac{ds}{s^2}\exp\left[-m^2 s-\frac{\sigma}{2s}\right]a^{ij}(x,x';s) , \tag{2.9}$$

where m is the mass of the field ϕ^i, $\sigma(x,x')$ is the biscalar of a geodetic interval,

$$\Delta(x,x')=g^{-1/2}(x)D(x,x')g^{-1/2}(x') ,$$

$D(x,x')\equiv-\det(-\sigma_{;\mu\nu'})$ is the Van Vleck—Morette determinant, and

$$a^{ij}(x,x';s)=\sum_{n=0}^{\infty}a_n^{ij}(x,x')s^n . \tag{2.10}$$

The matrices $a_n^{ij}(x,x')$ are defined by a chain of recursion relations.[15] Equation (2.9) allows one to obtain the following expression for the Euclidean effective action $W=-\frac{1}{2}\operatorname{Tr}\ln G^{ij}$:

$$W=\int dx\,|g|^{1/2}\mathscr{L} , \tag{2.11}$$

$$\mathscr{L}=\lim_{l\to\infty}\frac{1}{32\pi^2}\int_{1/l^2}^{\infty}\frac{ds}{s^3}e^{-sm^2}\sum_{n=0}^{\infty}s^n\operatorname{tr}[a_n^{ij}(x,x')] , \tag{2.12}$$

where the ultraviolet divergence is regularized by the introduction of a positive lower limit in the proper-time integral and tr means the trace over the matrix indices. As a result we find

$$W=W_{\text{div}}+W_{\text{ren}} , \tag{2.13}$$

$$W_{\text{div}}=\frac{1}{32\pi^2}\operatorname{tr}\int dx\,|g|^{1/2}[f_0 a_0(x,x)+f_1 a_1(x,x)+f_2 a_2(x,x)] , \tag{2.14}$$

$$W_{\text{ren}}=\frac{1}{32\pi^2}\sum_{n=3}^{\infty}\frac{(n-3)!}{(m^2)^{n-2}}\int \operatorname{tr}a_n(x,x)|g|^{1/2}dx , \tag{2.15}$$

where

$$\begin{aligned}f_0&=-m^2l^2+\frac{m^4}{2}\ln l^2+\frac{m^4}{2}(\tfrac{3}{2}-C-\ln m^2)+\tfrac{1}{2}l^4 ,\\ f_1&=l^2-m^2\ln l^2+m^2(C-1+\ln m^2) ,\\ f_2&=\ln l^2-C-\ln m^2 ,\end{aligned} \tag{2.16}$$

C being Euler's constant. Schwinger's renormalization prescription tells us that the renormalized effective action is $W_{\rm ren}=W-W_{\rm div}$. $W_{\rm div}$ can be absorbed into a classical gravitational action of the form

$$S_{\rm grav}=\int |g|^{1/2}dx(\lambda_0+\lambda_1 R+\lambda_2 R^2+\lambda_3 R_{\mu\nu}R^{\mu\nu}) . \tag{2.17}$$

In what follows we shall consider the Ricci-flat metrics so that there is no ambiguity in $\langle T^{\mu\nu}\rangle$ connected with the finite renormalization of the coupling constants λ_2 and λ_3.

Substituting (2.15) in (2.8) gives the series expansion for the normalized expectation value of the energy-momentum tensor in powers of the parameter $\epsilon=(\lambdabar/L)^2$, where $\lambdabar$ is the Compton length $\hbar/mc$ and L is the characteristic radius of the spacetime curvature.

We are interested in obtaining $\langle T^{\mu\nu}\rangle$ for scalar ($s=0$), Dirac spinor ($s=\frac{1}{2}$), and vector ($s=1$) massive fields satisfying the equations

$$(\nabla^\epsilon\nabla_\epsilon-\xi R-m^2)\Phi^{(0)}=0 , \tag{2.18a}$$

$$(\gamma^\epsilon\nabla_\epsilon+m)\Phi^{(1/2)}=0 , \tag{2.18b}$$

$$(\delta^\beta_\alpha\nabla^\epsilon\nabla_\epsilon-\nabla_\alpha\nabla^\beta-R^{\ \beta}_{\alpha}-\delta^\beta_\alpha m^2)\Phi^{(1)}_\beta=0 \tag{2.18c}$$

in the spacetime of a rotating black hole. The corresponding Kerr metric in Boyer-Lindquist coordinates is of the form

$$ds^2=-\left[1-\frac{2Mr}{\Sigma}\right]dt^2-\frac{4Mra}{\Sigma}\sin^2\theta\,dt\,d\phi+\left[r^2+a^2+\frac{2Mra^2}{\Sigma}\sin^2\theta\right]\sin^2\theta\,d\phi^2+\frac{\Sigma}{\Delta}dr^2+\Sigma\,d\theta^2 , \tag{2.19}$$

where $\Delta=r^2+a^2-2Mr$ and $\Sigma=r^2+a^2\cos^2\theta$. M and $J=aM$ are the mass and the angular momentum of a black hole.

Before applying the described approach to this particular problem we are to make the following remarks. First of all the Schwinger-DeWitt technique is directly applicable only to second-order operators F_{ij} with leading derivatives of the form

$$F_{ij}=Q_{ij}g^{\mu\nu}\nabla_\mu\nabla_\nu+\cdots , \tag{2.20}$$

where Q_{ij} is a local and invertible matrix and ∇_μ is a covariant derivative with any connection acting on any field. Only Eq. (2.18a) is of the form. In the spinor case one can introduce a new spinor variable $\tilde\Phi^{(1/2)}$ connected with $\Phi^{(1/2)}$ by the relation

$$\Phi^{(1/2)}=\gamma^\epsilon\tilde\Phi^{(1/2)}_{;\epsilon}-m\tilde\Phi^{(1/2)} , \tag{2.21}$$

so that Eq. (2.18a) takes the necessary form

$$(\nabla^\epsilon\nabla_\epsilon-\tfrac{1}{4}R-m^2)I\tilde\Phi^{(1/2)}=0 , \tag{2.22}$$

where I is the unit four-dimensional matrix. As to the vector field the nondiagonal term $\nabla_\alpha\nabla^\beta\Phi^{(1)}_\beta$ in Eq. (2.18c) is an obstacle to applying the Schwinger-DeWitt technique. The generalization of this technique which allows one to overcome this difficulty was developed by Barvinsky and Vilkovisky.[19] In our particular case the result can be obtained in the following way. (The authors are greatful to Dr. Vilkovisky and Dr. Barvinsky for indicating this possibility.) One can verify that the operators $\hat D$ and $\hat S$,

$$D^\beta_\alpha=\delta^\beta_\alpha\nabla^\epsilon\nabla_\epsilon-R^{\ \beta}_{\alpha},\quad S^\beta_\alpha=\nabla_\alpha\nabla^\beta , \tag{2.23}$$

satisfy the relations $\hat D\hat S=\hat S\hat D=\hat S\hat S$ and hence

$$(\hat D-\hat S-\hat m^2)^{-1}=\frac{1}{m^2}(\hat m^2-\hat S)(\hat D-\hat m^2)^{-1} . \tag{2.24}$$

Note that the operator $\hat F=\hat D-\hat S-\hat m^2$ coincides with the differential operator in Eq. (2.18c) so that omitting an inessential constant we have for the effective action

$$W=\frac{i}{2}\mathrm{Tr}\ln\hat F=\frac{i}{2}[\mathrm{Tr}\ln(\hat D-\hat m^2)-\mathrm{Tr}\ln(\Box-m^2)] . \tag{2.25}$$

This shows that the effective action for the massive vector field is equal to the effective action for the operator $\hat D-\hat m^2$ which is of the form (2.20) minus the effective action for a scalar field (2.18a) with $\xi=0$.

The general expression for a_3 [matrices for the operators of the form (2.20)] was obtained by Gilkey.[20,21] By using his results one can write

$$W^s_{\rm ren}=\frac{\alpha_s}{288\times 7!\pi^2m^2}\int dx\,|g|^{1/2}(-7R_{\alpha\beta\gamma\delta;\epsilon}R^{\alpha\beta\gamma\delta;\epsilon}+24R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\epsilon\zeta}R_{\epsilon\zeta}{}^{\alpha\beta}-8R^{\ \beta}_{\alpha}R^{\alpha\gamma\delta\epsilon}R_{\beta\gamma\delta\epsilon}+\mu_sRR_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})+\cdots+O(m^{-4}) , \tag{2.26}$$

where the ellipsis denotes the omitted terms which do not contribute to $\langle T_{\mu\nu}\rangle$ in the Ricci-flat ($R_{\mu\nu}=0$) spacetime and

$$\alpha_s=\begin{Bmatrix}\alpha_0\\ \alpha_{1/2}\\ \alpha_1\end{Bmatrix}=\begin{Bmatrix}1\\ -4\\ 3\end{Bmatrix},\quad \mu_s=\begin{Bmatrix}\mu_0\\ \mu_{1/2}\\ \mu_1\end{Bmatrix}=\begin{Bmatrix}42-252\xi\\ -21\\ -42\end{Bmatrix} . \tag{2.27}$$

We show now that Eq. (2.26) allows further simplification. Using the Bianchi identities one can write

$$R_{\alpha\beta\gamma\delta;\epsilon}R^{\alpha\beta\gamma\delta;\epsilon}=[\tfrac{1}{2}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})^{;\epsilon}+4R_{\alpha\beta\gamma}{}^{\epsilon}R^{\alpha\gamma;\beta}]_{;\epsilon}-4R^{\alpha\gamma;\beta}(R_{\alpha\gamma;\beta}-R_{\beta\gamma;\alpha})+4R_{\alpha\beta\gamma\delta}R^{\alpha\epsilon\gamma\zeta}R_{\epsilon}{}^{\beta}{}_{\zeta}{}^{\delta}+R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\epsilon\zeta}R_{\epsilon\zeta}{}^{\alpha\beta}-2R^{\alpha}{}_{\lambda}R_{\alpha\beta\gamma\delta}R^{\lambda\beta\gamma\delta} . \tag{2.28}$$

After inserting this relation in Eq. (2.26) we conclude that the term in the square brackets is a pure divergence and it can be omitted. In the Ricci-flat metric the second term on the right-hand side of Eq. (2.28) does not contribute to $\langle T_{\mu\nu}\rangle$ and thus it is also inessential. To simplify the expression for $W^s_{\rm ren}$ we take into account the relations between tensor invariants constructed from the Weyl tensor $C_{\alpha\beta\gamma\delta}$. Let us denote

$$
\begin{aligned}
&I^\nu_\mu = C_{\mu\beta\gamma\delta}C^{\nu\beta\gamma\delta}, \quad J^\nu_{1\mu} = C_{\mu\beta\gamma\delta}C^{\nu\beta\epsilon\zeta}C_{\epsilon\zeta}{}^{\gamma\delta}, \\
&J^\nu_{2\mu} = C_{\mu\beta\gamma\delta}C^{\nu\epsilon\gamma\zeta}C_\epsilon{}^\beta{}_\zeta{}^\delta, \quad J^\nu_{3\mu} = C_{\mu\beta\gamma\delta}C^{\nu\epsilon\gamma\zeta}C_\zeta{}^\beta{}_\epsilon{}^\delta, \\
&J^\nu_{4\mu} = C_{\mu\beta\gamma\delta}C^{\nu\gamma\epsilon\zeta}C_\epsilon{}^\beta{}_\zeta{}^\delta, \quad J^\nu_{5\mu} = C_{\mu\beta\gamma\delta}C^{\nu\gamma\epsilon\zeta}C_{\epsilon\zeta}{}^{\beta\delta},
\end{aligned} \tag{2.29}
$$

$$I \equiv I^\epsilon{}_\epsilon, \quad J \equiv J^\epsilon_{1\epsilon}, \quad J_i \equiv J^\epsilon_{i\epsilon} .$$

Using the symmetry properties of the Weyl tensor $C^\alpha{}_{\beta\alpha\delta}=0$ and $C_{\alpha\beta\gamma\delta}=C_{\gamma\delta\alpha\beta}=C_{[\alpha\beta][\gamma\delta]}$ and the identity $C^{\mu[\beta}{}_{\gamma\delta}C^{\gamma\delta}{}_{\epsilon\zeta}C^{\epsilon\zeta]}{}_{\nu\beta}=0$ gives the following relations:

$$J^\nu_\mu \equiv J^\nu_{1\mu} = 2J^\nu_{2\mu} = 4J^\nu_{3\mu} = 4J^\nu_{4\mu} = 2J^\nu_{5\mu} . \tag{2.30}$$

One can also verify that

$$I^\nu_\mu = \tfrac{1}{4}I\delta^\nu_\mu, \quad J^\nu_{i\mu} = \tfrac{1}{4}J_i\delta^\nu_\mu . \tag{2.31}$$

These relations allow us to rewrite Eq. (2.26) in the following final form:

$$W^s_{\rm ren} = \frac{1}{96\times 7!\pi^2 m^2}\int dx\,|g|^{1/2}(\alpha_s R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\epsilon\zeta}R_{\epsilon\zeta}{}^{\alpha\beta} + \tfrac{1}{12}\beta_s R R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}) + \cdots + O(m^{-4}) , \tag{2.32}$$

where

$$\alpha_s = \begin{Bmatrix} 1 \\ -4 \\ 3 \end{Bmatrix}, \quad \beta_s = \begin{Bmatrix} 216-1008\xi \\ 144 \\ -360 \end{Bmatrix} . \tag{2.33}$$

It should be remembered that the above consideration has a well-defined meaning only for a space with the positive-definite metric. In order to obtain the results in physical spacetime one can use analytical continuation. In the general case analytical continuation of $\langle T_{\mu\nu}\rangle$ gives

$$\langle T_{\mu\nu}\rangle = \frac{\langle 0;\text{out}\,|\,\hat{T}_{\mu\nu}\,|\,0;\text{in}\rangle}{\langle 0;\text{out}\,|\,0;\text{in}\rangle} . \tag{2.34}$$

In our case the Euclidean Kerr metric is described by Eq. (2.19) where the transformation

$$a = ib, \quad t = -i\tau \tag{2.35}$$

is performed. The Euclidean Kerr metric is regular if τ is periodic with the period $\kappa/2\pi$ where $\kappa=(r_+ - r_-)/4Mr_+$ and $r_\pm = M\pm(M^2+b^2)^{1/2}$. In this particular case analytical continuation of the Euclidean effective action to physical spacetime creates the mean value

$$\langle T_{\mu\nu}\rangle = \langle H\,|\,\hat{T}_{\mu\nu}\,|\,H\rangle , \tag{2.36}$$

where $|H\rangle$ is the Hartle-Hawking vacuum state.[22]

It is worth noting that the differences between this mean value and the mean values in the Boulware vacuum[23] $|B\rangle$ and the Unruh vacuum[3] $|U\rangle$ states are proportional to the factor $\exp(-m/T_{\rm BH})$. This difference can be neglected everywhere except the exponentially narrow strip around the horizon.

III. $\langle T^\nu_\mu\rangle^s_{\rm ren}$ IN VACCUM SPACETIME OF TYPE *D*

Functionally differentiating Eq. (2.32) with respect to $g_{\mu\nu}$ gives

$$\langle T^{\mu\nu}\rangle^s_{\rm ren} = \frac{2}{|g|^{1/2}}\frac{\delta W^s}{\delta g_{\mu\nu}} ,$$

$$\langle T^{\mu\nu}\rangle^s_{\rm ren} = \frac{1}{96\times 7!\pi^2 m^2}(\alpha_s T^{\mu\nu}_1 + \beta_s T^{\mu\nu}_2) + O(m^{-4}) ,$$

$$
\begin{aligned}
&T^\nu_{1\mu} = -6(C_{\mu\beta\gamma\delta}C^{\nu\alpha\gamma\delta} + C^\nu{}_{\beta\gamma\delta}C_\mu{}^{\alpha\gamma\delta})_{;\alpha}{}^{;\beta} - \tfrac{1}{2}\delta^\nu_\mu J , \\
&T^\nu_{2\mu} = \tfrac{1}{6}I_{;\mu}{}^{;\nu} - \tfrac{1}{6}\delta^\nu_\mu I_{;\epsilon}{}^{;\epsilon} .
\end{aligned} \tag{3.1}
$$

The Bianchi identities and Eqs. (2.30) and (2.31) allow one to rewrite the expression for $T^\nu_{1\mu}$ in the form

$$T^\nu_{1\mu} = 6C_{\alpha\beta\gamma\delta;\mu}C^{\alpha\beta\gamma\delta;\nu} - \tfrac{3}{2}\delta^\nu_\mu I_{;\epsilon}{}^{;\epsilon} - 5\delta^\nu_\mu J . \tag{3.2}$$

Let us introduce the following complex quantities:

$$
\begin{aligned}
I^{+\nu}_\mu &\equiv C^+_{\mu\beta\gamma\delta}C^{+\nu\beta\gamma\delta} = \tfrac{1}{2}(C_{\mu\beta\gamma\delta}C^{\nu\beta\gamma\delta} + iC^*_{\mu\beta\gamma\delta}C^{\nu\beta\gamma\delta}) , \\
J^{+\nu}_\mu &\equiv C^+_{\mu\beta}{}^{\gamma\delta}C^+_{\gamma\delta}{}^{\epsilon\zeta}C^+_{\epsilon\zeta}{}^{\nu\beta} \\
&= \tfrac{1}{2}(C_{\mu\beta}{}^{\gamma\delta}C_{\gamma\delta}{}^{\epsilon\zeta}C_{\epsilon\zeta}{}^{\nu\beta} + iC^*_{\mu\beta}{}^{\gamma\delta}C_{\gamma\delta}{}^{\epsilon\zeta}C_{\epsilon\zeta}{}^{\nu\beta}) , \\
K^{+\nu}_\mu &\equiv C^+_{\alpha\beta\gamma\delta;\mu}C^{+\alpha\beta\gamma\delta;\nu} \\
&= \tfrac{1}{2}(C_{\alpha\beta\gamma\delta;\mu}C^{\alpha\beta\gamma\delta;\nu} + iC^*_{\alpha\beta\gamma\delta;\mu}C^{\alpha\beta\gamma\delta;\nu}) ,
\end{aligned} \tag{3.3}
$$

where

$$
\begin{aligned}
&C^+_{\alpha\beta\gamma\delta} \equiv \tfrac{1}{2}(C_{\alpha\beta\gamma\delta} + iC^*_{\alpha\beta\gamma\delta}), \quad C^*_{\alpha\beta\gamma\delta} \equiv \tfrac{1}{2}\epsilon_{\alpha\beta\rho\sigma}C^{\rho\sigma}{}_{\gamma\delta} , \\
&C^*_{\alpha\beta\gamma\delta}C^{*\gamma\delta\epsilon\zeta} = -C_{\alpha\beta\gamma\delta}C^{\gamma\delta\epsilon\zeta}, \quad C^*_{\alpha\beta\gamma\delta}C^{\gamma\delta\epsilon\zeta} = C_{\alpha\beta\gamma\delta}C^{*\gamma\delta\epsilon\zeta} .
\end{aligned} \tag{3.4}
$$

Then we have

$$
\begin{aligned}
&T^\nu_{1\mu} = \mathrm{Re}(12K^{+\nu}_\mu - 3\delta^\nu_\mu I^{+;\epsilon}_{;\epsilon} - 10\delta^\nu_\mu J^+) , \\
&T^\nu_{2\mu} = \mathrm{Re}(\tfrac{1}{3}I^{+;\nu}_{;\mu} - \tfrac{1}{3}\delta^\nu_\mu I^{+;\epsilon}_{;\epsilon}) .
\end{aligned} \tag{3.5}
$$

We restrict ourselves by considering vacuum type-*D* background geometries. Let k^μ and l^μ be the principal null vectors normalized by the condition $l^\mu k_\mu = -1$. In the null complex tetrad

$$e^\mu_a = (k^\mu, l^\mu, m^\mu, \bar{m}^\mu) ,$$

$$e^\mu_a e_{b\mu} = \eta_{ab}, \quad \eta_{ab} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} , \tag{3.6}$$

the Weyl tensor $C_{\alpha\beta\gamma\delta}$ has only one essential component:

$$\psi \equiv \psi_2 = \tfrac{1}{2} C_{\alpha\beta\gamma\delta} k^{\alpha} l^{\beta} (k^{\gamma} l^{\delta} - m^{\gamma} \bar{m}^{\delta}) . \tag{3.7}$$

Denote by ζ_a^A the basis in a spinor space which is connected with e_a^{μ} by the relations

$$\begin{aligned} e_1^{\mu} &\equiv k^{\mu} = \sigma^{\mu}_{AA'} \zeta_0^A \bar{\zeta}_{0'}^{A'} \equiv \sigma^{\mu}_{00'} , \\ e_2^{\mu} &\equiv l^{\mu} = \sigma^{\mu}_{AA'} \zeta_1^A \bar{\zeta}_{1'}^{A'} \equiv \sigma^{\mu}_{11'} , \\ e_3^{\mu} &\equiv m^{\mu} = \sigma^{\mu}_{AA'} \zeta_0^A \bar{\zeta}_{1'}^{A'} \equiv \sigma^{\mu}_{01'} , \\ e_4^{\mu} &\equiv \bar{m}^{\mu} = \sigma^{\mu}_{AA'} \zeta_1^A \bar{\zeta}_{0'}^{A'} \equiv \sigma^{\mu}_{10'} , \end{aligned} \tag{3.8}$$

where $\sigma^{\mu}_{AA'}$ are the van der Waerden symbols satisfying the equations

$$\sigma^{\mu}_{AA'} \sigma_{\mu}^{BB'} = \delta_A^B \delta_{A'}^{B'}, \quad \sigma^{\mu}_{AA'} \sigma_{\nu}^{AA'} = \delta^{\mu}_{\nu} . \tag{3.9}$$

We shall use the following standard Newman-Penrose notation[24] for the spin coefficients $\Gamma_{abcd'} = \epsilon_{al} \sigma^{\mu}_{cd'} \zeta_A^l \nabla_{\mu} \bar{\zeta}_b^A$:

$cd' \backslash ab$	00	01 and 10	11
$00'$	κ	ϵ	π
$\Gamma_{abcd'} =$ $10'$	ρ	α	λ
$01'$	σ	β	μ
$11'$	τ	γ	ν

(3.10)

In type-D vacuum spacetime such a choice of a spinor basis exists in which the following statements are valid[25]: (i) The spin coefficients κ, σ, ν, λ, and ϵ are equal to zero, and (ii) the Weyl spin ψ_{abcd},

$$\psi_{abcd} = \tfrac{1}{4} \epsilon^{a'b'} \epsilon^{c'd'} \sigma^{\alpha}_{aa'} \sigma^{\beta}_{bb'} \sigma^{\gamma}_{cc'} \sigma^{\delta}_{dd'} C^{+}_{\alpha\beta\gamma\delta} , \tag{3.11}$$

is of the form $\psi_{abcd} = 6\psi \delta^0_{(a} \delta^0_b \delta^1_c \delta^1_{d)}$, where ψ is given by Eq. (3.7). Using this basis we find for the complex tensors (3.3)

$$\begin{aligned} &I_{\mu}^{+\nu} = 6\psi^2 \delta^{\nu}_{\mu}, \quad J_{\mu}^{+\nu} = -12\psi^3 \delta^{\nu}_{\mu} , \\ &K_{\mu}^{+\nu} = 24 \sigma^{\nu nn'} \sigma_{\mu}^{mm'} (\partial_{nn'} \psi \partial_{mm'} \psi + 2\Gamma_{abmm'} \Gamma^{ab}{}_{nn'} \psi^2 - 2\Gamma_{abmm'} \psi^{bd}{}_{ez} \Gamma_{cdnn'} \psi^{acez}) . \end{aligned} \tag{3.12}$$

Substituting these relations in Eq. (3.5) we finally obtain the following expression for the vacuum energy-momentum tensor in the vacuum type-D spacetime:

$$\begin{aligned} T^{\nu}_{1\mu} = -24\,\mathrm{Re}\{ &3\delta^{\nu}_{\mu} (\psi^2)_{;\epsilon}{}^{;\epsilon} - 20\delta^{\nu}_{\mu} \psi^3 - 12 g^{\nu\alpha} \partial_{\mu} \psi \partial_{\alpha} \psi - 72\psi^2 [\, \pi\tau (k_{\mu} l^{\nu} + l_{\mu} k^{\nu}) + \rho\mu (m_{\mu} \bar{m}^{\nu} + \bar{m}_{\mu} m^{\nu}) \\ &- \mu\tau (k_{\mu} \bar{m}^{\nu} + \bar{m}_{\mu} k^{\nu}) - \rho\pi (l_{\mu} m^{\nu} + m_{\mu} l^{\nu})]\} , \\ T^{\nu}_{2\mu} = 8\,\mathrm{Re}[&(\psi^2)_{;\mu}{}^{;\nu} - \delta^{\nu}_{\mu} (\psi^2)_{;\epsilon}{}^{;\epsilon}] . \end{aligned} \tag{3.13}$$

IV. VACUUM ENERGY-MOMENTUM TENSOR IN KERR SPACETIME

In this section we obtain the explicit expression for the vacuum energy-momentum tensor of massive fields in Kerr spacetime. It is convenient to choose the null complex tetrad e_a^{μ} in Kerr geometry which in the Boyer-Lindquist coordinates is of the form

$$\begin{aligned} k^{\mu} &= \left[\frac{r^2+a^2}{\Delta}, 1, 0, \frac{a}{\Delta} \right] , \\ l^{\mu} &= \frac{1}{2\Sigma} (r^2+a^2, -\Delta, 0, a) , \\ m^{\mu} &= -\frac{1}{\sqrt{2}} \bar{\rho} \left[ia \sin\theta, 0, 1, \frac{i}{\sin\theta} \right] , \\ \bar{m}^{\mu} &= -\frac{1}{\sqrt{2}} \rho \left[-ia \sin\theta, 0, 1, -\frac{i}{\sin\theta} \right] . \end{aligned} \tag{4.1}$$

The spin coefficients ρ, π, τ, and μ which enter Eq. (3.13) are

$$\begin{aligned} \rho &= -(r - ia \cos\theta)^{-1} , \\ \pi &= \rho^2 \bar{\tau}^0 , \\ \tau &= \tau^0 / \Sigma , \\ \mu &= \rho \Delta / 2\Sigma , \\ \tau^0 &= -ia \sin\theta / \sqrt{2} , \end{aligned} \tag{4.2}$$

and

$$\psi = M\rho^3 .$$

Rather long but straightforward calculations give

$$\begin{aligned} \langle T^{\nu}_{\mu} \rangle^{s}_{\mathrm{ren}} &= \frac{M^2}{10\,080 \pi^2 m^2 r_+^8} \tau_{\mu}^{(s)\nu} + O(m^{-4}) , \\ \tau_{\mu}^{(s)\nu} &= r_+^8 \,\mathrm{Re}(\alpha_s \tau^{\nu}_{1\mu} + \beta_s \tau^{\nu}_{2\mu}) , \\ \tau^{t}_{1t} &= \rho^8 \left[-45 + 106 \frac{Mr}{\Sigma} \right] + 8\rho^7 \frac{M}{\Sigma} , \\ \tau^{\phi}_{1t} &= 0 , \\ \tau^{t}_{1\phi} &= 36\rho^8 \frac{Mr}{a} \left[1 - \frac{r^2+a^2}{\Sigma} \right] , \\ \tau^{\phi}_{1\phi} &= \rho^8 \left[-27 + 70 \frac{Mr}{\Sigma} \right] + 8\rho^7 \frac{M}{\Sigma} , \\ \tau^{r}_{1r} &= \rho^8 \left[-27 + 36 \frac{r^2+a^2}{\Sigma} - 2 \frac{Mr}{\Sigma} \right] + 8\rho^7 \frac{M}{\Sigma} , \\ \tau^{\theta}_{1r} &= 36 i \rho^8 \frac{a \sin\theta}{\Sigma} , \\ \tau^{r}_{1\theta} &= \tau^{\theta}_{1r} \Delta , \\ \tau^{\theta}_{1\theta} &= \rho^8 \left[9 - 36 \frac{r^2+a^2}{\Sigma} + 70 \frac{Mr}{\Sigma} \right] + 8\rho^7 \frac{M}{\Sigma} , \end{aligned} \tag{4.3}$$

$$
\begin{aligned}
\tau^t_{2t} &= -5\rho^8 + \rho^8 \frac{Mr}{\Sigma^2}[12\Sigma + 2(r^2+a^2)] \\
&\quad + \rho^7 \frac{M}{\Sigma^3}[2\Sigma^2 - 2\Sigma r^2 - (r^2+a^2)(\Sigma - 4r^2)] , \\
\tau^\phi_{2t} &= 2\rho^8 \frac{Mra}{\Sigma^2} + \rho^7 \frac{Ma}{\Sigma^3}(-\Sigma + 4r^2) , \\
\tau^t_{2\phi} &= -2\rho^8 \frac{Mr}{a\Sigma^2}[\Sigma - (r^2+a^2)]^2 \\
&\quad + \rho^7 \frac{M}{a\Sigma^3}(r^2+a^2)[\Sigma - (r^2+a^2)](-\Sigma + 4r^2) , \\
\tau^\phi_{2\phi} &= -6\rho^8 + \rho^8 \frac{Mr}{\Sigma^2}[16\Sigma - 2(r^2+a^2)] \\
&\quad + \rho^7 \frac{M}{\Sigma^3}[\Sigma^2 + 2\Sigma r^2 + (r^2+a^2)(\Sigma - 4r^2)] , \\
\tau^r_{2r} &= -6\rho^8 + 8\rho^8 \frac{r^2+a^2}{\Sigma} + \rho^7 \frac{M}{\Sigma^2}(\Sigma + 2r^2) , \\
\tau^\theta_{2r} &= 8i\rho^8 \frac{a \sin\theta}{\Sigma} , \\
\tau^r_{2\theta} &= \tau^\theta_{2r}\Delta , \\
\tau^\theta_{2\theta} &= \rho^8 \left[2 - 8\frac{r^2+a^2}{\Sigma} + 14\frac{Mr}{\Sigma} \right] + \rho^7 \frac{M}{\Sigma^2}(2\Sigma - 2r^2) ,
\end{aligned}
\tag{4.4}
$$

where $\rho = -(r - ia\cos\theta)^{-1}$, $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 - 2Mr$,

$$
\alpha_s = \begin{Bmatrix} \alpha_0 \\ \alpha_{1/2} \\ \alpha_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -4 \\ 3 \end{Bmatrix} ,
\qquad
\beta_s = \begin{Bmatrix} \beta_0 \\ \beta_{1/2} \\ \beta_1 \end{Bmatrix} = \begin{Bmatrix} 216 - 1008\xi \\ 144 \\ -360 \end{Bmatrix} .
\tag{4.5}
$$

One can verify that $\tau_i^{\mu\nu}$ satisfies the conservation law $\tau^\nu_{i\mu;\nu} = 0$ and the following relations are valid at the event horizon:

$$
\begin{aligned}
&\tau^t_{it} - \tau^r_{ir} + \Omega_{\rm BH}\tau^t_{i\phi} = 0 , \\
&\tau^\phi_{i\phi} - \tau^r_{ir} + \frac{1}{\Omega_{\rm BH}}\tau^\phi_{it} = 0 ,
\end{aligned}
\tag{4.6}
$$

where $\Omega_{\rm BH} = a/(r_+^2 + a^2)$ is the angular velocity of the black hole. Equations (4.6) are the consequence of the regularity of $\tau^\nu_{i\mu}$ in a regular map covering the event horizon. For $s=0$ and $a=0$, Eqs. (4.3) and (4.4) coincide with the result obtained by the different method in Ref. 17.

V. PROPERTIES OF THE VACUUM ENERGY-MOMENTUM TENSOR IN THE KERR METRIC

In this section the properties of $\langle T^\nu_\mu \rangle^s_{\rm ren}$ for massive fields in the spacetime of a stationary black hole are discussed and the contribution of vacuum polarization to shifts of the mass and the angular momentum of a black hole are calculated.

We begin by considering vacuum polarization near a nonrotating black hole. The dependences of the components $\langle T^t_t \rangle^s_{\rm ren}$ (firm lines), $\langle T^r_r \rangle^s_{\rm ren}$ (dashed and dotted lines), and $\langle T^\theta_\theta \rangle^s_{\rm ren} = \langle T^\phi_\phi \rangle^s_{\rm ren}$ (broken lines) on the radius are shown in Fig. 1. The other components of the energy-momentum tensor for massive scalar, spinor, and vector fields are equal to zero. The numerical calculations show that the contribution of $T^\nu_{1\mu}$ to $\langle T^\nu_\mu \rangle^s_{\rm ren}$ is much smaller than the contribution of $T^\nu_{2\mu}$. This explains why after dividing by the factor β_s the quantities $\langle T^\nu_\mu \rangle^s_{\rm ren}$ for different spins are very similar. Because of this property, which is also valid for a rotating black hole, we shall give later only the plots of $\langle T^\nu_\mu \rangle^0_{\rm ren}$ for the conformal ($\xi = \frac{1}{6}$) scalar ($s=0$) field.

The vacuum energy density $\epsilon^s = -\langle T^t_t \rangle^s_{\rm ren}$ is negative at (and under) the event horizon for the scalar and spinor fields and is positive for the vector field. The vacuum energy density ϵ^s vanishes at $r \simeq 2.2M$, achieves its max-

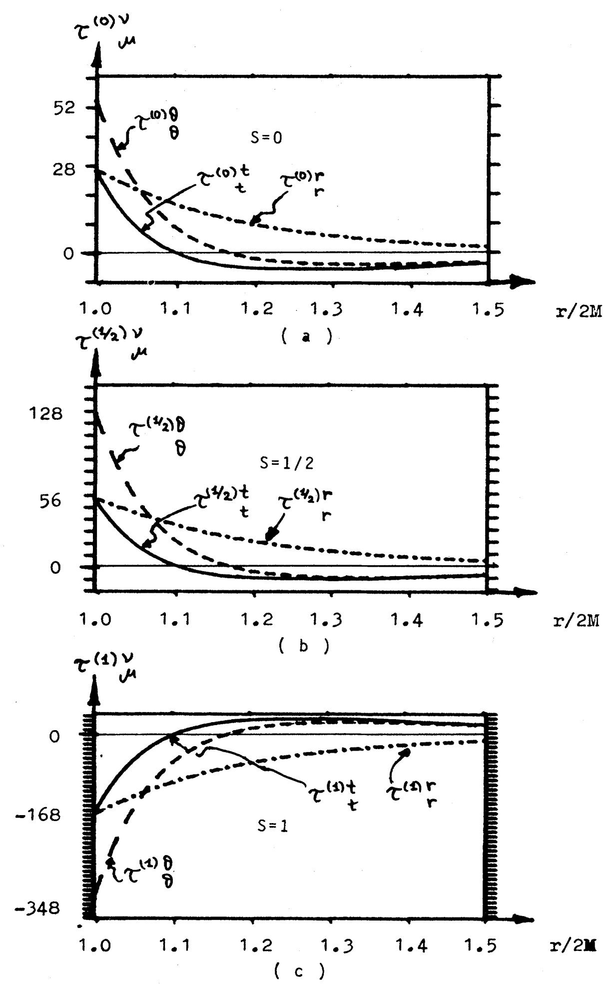

FIG. 1. The dependence of $\langle T^t_t \rangle^s_{\rm ren}$ (firm lines), $\langle T^r_r \rangle^s_{\rm ren}$ (dashed and dotted lines), and $\langle T^\theta_\theta \rangle^s_{\rm ren}$ (broken lines) on $r/2M$ for scalar ($s=0$), spinor ($s=\frac{1}{2}$), and vector ($s=1$) massive fields in Schwarzschild spacetime.

imum (for $s=0$ and $s=\frac{1}{2}$) or minimum (for $s=1$) at $r\simeq 2.5M$, and tends to zero as Ar^{-8} at large distances. This energy density does not indicate any peculiarity near $r=r_+$ which might be connected with the conjectural high concentration of real massive particles near the event horizon discussed by Zel'dovich.[26] The components $\langle T_\theta^\theta\rangle_{\rm ren}^s=\langle T_\phi^\phi\rangle_{\rm ren}^s$ behave analogously while $\langle T_r^r\rangle_{\rm ren}^s$ has no extremum outside the horizon. Comparing the plots given in Fig. 1(a) with the plots for the components of the vacuum energy-momentum tensor for the scalar massless field near the black hole obtained by Fawcett[9] shows that they are qualitatively the same up to the common factor $M^2m^2/m_{\rm Pl}{}^4$).

Because of vacuum polarization the mass M of a black hole measured by a distant observer,[27]

$$M=-\frac{1}{8\pi}\int_{s_\infty}\nabla^\alpha\xi^\beta_{(t)}ds_{\alpha\beta}\ ,\qquad \xi^\alpha_{(t)}\partial_\alpha=\partial_t,\ \ \xi^{(\alpha;\beta)}_{(t)}=0\ , \tag{5.1}$$

differs from the mass $M_{\rm BH}$ determined as the surface integral over the horizon:

$$M_{\rm BH}=-\frac{1}{8\pi}\int_{s_{\rm BH}}\nabla^\alpha\xi^\beta_{(t)}ds_{\alpha\beta}\ . \tag{5.2}$$

This difference is equal to

$$\Delta M^s\equiv M^s-M_{\rm BH}=-\int_{\substack{\Sigma\\ r>r_+}}(2\langle T^\alpha_\beta\rangle^s_{\rm ren}\xi^\beta_{(t)}-\langle T^\epsilon_\epsilon\rangle^s_{\rm ren}\xi^\alpha_{(t)})d\Sigma_\alpha\ . \tag{5.3}$$

For the black hole with $a\ll M$ calculations give

$$\Delta M^s=\frac{M}{96\times 7!\pi m^2M^4}\left[\begin{Bmatrix}224-1008\xi\\ 112\\ -336\end{Bmatrix}+\begin{Bmatrix}-1144+5292\xi\\ -716\\ 1186\end{Bmatrix}\frac{a^2}{M^2}+\cdots\right]. \tag{5.4}$$

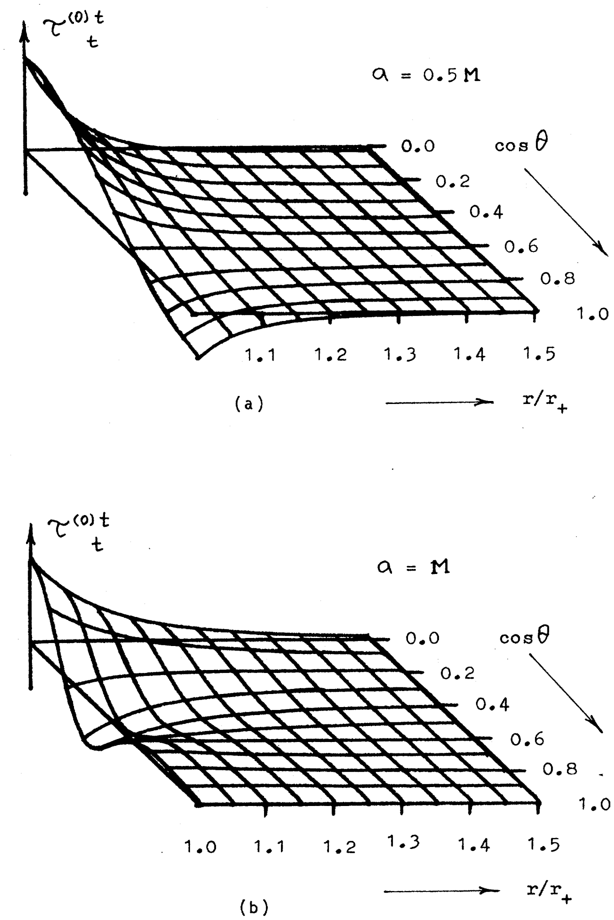

FIG. 2. The dependence of $\tau_t^{(0)t}$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau_t^t)_{\max}=48.62$, $(\tau_t^t)_{\min}=-24.62$; (b) $a/M=1.0$, $(\tau_t^t)_{\max}=293$, $(\tau_t^t)_{\min}=-169.63$.

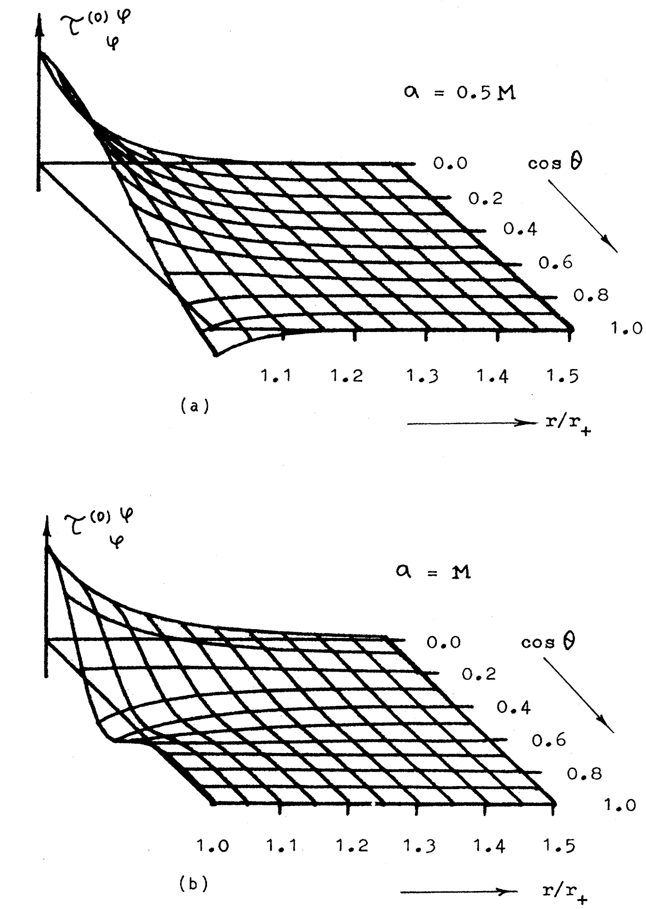

FIG. 3. The dependence of $\tau_\phi^{(0)\phi}$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau_\phi^\phi)_{\max}=80.19$, $(\tau_\phi^\phi)_{\min}=-19.75$; (b) $a/M=1.0$, $(\tau_\phi^\phi)_{\max}=467$, $(\tau_\phi^\phi)_{\min}=-191.16$.

For rotating black holes, besides the dependence of ΔM^s on a, the rotation leads also to the effect of "(anti)screening" of the angular momentum of the black hole. Define the angular momentum J of the black hole measured by a distant observer and its proper angular momentum $J_{\rm BH}$ by the relations[27]

$$J=\frac{1}{16\pi}\int_{s_\infty}\nabla^\alpha\xi^\beta_{(\phi)}ds_{\alpha\beta}\,,\qquad J_{\rm BH}=\frac{1}{16\pi}\int_{s_{\rm BH}}\nabla^\alpha\xi^\beta_{(\phi)}ds_{\alpha\beta}\,, \tag{5.5}$$

where $\xi^\beta_{(\phi)}\partial_\beta=\partial_\phi$ and $\xi^{(\alpha;\beta)}_{(\phi)}=0$. Then the vacuum-polarization contribution to the total angular momentum is

$$\Delta J^s=J^s-J_{\rm BH}=\int_{\substack{\Sigma\\ r>r_+}}\langle T^\alpha_\beta\rangle^s_{\rm ren}\xi^\beta_{(\phi)}d\Sigma_\alpha\,. \tag{5.6}$$

For $a \ll M$ the calculation gives

$$\Delta J^s=\frac{aM}{96\times 7!\pi m^2M^4}\begin{Bmatrix}204-1008\xi\\ 192\\ -396\end{Bmatrix}+O\left[\frac{a^3}{M^3}\right]. \tag{5.7}$$

This effect is a consequence of the existence of nonzero fluxes of the energy density $\langle T^t_\phi\rangle^s_{\rm ren}$ around a black hole. The angular velocity ω of the observer for which this flux vanishes coincides near the event horizon with the angular velocity of the black hole: $\Omega_{\rm BH}=a/(r_+^2+a^2)$. At far distances the angular velocity ω of such an observer is proportional to Bardeen's angular velocity $\Omega=-g_{t\phi}/g_{\phi\phi}$:

$$\omega\underset{r\to\infty}{\simeq}\begin{Bmatrix}(6-28\xi)/(-11+56\xi)\\ -\frac{1}{3}\\ -\frac{10}{23}\end{Bmatrix}\Omega\,,\qquad \Omega\underset{r\to\infty}{\simeq}2aM/r^3\,. \tag{5.8}$$

In the general case such a reference system which is comoving with the vacuum-energy-density fluxes does not necessarily exist for all values of r. In Figs. 2—7 the dependences of $\tau^{(0)\nu}_\nu$ on the angles θ and the radius r for the massive scalar ($\xi=\frac{1}{6}$) field are shown for the values 0.5 and 1.0 of the parameter a/M. The maximal $(\tau^\nu_\mu)_{\max}$ and minimal $(\tau^\nu_\mu)_{\min}$ values of $\tau^{(0)\nu}_\mu$ are given in the figure captions.

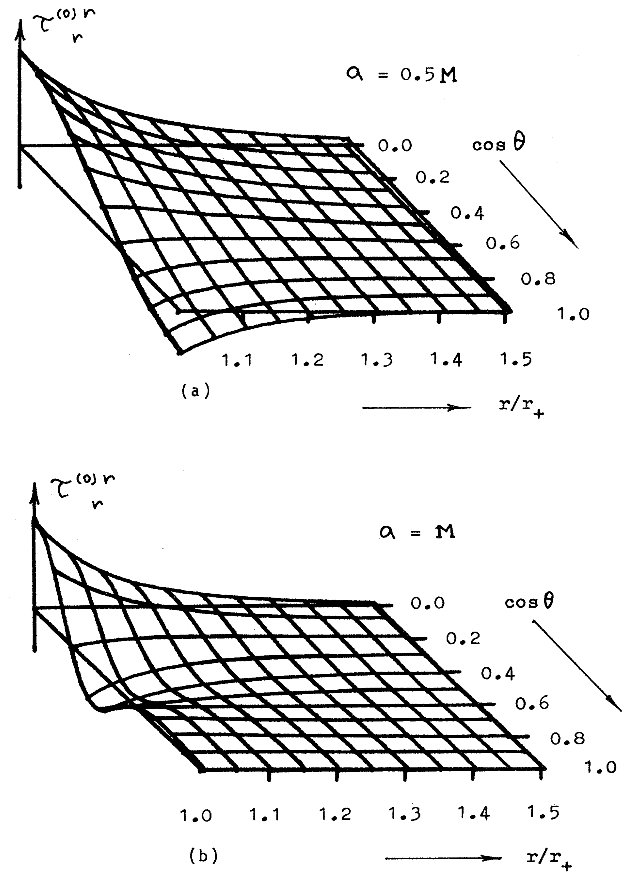

FIG. 4. The dependence of $\tau^{(0)r}_r$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau^r_r)_{\max}=52.62$, $(\tau^r_r)_{\min}=-24.62$; (b) $a/M=1.0$, $(\tau^r_r)_{\max}=371$, $(\tau^r_r)_{\min}=-185.45$.

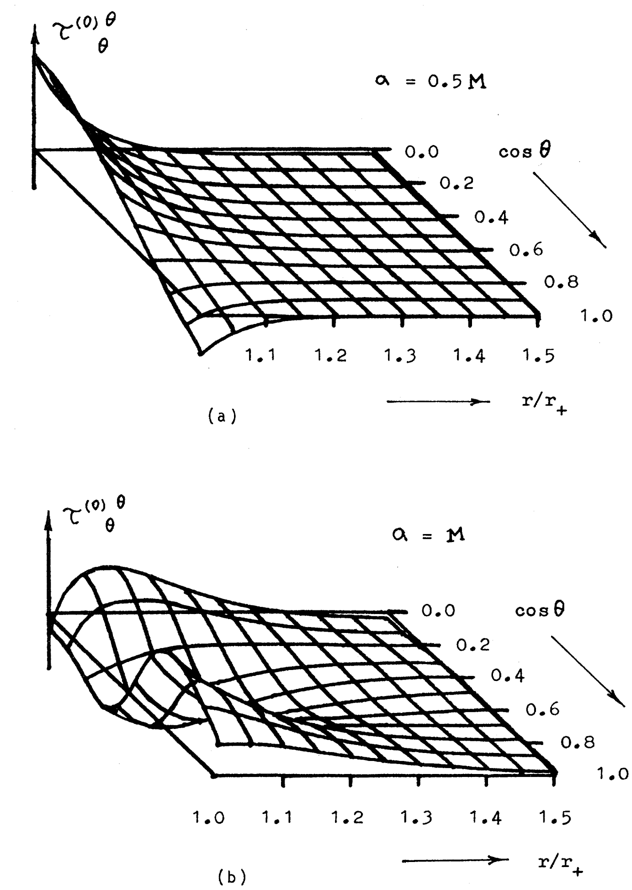

FIG. 5. The dependence of $\tau^{(0)\theta}_\theta$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau^\theta_\theta)_{\max}=48.19$, $(\tau^\theta_\theta)_{\min}=-19.75$; (b) $a/M=1.0$, $(\tau^\theta_\theta)_{\max}=4.17$, $(\tau^\theta_\theta)_{\min}=-3.09$.

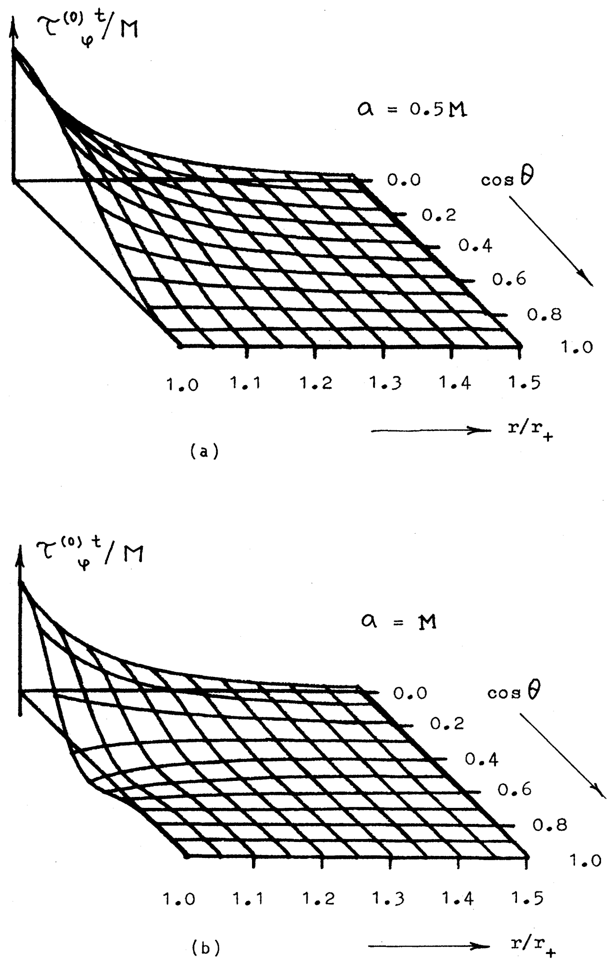

FIG. 6. The dependence of $\tau_\phi^{(0)t}/M$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau^t_\phi/M)_{\max}=29.86$, $(\tau^t_\phi/M)_{\min}=-0.17$; (b) $a/M=1.0$, $(\tau^t_\phi/M)_{\max}=156$, $(\tau^t_\phi/M)_{\min}=-35.49$.

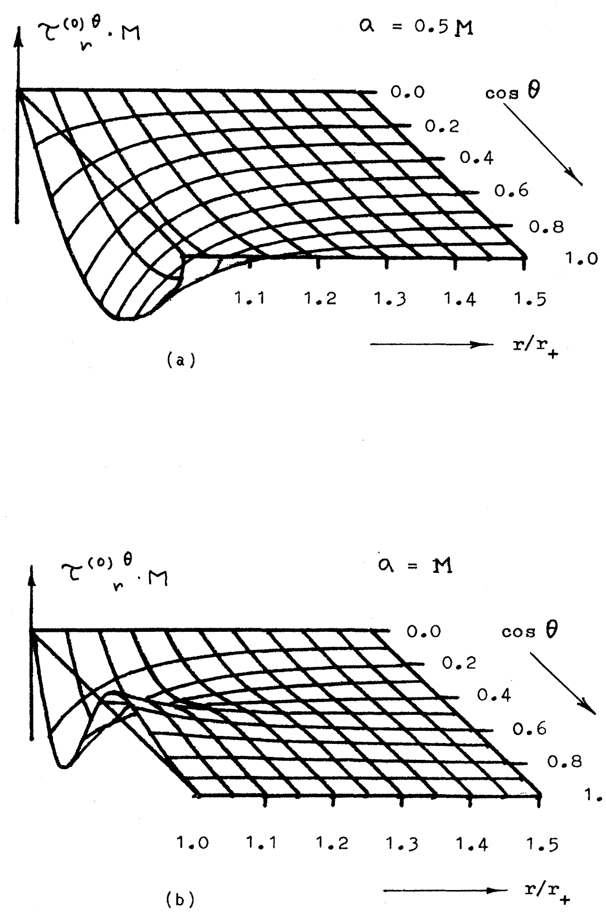

FIG. 7. The dependence of $\tau_r^{(0)\theta}M$ on r and θ in Kerr spacetime. (a) $a/M=0.5$, $(\tau_r^\theta M)_{\max}=0$, $(\tau_r^\theta M)_{\min}=-41.81$; (b) $a/M=1.0$, $(\tau_r^\theta M)_{\max}=72.19$, $(\tau_r^\theta M)_{\min}=-345.07$.

One can see that rotation of the black holes leads to the variation of $\langle T_\mu^\nu\rangle^s_{\rm ren}$ with θ, so that the sign of $\langle T_\mu^\nu\rangle^s_{\rm ren}$ can change along the meridian.

For rotating black holes one can expect that the main features of the behavior of $\langle T_\mu^\nu\rangle^s_{\rm ren}$ which are connected with the effect of dragging the surrounding spacetime into rotation are also inherent in the massless case. Namely, the massless fields must contribute to the energy-density flux around the rotating black hole and to the shift of its angular momentum. These contributions are larger by the factor $M^2m^2/m_{\rm Pl}{}^4$ than the contributions of the massive fields. One can also expect that another important property of vacuum polarization of massive fields, namely its essential spin dependence, is valid in the case of massless fields. It is interesting to note that there exists deep analogy between gravitation and electromagnetism.[28] In the framework of this analogy gravitational interaction of masses and angular momenta is analogous to electromagnetic interaction of charges and magnetic moments. The main difference between these two cases lies in an additional minus sign which is present in the gravitational theory. Using this analogy we may speculate that the obtained spin dependence of the shift of the angular momentum of a black hole is analogous to the spin dependence of the shift of the magnetic moment in quantum electrodynamics and that there exists the gravitational analog of diamagnetism and paramagnetism.

It is interesting to note that the contribution of scalar and spinor massive fields make the observable (at far distances) mass and angular momentum of the black hole larger than its proper mass and angular momentum measured at the horizon. This effect can become important for the existence and properties of elementary black holes which have been discussed by Markov[29,30] and by Hawking.[31]

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