PHYSICAL REVIEW D VOLUME 29, NUMBER 5 1 MARCH 1984

Glueball candidate $\iota(1440)$, anomalous Ward identities, and two-photon decays

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(Received 22 August 1983)

Anomalous Ward identities are given for the U(1) problem, showing that some recent papers have neglected the large topological susceptibility coming from the pure Yang-Mills sector of QCD. A reanalysis of the Ward identities is given, including the pseudoscalar glueball candidate $\iota(1440)$ with the pseudoscalar nonet. It is shown that positivity of the topological susceptibility together with other constraints is sufficient to narrow down the permitted range of pseudoscalar axial couplings. In particular the $\iota(1440)$ couplings are consistent wtih those expected for a glueball with the decay $\iota \rightarrow \gamma\gamma$ probably immeasurably small. Contrary to a recent claim, the results are not sensitive to the branching ratio for $\iota \rightarrow K\bar{K}\pi$, which may be as large as 100%.

One of the most important successes of QCD has been the possibility that it contains all the ingredients for solving the U(1) problem.[1] The anomalously large mass of the η' compared with the remaining eight members of the pseudoscalar nonet may be attributed to the existence of gluons, their axial anomaly, and the consequent large topological susceptibility in the underlying pure Yang-Mills sector. So far the most general framework used for studying this problem has been that of meson-saturated anomalous Ward identities,[2–9] although some work has also been done using the sum rules of the ITEP group.[10] There are also several so far unsuccessful attempts to derive the same results from QCD on the lattice.[11]

The recent discovery of a tenth pseudoscalar meson, the $\iota(1440)$,[12] has led several authors[13–15] to study its impact on the saturation of the anomalous Ward identities. Here we intend to comment primarily on Refs. 14 and 15 since their approach is closest to our own. First we show that these papers fail to take account of the large and important pure Yang-Mills topological susceptibility; we then solve the corrected set of saturated Ward identities, including the $\iota(1440)$, using an extension of the techniques developed in Refs. 8 and 9; finally we present a few sample solutions including predictions on the decay $\iota \rightarrow \gamma\gamma$ and comment on the differences from and similarities to other work. Our approach also includes a constraint on the positivity of the topological susceptibility which provides a strong restriction on otherwise unknown parameters. In order to clarify the point of disagreement with Refs. 14 and 15, we begin with a brief resume of the steps in deriving the Ward identities.

The anomalous Ward identities for the chiral $U(3)\times U(3)$ algebra may be obtained by considering axial-vector current-current correlation functions at zero momentum. The assumption that there are no zero-mass pseudoscalar states coupling to *gauge-invariant* axial-vector currents leads to[3,8,9]

$$\int d^4x\,\langle 0|T[\partial^\mu A^i_\mu(x)\partial^\nu A^j_\nu(0)]|0\rangle - \delta_{j0}\int d^4x\,\langle 0|T[\partial^\mu A^i_\mu(x)G\tilde{G}(0)]|0\rangle = -\langle 0|[Q^i_5(0),D^j(0)]|0\rangle \tag{1a}$$

and

$$\int d^4x\,\langle 0|T[\partial^\mu A^i_\mu(x)G\tilde{G}(0)]|0\rangle = 0,\quad i,j=0,\ldots,8\ . \tag{1b}$$

The notation is that of Refs. 8 and 9, where the axial-vector-current divergences have a soft ($m\bar{\psi}\gamma_5\lambda_i\psi$) and anomalous hard ($G\tilde{G}$) piece:

$$\partial^\mu A_i(x) = D^i(x) + \delta_{i0}G\tilde{G}(x)\ , \tag{1c}$$

$$G\tilde{G}(x) = \left(\frac{3}{2}\right)^{1/2}\left(\frac{g^2}{32\pi^2}\right)\epsilon^{\mu\nu\alpha\beta}G^a_{\mu\nu}G^a_{\alpha\beta}\ , \tag{1d}$$

with $G^a_{\mu\nu}$ the gluon field tensor.

Assuming these to be adequately saturated with the lowest-lying pseudoscalar nonet *and* the $\iota(1440)$ leads in the standard way, after elimination of the quark parameters appearing on the right-hand side of (1a), to[16]

$$m_a{}^2F_{8a}{}^2 = \tfrac{1}{3}(4m_K{}^2F_K{}^2 - m_\pi{}^2F_\pi{}^2)\ , \tag{2a}$$

$$m_a{}^2F_{8a}F_{0a} = -\frac{2\sqrt{2}}{3}(m_K{}^2F_K{}^2 - m_\pi{}^2F_\pi{}^2)\ , \tag{2b}$$

$$m_a{}^2F_{0a}(F_{0a}-A_a) = \tfrac{1}{3}(2m_K{}^2F_K{}^2 + m_\pi{}^2F_\pi{}^2)\ , \tag{2c}$$

$$m_a{}^2F_{8a}A_a = 0\ , \tag{2d}$$

where the repeated index a is summed over $a=\eta,\ \eta',\ \iota$. The last equation follows from (1b) with $j=8$; but with $j=0$ the naive form of saturation violates positivity of the topological susceptibility

$$\chi_t \equiv -i\int d^4x\,\langle 0|T[G\tilde{G}(x)G\tilde{G}(0)]|0\rangle\ . \tag{3}$$

This is clearly positive[2] in the Euclidean region because it is simply the average of the square of the topological density ($6\langle\langle \nu^2\rangle\rangle$ in Crewther's notation[2,3]). This failure of positivity is remedied by introducing a Kogut-Susskind-type ghost into non-gauge-invariant matrix elements in the phenomenological-Lagrangian approach,[5] while Witten[4] identifies the extra term as a contact term representing the topological susceptibility in the pure Yang-Mills sector arising from the need to carefully define the meaning of the singular operator product in (3):

$$\chi_t = \chi_t^{\rm YM} - m_a{}^2A_a{}^2\ . \tag{4}$$

The fifth Ward identity[9] now follows from (1b) with $j=0$:

$$\chi_t = m_a{}^2 A_a (F_{0a} - A_a) \quad , \tag{5}$$

thus yielding a value[2,4,5,9] for the Yang-Mills susceptibility which is large, of order $m_{\eta'}{}^2 F_\pi{}^2$:

$$\chi_t^{YM} = m_a{}^2 F_{0a} A_a \quad . \tag{6}$$

Although (2a) and (2d) are precisely Eqs. (2.5) and (2.8) of Ref. 15, their other Ward identities must be obtained from linear combinations of (2): Their Eq. (2.6) follows from (2a) $+\sqrt{2}$(2b)$+\sqrt{2}$(2d); but (2.7), which comes from (2a)$+2\sqrt{2}$(2b)$+2$(2c)$-2\sqrt{2}$(2d), should read

$$3m_\pi{}^2 F_\pi{}^2 = m_a{}^2 (F_{8a} + \sqrt{2}\tilde{F}_{0a})^2 - 2m_a{}^2 A_a{}^2 + 2m_a{}^2 F_{0a} A_a \quad . \tag{7}$$

The first two terms on the right are present in (2.7); but the third [identified in (6) as the large quantity $2\chi_t^{YM}$] is absent. We are therefore forced to conclude that one of the Ward identities used in Refs. 14 and 15 is in error by a very large term of order $2m_{\eta'}{}^2 F_\pi{}^2$. Thus, although their solutions do satisfy Eqs. (2a), (2b), and (2d), they violate (2c) by a factor of about 5. It should be noted that a related error appears in Ref. 6 as already noted in Refs. 7 and 8; the authors of Ref. 7 seem to have included the contact term correctly there but have omitted it in their more recent work.[15] In general, authors using the equivalent phenomenological-Lagrangian technique have correctly included the contact term by means of a "ghost."

We now turn to a brief discussion of our numerical solutions, which will be presented in greater detail elsewhere. First a summary of the methodology, which differs considerably from Ref. 15. In all there are nine unknown parameters: The axial-vector couplings F_{8a}, F_{0a} and topological charges A_a for $a=\eta$, η', and ι; but there are only four Ward identities (2) with the fifth, Eq. (6), serving to determine the unknown χ_t^{YM}. Additional constraints come from the two ratios of measured[12,17] $\psi\to\gamma(\eta,\eta',\iota)$ widths in the usual way,[6–9] and two sum rules for the η, $\eta',\iota\to\gamma\gamma$ widths, two of which are measured.[17] In principle this gives three equations, although it is hard to be confident of the 2γ sum rules because of the large off-mass-shell extrapolations involved. In practice we have therefore taken all three 2γ widths as output, allowing the η and η' widths to deviate from experimental values by amounts considered reasonable if the extrapolations off mass shell are smooth. We will discuss this rather ticklish problem in more detail elsewhere. A final constraint which turned out to be quite powerful in restricting the difference between $F_{0\iota}$ and A_ι is the positivity of the topological susceptibility[2] χ_t given by Eq. (5). In addition, our search for solutions imposed near SU(3) symmetry on the axial-vector couplings of η and η'.

An extension of the solution technique used previously in Refs. 8 and 9 made it possible to search efficiently through a wide range of parameter space and rapidly delineate quite a narrow region of physically reasonable solutions. A few of these are shown in Table I together with a comparison solution presented earlier without $\iota(1440)$. We note in passing that the no-glueball solution presented in the table of Ref.

TABLE I. Solutions to anomalous Ward identities. All axial-vector couplings are measured in units of $F_\pi=93$ MeV. We use $F_K=1.12$; $m_K=3.54$, $m_\eta=3.93$, $m_{\eta'}=6.86$, and $m_\iota=10.32$ in units of m_{π^+}.

	No $\iota(1440)$[a]	A	B	C	D	Experimental value
$F_{8\eta}$	0.97	1.01	0.99	1.01	1.02	
$F_{8\eta'}$	−0.35	−0.13	−0.13	−0.13	−0.11	
$F_{8\iota}$		−0.20	−0.22	−0.20	−0.20	
$F_{0\eta}$	0.38	0.14	0.20	0.33	0.10	
$F_{0\eta'}$	1.18	0.91	0.91	0.90	0.96	
$F_{0\iota}$		0.50	0.50	0.65	0.50	
A_η	1.01	0.87	0.90	0.99	0.85	
$A_{\eta'}$	0.91	0.70	0.72	0.79	0.77	
A_ι		0.44	0.40	0.51	0.90	
$A_{\eta'}/A_\eta$	0.9	0.8	0.8	0.8	0.9	0.8 ± 0.1[b]
$A_{\eta'}/A_\iota$		1.59	1.80	1.57	1.73	1.76 ± 0.44[c] (1.25 ± 0.31)
$\Gamma_{\eta\to2\gamma}$ (keV)[d]	0.49	0.36 (0.32)	0.36 (0.32)	0.32	0.33 (0.32)	0.32 ± 0.05[e]
$\Gamma_{\eta'\to2\gamma}$ (keV)	3.1	6.1 (8.1)	6.3 (7.9)	5.7	5.7 (6.1)	5.3 ± 1.6[e]
$\Gamma_{\iota\to2\gamma}$ (keV)		1.05 (0.02)	0.45 (0.10)	0.22	1.3 (0.70)	< 10 keV[f]
χ_t[d]	1.35	0	0.6	1.36	0	
χ_t^{YM}	57.5	55.2	55.2	73.6	59.5	

[a] This is solution D of Ref. 9.
[b] From measurements of $\psi\to\gamma(\eta,\eta')$, Ref. 17.
[c] From measurements (Ref. 12) of the product $\Gamma_{\psi\to\gamma\iota}B(\iota\to K\bar{K}\pi)$ assuming $B(\iota\to K\bar{K}\pi)\cong100\%$; the figure in parentheses assumes a branching ratio of 50%.
[d] In most cases these are input values. The widths in parentheses correspond to the input $\Gamma_{\eta\to2\gamma}=0.32$ keV, the experimental value (Ref. 17).
[e] Reference 17.
[f] Quoted in Ref. 15.

15 does not solve the Ward identities (2), with (2c) and (2d) particularly badly violated. Solutions A and D are probably the best overall, with the ratio $A_{\eta'}/A_\eta=0.8$ and 0.9, respectively; B and C show that vlaues of $\chi_t\neq 0$ lead to progressively larger SU(3) breaking in $F_{0\eta}$. The near equality of $F_{0\iota}$ and A_ι in all our solutions is imposed by the positivity of χ_t: the more they differ the smaller is χ_t, with a maximum value of 1.36.

We end with a few observations:

(i) The solutions presented are in fact difficult to improve upon, because outside the region $F_{8\iota}\cong -0.2$, $F_{0\iota}\cong 0.5\sim A_\iota$ either the susceptibility χ_t goes negative, or the output width $\Gamma_{\eta'}\rightarrow 2\gamma$ differs greatly from its experimental value, or SU(3) is badly violated, especially by the axial-vector coupling $F_{0\eta}$.

(ii) Inclusion of $\iota(1440)$ greatly improves the overall quality of the solutions compared with those leaving it out, particularly with regard to the 2γ widths and SU(3) symmetry in $F_{8\eta'}$ and $F_{0\eta}$. Of course this may simply be due to the magnifying effect of the large $\iota(1440)$ mass squared in the Ward identities, but our experience of searching for solutions suggests otherwise.

(iii) The ratio $A_{\eta'}/A_\iota$ is directly related to the width for $\psi\rightarrow\iota\gamma$ only determined if we assume we know the branching ratio of the detected mode $\iota\rightarrow K\bar{K}\pi$. All the experimental evidence suggests that this branching ratio is near 100%;[12] but the authors of Ref. 15 found no solutions with a value $\gtrsim 30\%$. We find no such difficulty, and are able to obtain solutions for a wide range of branching ratios, including 100%.

(iv) The output width for $\iota\rightarrow 2\gamma$ always turns out to be small, usually immeasurably so; but there are viable solutions with values as high as 5 or even 10 keV, although they are less attractive for one reason or another. This is largely due to the *octet* sum rule for the 2γ widths where the near saturation by η and η' contributions, the non-negligible size of $F_{0\iota}$, and the large $\iota(1440)$ mass all conspire to force a small $\iota\rightarrow 2\gamma$ width.

(v) Perhaps the most interesting observation is that $F_{8\iota}\cong -0.2$ is small compared to $F_{0\iota}\cong +0.5$: this is certainly consistent with an SU(3)-singlet glueball $\iota(1440)$; moreover, the $1/N_c$ expansion does indeed give a suppressed glueball axial-vector coupling, $F_{0G}\sim 1/\sqrt{N_c}$. So far this appears to be the firmest evidence for a glueball, meager though it is.

(vi) A surprising feature of our solutions is that small values of the topological susceptibility χ_t are favored, with a large pure Yang-Mills (no quarks or $\bar{q}q$ mesons) susceptibility x_t^{YM}. This provides a nice challenge for QCD: The underlying Yang-Mills structure should give a large susceptibility; but the introduction of quarks into the theory ought to give rise to large cancellations. This property, originally noted by Witten,[4] could also furnish not only a difficult test of pure Yang-Mills on the lattice, but also a sensitive check on the treatment of fermions on the lattice and the complicated relationship between chiral-symmetry breaking and topology.[2,3]

I would like to thank Rod Crewther for enlightening correspondence concerning the topological susceptibility.

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[16]We recall that $\langle 0|\partial^\mu A^j_\mu(0)|a\rangle \equiv m_a{}^2F_{ja}$ and $\langle 0|G\tilde{G}(0)|a\rangle \equiv m_a{}^2A_a$ with $\tilde{F}_{0a}\equiv F_{0a}-A_a$. We always express these couplings in units of $F_\pi=93$ MeV.
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