Reply to "Note on the $\gamma\gamma$ contribution to $\pi^0 \rightarrow e^+e^-$ and $\eta \rightarrow \mu^+\mu^-$"

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(Received 13 October 1983)

We briefly review the essential aspects of our model for the rare decay of neutral pseudoscalar mesons into lepton pairs and show that the criticisms of Bergström and Ma derive from assumptions not made in our work.

It has been known for several years that the rare decay $\pi^0 \rightarrow e^+e^-$ presents a problem since all standard calculations give a branching ratio $[\Gamma(\pi^0 \rightarrow e^+e^-)/\Gamma(\pi^0 \rightarrow \text{all})]$ near the unitarity bound, whereas current experiments[1,2] favor a value three to five times larger.

Recently we have proposed a model for resolving this discrepancy.[3] The essential element is to define a physical amplitude $K(q^2)$ which reflects the neutral-current sector of the effective low-energy weak Lagrangian for $q^2=0$; as $K(0)$ is then fixed, $K(q^2)$ must be given by a once-subtracted dispersion relation

$$\mathrm{Re}K(q^2) = K(0) + \frac{q^2}{\pi}\int_0^\infty \frac{\mathrm{Im}K(t)}{t(t-q^2)}dt \quad . \tag{1}$$

Since in general

$$\mathrm{Im}K(t) = \mathrm{Im}K_{\gamma\gamma}(t) + \sum_X \mathrm{Im}K_X(t) \quad , \tag{2}$$

where the sum extends over all intermediate states excluding the two-photon contribution, we can also write $\mathrm{Re}K(q^2)$ as

$$\mathrm{Re}K(q^2) = K(0) + \mathrm{Re}K_{\gamma\gamma}(q^2) + \sum_X \mathrm{Re}K_X(q^2) \quad . \tag{3}$$

Also, in general, both $K_{\gamma\gamma}(q^2)$ and $K_X(q^2)$ will reflect the form factor $f(k_1^2,k_2^2,q^2)$ which enters at the $\pi^0(q) \rightarrow \gamma^*(k_1)+\gamma^*(k_2)$ vertex; if, however, we take

$$f(0,0,q^2) = 1 \quad , \tag{4}$$

as was done in our earlier work,[3] then all dependence of $K(q^2)$ on the π^0 structure is contained in $K_X(q^2)$ because $\mathrm{Im}K_{\gamma\gamma}(q^2)$ depends only upon $f(0,0,q^2)$.

Let us now consider the issues raised in the preceding Comment[4] by Bergström and Ma. First, comparing Eq. (2) above to Eq. (5) of Ref. 4, we find Bergström and Ma ascribing to us not only the assumption $f(0,0,q^2)=1$ but also $\mathrm{Im}K_X(q^2)=0$. Inasmuch as in Ref. 3 we examined the contribution of $K_X(q^2)$ in general, and for a specific model, this claim is clearly unjustified. Indeed, as a check of our previous remarks[3] concerning the suppression of the contribution of higher-mass intermediate states, we have used the relation

$$\mathrm{Re}K(q^2) = K(0) + 2\mathrm{Re}[R(q^2) - R(0)] \tag{5}$$

connecting our $K(q^2)$ with $R(q^2)$, as defined in Bergström and Ma, to calculate $K(q^2)$ for the form factor given in Eq. (3) of Ref. 4. We find

$$\mathrm{Re}K(q^2) = K(0) + \mathrm{Re}K_{\gamma\gamma}(q^2) + O(q^2/M_V^2) \quad , \tag{6}$$

again showing the model insensitivity of $K(q^2)$. Note that for both quantum-chromodynamics-inspired form factors in Ref. 4, as well as for the usual vector-meson-dominance model, Eq. (4) is an exact relation.

Second, as noted above, the choice of dispersion relation is determined by the condition on $K(0)$, not by the choice of $\mathrm{Im}K(q^2)$. From Eq. (1) it follows that $\mathrm{Re}K_{\gamma\gamma}(0)$ and $\mathrm{Re}K_X(0)=0$ but *not* that $R(0)=0$ "by definition," as witnessed by Eq. (5). Although Bergström and Ma describe our choice of $K(0)$ as "arbitrary" we would argue that it is quite natural from the point of view of low-energy phenomenology.

Third, since we have assumed neither $R(0)=0$ nor $K_X(q^2)=0$, we observe that the form factor given in Eqs. (9) and (10) of Ref. 4 cannot be considered equivalent to our model. The most that may be said for this form factor is that it reproduces our result for $K_{\gamma\gamma}(q^2)$. In contrast our model is able to accommodate a variety of form factors while still giving predictions in agreement with experiment, both for $\pi^0 \rightarrow e^+e^-$ and for $\eta \rightarrow \mu^+\mu^-$.[3] Our conclusion is that the criticisms of Bergström and Ma cannot be taken as serious objections to our model, which then must still stand as a leading candidate for describing these decays.

The authors wish to acknowledge a valuable discussion with our colleague Kimball Milton. This work was supported by the U.S. Department of Energy under Contract No. EY-76-S-05-5074.

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