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Magnetic-monopole spin resonance

David Akers
Space Systems Division, Lockheed Missiles and Space Company, Sunnyvale, California 94086

Donald O. Akers
402 E. Main Street, Boswell, Indiana 47921

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The magnetic monopole of charge g is given an intrinsic spin $\frac{1}{2}$ and is bound to a spin-0 nucleus with a repulsive core potential. Bound-state energies are calculated. Speculation regarding the existence of magnetic-monopole spin resonance is examined.

In a paper by Sivers,[1] it was pointed out that a spin-0 magnetic monopole can be bound to a naturally occurring free nucleus with magnetic dipole moment. By introducing an infinite repulsive core of nuclear radius $r = r_0 A^{1/3}$ ($r_0 = 31.5 M_p{}^{-1}$, where M_p is the proton mass), he is able to calculate bound-state energies up to the 10^5-eV range.

We can look at the symmetric case of a spin-0 nucleus binding to a magnetic monopole with electric dipole moment. We introduce the idea that a magnetic monopole may have an intrinsic spin $\frac{1}{2}$ and, hence, an electric dipole moment.[2] We postulate the existence of magnetic-monopole spin resnoance and derive a simple analytical expression for the absorption energy.

First, let us derive the electric dipole moment of a spin-$\frac{1}{2}$ magnetic monopole. The electric dipole moment of a system of magnetic charges is

$$\vec{m} = \frac{1}{2c} \int \vec{x}' \times \vec{J}(x')\, d^3x' \ , \tag{1}$$

where

$$\vec{J}(x) = \sum_i g_i \vec{v}_i \delta(\vec{x} - \vec{x}_i) \tag{2}$$

is the volume current density, g_i are the magnitudes of the magnetic charges, and $\vec{v}_i$ are the velocities of the charges. Substituting Eq. (2) into Eq. (1) gives

$$\vec{m} = \frac{1}{2c} \sum_i g_i \vec{x}_i \times \vec{v}_i \ . \tag{3}$$

For a single magnetic monopole of mass M and charge g,

$$\begin{aligned} \vec{m} &= \frac{1}{2c} g \vec{x} \times \vec{v} \\ &= \frac{1}{2Mc} g \vec{L} \ , \end{aligned} \tag{4}$$

where $\vec{L}$ is the orbital angular momentum.

In addition to the orbital angular momentum $\vec{L}$, consider that the monopole has an intrinsic spin $\vec{S}$ of $\frac{1}{2}$. The additional angular momentum $\vec{S}$ will give rise to an additional electric moment

$$\vec{m}_{\text{spin}} = \frac{g}{2Mc} \delta \vec{S} \ , \tag{5}$$

where δ is a constant, which is characteristic of the state of the monopole.[3] Thus, the total electric moment of the magnetic monopole is then

$$\vec{m} = \frac{g}{2Mc} (\vec{L} + \delta \vec{S}) \ . \tag{6}$$

For our case of a spin-$\frac{1}{2}$ magnetic monopole in the field of a nucleus of charge Ze and of nuclear spin 0, we have the nonrelativistic, quantum-mechanical Hamiltonian of the system as

$$H = \frac{(\vec{p} - Ze\vec{A})^2}{2\mu} - \vec{m} \cdot \vec{E} + U(r) \ , \tag{7}$$

where μ is the reduced mass, $\vec{A}$ is the vector potential of the monopole charge, and $\vec{E}$ is the electric field of the nucleus. $U(r)$ is an undetermined potential which is assumed to be appreciable only at short distances.

Inserting Eq. (6) into Eq. (7), we have

$$H = \frac{(\vec{p} - Ze\vec{A})^2}{2\mu} - \frac{g}{2Mc} (\vec{L} \cdot \vec{E} + \delta \vec{S} \cdot \vec{E}) + U(r) \ . \tag{8}$$

We note that the interaction term $\vec{S} \cdot \vec{E}$ was first considered by Bauer.[4] The electric field of the nucleus is

$$\vec{E} = Ze \frac{\vec{r}}{r^3} \ . \tag{9}$$

Substituting for $\vec{E}$ into Eq. (8), we obtain

$$\begin{aligned} H &= \frac{(\vec{p} - Ze\vec{A})^2}{2\mu} - \frac{Zeg}{2Mcr^3} (\vec{L} \cdot \vec{r} + \delta \vec{S} \cdot \vec{r}) + U(r) \\ &= \frac{(\vec{p} - Ze\vec{A})^2}{2\mu} - \frac{Zeg}{2Mcr^3} \delta \vec{S} \cdot \vec{r} + U(r) \ , \end{aligned} \tag{10}$$

since $\vec{L}\cdot\vec{r}=0$. Now $\hat{r}=\vec{r}/r$ and for $\vec{S}=\frac{1}{2}\hbar\vec{\sigma}$, where $\vec{\sigma}$ is the Pauli spin operator, we have

$$H=\frac{(\vec{p}-Ze\vec{A})^2}{2\mu}-\frac{Zeg\hbar}{4Mc}\frac{\delta\vec{\sigma}\cdot\hat{r}}{r^2}+U(r) \quad . \tag{11}$$

We follow the mathematical treatment given by Sivers, introduce the two possibilities for the vector potential $\vec{A}$ as

$$A_r^{(1)}=A_\theta^{(1)}=0,\quad A_\phi^{(1)}=g(1-\cos\theta)/\sin\theta \ , \tag{12}$$

$$A_r^{(2)}=A_\theta^{(2)}=0,\quad A_\phi^{(2)}=-g(\cos\theta/\sin\theta) \ , \tag{13}$$

where $\vec{A}^{(1)}$ is singular along the negative z axis and $\vec{A}^{(2)}$ is singular along the entire z axis, and take the potential $U(r)$ as

$$U(r)=\begin{cases}\infty, & 0<r<r_0A^{1/3}\\ 0, & r_0A^{1/3}<r<\infty\ .\end{cases} \tag{14}$$

$U(r)$ is an infinite-repulsive-core potential[5] of nuclear radius $r_0A^{1/3}$.

The solution of Schrödinger's equation with the Hamiltonian operator of Eq. (11) is given by Sivers, and we quote his solution for our case (spin-$\frac{1}{2}$ monopole and spin-0 nucleus). The angular eigenvalues are given by

$$\beta=-Z\left[\frac{1}{2}+\frac{\delta}{8}\left(\frac{m_A}{M+m_A}\right)\right] \ , \tag{15}$$

where m_A is the mass of the nucleus.[6] The binding energy of the monopole-nucleus system is given by

$$E_0=\frac{1}{2\mu r_0^2A^{2/3}}(\beta+\tfrac{1}{4}) \ . \tag{16}$$

Sivers has shown that the binding energies exist only for $\beta<-\frac{1}{4}$. We have calculated binding energies for our case, and the results are tabulated in Table I.

In Table I, we have chosen the magnetic g factor $\delta=2$ and the monopole mass $M=100M_p$ in order to estimate typical binding energies.[7] The energies are of the order 10^4 eV. This is a factor 10 less than the highest value reported by Sivers (for spin-$\frac{1}{2}$ nucleus and spin-0 monopole). We note that the $\beta<-\frac{1}{4}$ condition for bound states is satisfied. If bound states exist and if the magnetic monopole has a spin $\vec{S}=\frac{1}{2}\hbar\vec{\sigma}$, then an additional effect may be observed: namely, the probability of a spin up flipping to a spin down in an actual spin-resonance experiment.

TABLE I. Typical values of the binding energy, Eq. (16), for spin-0 nuclei with infinite repulsive core and for spin-$\frac{1}{2}$ magnetic monopole.

Nucleus	Z	β	Binding energy (eV)
^{4}He	2	−1.02	3.8×10^4
^{12}C	6	−3.16	2.4×10^4
^{16}O	8	−4.28	2.2×10^4
^{28}Si	14	−7.77	1.8×10^4
^{40}Ca	20	−11.43	1.6×10^4
^{56}Fe	26	−15.33	1.4×10^4
^{208}Pb	82	−54.84	1.1×10^4

We recall that the spin $\vec{S}$ of a magnetic monopole gives rise to a spin contribution $\vec{m}_{\rm spin}$ to the electric moment and that its interaction Hamiltonian is

$$H_{\rm spin}=-\frac{g\delta}{2Mc}\vec{S}\cdot\vec{E} =-\frac{g\delta\hbar}{4Mc}\vec{\sigma}\cdot\vec{E} \ . \tag{17}$$

By applying an oscillating electric field in addition to a static field, we could induce a spin pointing up to flip pointing down. We choose an electric field

$$\vec{E}=\mathscr{E}_1\cos\omega t\,\hat{x}+\mathscr{E}_0\hat{z} \ , \tag{18}$$

and the interaction Hamiltonian is then

$$H_{\rm spin}=-\frac{g\delta\hbar}{4Mc}(\mathscr{E}_0\sigma_z+\mathscr{E}_1\cos\omega t\,\sigma_x) \ , \tag{19}$$

where $\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli spin matrices.

Letting the Hamiltonian (19) operate on a spin state $|\Psi(t)\rangle$, we have

$$i\hbar\frac{d|\Psi(t)\rangle}{dt}=-\frac{g\delta\hbar}{4Mc}(\mathscr{E}_0\sigma_z+\mathscr{E}_1\cos\omega t\,\sigma_x)|\Psi(t)\rangle \ . \tag{20}$$

We may write

$$|\Psi(t)\rangle=e^{i\omega t\sigma_z/2}|\psi(t)\rangle \ . \tag{21}$$

Substituting this into Eq. (20) and multiplying on the left by $e^{-i\omega t\sigma_z/2}$, we obtain

$$i\frac{d|\psi(t)\rangle}{dt}=\left[\left(\frac{\omega-\omega_0}{2}\right)\sigma_z-\omega_1\cos\omega t(e^{-i\omega t\sigma_z/2}\sigma_xe^{i\omega t\sigma_z/2})\right]|\psi(t)\rangle \ , \tag{22}$$

where

$$\omega_0=\frac{g\delta}{2Mc}\mathscr{E}_0 \ \text{and}\ \omega_1=\frac{g\delta}{4Mc}\mathscr{E}_1 \ .$$

The second term in Eq. (22) may be expressed as

$$\cos\omega t(e^{-i\omega t\sigma_z/2}\sigma_xe^{i\omega t\sigma_z/2})=\sigma_x/2+\tfrac{1}{2}(\sigma_x\cos2\omega t+\sigma_y\sin2\omega t) \ .$$

Introducing this into Eq. (22) and solving the equation, we find

$$|\psi(t)\rangle=e^{-i\left\{\left[\left(\frac{\omega-\omega_0}{2}\right)\sigma_z-\frac{\omega_1\sigma_x}{2}\right]t-\frac{\omega_1\sigma_x}{4\omega}\sin2\omega t+\frac{\omega_1\sigma_y}{4\omega}(\cos2\omega t-1)\right\}}|\psi(0)\rangle \ . \tag{23}$$

The last two terms in the exponent produce high-frequency wiggles in $|\psi(t)\rangle$ and they may be neglected for our purpose.

We let

$$\lambda=\frac{\omega-\omega_0}{\Omega}\sigma_z-\frac{\omega_1}{\Omega}\sigma_x \ , \tag{24}$$

where

$$\Omega=[(\omega-\omega_0)^2+\omega_1^2]^{1/2} \ . \tag{25}$$

Then

$$|\psi(t)\rangle=e^{i\Omega t\lambda/2}|\psi(0)\rangle \ . \tag{26}$$

If the initial spin state is

$$|\psi(0)\rangle=|\chi_1\rangle=\begin{pmatrix}1\\0\end{pmatrix} \ ,$$

then

$$|\psi(t)\rangle=e^{-i\Omega t\lambda/2}|\chi_1\rangle \ , \tag{27}$$

and Eq. (21) becomes

$$|\Psi(t)\rangle=e^{i\omega t\sigma_z/2}e^{-i\Omega t\lambda/2}|\chi_1\rangle \ . \tag{28}$$

The probability amplitude for inducing a spin flip to the

$$|\chi_2\rangle=\begin{pmatrix}0\\1\end{pmatrix}$$

state is

$$\begin{aligned}\langle\chi_2|\Psi(t)\rangle&=\langle\chi_2|e^{i\omega t\sigma_z/2}e^{-i\Omega t\lambda/2}|\chi_1\rangle\\&=e^{-i\omega t/2}\langle\chi_2|e^{-i\Omega t\lambda/2}|\chi_1\rangle\\&=-ie^{-i\omega t/2}\sin\frac{\Omega t}{2}\langle\chi_2|\lambda|\chi_1\rangle\\&=ie^{-i\omega t/2}\frac{\omega_1}{\Omega}\sin\frac{\Omega t}{2} \ .\end{aligned} \tag{29}$$

Finally, the probability of a spin flip at time t is

$$\begin{aligned}P(t)_{1\to2}&=|\langle\chi_2|\Psi(t)\rangle|^2\\&=\frac{\omega_1^2}{\Omega^2}\left(\sin\frac{\Omega t}{2}\right)^2 \ .\end{aligned} \tag{30}$$

Equation (30) has a maximum probability 1 when on resonance, $\omega=\omega_0$, and when $\Omega t=k\pi$ (k = odd integer). In the process of the spin flipping, an energy $\hbar\omega_0$ is absorbed from the oscillating electric field and is given by

$$E=\frac{g\delta\hbar}{2Mc}\mathscr{E}_0 \ . \tag{31}$$

We dare to present our speculation that the magnetic monopole may be detected in a spin-resonance-type experiment. This idea must be considered in the evaluation of monopole searches.[8,9] In our treatment of monopole spin, we assumed that the spin $\vec{S}$ satisfies properties of the Pauli spin operator $\vec{\sigma}$; this assumption may not be valid.[10] The correct identification of $\vec{S}$ is given by Osborn.[2]

In summary, we have shown that if magnetic monopoles exist, they can be bound to nuclei with repulsive-core potentials. The inclusion of monopole spin adds to this binding. The bound-state energies are of a magnitude which makes extracting magnetic monopoles difficult with present techniques. Even though detecting free magnetic monopoles may be difficult, we suggest that it could be possible to detect monopoles by measuring the spin absorption energy, Eq. (31). The result of such a measurement is the experimental verification of monopole spin and a way for determining the magnetic g factor, provided the monopole mass is known.

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[1]D. Sivers, Phys. Rev. D 2, 2048 (1970).

[2]H. Osborn, Phys. Lett. 115B, 226 (1982); J. Ficenec and V. Teplitz, in *Electromagnetism*, edited by D. Teplitz (Plenum, New York, 1982), p. 297.

[3]A relative minus sign may be introduced in Eq. (5) to satisfy electromagnetic duality $\vec{E}\to\vec{B}$, $\vec{B}\to-\vec{E}$; however, we carry this along in the constant δ.

[4]E. Bauer, Proc. Cambridge Philos. Soc. 47, 777 (1951).

[5]The use of a repulsive-core potential, Eq. (14), is not physically unreasonable since hadronic effects may arise; see C. Montonen and D. Olive, Phys. Lett. 72B, 117 (1977).

[6]The constant δ is now positive definite in Eq. (15), where the plus sign satisfies electromagnetic duality.

[7]We identify the constant δ with the magnetic g factor $g_M=2$; see Osborn (Ref. 2). The monopole mass is arbitrarily selected to be 100 proton masses; see Sivers (Ref. 1).

[8]Recently, B. Cabrera [Phys. Rev. Lett. 48, 1378 (1982)] reported the possible detection of a magnetic monopole in a superconducting-ring experiment.

[9]B. Cabrera and W. P. Trower, Found. Phys. 13, 195 (1983).

[10]M. M. Ansourian, Phys. Rev. D 14, 2732 (1976).