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Decay $\pi^+ \to e^+e^+e^-\nu$ and the γ parameter

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A possible way to extract the value for the γ parameter in the radiative pion decay is presented through the study of the decay process $\pi^+ \to e^+e^+e^-\nu$. A particular kinematic configuration is suggested to select those events dominated by the structure-dependent contribution without having to perform the difficult measurement on particle polarizations, but retaining the favored picture from the study of a polarized electron in the radiative pion decay $\pi^- \to e^-\bar{\nu}\gamma$. A specific positron spectrum is given for this particular configuration to serve as a tentative way of determining an exact value for γ between the existing two possible ones.

I. INTRODUCTION

The parameter γ, the ratio between axial-vector and vector pion form factors in radiative pion decay, has drawn considerable attention for more than two decades,[1–7] because this parameter is believed to give some important information concerning the quark masses and, also, some clue to the mystery of chiral-symmetry breaking. So far, two possible values for γ are obtained from experiments.[7–9] It has been a difficult problem for experimentalists to distinguish between these two values due to various reasons involved in a realistic experiment. We have suggested a way to avoid these difficulties by selecting those events with a left-handed electron in the $\pi^- \to e^-\bar{\nu}\gamma$ decay.[10] In so doing, the chirality argument enables us to pick the events dominated by the structure-dependent (SD) contribution, and hence come to a resolution for the ambiguity of the values for γ. However, the conventional way of measuring electron helicity involves an additional scattering process, which makes this experiment still not feasible at present. Therefore, we have looked into the decay process $\pi^+ \to e^+e^+e^-\nu$, hoping that somehow we may be able to find a suitable kinematic configuration that would retain the favored picture we have suggested in the radiative pion decay, without appealing to the difficult measurement of the electron or positron helicity.

Bardin, Bilen'kii, Mitsel'makher, and Shumeiko,[11] have given this process some detailed study. But no really specific kinematic configuration was given that would give a sensitive resolution for the parameter γ, other than that of small k^2 ($< 0.01 m_\pi{}^2$) for the virtual photon. Actually, it turns out that a reasonably distinguishable spectrum with respect to different values of γ comes from a sensitive dependence upon different spin-state configurations. The condition of small k^2 alone still allows for comparable contributions from various spin-state possibilities. This will become clear in our discussions later, and is one of the reasons why their result, when realized in particle spectra, will not be very sensitive to the parameter γ.

Because of the various reasons mentioned above, it seems necessary to make a more careful study of this decay process $\pi^+ \to e^+e^+e^-\nu$ to see if there is some other specific kinematic configuration which may be found to have a more sensitive dependence upon the parameter γ.

With the favored picture from the previous study of the radiative pion decay $\pi^- \to e^-\bar{\nu}\gamma$ in mind, we proceeded to calculate the particular process $\pi^+ \to e^+e^+e^-\nu$ without introducing any new parameter besides those in the former decay. Essentially because of the sensitive dependence upon the opening angle between the e^+e^- pair converted from the virtual photon, and the chirality argument employed in the previous work, we find, indeed, there does exist a more specific configuration where the structure-dependent contribution dominates. The result is consistent with our previous analysis concerning a left-handed electron in the decay $\pi^- \to e^-\bar{\nu}\gamma$. This will be discussed in detail in the following sections.

In Sec. II, we shall give first the detailed decay amplitude and its properties that are crucial for our analysis of this process concerning the parameter γ. The numerical results of our calculation are presented in Sec. III, along with some further discussions.

II. THE DECAY AMPLITUDE

We start by summing up the Feynman diagrams as shown in Fig. 1, and their corresponding antisymmetrized parts. We have, for the $\pi^+ \to e^+e^+e^-\nu$ decay, the following amplitude with a massless neutrino:

$$\langle e^+e^+e^-\nu|\pi^+\rangle = \mathrm{IB} + \mathrm{SDA} + \mathrm{SDV} \ , \qquad (2.1)$$

where IB is the inner-bremsstrahlung term, SDA is the structure-dependent axial-vector term, and SDV is the structure-dependent vector term. They are, separately,

$$\mathrm{IB} = \frac{m_l f_\pi e^2}{k^2}\bar{u}(q)\left[\frac{2t\cdot j + \not{k}\not{j}}{2t\cdot k + k^2} - \frac{(2P-k)\cdot j}{2P\cdot k - k^2}\right](1+\gamma_5)v(t) \ , \qquad (2.2)$$

$$\mathrm{SDA} = \frac{Ge}{\sqrt{2}k^2} b[(P\cdot k)(L\cdot j) - (k\cdot L)(P\cdot j)] \ , \qquad (2.3)$$

$$\mathrm{SDV} = \frac{Ge}{\sqrt{2}k^2} a i \epsilon_{\mu\nu\lambda\rho} P^\nu k^\rho L^\mu j^\lambda \ . \qquad (2.4)$$

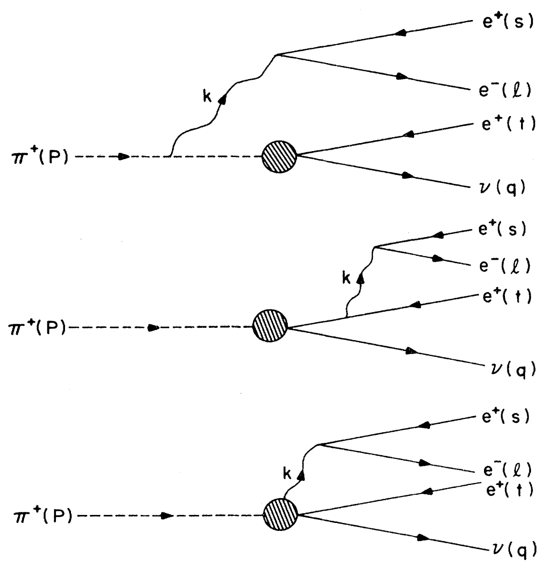

FIG. 1. The Feynman diagrams for the $\pi^+ \to e^+e^+e^-\nu$ decay. It is understood that the corresponding antisymmetrized diagrams should be taken into account.

As before, a and b are the vector and axial-vector form factors. The parameter $\gamma \equiv b/a$,

$$L^\mu = \bar{u}(q)\gamma^\mu(1-\gamma_5)v(t) \quad ,$$

is the usual $V-A$ charged weak current, and

$$j^\lambda = \bar{u}(l)\gamma^\lambda v(s)$$

is the electromagnetic current. It is understood that the corresponding antisymmetrized parts should be taken into account.

We shall not present the explicit algebraic result in this report, but only give some main observations that are important for our purpose here after the calculation has been finished.

The first thing worth noticing in the results is the familiar property for the pair conversion of the virtual photon in this process. The e^+e^- pair is well known to be dominated by the small-opening-angle regime. The contribution is suppressed for the situation where this converted e^+e^- pair is antiparallel in momentum relative to each other. Because of the factor $1/k^2$ from the photon propagator that appears in the decay amplitude (see Fig. 1), the ratio between the contribution from an antiparallel converted e^+e^- pair and that from a parallel one with the same e^+,e^- energies E_+,E_- is of the order $(m_l^2/E_+E_-)^2$, which is very small for relevant energy scales. Therefore, when we look at the particular kinematic configuration where a positron is emitted antiparallel to the electron and the other positron, we know that this single positron essentially comes from the weak-coupling vertex instead of from the virtual-photon conversion. The chiral structure of the $V-A$ weak coupling then tells us that this singly emitted positron is basically right-handed. Just as in the case of π_{l2} decays with right-handed positrons, which are forbidden by chirality selection rules, the associated IB amplitude here is similarly suppressed, and is proportional to m_l. Hence, this particular configuration picks up those events coming from the SD contribution, and is sensitive to the value of γ.

As could be seen now, if we only impose the kinematic condition of small k^2, e.g., $k^2 < 0.01 m_\pi^2$, as Bardin *et al.* have done, we would be picking up events where the weak-vertex positron comes out in all directions. Under such a circumstance, the large mixing between this positron and the other one from the virtual photon would wash away the favored picture we got above in selecting the events coming from the SD contribution. That is essentially the reason why their prediction is still not as sensitive as it could be in this decay process.

Up to the present point, after taking into account the left-handedness of the neutrino, there exist two spin-state subconfigurations. The one with a positive-helicity virtual photon is accompanied by a neutrino which is emitted opposite in momentum to the e^+, and the other, with a zero-helicity photon, will be accompanied by a neutrino parallel in momentum to the e^+. As indicated in Refs. 6 and 10, we have seen that SD contributions with positive-helicity photons are particularly sensitive to γ. In the present reaction, the kinematic configuration defined above picks up the positive-helicity-photon contribution. This is because of the nature of the photon coupling in the SD term, which is of the $F_{\mu\nu}$ type. For small values of k^2, the coupling involves predominantly transversely polarized photon states. Angular momentum conservation now implies that in fact only the positive-helicity photons will contribute. As a result, this particular kinematic configuration is really dominated by the situation where both the singly-emitted positron and the virtual photon are right-handed. As we shall see later, this particular spin state of the virtual photon enhances one step further the resolution capability for γ in this specific configuration.

III. THE NUMERICAL RESULTS

Bearing in mind the observations described in the last section, here we shall look at some numerical results from this calculation as an example to illustrate how in an experiment the extraction of an exact value for γ could be achieved. Referring to Fig. 2, here we collect those events satisfying the conditions

$$-1 \le \cos\alpha \le -0.8, \;\; 0.8 \le \cos\beta \le 1 \quad ,$$

where α and β refer to the opening angles from each of the two positrons to the electron, respectively. In other words, to make it practical for an experiment, we collect those events near the specific configuration suggested above. Also, a cutoff energy around 7 MeV is imposed on the charged particles. Our numerical study shows that the

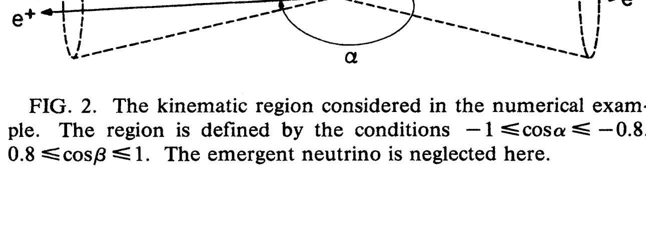

FIG. 2. The kinematic region considered in the numerical example. The region is defined by the conditions $-1 \le \cos\alpha \le -0.8$, $0.8 \le \cos\beta \le 1$. The emergent neutrino is neglected here.

branching ratio subjected to these conditions is

$$R=\frac{W(\pi^+\to e^+e^+e^-\nu)}{W(\pi^+\to\mu^+\nu)}\simeq\begin{cases}3.3\times10^{-9}, & \text{if } \gamma=-1.98\ ,\\ 0.8\times10^{-9}, & \text{if } \gamma=0.26\ .\end{cases}$$

These numbers may, at first, look quite large when we look at the existing branching ratio from the experiment performed by Korenchenko *et al*;[12] their result, however, has been underestimated. This is because they obtained the branching ratio by assuming that the decay matrix element is a constant. Thus they have left out a major contribution of the decay rate from the kinematic region that was not measured, which is actually dominated by the growing IB contribution.

The detailed spectrum of the singly emitted positron is shown in Fig. 3. In addition to the apparent different counting rates (about 4.2 to 1), the slopes of the curves are also very distinctive (about 20 to 1 at the high-energy regime). From the present capacity of the pion factory and the branching ratios given above, we should be able to expect a reasonable amount of events to serve the purpose.

We also notice that the ratio between these two peaks in Fig. 3 is of the order 10. This could be understood if we recall that in the $\pi^-\to e^-\bar{\nu}\gamma$ decay, for the SD contribution, the number of events with a right-handed photon is proportional to $(1-\gamma)^2$, while for those with a left-handed one, it is proportional to $(1+\gamma)^2$. Hence, just referring to the SD contribution with a right-handed photon, the relative counting rate between the two results corresponding to the two possible values of γ, here namely -1.98 and 0.26, would be

$$(1+1.98)^2/(1-0.26)^2\simeq16\ ,$$

while for a left-handed photon, we would get a relative counting rate

$$(1-1.98)^2/(1+0.26)^2\simeq0.6\ .$$

Thus, the ratio between these two peaks here in the present decay process reflects the dominance of a right-handed virtual photon, as we have expected for the particular kinematic configuration suggested. Actually, when we narrow further the above kinematic region, i.e., letting $\cos\alpha\to-1$ and $\cos\beta\to1$, in other words, when this region really reduces closer to the specific linear configuration mentioned above, the ratio of the two peaks becomes closer to the value 16. This particular character is maintained when different cutoff energies are used.

To summarize our discussions above, the reason for this fairly distinctive resolution power with respect to the values of γ in this process is basically twofold. First of all, in the specific configuration suggested, the essentially right-handed single positron foretells that the SD contribution dominates.

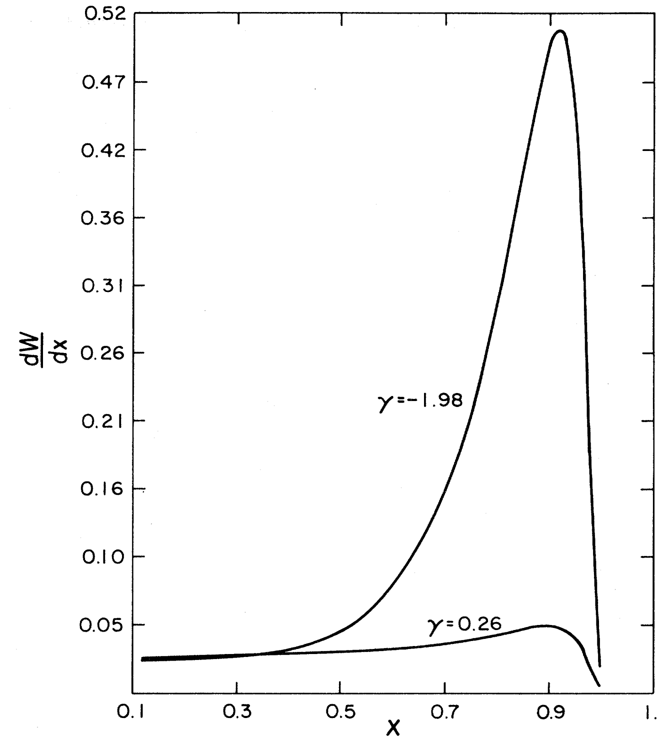

FIG. 3. The spectrum of the particular positron that is emitted opposite to the electron in the example. Here $x=2E/m_\pi$, where E is the energy of the positron. A cutoff energy for the charged particles at 7 MeV is used.

Secondly, the existence of a dominant right-handed virtual photon enhances one step further the capability to resolve the ambiguity concerning the value of γ. We also conclude that the favored situation of a left-handed electron in the radiative pion decay $\pi^-\to e^-\bar{\nu}\gamma$ is indeed retained and realized in the particular configuration discussed here, though implicitly it is referring to a right-handed positron in this configuration for the present $\pi^+\to e^+e^+e^-\nu$ decay process.

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