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Low-lying bound states of three gluons: Their spectrum and production and decay properties

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In a potential model for glueballs, with a dynamical gluon mass $m \simeq 500$ MeV, the masses of the low-lying three-gluon glueballs (with $J^{PC}=0^{-+}$, 1^{--}, and 3^{--}) are found to be approximately degenerate $\simeq 4.8m \simeq 2.4$ GeV. The $\eta_c \to \eta\pi\pi$, $\psi \to \rho\pi$ decays are used to extract mixing of three-gluon 0^{-+} and 1^{--} glueballs with η_c and ψ, respectively. The decays $\psi(\psi') \to h+X$ ($h=\pi\pi,\eta,\eta'$) are suggested for experimentally studying the three-gluon continuum and three-gluon bound states. The three-gluon glueballs production rate via that mechanism as well as in pp, γp, and $\gamma\gamma$ collisions is estimated. The 1^{--} decays dominantly into $\rho\pi$, but also has reasonable decay widths for $K^*\bar{K}$ channels. The 0^{-+} glueball state decays dominantly into VV ($V=\rho,\omega,\phi$). Its other important decay modes are found to be $\delta\pi$, $V\gamma$, and $\gamma\gamma$.

I. INTRODUCTION

In the past few years considerable effort has been devoted to an understanding of the physics of glueballs.[1] Although most models of QCD imply the existence of such bound states of gauge fields, numerous technical difficulties have prevented a clear understanding of their properties so far. Experimentally[2] there are now at least two serious candidates for glueballs, the $\iota(1440$ MeV) and the $\theta(1660$ MeV) with $J^{PC}=0^{-+}$ and 2^{++}, respectively, whose properties are not inconsistent with theoretical expectations of glueballs. However, considerably more experimental information is still necessary before alternate explanations for these candidates can be entirely ruled out.

This paper addresses itself to some of the phenomenological issues involving the low-lying bound states of three gluons. Taking the relative angular momentum between any pair of gluons to be zero one has three possible S states with $J^{PC}=0^{-+}$, 1^{--}, and 3^{--}. The masses of these states are calculated using the potential model for glueballs developed in Ref. 3. In that model the gluons are taken to have a dynamical mass which presumably arises as a result of nonperturbative strong quantum corrections to the basic QCD Lagrangian.[4,5] The short-distance part of the glueball potential is extracted, then, from massive QCD. The long-distance part of the potential is assumed to have the form (of a breakable string) which results for the $d=2$ Schwinger model[6] and is also supported by studies on the lattice[7] of the intergluonic potential between static gluons and further by works on massive QCD.[5] The gluonium S state of three gluons are found to be essentially degenerate (within our approximation) with their mass $\simeq 4.8\,m$ where m is the effective mass of the gluon, expected to be $\simeq 500 \pm 200$ MeV.[5]

The production and decay properties of these three-gluon glueball states are studied using pole models.[8–16] The mixing parameters of the 0^{-+} and 1^{--} glueballs with pseudoscalars (η,η',η_c) and with vectors (ω,ϕ,ψ) are extracted by assuming that the decays $\eta_c \to \eta\pi\pi$ and $\psi \to \rho\pi$, are dominated by the 0^{-+} and the 1^{--} three-gluon glueballs, respectively.

We suggest that the ψ (ψ') decays

$$\psi(\psi') \to h+X, \quad h=\pi\pi,\eta,\eta' \tag{1.1}$$

may be used for the production and the study of the three-gluon continuum and the three-gluon bound states [represented by X in (1.1)]. Detection of h allows one to probe the distribution of the missing invariant mass of X. The rates for such reactions are expected to be at the level of a few percent.

We also present estimates for the cross sections for producing these three-gluon glueballs in pp, γp, and $\gamma\gamma$ collisions.

We suggest that the total decay widths of glueballs are Zweig-rule allowed and are characteristic of ordinary hadrons decays. We argue that the glueball widths must be much larger than the width (12 ± 4 MeV) for $\eta_c \to$ hadrons and expect them to be $\gtrsim 50$ MeV.

The partial decay widths of the $\tilde{O}$ (i.e., the $J^{PC}=1^{--}$ glueball[8]) and the ι_3 (i.e., the $J^{PC}=0^{-+}$ three-gluon glueball) are calculated. We find $O \to \rho\pi$ is a dominant decay with $\Gamma_{O\to\rho\pi} \simeq 50$ MeV. Furthermore $\Gamma(O \to K^*\bar{K})$ is found to be about 8 MeV. For the ι_3 we find $\iota_3 \to VV$, $V=\rho,\omega,\phi$, to be the dominant with a decay width $\simeq 360 \pm (\geq 140)$ MeV.[17] Also $\Gamma_{\iota_3\to\delta\pi} \simeq 15$ MeV, $\Gamma_{\iota_3\to V\gamma} \simeq 6$ MeV, and $\Gamma_{\iota_3\to\gamma\gamma} \simeq 1.5$ keV.

In Sec. II we calculate the masses of the three-gluon states. In Sec. III we extract the mixing parameters of the 0^{-+} and 1^{--} three-gluon glueballs with the quarkonia. Section IV discusses production of these glueballs via $\psi(\psi') \to \pi\pi X$. Section V contains brief remarks on the total width of glueballs. Their production cross sections via pp, γp, and $\gamma\gamma$ collisions are estimated in Sec. VI and Sec. VII deals with their decays. Section VIII presents a summary of the paper.

II. A POTENTIAL MODEL FOR GLUEBALLS AND ITS IMPLICATIONS FOR THE SPECTRUM OF LOW-LYING THREE-GLUON GLUEBALLS

We use the potential model for glueballs that was proposed in Ref. 3. In this model the gauge field is taken to be massive. The gluon mass is considered to arise dynamically and is estimated to lie in the range of 500 ± 200 MeV.[5] The short-distance part of the potential is calculated using massive QCD. In the one-gluon-exchange approximation the relevant Feynman graphs are shown in Fig. 1.

For the case of three-gluon bound states, pairs of gluons form a color octet and are coupled via color f-type or color d-type coupling. For the case of a two-gluon bound state the two gluons, by definition, form a color singlet. This has the effect that the sea gull graph contributes differently for the case of gluon pairs in a three-gluon gluonium from the case of a two-gluon gluonium. We are thus led to the amplitude

$$T=-\Big[g^2 f_{ace}f_{bde}\langle 4\,|\,J_\sigma\,|\,1\rangle\frac{1}{t-m^2}\langle 5\,|\,J^\sigma\,|\,2\rangle + f_{ace}f_{bde}(\epsilon_1\cdot\epsilon_2\epsilon_4^*\cdot\epsilon_5^*-\epsilon_5^*\cdot\epsilon_1\epsilon_4^*\cdot\epsilon_2) + f_{abe}f_{cde}\epsilon_4^*\cdot\epsilon_1\epsilon_5^*\cdot\epsilon_2+\text{exch.}\Big]\,, \tag{2.1}$$

where $t=(p_1-p_4)^2$, exch implies interchange of lines 4 and 5, and

$$\langle 4\,|\,J_\sigma\,|\,1\rangle=\epsilon_4^*\cdot\epsilon_1(p_1+p_4)_\sigma-2\epsilon_{4\sigma}^*p_4\cdot\epsilon_1-2\epsilon_{1\sigma}p_1\cdot\epsilon_4^*\,. \tag{2.2}$$

As has been emphasized[3] the on-shell amplitude given in (2.1) is fully gauge-invariant as long as all the internal and external gluon lines have the same mass.

Retaining lowest order relativistic corrections the one-gluon-exchange (OGE) amplitude (2.1) leads to the following space-time potential for a gluon pair in a three-gluon glueball:

$$V_{\rm OGE}=-\frac{\lambda}{2}\left\{\left[1+\frac{4\vec{p}^{\,2}-3m^2}{4m^2}+\frac{3}{2m^2}\vec{L}\cdot\vec{S}\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{2m^2}(\vec{S}\cdot\vec{\nabla}\,\vec{S}\cdot\vec{\nabla}-\tfrac{1}{3}S^2\vec{\nabla}^2)+\tfrac{1}{3}\vec{S}^2\right]\frac{e^{-mr}}{r}-(1+\tfrac{5}{6}S^2)\frac{\pi}{m^2}\delta^3(r)\right\}. \tag{2.3}$$

In (2.3) $\lambda=3g^2/4\pi$ characterizes the QCD coupling strength and the factor of $\frac{1}{2}$ arises because the gluon pair is in an octet in a three-gluon glueball. As expected, the potential consists of a Yukawa term and various spin-spin, spin-orbit, and tensor terms. We note that, unlike was the case for two-gluon glueballs, for the three-gluon case the δ-function term is always repulsive and as such causes no special problems.

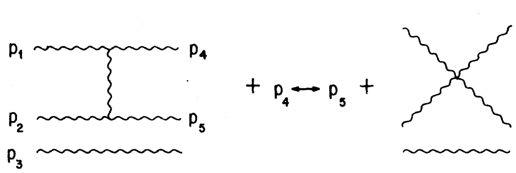

FIG. 1. Feynman graphs used to extract the short-distance part of the gluon-gluon potential.

Let us next discuss the long-distance part of the potential which is assumed to be spin independent. The color screening of gluons is brought about by a breakable string, that is, the gluon string breaks when sufficient energy has been stored in it to materialize a gluon pair. This is attained by taking the following simple form of the string potential for a two-gluon glueball:

$$V_s(r)=2m(1-e^{-\beta mr})\,. \tag{2.4}$$

We recall that this form for the string potential results in the $d=2$ Schwinger model where the string breaks by formation of fermion-antifermion pairs.[6] This form of the string potential also simulates the intergluonic potential as seen in lattice calculations.[7] In addition $V_s(r)$ can also be justifed for gluons by consideration of a condensate of vortices that are thought to exist for massive QCD.[5]

For three-gluon glueballs the complete potential is obtained by summing over pairs of two-body potentials. Thus,

$$V_{3\ \rm gluon}=\sum_{i<j}[V_{\rm OGE}(r_{ij})+\tfrac{1}{2}V_s(r_{ij})]\,. \tag{2.5}$$

The factor of half in the string potential is there because one needs to remove three (and not six) gluons from the vacuum to screen the three gluons that are originally there in the glueball.

This glueball potential has three semifree parameters which are m, β, and λ. As stated earlier the gluon mass m is estimated to have a range of 500 ± 200 MeV.[5] β is related to the string tension (K_A) for gluons: $\beta=K_A/2m^2$. From the work on the lattice of Ref. 7 one expects $\beta\sim(1\pm0.7)$. Finally $\lambda(=3g^2/4\pi)$ is related to the gauge coupling constant and controls the strength of the effective one-gluon-exchange potential. λ is expected to be in the range $\sim(2\pm0.5)$. The potential with the values of the parameters within the stated ranges yields two-gluon glueballs with masses 1 to 2 GeV. However $J^{PC}=0^{\pm+}$ glueballs do not bind unless $\beta\lesssim0.6$. Taking β, m, and λ to have the central values of 0.3, 500 MeV, and 2, respectively, one finds that $J^{PC}=0^{-+}$ and 2^{++} glueballs have masses ~1400 and ~1600 MeV, respectively, that is, in the vicinity of the experimental candidates $\iota(1440)$ and $\theta(1660)$ seen at SLAC. While we understand that considerable experimental and theoretical work need to be done before the glueball interpretation of these two candidates becomes fully acceptable we will estimate the mass of low-lying three-gluon gluonia with the parameters held fixed at the values mentioned above. Needless to say the simplification and the approximations used (to cite a few—nonrelativistic potential picture, mixing of glueballs with quark-antiquark states, mixing of two-gluon and three-gluon states, higher-order corrections, etc.), lead us to believe that the numerical values of the glueball masses that we obtain will necessarily have an uncertainty of a few hundred MeV.

We concentrate on the low-lying three-gluon glueball states which are obtained by taking the relative angular momenta between any pair to be 0. Then the space part of the glueball wave function for such S states is symmetric and their parity P=odd. Now the color part of the wave function is either totally symmetric (d-type coupling, C=odd), or antisymmetric (f-type coupling, C=even). Thus, for the S states mixed symmetry in the spin part cannot be accommodated. Note that for three spin-1 objects

$$1\oplus 1\oplus 1=(3_s+2_m+1_s)+(2_m+1_m+0_a)+1_s\ , \qquad (2.6)$$

Bose statistics then constrains that the state antisymmetric in spin ($J=0$) combines with f-type color to yield $J^{PC}(S_{\rm pair}=\text{total spin of any pair})=0^{-+}(1)$ and similarly spin symmetric states combine with d-type color coupling to yield $J^{PC}(S_{\rm pair})=3^{--}(2)$, $1^{--}(2)$, and $1^{--}(0)$.

As usual we introduce the center-of-mass and relative coordinates[18]:

$$R=\frac{r_1+r_2+r_3}{\sqrt{3}}\ ,$$
$$\rho=\frac{r_1-r_2}{\sqrt{2}}\ , \qquad (2.7)$$
$$r=\frac{1}{\sqrt{6}}(r_1+r_2-2r_3)\ .$$

Then the expression for the total kinetic energy of the three-body bound state takes the form:

$$-\frac{1}{2m}\sum_{i=1}^{3}\nabla_i{}^2=-\frac{1}{2m}\left[\frac{\partial}{\partial R}\right]^2-\frac{1}{2m}\left[\frac{\partial}{\partial r}\right]^2-\frac{1}{2m}\left[\frac{\partial}{\partial \rho}\right]^2 . \qquad (2.8)$$

We shall use a variational method to search for energy eigenvalues. For that purpose we take as the trial wave function:

$$\phi=\alpha^3\left[\frac{2}{\pi}\right]^{3/2}e^{-\alpha^2(r^2+\rho^2)}\ . \qquad (2.9)$$

For $\lambda=2$, $\beta=0.3$ we find $M_{1^{--}(0)}\simeq 4.80\ m$, $M_{0^{-+}(1)}\simeq 4.81\ m$, $M_{1^{--}(2)}\simeq M_{3^{--}(2)}\simeq 4.79\ m$, i.e., masses of the low-lying three-gluon states are degenerate within our approximation and approximately equal 4.8 m = 2.4 GeV for m = 500 MeV. In fact for $\lambda=2\pm 0.5$ and $\beta=0.6\pm 0.3$ we find that the mass splitting between these states is always $<0.1\ m\simeq 50$ MeV. Figure 2 shows the dependence on β of the average mass of these states. For β=0.3, 0.6, and 0.9 the average mass is 4.9 m, 5.6 m, and 5.9 m, respectively.

This approximate degeneracy between these S states is likely to be removed once corrections to the various approximations and simplifications that we have made (such as higher-order and relativistic corrections and mixings) are included. If the long-distance part of the potential should have any spin dependence that could also cause splitting between these states. Furthermore since the 0^{-+} state is available to both two- and three-gluon channels whereas 1^{--} and 3^{--} occur only in three-gluon states the mixing among the 0^{-+}'s would certainly tend to raise the mass estimate for the three-gluon 0^{-+} state made by ignoring that mixing.

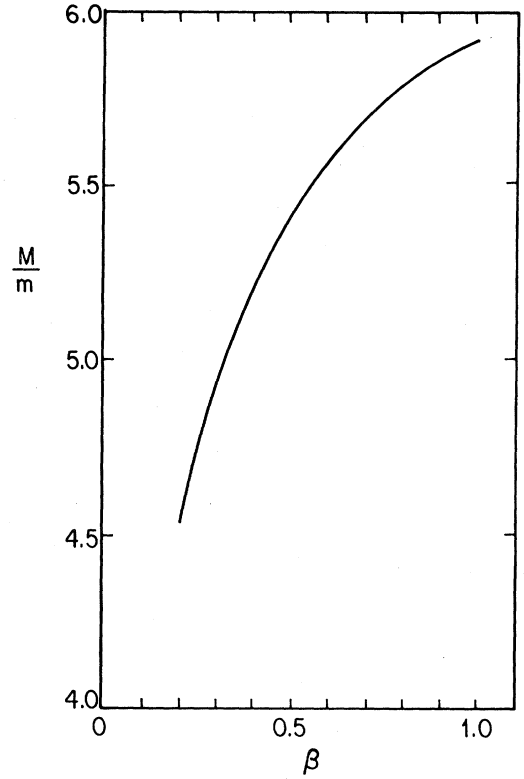

FIG. 2. Dependence of the average mass (M) of the three-gluon glueball S states 0^{-+}, 1^{--}, and 3^{--} on the screening parameter β [see Eq. (2.4)]. Note that m is the gluon effective mass and λ the QCD coupling constant ($\lambda=3g^2/4\pi$) is held fixed at $\lambda=2$. The splitting among the three states is very small ($\lesssim 50$ MeV) within our approximation.

III. MIXING PARAMETERS OF THE $J^{PC}=0^{-+}$ AND 1^{--} THREE-GLUON GLUEBALLS WITH QUARKONIA

Mixings of quarkonia with neighboring glueballs can be used as a simple method to relate the decays of quarkonia and glueballs.[8–16] We shall use this approach for the $J^{PC}=0^{-+}$ three-gluon glueball (which we call ι_3 hereafter) and for the $J^{PC}=1^{--}$ glueball (which, following Freund and Nambu,[8] we are continuing to denote by the letter O).

Let us consider the Zweig-rule-forbidden decay mode

$$\eta_c\to\eta\pi\pi\ .$$

We view this decay (see Fig. 3) to be dominated by the ι_3 glueball or by the $\iota(1440)$ glueball. [We take the $\iota(1440)$ to be made up of two constituent gluons so we designate it ι_2.] The figure shows the sequence $\eta_c\to\iota_K\to\eta'\to\eta\pi\pi$, where $K=2,3$. The vertex $\eta'\to\eta\pi\pi$ is normalized through a dimensionless effective coupling to account for the observed decay rate for the process $\eta'\to\eta\pi\pi$. For simplicity we assume ι_3 and ι_2 mixing with η, η', and η_c are identical. Flavor blindness of the glueball interactions

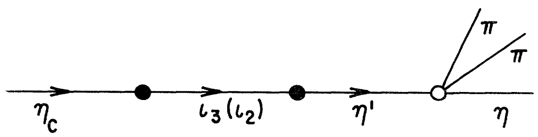

FIG. 3. The decay $\eta_c \to \eta\pi\pi$ via the mixings with ι_3 or ι_2.

then suggests that the mixing obey the following relation:

$$f_{\iota_K\eta_c}:f_{\iota_K\eta'}:f_{\iota_K\eta}=(1:\cos\theta_P\sqrt{3}:-\sin\theta_P\sqrt{3})f_{\iota_K}\ , \tag{3.2}$$

where $\theta_P=-11°$. Taking the measured branching ratio for $\eta_c \to \eta\pi\pi$ to be $\simeq 3\%$ and its total decay width to be $\simeq 12$ MeV we find

$$f_{\iota_K}\simeq 1\ \mathrm{GeV}^2,\quad K=2,3\ . \tag{3.3}$$

Simple as it is such a pole model for (3.1) through ι_K lends us with considerable amount of predictive power for estimating various production cross sections (via, e.g., $\gamma\gamma$, γp, and hadron-hadron collisions, and via ψ, ψ' decays, etc.) and for the decay widths (for $\iota_3 \to \eta\pi\pi$, $\iota_3 \to \rho\rho$, $\iota_3 \to \rho\gamma$, etc.) of ι_3.

Let us next consider the mixing of 1^{--} glueball (i.e., the O) with flavor-singlet vector quarkonia: ω,ϕ,ψ. A pole model involving the O to account for the Zweig-rule-forbidden decay modes of vector quarkonia has actually been discussed in the literature.[8,9,15] There are important differences between those works and ours. First, in our potential model the mass ($\simeq 2.4$ GeV) of the O is much larger than $\sim(1-1.5$ GeV) taken in the literature. In addition, rather than assuming any specific form and value(s) for the mixing as has been done by previous authors we shall use the experimental information on quarkonium decays and evaluate the mixing parameter. Furthermore, we take the mixing parameter of the O with the vector mesons to be a function of the invariant mass.[19] Thus,

$$f_{O\omega}(q^2):f_{O\phi}(q^2):f_{O\psi}(q^2)=(\sqrt{2}:-1:1)f_O(q^2) \tag{3.4}$$

and $f_O(q^2)$ will be determined from experiment for various values of q^2.

Consider the decay $\phi \to \rho\pi$. This is viewed to proceed via the sequence $\phi \to O \to \omega \to \rho\pi$ so that

$$\Gamma(\phi\to\rho\pi)=3\frac{f_{\omega\rho\pi}{}^2}{96\pi}f_{O\phi}{}^2(m_\phi)f_{O\omega}{}^2(m_\phi)\frac{m_\phi{}^2-m_\omega{}^2}{(m_O{}^2-m_\phi{}^2)^2}$$

$$\times\frac{1}{m_\phi{}^3}\left[1-\frac{m_\pi{}^2}{(m_\phi-m_\rho)^2}\right]^{3/2}. \tag{3.5}$$

Using $f_{\omega\rho\pi}{}^2=(4/m_\rho{}^2)f_{\rho\pi\pi}{}^2$, $f_{O\omega}{}^2(m_\phi)=2f_{O\phi}{}^2(m_\phi)$, $M_O=2.4$ GeV, and the measured decay width for $\phi\to\rho\pi$ we obtain

$$f_O(m_\phi)\simeq 0.28\ (\mathrm{GeV})^2\ . \tag{3.6}$$

Next consider the decay $\psi\to\rho\pi$. It takes place in the model via $\psi\to 0\to\omega\to\rho\pi$. Thus,

$$\Gamma(\psi\to\rho\pi)=\frac{3}{96\pi}f_{\omega\rho\pi}{}^2[2f^4(m_\psi)]$$

$$\times\frac{m_\psi{}^2-m_\omega{}^2}{(m_O{}^2-m_\psi{}^2)^2}\frac{1}{m_\psi{}^3}\left[1-\frac{m_\pi{}^2}{(m_\psi-m_\rho)^2}\right]^{3/2} \tag{3.7}$$

yielding

$$f_O^{\rho\pi}(m_\psi)\simeq 0.05\ \mathrm{GeV}^2\ , \tag{3.8}$$

which we will use in later application. We will also need the mixing parameter at the mass of the O, i.e., $f_O(M_O)$. In keeping with our philosophy that the mixing parameter f_0 is independent of flavor but depends only on the mass relevant to a given transition we shall take $f_O(M_O)$ to be the geometric mean of (3.6) and (3.8), i.e.,

$$f_O(M_O)\simeq 0.13\ \mathrm{GeV}^2\ . \tag{3.9}$$

IV. THE REACTIONS $\psi(\psi')\to\pi\pi$ (or η,η')$+X$ AS POSSIBLE FACTORIES FOR PRODUCING THREE-GLUON BOUND STATES

Let us recall that the reaction $\psi\to\gamma+X$ is a very important means for probing the spectrum of two-gluon-glueballs. The two experimental candidates for glueballs, namely, $\iota(1440)$ and $\theta(1660)$ were discovered through this process. The total rate for this inclusive reaction is estimated to be approximately 8%.[20]

It is clearly important to think of processes in which the bound states of three gluons could be produced and be accessible to experimental investigation. For this purpose we propose the reaction:

$$\psi(\psi')\to h+X;\quad h=\pi\pi,\eta,\eta'\ldots\ , \tag{4.1}$$

where h is a specific hadronic final state that couples to two gluons. Thus h must be a color and flavor singlet, e.g., $\pi\pi$, η, or η'. The pion pair thus coupled must transform as an isoscalar. Detection of h allows one to reconstruct and thereby directly probe the spectrum of the missing invariant mass of X. Reactions of the type (4.1) arise via Fig. 4 which represents decays of vector quarkonia via emission of five gluons. These are corrections to the basic lowest-order decay of quarkonia via three gluons and as such their branching ratios are expected to be of order $(\alpha_s)^2\sim$ a few percent.

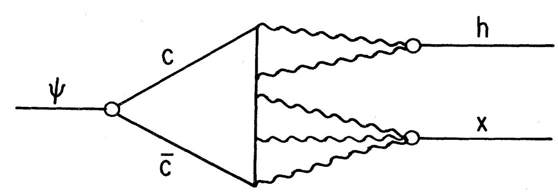

FIG. 4. The process $\phi\to h+X$ where X is a three-gluon glueball (or continuum) state and h represents a color-singlet state obtained from the convolution of two gluons, e.g., $h=\pi\pi,\eta,\eta'\ldots$.

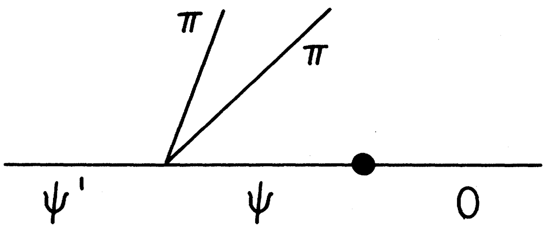

FIG. 5. The reaction $\phi' \rightarrow \pi\pi O$ resulting from the observed decay $\psi' \rightarrow \pi\pi\psi$ and $\psi \leftrightarrow O$ mixing.

A reliable determination of these rates is rather difficult. For the specific case of the vector glueball (O), however, the observed rate for $\psi' \rightarrow \psi\pi\pi$ can be used to determine directly $\psi' \rightarrow \pi\pi O$ via the $O \rightarrow \psi$ mixing as shown in Fig. 5. We find

$$\frac{B(\psi' \rightarrow \pi\pi O)}{B(\psi' \rightarrow \pi\pi\psi)} \simeq 2\% \tag{4.2}$$

for M_O between 2.4 to 3.0 GeV.

V. GENERAL REMARKS ON THE TOTAL DECAY WIDTHS OF GLUEBALLS

We recall that the total decay width of $q\bar{q}$ bound states (such as η_c) into hadrons can be calculated by considering the process:

$$\eta_c \rightarrow g + g \ . \tag{5.1}$$

One thus finds that[21]

$$\Gamma(\eta_c \rightarrow \text{hadrons}) = \frac{32\pi}{3} \frac{\alpha_s{}^2}{m_{\eta_c}{}^2} \, |\, \psi(0)\, |^2 \simeq 5 \pm 1 \text{ MeV} \tag{5.2}$$

to be compared to the measured width $\sim(12 \pm 5)$ MeV. The total decay width of a glueball [say a two-gluon bound state such as $\iota(1440)$] would be given similarly by the process

$$\iota_2 \rightarrow g + g \ . \tag{5.3}$$

Now the factor of $\alpha_s{}^2$ in (5.2) which is a measure of the rate for $c\bar{c} \rightarrow gg$ is clearly absent. One is therefore led to expect that the total decay width of glueballs into hadrons is likely to be appreciably larger than that of $\eta_c \rightarrow$ hadrons. Indeed as has been noted,[3] in the large-N limit the process glueball$\rightarrow g + g$, which characterizes the total decay width of glueballs, exhibits no $1/N$ suppression and is completely Zweig-rule-allowed whereas the decay $\eta_c \rightarrow g + g$ is suppressed by a factor $1/N$. We thus expect glueball widths to be similar to the decay widths of ordinary hadrons, i.e., $\gtrsim 50$ MeV. Later in this paper we shall need to use this estimate for glueball widths for a determination of their production cross sections.

VI. ESTIMATES OF THE CROSS SECTIONS FOR PRODUCING THREE-GLUON GLUEBALLS

A. Hadron collisions

Let us first consider the case of producing any two-gluon glueball (denoted by G_2) in hadronic collisions. The cross section can be estimated using gluon-gluon fusion (see Fig. 6). In the literature a similar approach was used for estimating the cross section of the η_c.[22] Then

$$\sigma(A + B \rightarrow G_2 + X) = \frac{8\pi^2 \Gamma_{G_2}}{M_{G_2}{}^3} \tau \int_\tau^1 \frac{dx}{x} F_g^A(x) F_g^B(\tau/x) \ , \tag{6.1}$$

where $\tau = M_{G_2}{}^2/s$, F_g's are, as usual, the gluon distribution functions and Γ_{G_2} is the total decay width of G_2, i.e., of the process $G_2 \rightarrow g + g$.

Figure 7 shows (see the dashed curve) the cross section as a function of s for producing G_2 in pp collision. For G_2 mass of 1.5 (2.5) GeV the cross section increases from $\simeq$08 mb (0.009 mb) at $s = 400$ GeV^2 to $\simeq$0.2 mb (0.03 mb) at $s = 3000$ GeV^2 where we have used $\Gamma_{G_2} = 50$ MeV.

Now the cross section for producing three-gluon glueballs (denoted generically by G_3) is expected to be smaller (i.e., by about a factor of $\simeq \alpha_s$). The estimates given above (for a mass of 2.4 GeV) should therefore serve as an upper bound for three-gluon glueballs.

For ι_3, a lower bound to the cross section can be estimated by using the mixing of ι_3 and ι_2. Since the gluon-gluon interactions are stronger than gluon-quark interactions we expect the $\iota_3 \leftrightarrow \iota_2$ mixing to be larger than ι_3 mixing with pseudoscalars (η_c, η', η), i.e.,

$$f_{\iota_3 \iota_2} > f_{\iota_3} \simeq 1 \text{ GeV}^2 \ . \tag{6.2}$$

This lower bound for the $\iota_3 \leftrightarrow \iota_2$ mixing therefore gives rise to a lower bound for the cross section for producing ι_3 via the mixing (see Fig. 8). The upper and lower bounds for the cross sections for ι_3 are represented by solid curves in Fig. 7. The lower bounds are smaller by about a factor of 13 than the upper bounds.

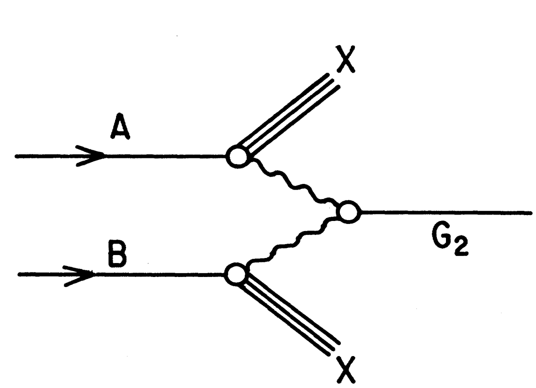

FIG. 6. Formation of a two-gluon glueball (generically denoted by G_2) via gluon fusion in the reaction $A + B \rightarrow G_2 + X$.

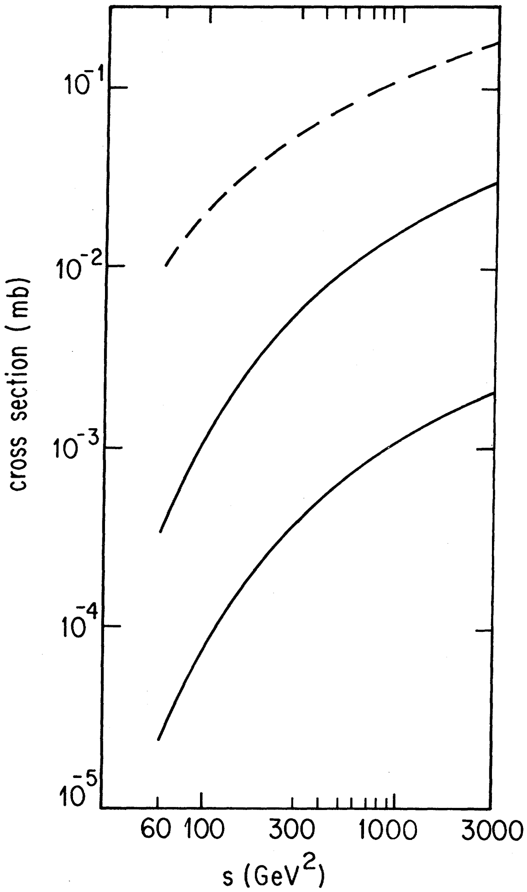

FIG. 7. Cross section for producing glueballs in pp collisions via gluon fusion. Gluon distribution in the nucleon is taken to have the form $(1-x)^5/x$. Dashed curve and the top solid curve are for a two-gluon glueball of mass 1.5 GeV and 2.5 GeV, respectively. Bottom solid curve is for a three-gluon glueball of mass 2.5 GeV formed by mixing with a two-gluon state. (See Fig. 8.) Solid curves represent upper and lower bounds for producing ι_3. Glueball total width of 50 MeV is used in these calculations.

B. Photoproduction

In γN collision gluon-gluon fusion can be an important means for producing bound states.[23] Here one of the gluons originate from the photon beam. The distribution function for the gluons in the photon has two components to it, i.e., the vector-meson-dominance component and the perturbative component. Using these we calculate the cross sections for producing a two-gluon bound state with mass $\simeq 2.4$ GeV. We find that the numbers for that process are $\simeq 0.2\ \mu$b for $s \gtrsim 200$ GeV^2 with very little energy dependence. Once again this estimate represents an indication of the upper bound on the cross section for producing ι_3. Next, as in the last section, we calculate the cross sections for producing ι_3 via mixing with ι_2. The corresponding numerical value for this is $\simeq 15$ nb for $s \gtrsim 200$ GeV^2. This should represent an indicative lower bound for photoproducing ι_3.

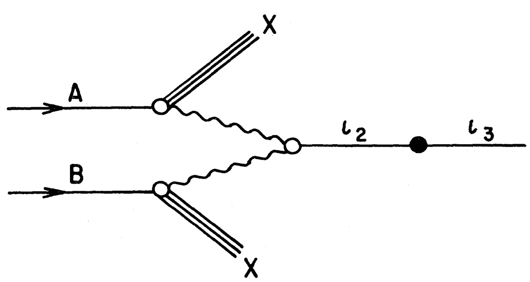

FIG. 8. Formation of a three-gluon glueball (ι_3) via its mixing with a two-gluon glueball (ι_2).

We next consider photoproduction of the O meson. Here we use the mixing of the O meson with $\omega, \phi, \psi, \ldots$. Then the amplitude for

$$\gamma + p \rightarrow O + p$$

is related to

$$\gamma + p \rightarrow V + p, \quad V = \omega, \phi, \psi, \ldots,$$

via

$$\mathcal{M}_{\gamma p \rightarrow Op} = \sum_V \frac{f_{OV}(M_O)}{m_O{}^2 - m_V{}^2} \mathcal{M}_{\gamma p \rightarrow Vp} \; . \tag{6.3}$$

This yields

$$\frac{\sigma_{\gamma p \rightarrow Op}}{\sigma_{\gamma p \rightarrow \omega p}} = f^2(m_O) \left[\frac{\sqrt{2}}{(m_O{}^2 - m_\omega{}^2)} - \frac{1}{(m_O{}^2 - m_\phi{}^2)} \left[\frac{\sigma_{\gamma p \rightarrow \phi p}}{\sigma_{\gamma p \rightarrow \omega p}} \right]^{1/2} + \frac{1}{(m_O{}^2 - m_\psi{}^2)} \left[\frac{\sigma_{\gamma p \rightarrow \psi p}}{\sigma_{\gamma p \rightarrow \omega p}} \right]^{1/2} \right]^2 \tag{6.4}$$

leading to $\sigma_{\gamma p \rightarrow Op} \simeq 0.2$ nb, where we have assumed total coherence.

C. Production of ι_3 via $\gamma\gamma$ collision

The decay width for $\iota_3 \rightarrow \gamma\gamma$ via $\iota_3 \leftrightarrow \eta_c$ mixing is found to be [see Eq. (7.21)] $\simeq 1.5$ keV. Using standard methods[24] we then estimate the cross section for the reaction

$$e + e \rightarrow e + e + \iota_3 \tag{6.5}$$

to be $\simeq (1.5, 2.4, 3.1, 5.8) \times 10^{-36}$ cm^2 for $s = (100, 400, 1000, 3000)$ GeV^2.

VII. SELECTED DECAYS OF THE O AND THE ι_3

A. Decays of the O

We use the parameters obtained in Sec. III for the mixing of the O with ω, ϕ, ψ to calculate the decays of the O into some selected final states.

$O \rightarrow \rho\pi$: Using the $O \leftrightarrow \omega$ mixing one has for this mode

$$\Gamma_{O\to\rho\pi}=\frac{3f_{\omega\rho\pi}^{2}}{96\pi}f_{O\omega}^{2}(m_O)(m_O^{2}-m_\omega^{2})\left[1-\frac{m_\pi^{2}}{(m_O-m_\rho)^2}\right]^{3/2}\Big/m_O^{2}\simeq 50\text{ MeV}\ . \tag{7.1}$$

$O\to K^*\bar{K}$: This decay proceeds through the $O\leftrightarrow\phi$ mixing. The decay vertex for $\phi\to K^*\bar{K}$ is analogous to that of $\omega\to\rho\pi$. Thus $f_{\omega\rho\pi}{}^2=4f_{\rho\pi\pi}{}^2/m_\rho{}^2$ implies

$$f_{\phi K^*\bar{K}}{}^2=\frac{4f_{\phi K\bar{K}}{}^2}{m_\phi{}^2}=\frac{2f_{\rho\pi\pi}{}^2}{m_\phi{}^2}\ . \tag{7.2}$$

Using Eqs. (7.1) and (7.2) we arrive at the estimate

$$\Gamma_{O\to K^*\bar{K}}\simeq 8\text{ MeV}\ . \tag{7.3}$$

$O\to K\bar{K}$: This decay mode proceeding via $O\to\phi\to K\bar{K}$ is suppressed. We find

$$\Gamma_{O\to K\bar{K}}=\frac{2f_{O\phi}{}^2(m_O)}{(m_O{}^2-m_\phi{}^2)^2}\frac{2}{3}\frac{f_{\phi K\bar{K}}{}^2}{4\pi}\frac{(m_O{}^2-4m_K{}^2)^{3/2}}{8m_O{}^2}$$
$$\simeq 0.4\text{ MeV}\ . \tag{7.4}$$

$O\to e^+e^-$: The mixing of O with ω,ϕ,ψ leads to

$$\frac{\Gamma_{O\to e^+e^-}}{\Gamma_{\omega\to e^+e^-}}=2f^2(m_O)\left[\frac{m_\omega}{m_O}\right]^3\left[\frac{1}{m_O{}^2-m_\omega{}^2}-\frac{1}{m_O{}^2-m_\phi{}^2}+\frac{2}{m_O{}^2-m_\psi{}^2}\right]^2 \tag{7.5}$$

yielding

$$\Gamma_{O\to e^+e}\simeq 0.3\text{ eV}\ , \tag{7.6}$$

very very small indeed.

B. Decays of the ι_3

We recall that the mixing of ι_3 with pseudoscalar quarkonia η,η',η_c is related by

$$f_{\iota_K\eta_c}:f_{\iota_K\eta'}:f_{\iota_K\eta}=(1:\cos\theta_P\sqrt{3}:-\sin\theta_P\sqrt{3})f_{\iota_K}\ . \tag{7.7}$$

Using the experimental rate for $\eta_c\to\eta\pi\pi$ we deduced (see Sec. III) that

$$f_{\iota_3}\simeq 1\text{ GeV}^2\ . \tag{7.8}$$

We shall now use these mixings of ι_3 to discuss some of its decay modes.

$\iota_3\to\eta\pi\pi$: We take the dominant contribution to this mode to arise from the sequence $\iota_3\to\eta'\to\eta\pi\pi$. The decay is related to $\eta_c\to\eta\pi\pi$ (see Fig. 3). We thus find

$$\Gamma(\iota_3\to\eta\pi\pi)=\Gamma(\eta_c\to\eta\pi\pi)\left[\frac{m_{\eta_c}{}^2-m_{\iota_3}{}^2}{f_{\iota_3\eta_c}}\right]^2, \tag{7.9}$$

$$\Gamma(\iota_3\to\eta\pi\pi)\simeq 8\text{ MeV}\ . \tag{7.10}$$

$\iota_3\to\delta\pi$: The $\iota_3\leftrightarrow\eta,\eta'$ mixing allows one to relate $\iota_3\to\delta\pi$ to $\delta\to\eta\pi$.[25] (See Fig. 9.) Taking the dimensional coupling at the $\delta\eta\pi$ vertex to be proportional to the initial mass:

$$\frac{\Gamma(\iota_3\to\delta\pi)}{\Gamma(\delta\to\eta\pi)}=3\frac{P_{\iota_3}}{P_\delta}\frac{m_\delta{}^2}{m_{\iota_3}{}^2}\frac{f_{\iota_3\eta}{}^2}{(m_{\iota_3}{}^2-m_{\eta'}{}^2)^2}\left[\frac{g_{\eta'\pi}}{g_{\eta\pi\delta}}+\frac{f_{\iota_3\eta}}{f_{\iota_3\eta'}}\frac{m_{\iota_3}{}^2-m_{\eta'}{}^2}{m_{\iota_3}{}^2-m_\eta{}^2}\right]^2, \tag{7.11}$$

where P_ι and P_δ are the phase-space factors and

$$\frac{g_{\eta'\pi\delta}}{g_{\eta\pi\delta}}\simeq\frac{\langle\eta'\,|\,\text{light quarks}\rangle}{\langle\eta\,|\,\text{light quarks}\rangle}\simeq 0.8\ . \tag{7.12}$$

Thus

$$\Gamma(\iota\to\delta\pi)\simeq 0.5\Gamma(\delta\to\eta\pi)\simeq(15\pm 10)\text{ MeV}\ . \tag{7.13}$$

$\iota_3\to V\gamma$: The mixing of ι_3 with η,η' allows one to do a calculation very similar to the preceding one to yield

$$\Gamma(\iota_3\to\rho\gamma)\simeq 80\Gamma(\eta'\to\rho\gamma)\simeq 6\pm 2\text{ MeV} \tag{7.14}$$

and

$$\Gamma(\iota_3\to\omega\gamma)\simeq 0.7\text{ MeV}\ , \tag{7.15}$$

$$\Gamma(\iota_3\to\phi\gamma)\simeq 0.3\text{ MeV}\ . \tag{7.16}$$

$\iota_3\to VV$: Vector-meson dominance now allows one to relate $\iota_3\to VV$ to $\iota_3\to V_\gamma$. Thus,

$$\frac{\Gamma(\iota_3\to\rho\rho)}{\Gamma(\iota_3\to\rho\gamma)}=\frac{\alpha m_\rho}{3\Gamma_{\rho\to ee}}\frac{\text{phase space for }\rho\rho}{\text{phase space for }\rho\gamma}\times\frac{3}{2}\times\frac{1}{4}\simeq 60 \tag{7.17}$$

$$\Longrightarrow\ \Gamma(\iota_3\to\rho\rho)\simeq 360\pm(>140)\text{ MeV (Ref. 17)}\ . \tag{7.18}$$

Similarly, one finds

$$\Gamma(\iota_3\to\omega\omega)\simeq 3\text{ MeV}\ , \tag{7.19}$$

$$\Gamma(\iota_3\to\phi\phi)\simeq 12\text{ MeV}\ . \tag{7.20}$$

$\iota_3\to\gamma\gamma$: The mixing with η,η',η_c contributes to this mode. (See Fig. 10.) However the decay width for $\eta_c\to\gamma\gamma$ is very much larger than that for $\eta\to\gamma\gamma$ or $\eta'\to\gamma\gamma$. So that to a good approximation only the η_c pole need be taken into account. Thus,

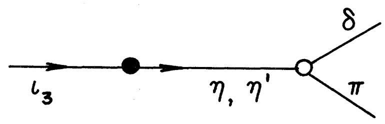

FIG. 9. $\iota_3 \rightarrow \delta\pi$ via $\iota_3 \rightarrow \eta,\eta'$ mixing.

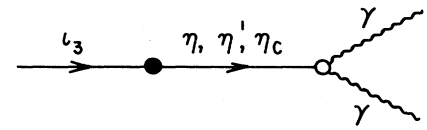

FIG. 10. $\iota_3 \rightarrow \gamma\gamma$ via $\iota_3 \leftrightarrow \eta,\eta',\eta_c$ mixing.

$$\frac{\Gamma(\iota_3 \rightarrow \gamma\gamma)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = \frac{f_{\iota_3\eta_c}{}^2}{(m_{\iota_3}{}^2 - m_{\eta_c}{}^2)^2} \simeq 3\%$$

$$\Longrightarrow \Gamma(\iota_3 \rightarrow \gamma\gamma) \simeq 1.5 \text{ keV} . \qquad (7.21)$$

VIII. SUMMARY

We have presented a study of the properties of the low-lying three-gluon glueballs with $J^{PC}=0^{-+}$, 1^{--}, and 3^{--}. Using the potential model of Ref. 3 with dynamical gluon mass we find these three-gluon states to be essentially degenerate at approximately 2.4 GeV.

To study their production and decay properties we have extracted the mixing of the 0^{-+} and 1^{--} glueballs with (η,η',η_c) and with (ω,ϕ,ψ), respectively. The decays $\psi(\psi') \rightarrow h+X, h=\pi\pi,\eta,\eta'$ may be a copious means for producing three-gluon continuum, and three-gluon bound states (represented by X). The cross sections for producing these states via pp, γp, and $\gamma\gamma$ collisions are estimated.

Pole models suggest that the vector glueball decays predominantly into $\delta\pi, K^*\bar{K}$ and the pseudoscalar three-gluon glueball decays predominantly into $VV\,(V=\rho,\omega,\phi)$. Estimates for their respective decays to $(\omega\pi\pi,\phi\pi\pi, K^*\bar{K},K\bar{K})$ and to $(\eta\pi\pi,\delta\pi,V\gamma,\gamma\gamma)$ are also given.

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