On the correlation between transverse momentum and multiplicity of secondaries at collider energy

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The correlation between $\langle p_t \rangle$ and multiplicity is examined in the framework of conventional hadron dynamics. A plausible estimate of the effect at collider energy close to the experimental value is given. We predict a very specific rapidity structure of the correlation.

An interesting correlation has been observed[1] at the CERN $p\bar{p}$ collider: in the center-of-mass rapidity window $y \in (-2.5, 2.5)$ the transverse momentum $\langle p_t \rangle$ of a secondary particle increases from roughly 0.33 GeV/c to about 0.47 GeV/c, when the multiplicity per unit rapidity rises from 1 to 20. An early discussion of this correlation can be found in Ref. 2. In this paper we examine carefully the compatibility of the effect with the most standard low-p_t phenomenology.

Anticipating the discussion to follow, our conclusion can be stated as follows: The correlation observed so far is (perhaps) compatible with the conventional wisdom. However, a conventional explanation implies a very specific and easily observable rapidity structure of the correlation, as shown in Fig. 1. We urge the experimenters to check this point. Logically, it seems to be a necessary step before more speculative considerations are seriously taken into account.

Let us recall the conventional picture: The forward elastic scattering amplitude is a sum of single- and multi-Pomeron-exchange terms. Although the single-Pomeron exchange dominates, the multi-Pomeron exchanges are necessarily there as a consequence of unitarity. Cross sections for production processes are obtained by "cutting" through the Pomerons. We neglect the interactions between the latter and we call a "chain" the ensemble of particles belonging to the same cut Pomeron. The very existence of the Pomeron reflects the short range in rapidity of the effective hadronic forces. Thus, particles are produced in chains and there are only short-range rapidity correlations within each chain. However, fluctuations in the number of superposed chains produce positive long-range rapidity correlations.[3]

Notice that we use the expression "conventional picture" in a rather restrictive sense, as synonomous with the ensemble of dynamical postulates and concepts underlying the Reggeon theory (occasionally supplemented by a more specific model when numerical estimates are sought). This is partly for definiteness and partly because we believe that the dynamical picture in question represents to date the most elaborate, albeit purely phenomenological, description of hadronic production and diffraction.

Chains result from interactions between virtual constituents of the incoming hadrons. In a frame where virtual consitituents are collinear, the secondary particles have the "canonically' small transverse momentum, $\langle p_t \rangle = 0.33$ GeV/c or so. However, this frame does not coincide with the one where the *incident hadrons* are collinear. Chains are "tilted" with respect to the latter frame and there results a broadening of the observed transverse-momentum spectrum. As we shall argue below, this broadening is expected to be the largest in events where many chains are formed. This, in turn, implies an increase of $\langle p_t \rangle$ with multiplicity.

The effect is necessarily present in the conventional picture. The question is whether it is relevant to the data. Is

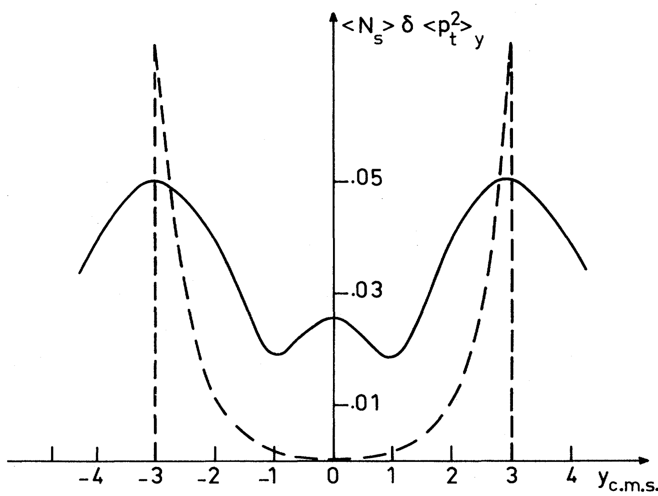

FIG. 1. The rapidity structure of the $\langle p_t \rangle$-multiplicity correlation expected at the collider energy from conventional hadron dynamics. The value of $\langle N_s \rangle \delta \langle p_t^2 \rangle_y$ (in units of $\langle k_t^2 \rangle_\infty$) is plotted as a function of the rapidity. Here $\langle N_s \rangle$ is the average multiplicity of the short chain in the considered rapidity window and $\delta \langle p_t^2 \rangle$ is defined in Eq. (11). Two situations are represented: (a) Assuming that the short chains, with a flat rapidity distribution and a length of 6 units, are at rest in the c.m.s. (dashed curve). In this case, the vertical scale is arbitrary. The values at $y = \pm 3$ are equal to the plateau height of the short chain, and Eq. (13) is obtained after integration in y. (b) Here chain fluctuations are taken into account (solid curve). The calculations are performed in the dual parton model (Ref. 4) with flat fragmentation functions $\bar{z}D(z) = 1$. Note the smearing produced by the chain fluctuations. The small local maximum near $y = 0$ is not to be taken seriously. The values of $\langle N_s \rangle$ obtained in the model are 0.87, 1.92, and 2.57 for $(-1, 1)$, $(-2.5, 2.5)$, and $(-4, 4)$, respectively. The resulting value of $\delta \langle p_t^2 \rangle$ in the experimental window $(-2.5, 2.5)$ obtained by integrating the curve in this rapidity interval and dividing by $\langle N_s \rangle$ is close to the value of 0.1 obtained from Eqs. (14) and (15). More interesting, the predicted value of $\delta \langle p_t^2 \rangle$ in the largest window $(-4 < y < 4)$ is twice the corresponding value in the smallest one $(-1 < y < 1)$.

it strong enough to explain the collider data? Why does it not show up with the same strength at lower energies?

In this Brief Report, we shall present a rough but very transparent estimate of the effect in question. We have used the model of Ref. 4 to check this estimate.

Consider a production process

$$a(k_1)+b(k_2)\to c(p)+X \quad . \tag{1}$$

The letter in parentheses indicates the four-momentum of the particle. Let us have $k_i^2=M_i^2$, $p^2=m^2$, $(k_1+k_2)^2=W^2$, and let $\vec{p}_{t'}$ denote the transverse component of $\vec{p}$, in a frame where $\vec{k}_1$ and $\vec{k}_2$ are collinear. It is easy to check that for $W\to\infty$ the transverse mass $m_{t'}=(m^2+p_{t'}^2)^{1/2}$ is given by the following invariant formula,

$$m_{t'}^2=4(k_1\cdot p)(k_2\cdot p)/W^2 - 4[M_1^2(k_2\cdot p)^2+M_2^2(k_1\cdot p)^2]/W^4 \quad . \tag{2}$$

We assume that a and b are virtual constituents of some incident hadrons. We work in the center-of-mass system (c.m.s.) of the hadronic collision using the familiar light-cone parametrizations:

$$\begin{aligned} p &= (|z|+m_t^2/4|z|P,\ \vec{p}_t, zP-m_t^2/4zP) \ , \\ k &= (|x|+M_{1t}^2/4|x|P,\ \vec{k}_t, xP-M_{1t}^2/4xP) \ , \end{aligned} \tag{3}$$

and similarly for k_2. Here P, assumed very large, is the momentum of the incident hadrons and "transverse" refers to the direction of motion of these hadrons. Without any loss of generality we assume $z,x>0$. Energy-momentum conservation implies $z<x$.

After some algebra one finds from (2) and for $P\to\infty$

$$p_{t'}^2=\left(\vec{p}_t-\frac{z}{x}\vec{k}_t\right)^2 \quad . \tag{4}$$

Since $\vec{p}_{t'}$ cannot depend on $\vec{k}_t$ one must have

$$\vec{p}_t=\vec{p}_{t'}+\frac{z}{x}\vec{k}_t \quad . \tag{5}$$

Thus, going from the "chain frame" to the c.m.s. corresponds to an effective rotation of momenta by an angle $\theta\simeq k_t/P$, as one might expect. Squaring both sides of the above equation and averaging with respect to $\vec{p}_{t'}$ and $\vec{k}_t$, one obtains

$$\langle p_t^2\rangle=\langle p_{t'}^2\rangle+\left(\frac{z}{x}\right)^2\langle k_t^2\rangle_n \quad . \tag{6}$$

Here z, x, and n = (number of chains) are kept fixed. We attach the subscript n to $\langle k_t^2\rangle$ to emphasize that this quantity is expected to depend on n. For simplicity we assume it is independent of x. The average $\langle p_{t'}^2\rangle$ is an intrinsic property of a chain and is independent of x and n. We shall neglect its correlation with z. We expect $\langle p_{t'}^2\rangle=0.14$ $(\mathrm{GeV}/c)^2$ or so.

Assume further that the transverse momenta of the virtual particles that "initiate" chains are statistically independent, except for momentum conservation, and normally distributed. The distribution of one of them is then

$$P(\vec{k}_t,\vec{k}_t+d^2k_t)=\mathrm{const}\times e^{-ck_t^2}\int\prod_{j=2}^{n}d^2k_{jt}\exp\left(-c\sum_{j=2}^{n}k_{jt}^2\right)\delta_2\left(\vec{k}_t+\sum_{j=2}^{n}\vec{k}_{jt}\right)d^2k_t \ , \tag{7}$$

where P stands for probability, and c is some constant, which we assume independent of n. Hence

$$\langle k_t^2\rangle_n=\left(\frac{n-1}{n}\right)\langle k_t^2\rangle_\infty \quad . \tag{8}$$

The average $\langle k_t^2\rangle_n$ increases with n because the constraint of momentum conservation becomes irrelevant when this momentum is shared by many consitituents. Combining (6) and (8) we find

$$\langle p_t^2\rangle=\langle p_{t'}^2\rangle+\left(\frac{z}{x}\right)^2\left(\frac{n-1}{n}\right)\langle k_t^2\rangle_\infty \quad . \tag{9}$$

Hence for finite z/x, $\langle p_t^2\rangle$ increases by $(z/x)^2\ \langle k_t^2\rangle_\infty$ when n changes from $n=1$ to $n>>1$. For $z/x\to 0$ the correlation disappears.

At this point in the discussion we have to inject more empirical information. As already mentioned, the multichain structure of production events implies a correlation between multiplicities in widely separated rapidity intervals. Consider the rapidity intervals $(\Delta,\Delta+1)$ and $(-\Delta-1,-\Delta)$ and let the corresponding multiplicities be N_F and N_B, respectively. One studies the linear regression $\langle N_F(N_B)\rangle = a+bN_B$ for values of $\Delta>1$, so that the short-range correlations do not contribute. Using the general formulas of Ref. (3) one finds that the parameter b is proportional to $[\langle n^2\rangle-\langle n\rangle^2]\langle N_0\rangle^2$, where $\langle N_0\rangle$ is the average multiplicity in $(\Delta,\Delta+1)$ of a single chain. At the collider energy the data are $b=0.45$, 0.27, and 0.04 for $\Delta=1$, 2, and 3, respectively.[5] This falloff of b with Δ can be reproduced by a multichain model if one assumes that all chains but one are much shorter than the full kinematically accessible interval. One can replace $\langle N_0\rangle$ by $\langle N_s\rangle$, the average multiplicity of a short chain, and b is nonzero only within the restricted interval where the short chains contribute. This particular chain structure can be justified within the framework of the "dual parton model." We refer the reader to Ref. 4, which contains a discussion of the long-range correlation and of other collider results supporting this picture. In the dual parton model each cut Pomeron is represented by two chains of hadrons. This subtlety is irrelevant for the following discussion. For our purposes it is sufficient to note that the rapidity extension of the short chains suggested by the above-mentioned data is about 6 units (at the collider energy). This is comparable to the rapidity size of the window where the $\langle p_t\rangle$-multiplicity correlation has been observed.

Let us now estimate the rise of $\langle p_t\rangle$ with increasing multiplicity. One can argue that the contribution of the long chain to this rise is negligible. In the central region it is killed by the factor $(z/x)^2$. For z closer to x, kinematics works against the effect: the larger the overall multiplicity is, the smaller the likelihood is that a particle has z close to

$x \simeq 1$. Thus, in the first approximation, we neglect the long chain altogether. We assume that only particles whose rapidities fall within the interval $(-y_0, +y_0)$ are recorded. Equation (9) holds for each chain separately. Averaging over the n chains and neglecting the contribution of the long one we find

$$\langle p_t^2 \rangle = \langle p_{t'}^2 \rangle + \frac{(n-1)R}{(n-1)R+1} \left\langle \left| \frac{z}{x} \right|^2 \right\rangle \frac{n-1}{n} \langle k_t^2 \rangle_\infty \ . \quad (10)$$

Here R is the ratio of the number of particles emitted within the interval $(-y_0, +y_0)$ in a short chain and in the long chain (on the average). For very high energy, $R=1$. In the model of Ref. 4, the value of R at collider energy, and for $y_0 = 2.5$, is $R = 0.35$. At CERN ISR energies, R is smaller and n is no longer simply correlated to multiplicity (for kinematic reasons) so that effectively n is never very large. Hence the rise of $\langle p_t \rangle$ with multiplicity is expected to be appreciably smaller at ISR than at collider energies. In the latter case, n is very large for high-multiplicity events and one has

$$\delta\langle p_t^2 \rangle \equiv \langle p_t^2 \rangle - \langle p_{t'}^2 \rangle \simeq \left\langle \left| \frac{z}{x} \right|^2 \right\rangle \langle k_t^2 \rangle_\infty \ . \quad (11)$$

Let us now compute $\langle (z/x)^2 \rangle$. Notice that $z' = z/x$ is essentially the longitudinal momentum of the secondary scaled by the momentum of the incident constituent. Take the simplest among the plausible distributions of z':

$$P(z', z'+dz') = \text{const} \times dz'/z', \quad 0 < z' < 1 \ . \quad (12)$$

Let us consider first the case in which the short chain, with a rapidity extension $2Y_0 = 6$ units, is fixed in rapidity space. In this simple case the rapidity dependence of $\delta\langle p_t^2 \rangle$ is obviously given by $(x/z)^2 = (z')^2$. Thus, one has a dramatic minimum at $y=0$, and two maxima at $y = \pm 3$ (see dashed curve in Fig. 1). If the rapidity window is large enough $(y_0 > Y_0)$, one gets

$$\langle (z')^2 \rangle = (2Y_0)^{-1} \simeq 0.17 \ . \quad (13)$$

For $y_0 < Y_0$ this value is reduced. In this case the value of $\langle (z')^2 \rangle$ depends in a rather crucial way on the fluctuations in the chain position, which therefore can no longer be ignored. Obviously, the value of $\langle (z')^2 \rangle$ can only be computed in this case for a well-defined form of the fluctuation of the chain edges in rapidity space. We have computed this value using the model of Ref. 4. The rapidity dependence of $\delta\langle p_t^2 \rangle$ is shown in Fig. 1 (full line) and the obtained value of $\langle (z')^2 \rangle$ is given in the figure caption. A similar value is obtained using the following crude but transparent estimate. Let Y denote the rapidity of the fastest particle in the chain and let $P(Y, Y+dY) = f(Y)dY$. Assuming Y is never too small one finds

$$\langle (z')^2 \rangle = \tfrac{1}{2} \int \frac{dY f(Y)}{Y} [\theta(y_0 - Y) + e^{2(y_0 - Y)} \theta(Y - y_0)] \ . \quad (14)$$

Hence, roughly,

$$\langle (z/x)^2 \rangle \simeq \frac{1}{2Y_0} \left[1 - [1 - e^{2(y_0 - Y_0)}] \int_{y_0}^{\infty} f(Y) dY \right] \ , \quad (15)$$

where Y_0 is the average value of Y. The above estimate yields the exact result in the two extreme situations: (i) the edges of the experimental window are always outside the interval $(-Y, Y)$ [in this case the integral in (15) is zero and one recovers the result in Eq. (13)], and (ii) the window is always within $(-Y, Y)$ (the integral is then 1). One can hope that Eq. (15) also provides a reasonable interpolation in intermediate situations. At the collider energy we have roughly $Y_0 = 3$, and in Ref. 1 $y_0 = 2.5$. For Y_0 and y_0 rather close to each other the integral in (15) is presumably not very different from $\frac{1}{2}$. With the latter value, one finds $\langle (z/x)^2 \rangle \simeq 0.11$. The largest uncertainty in the calculation of $\delta\langle p_t^2 \rangle$ is due to the presence in Eqs. (10) and (11) of an unknown strong-interactions parameter, viz., $\langle k_t^2 \rangle_\infty$. A value $\langle k_t^2 \rangle_\infty \simeq 1$ (GeV/c)^2 seems plausible. Think of the "intrinsic" transverse momenta used in the parton-model phenomenology. With $\langle (z/x)^2 \rangle = 0.11$ and $\langle k_t^2 \rangle_\infty = 1$ (GeV/c)^2 one finds that $\langle p_t^2 \rangle$ rises from about 0.14 to 0.25 (GeV/c)^2 (corresponding to a rise in $\langle p_t \rangle$ from 0.33 to 0.44 GeV/c), when one goes from the smallest to the highest multiplicities. This is close to the observed rise. Also, the predicted saturation of the correlation at high multiplicities is borne out by the data.

It is difficult to say how seriously the above estimate should be taken. Our only claim is that the discussed mechanism is plausible. One can easily get a smaller effect by taking a different (more realistic) longitudinal-momentum distribution within a chain. One can argue that $\langle k_t^2 \rangle_\infty$ should be taken smaller, etc. However, these are not directly measurable quantities and, therefore, such objections do not rule out the conventional mechanism. This is why we suggest that experimenters check the rapidity structure of the $\langle p_t \rangle$-multiplicity correlation. Here the prediction is qualitative and therefore clean: the absence of a two-maximum signal, such as the one shown in Fig. 1, would ruin the conventional explanation of the correlation between transverse momentum and multiplicity. An easy way of testing this rapidity structure consists in completing the available experimental data (in the window $-2.5 < y < 2.5$), with similar data in a smaller $(-1 < y < 1)$ and a larger $(-4 < y < 4)$ rapidity window. It follows from the results shown in Fig. 1 that with the largest window the value of $\delta\langle p_t^2 \rangle$ should be twice the value obtained with the shortest one.

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