Particle ratios at the CERN $\bar{p}p$ collider

C. K. Chew and T. Y. Liang
Department of Physics, National University of Singapore, Singapore 0511

S. Y. Lo
Department of Physics, University of Melbourne, Parkville, Victoria, Australia 3052

(Received 21 April 1983; revised manuscript received 26 September 1983)

By applying Chao-Yang statistics to multiplicities of different particles in pp collisions, we obtain good quantitative agreement with the data obtained from CERN ISR experiments. What is especially significant in this work is that the π/p ratios are given correctly for all energies. Predictions for various particle ratios for $\bar{p}p$ collisions are also calculated for confirmation by future experiments.

In hadron collisions at energy beyond a few GeV, hadrons are copiously produced. The dominant cross section generally involves on the order of ten or more particles when one reaches $\sqrt{s} \sim 10$ GeV and above. To date, normal field-theoretical calculation is hopeless when facing such large numbers of particles. Besides, most of these hadrons generally have small transverse momentum (~ 0.5 GeV), and hence they are not produced in a short distance. Large-distance behavior, according to QCD, has a very strong interaction, and does not yield to perturbative calculation. However, the strength of the interaction has the advantage that the multiparticle production probably has gone through a great many different states, and some statistical-ensemble argument may be applicable.

The crucial question is not whether one should use statistics but rather what sort of statistics is relevant. In the following we shall investigate the statistics first proposed by Chao and Yang[1] for bosons, and later generalized to quarks[2] by us. The most significant physical ingredient of this statistics is to preserve the quark number

$$N = N_q - N_{\bar{q}} \ , \tag{1}$$

where N_q is the number of quarks and $N_{\bar{q}}$ is the number of antiquarks, and q can be up quark, down quark, or quark of any flavor. Then the combinations of l quarks could be given by the generating function

$$\left(x + y + \frac{1}{x} + \frac{1}{y}\right)^l = \sum_{a,b} N^l_{a,b} x^a y^b \ , \tag{2}$$

where x and y stand for u and d quarks, respectively. The coefficient $N^l_{a,b}$ on the right-hand side indicates the number of combinations that would have the quantum numbers of a u quarks and b d quarks. From the $N^l_{a,b}$ it is easy to calculate the particle ratios of π, K, p, $\bar{p}$, etc.

In this paper we shall illustrate the results of applying this concept to pp collisions where there is a large amount of accurate data available. It is found that the π/p ratios can be explained purely by the functions $N^l_{a,b}$ from the lowest energy to the highest energy available without any additional assumptions or parameters. We then make the calculation for $\bar{p}p$ collisions; we hope the prediction of the quark statistics can be tested more severely because the two colliding hadrons are different and the number of particles produced is even larger.

The probability of the occurrence of a π^+ meson in an ensemble of l quarks with the quantum number of a u quarks and b d quarks is

$$P_{\pi^+} = N^{l-2}_{a-1,b+1}/N^l_{a,b} \ . \tag{3}$$

The corresponding probability for π^-, p, and $\bar{p}$ are

$$P_{\pi^-} = N^{l-2}_{a+1,b-1}/N^l_{a,b} \ , \tag{4}$$

$$P_p = N^{l-3}_{a-2,b-1}/N^l_{a,b} \ , \tag{5}$$

$$P_{\bar{p}} = N^{l-3}_{a+2,b+1}/N^l_{a,b} \ . \tag{6}$$

For a given set of l quarks, the ratios of $\pi^{\pm}$, p, and $\bar{p}$ are

$$R_l(\pi^+/p) = P_{\pi^+}/P_p \ , \tag{7}$$

$$R_l(\pi^-/p) = P_{\pi^-}/P_p \ , \tag{8}$$

$$R_l(\bar{p}/p) = P_{\bar{p}}/P_p \ . \tag{9}$$

These ratios are experimentally measurable in principle because the total number of hadrons produced in each given event can be counted and hence the value of l can be calculated from the number of $\pi^{\pm}$, $\bar{p}$, etc. In practice we do not have such detailed information and can only substitute the average multiplicity of hadrons for the exact values. We take the average quark number l to be

$$l = 3\langle n_{\rm ch}\rangle \ , \tag{10}$$

where $\langle n_{\rm ch}\rangle$ is the average charge multiplicity in pp collisions at any given energy. The charge multiplicity would be an experimental input.

It is commonly accepted that there are several mechanisms at work in pp collisions. There are the following.

(1) Central collision. In this type of collision, the two hadrons mix up very well before the final particles are produced. The quark content of the hadron system is four u quarks, $a = 4$, and two d quarks, $b = 2$. Thus Eq. (7) becomes

$$R_l(\pi^+/p) = N^{l-2}_{3,3}/N^{l-3}_{2,1} \ . \tag{11}$$

The other equations (3)–(9) can similarly be obtained.

(2) Single-diffractive processes. In this case, one of the protons goes straight through without being excited in any way while the other proton undergoes diffractive dissociation. Then $\langle l\rangle = 3\langle n_{\rm ch}\rangle - 3$, and Eqs. (3)–(9) would have

to be changed correspondingly. Typically, Eq. (3) becomes

$$P(\pi^+) = N^{l-5}_{1,2} / N^{l-3}_{2,1} \ . \tag{12}$$

(3) Double-diffractive processes. In this process both protons undergo diffractive dissociation. The number of particles produced by each proton in general would be different, but on the average would be $l/2$ quarks for each. Then Eq. (3) becomes

$$P(\pi^+) = N^{l/2-2}_{1,2} / N^{l/2}_{2,1} \ . \tag{13}$$

Surprisingly, the particle ratios calculated from these three mechanisms do not differ very much (10–20%). We plot them against the experimental data[3] on Fig. 1 and the latter fall within the range of the three theoretical curves. It is clear that some linear combination of these three mechanisms can represent the experimental data very well.

For the production of the K mesons, the s quarks have to be included and the generating function becomes

$$\left[x + y + \gamma z + \frac{1}{x} + \frac{1}{y} + \gamma \frac{1}{z}\right]^l = \sum_{abc} N^l_{abc}(\gamma) x^a y^b z^c \ . \tag{14}$$

The K-meson-to-proton ratios are then given by

$$R_l(K^+/p) = N^{l-2}_{a-1,b,c+1} / N^{l-3}_{a-2,b-1,c} \ , \tag{15}$$

$$R_l(K^-/p) = N^{l-2}_{a+1,b,c-1} / N^{l-3}_{a-2,b-1,c} \ . \tag{16}$$

We have introduced the suppression factor γ because of the heavier mass of the s quark when compared to the u and d quarks. The results of our theoretical calculation for these mechanisms for $\gamma = 0.02$ is shown in Fig. 2. The ratios depend greatly on the parameter γ, but does not change much for the different production mechanisms. Because of the small value of γ, it has no effect on the values of the π/p and $\bar{p}/p$ ratios as calculated previously without the s quarks.

From energy considerations alone, one would also expect that the creation of a $\bar{p}p$ pair for $E \sim 2$ GeV, which is five times the rest mass of three pions (~ 0.42 GeV), to be relatively suppressed even though they have the same number of quarks ($=6$). This suppression would become much smaller at higher energy because the energy of the three pions would then be closer to that of the $\bar{p}p$ pair. In our calculation it is observed that our theoretical values become closer to the experimental values as energy increases. Since the antiproton data, in general, is relatively of an inferior quality to the other data, the fit as shown in Fig. 1 is not too bad.

The most interesting number noticed during the computation is the asymptotic value of the π/p particle ratio. It has the value

$$\pi/p \to 4 + 2\gamma \text{ as } l \to \infty \ . \tag{17}$$

At the CERN ISR energy, the π/p particle ratio is about three. It is important to measure this ratio in the future because it is a unique unambiguous prediction of this particular kind of quark counting. It is clean and probably not so easy to obtain in any other method.

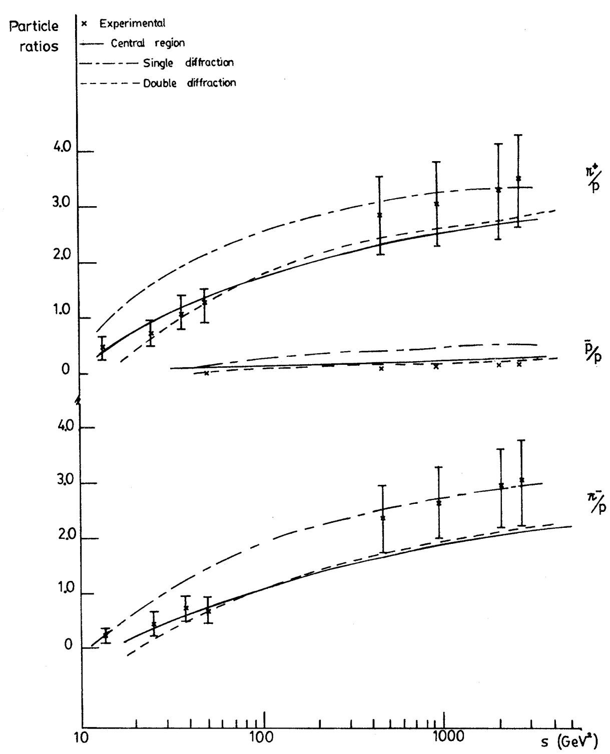

FIG. 1. Comparison of the variation of the theoretical and experimental values of π^+/p, π^-/p, and $\bar{p}/p$ with s. The theoretical curves are for violent collisions and for single and double diffraction.

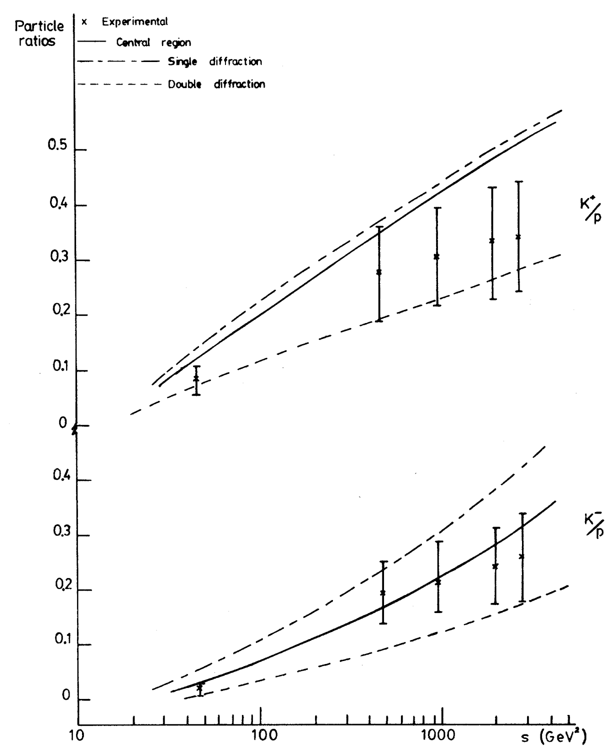

FIG. 2. Comparison of the variation of the theoretical and experimental values of K^+/p and K^-/p with s. The theoretical curves are for violent collisions and for single and double diffraction.

For higher energy, the next round of particle-ratio data would presumably come from the CERN $\bar{p}p$ collider. The $\bar{p}p$ collider has the advantage that the two colliding hadrons are different. The different mechanisms mentioned, the central production, single-diffractive, and double-diffractive processes, can perhaps be separated. There would, therefore, be different values for particle ratios for the different mechanisms.

For a given quark number l, the probabilities for the production of $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ with the initial state having a u quarks, b d quarks, and c s quarks are the same as that given by Eqs. (3)–(9), (15), and (16). For the central collision, the initial state would be $a=0$, $b=0$, and $c=0$, and the normalization function is $N^{l}_{0,0,0}$ with $l \sim 3\langle n_{\rm ch}\rangle$. For the single-diffractive process on the proton side the normalization function is $N^{l-3}_{2,1,0}$ and on the antiproton side it is $N^{l-3}_{-2,-1,0}$. For the double-diffractive case the normalization functions are $N^{l_1}_{2,1,0}$ and $N^{l_2}_{-2,-1,0}$, with $l_1+l_2=l$, and for simplicity we take $l_1 \sim l_2 \sim l/2$. The particle statistical probabilities for these three mechanisms are plotted in Figs. 3 to 5. These figures may help experimentalists to obtain the particle ratios immediately. We suggest that for precise comparison with the theoretical predictions one should look for particle ratios in exclusive processes where the number of hadrons in the final state is precisely known. With this, the value of quark number l can also be determined precisely. For inclusive production in the $\bar{p}p$ collision one should also obtain the average charge multiplicity and all average particle ratios, that is, $\langle n_{\rm ch}\rangle$, $\langle n_{\pi^{\pm}}\rangle$, $\langle n_{K^{\pm}}\rangle$, $\langle n_p\rangle$, and $\langle n_{\bar{p}}\rangle$ (and perhaps $\langle n_\Lambda\rangle$ and $\langle n_{\Sigma^{\pm}}\rangle$), as we have done in

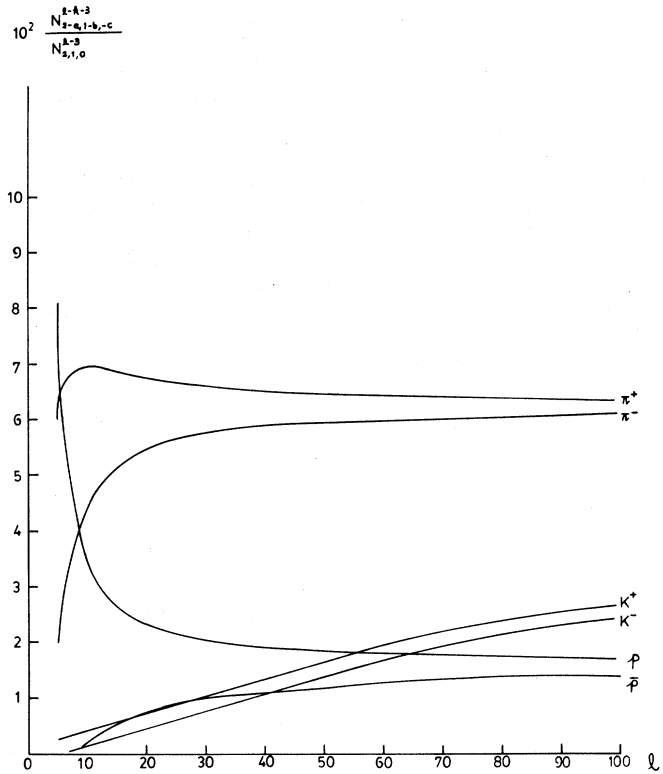

FIG. 4. The variation of the quark statistical probabilities of $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ with l for pp single diffraction.

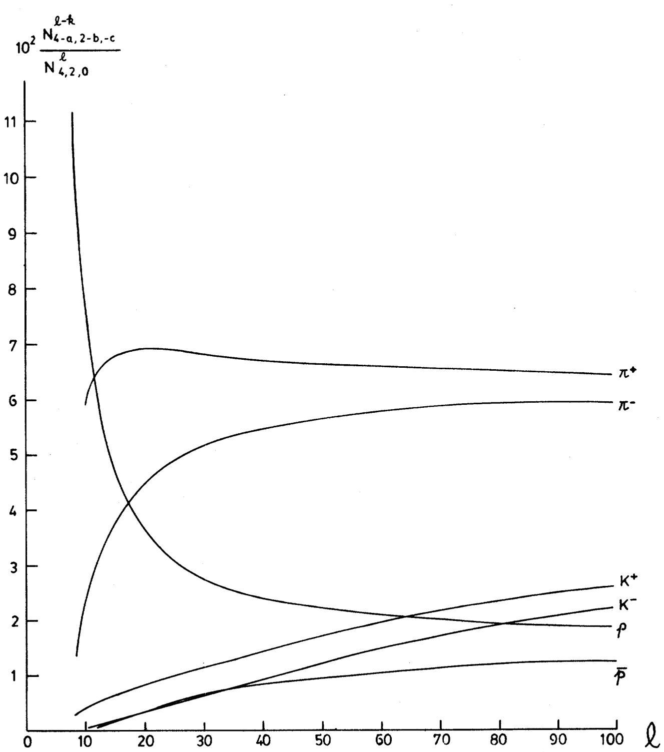

FIG. 3. The variation of the quark statistical probabilities of $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ with l for pp collisions in the central region.

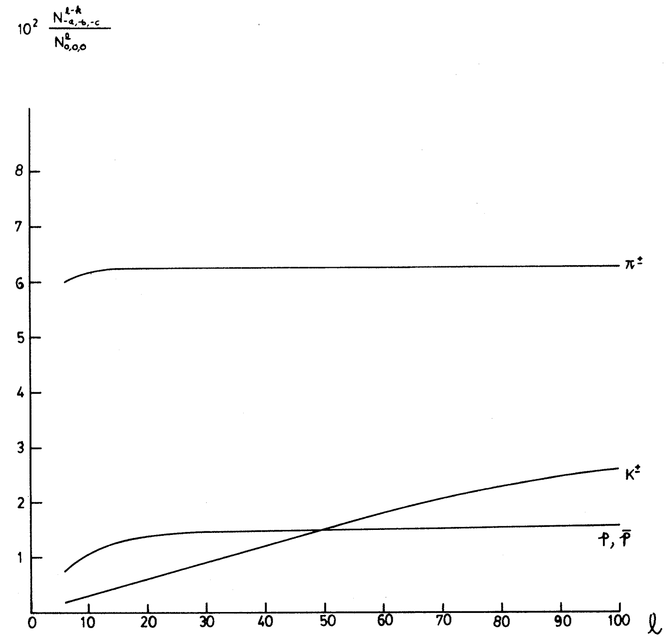

FIG. 5. The variation of the quark statistical probabilities of $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ with l for $p\bar{p}$ collisions in the central region.

the pp reactions.

After having applied Chao-Yang statistics to particle ratios in pp collisions with the number of quarks running from 8 to 40, which is very small in any sense of statistics, it is rather surprising to obtain very good agreement for the π/p ratios. However, since the numbers of pions and protons are the largest in the pp multiparticle production, they should be the first to be given correctly by any statistical argument. We find that the experimental π/p ratios, which run from 0.4 to 3.4 at energies $s \sim 10$ to 2800 GeV^2, indeed agree with the span of the theoretical values. The actual average number of pions is 3 to 4 and the average number of protons is approximately 1.6. They are small, but are nevertheless explainable by the statistical calculations. It would be most interesting to test such predictions in the coming $\bar{p}p$ collider experiments. We have indicated all such predictions in Figs. 3 to 5. They should be tested both in inclusive reactions as well as in exclusive processes.

In this analysis, we have included only the u, d, and s quarks in our treatment. It is easy to extend to heavier quarks such as the charm and the bottom quarks.[4] Since the addition of the s quark does not effect the π/p ratios, the addition of the c and b quarks will not change our numerical values presented here because of their heavier masses.

After the completion of the present paper, the multiplicity data for the $\bar{p}p$ collider at CERN[5] were published. However, the data are only limited to the central region and the multiplicity of proton and antiproton to pion ratios is 1.5/22.3 as compared to our theoretical prediction of 1.5/6.5. We expect that the inclusion of multiplicity in the fragmentation region would greatly increase this ratio.

---

[1]A. W. Chao and C. N. Yang, Phys. Rev. D 9, 2505 (1974); 10, 2119 (1974).

[2]C. K. Chew, H. B. Low, S. Y. Lo, and K. K. Phua, J. Phys. G 6, 17 (1980).

[3]A. M. Rossi, G. Vannini, A. Bussiere, E. Albini, D. D'Alessandro, and G. Giacomelli, Nucl. Phys. B84, 269 (1975); E. Albini, P. Capiluppi, G. Giacomelli, and M. Jacob, Phys. Rep. 55, 1 (1979).

[4]K. C. Chang, C. K. Chew, T. Y. Liang, K. K. Phua, and H. B. Low, J. Math. Phys. 24, 173 (1983).

[5]K. Alpgard *et al.*, Phys. Lett. 121B, 209 (1983).