Photon mass in a background of thermal particles

José F. Nieves
Department of Physics, University of Puerto Rico, Rio Piedras, Puerto Rico 00931

Palash B. Pal and David G. Unger
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 5 April 1983)

The mass of the photon in the presence of thermal background particles is studied. We present a general proof that all contributions to the photon's thermal mass from low-temperature thermal backgrounds are suppressed by a huge exponential factor at least as large as $e^{m_e/2kT}$. We explicitly show that there is no contribution to the photon's thermal mass of order $(\alpha G_F)^{1/2}(kT_\nu)^2$ from two-loop diagrams involving thermal neutrinos at temperature T_ν. The largest contribution to the photon's thermal mass from the observed 3°K background is also calculated.

I. INTRODUCTION

The mass of the photon is not constrained to be zero in finite-temperature field theory,[1] unlike zero-temperature field theory where gauge invariance guarantees that the photon is massless.[2] Recently, there have been several discussions about the effects of low-temperature thermal backgrounds on the photon mass. Abbott and Gavela[3] have argued that temperature mass effects will be suppressed by a factor at least as large as $e^{m_e/kT}$, which, for T of the order of room temperature or less, leads to a gigantic suppression. They asserted that this follows because the photon mass is influenced by temperature effects only through loops involving charged particles, the lightest of which is the electron. For all practical purposes, they conclude, the mass of the photon is unaffected by the presence of thermal backgrounds. Subsequently, however, Woloshyn[4] pointed out that the photon can interact with zero-mass thermal particles via diagrams of more than one loop and that large contributions to the photon mass come from terms in which only the zero-mass particles are treated as being thermal. These terms avoid a huge suppression factor like $e^{m_e/kT}$ since the thermal line has zero mass. In particular he considered a 1.9°K thermal background of neutrinos in the standard model[5] and obtained from two-loop diagrams involving a neutrino an "electric" mass of the photon of the order of $\sqrt{\alpha G_F}(m_l/M_W)(kT_\nu)^2$, where m_l is the mass of the heaviest standard-model charged lepton and T_ν is the thermal, background-neutrino temperature. For $T_\nu \approx 1.9$°K the "electric" photon mass is of the order of 10^{-21} eV, which is much smaller than the experimental limit[6] of 9×10^{-15} eV. While this example is of no immediate experimental interest, it leaves open the possibility of constructing models with much larger photon masses. Examples could be models in which thermal neutrinos are replaced by massless thermal scalars, such as Majorons,[7] or alternative cosmologies where the temperature of the background neutrinos are much greater than 1.9°K. Woloshyn argues that replacing the thermal neutrino with a thermal photon results in no contribution to the mass of the photon due to gauge invariance.

In this paper, in Sec. II, we repeat the calculation of the contribution to the photon mass from thermal neutrinos, initially performed in Ref. 4. In contrast to Ref. 4 we obtain no contribution of order $\sqrt{\alpha G_F}(kT_\nu)^2$. We perform our calculation in two different gauges. Also we calculate the largest contribution to the photon's electric mass, which comes from the electron one-loop diagram.

We present in Sec. III a general proof that thermal contributions to the thermal mass of the photon are suppressed by a factor at least as large as $e^{m_e/2kT}$, when $kT \ll m_e$, with the mass of the electron appearing in the suppression factor since the electron is the lightest charged particle. The general proof is based on electromagnetic gauge invariance and the singular or nonsingular nature of the constituent subdiagrams of the photon self-energy.

A summary and our conclusions are given in Sec. IV.

II. LOW-ORDER CONTRIBUTIONS TO THE THERMAL MASS OF THE PHOTON

In this section we shall compute the low-order contributions to the thermal mass of the photon in the standard model.[5] Initially we will discuss how in zero-temperature field theory electromagnetic gauge invariance constrains the photon's mass to be zero and how, in contrast, in finite-temperature field theory the photon's "electric" mass is no longer constrained to be zero although the photon's "magnetic" mass is still zero. We will outline how finite-temperature calculations are performed and, for illustration, calculate the one-loop electron contribution to the photon's thermal mass for a low-temperature background, $kT \ll m_e$. This contribution is the largest perturbative contribution but nevertheless is suppressed by the huge factor $e^{m_e/2kT}$. Next we calculate the contribution from two-loop diagrams involving low-temperature, thermal, massless neutrinos. Since each of these diagrams involves a *massless* thermal particle each contributes a term to the photon's thermal mass that is not suppressed by a huge exponential factor. However, upon summing all the diagrams, we find that the contributions that are not exponentially suppressed cancel, in disagreement with Ref. 4. As checks on our calculation we present diagram-by-diagram results in two gauges and show the electromagnetic gauge invariance in one gauge.

The photon mass in zero-temperature field theory is determined by calculating the photon self-energy

$$\pi_{\mu\nu}(P)=\int d^4x\, e^{iP\cdot x}\langle 0\,|\,T(j_\mu(x)j_\nu(0))\,|\,0\rangle \tag{2.1}$$

in the limit $P^\rho\to 0$, where P^ρ is the photon's four-momentum. Electromagnetic gauge invariance implies that

$$P^\mu\pi_{\mu\nu}(P^\rho)=0\ . \tag{2.2}$$

When Eq. (2.2) is combined with Lorentz covariance we obtain

$$\pi_{\mu\nu}(P^\rho)=\pi(P^2)(P^2g_{\mu\nu}-P_\mu P_\nu)\ . \tag{2.3}$$

To any finite order in e^2, $\pi(P^2)$ has no pole at $P^2=0$. This follows from investigating the analytic properties[8] of $\pi(P^2)$. Hence

$$\pi_{\mu\nu}(P^\rho\to 0)=0 \tag{2.4}$$

and therefore the photon has zero mass to any finite order in zero-temperature field theory.

Let us now consider the situation at finite temperature. In the covariant formulation of finite-temperature field theory[9] a fluid in thermodynamic equilibrium is characterized by the four-velocity of the center of mass of the fluid u_μ and a Lorentz-invariant parameter T. In the fluid's rest frame $u_\mu=(1,0,0,0)$, and T is the usual temperature of the fluid. At finite temperature, radiative corrections introduce a dependence on u_μ for physical quantities. For example, the photon self-energy is now given by

$$\pi_{\mu\nu}(P,u)=\int d^4x\, e^{iP\cdot x}\mathrm{Tr}[Z_G T(j_\mu(x)j_\nu(0))] \tag{2.5}$$

$$=\int d^4x\, e^{iP\cdot x}\sum_n e^{-u\cdot P_n/kT}\langle n\,|\,T(j_\mu(x)j_\nu(0))\,|\,n\rangle\ , \tag{2.6}$$

where Z_G is the grand partition density operator and where the trace has been evaluated in the basis of physical on-shell eigenstates of the Hamiltonian.[9] We have assumed that the densities are low so that the chemical potentials can be taken to be zero. As above for the zero-temperature case, electromagnetic gauge invariance still implies that[9]

$$P^\mu\pi_{\mu\nu}(P,u)=0\ . \tag{2.7}$$

One can construct two independent Lorentz scalars from P and u:

$$\begin{aligned}\omega&\equiv P^\alpha u_\alpha\ ,\\ p&\equiv[(P^\alpha u_\alpha)^2-P^2]^{1/2}\ .\end{aligned} \tag{2.8}$$

Since $P^2=\omega^2-p^2$ we may interpret ω and p as the "Lorentz-invariant" energy and the three-momentum of the photon.[9] In the rest frame of the fluid, ω is the usual energy while p is the magnitude of the usual three-momentum. The general solution of Eq. (2.7) is

$$\pi_{\mu\nu}(P,u)=\pi_T(p,\omega)R_{\mu\nu}+\pi_L(p,\omega)Q_{\mu\nu}\ , \tag{2.9}$$

where

$$\begin{aligned}R_{\mu\nu}&\equiv g_{\mu\nu}-u_\mu u_\nu+\tilde{P}_\mu\tilde{P}_\nu/p^2\ ,\\ Q_{\mu\nu}&\equiv-\frac{1}{P^2p^2}(p^2u_\mu+\omega\tilde{P}_\mu)(p^2u_\nu+\omega\tilde{P}_\nu)\ ,\end{aligned} \tag{2.10}$$

with

$$\tilde{P}_\mu\equiv P_\mu-\omega u_\mu\ .$$

While $R_{\mu\nu}$ and $Q_{\mu\nu}$ are not unique as a basis for $\pi_{\mu\nu}$, they have the useful properties

$$\begin{aligned}&R^\mu{}_\rho R^\rho{}_\nu=R^\mu{}_\nu\ ,\quad Q^\mu{}_\rho Q^\rho{}_\nu=Q^\mu{}_\nu\ ,\\ &R^\mu{}_\rho Q^\rho{}_\nu=Q^\mu{}_\rho R^\rho{}_\nu=0\ .\end{aligned} \tag{2.11}$$

The connection to the zero-temperature case is as follows. For $T=0$,

$$\pi_{\mu\nu}(P,u)\to\pi_{\mu\nu}(P)=\pi(P^2)(P^2g_{\mu\nu}-P_\mu P_\nu)\ , \tag{2.12}$$

but

$$R_{\mu\nu}+Q_{\mu\nu}=g_{\mu\nu}-P_\mu P_\nu/P^2\ . \tag{2.13}$$

Hence at $T=0$,

$$\pi(P^2)P^2=\pi_L(P^2)=\pi_T(P^2)\ . \tag{2.14}$$

The full photon propagator is obtained from the $T=0$ free-field propagator

$$D_{\mu\nu}(P)=i\left[-g_{\mu\nu}+(1-\xi)\frac{P_\mu P_\nu}{P^2}\right]\frac{1}{P^2}\ , \tag{2.15}$$

where ξ is the gauge parameter, by summing all the vacuum polarization insertions to obtain

$$\tilde{D}_{\mu\nu}(P)=-\frac{iR_{\mu\nu}}{P^2-\pi_T}-\frac{iQ_{\mu\nu}}{P^2-\pi_L}-\xi\frac{P_\mu P_\nu}{P^4}\ . \tag{2.16}$$

In the rest frame of the fluid ($\vec{u}=0$) and for $\omega=P^0=0$ in the Landau gauge ($\xi=0$) this reduces to

$$\begin{aligned}\tilde{D}_{00}&=\frac{i}{P^2+\pi_L}\ ,\\ \tilde{D}_{ij}&=\frac{i}{P^2+\pi_T}(-\delta_{ij}+P_iP_j/p^2)\ ,\end{aligned} \tag{2.17}$$

and $\tilde{D}_{0j}=0$. The limit $P^0=0$ is of interest for it is the relevant limit to determine the static correlation lengths of the electric and magnetic fields. The inverse of these correlation lengths for $\vec{P}\to 0$ are identified as the "electric" and "magnetic" mass of the photon. In particular the quantity $\pi_L(P^0=0,\vec{P}\to 0)$, evaluated in the rest frame of the fluid, is identified as the "electric" mass

$$(m_\gamma^{\mathrm{el}})^2\equiv\pi_L(P_0=0,\vec{P}\to 0)=\pi_{00}(P_0=0,\vec{P}\to 0)\ . \tag{2.18}$$

This identification follows since

$$\langle A_0(x)A_0(y)\rangle\sim e^{-m_\gamma^{\mathrm{el}}|\vec{x}-\vec{y}|} \tag{2.19}$$

as $|\vec{x}-\vec{y}|\to\infty$, so it is clear that $(m_\gamma^{\mathrm{el}})^{-1}$ is the correlation length of the electric field. There is no reason for m_γ^{el} to vanish. Similarly the quantity $\pi_T(P^0=0,\vec{P}\to 0)$, evaluated in the rest frame of the fluid, is identified as the magnetic mass

$$(m_\gamma^{\mathrm{mag}})^2=\pi_T(P_0=0,\vec{P}\to 0)=\tfrac{1}{2}\sum_i\pi_i^i(P_0=0,\vec{P}\to 0) \tag{2.20}$$

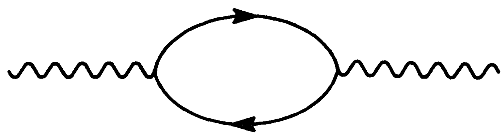

FIG. 1. The diagram for the electron one-loop photon self-energy.

since it defines the correlation length of the magnetic field. However it can be shown[10] that for all orders in perturbation theory $\pi_T(P_0=0,\vec{P}\to 0)\sim\vec{P}^2$ as $|\vec{P}|\to 0$ so that the photon's magnetic mass vanishes for finite temperatures.

To perform finite-temperature calculations in the fluid's rest frame there are two equivalent formulations[11]: the imaginary-time formalism and the real-time formalism. When particle densities are low, so that it is a good approximation to take the chemical potential equal to zero, Feynman diagrams can be calculated for a system at temperature T if the following changes are made.

(i) Imaginary Time:

$$\int\frac{d^4k}{(2\pi)^4}\to\frac{1}{(-i\beta)}\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\,, \tag{2.21}$$

where n is integer and $\beta=1/(kT)$. For boson propagators

$$k^0\to\omega_n=2\pi n/(-i\beta)\,, \tag{2.22}$$

while for fermion propagators

$$k^0\to\omega_n=\pi(2n+1)/(-i\beta)\,. \tag{2.23}$$

(ii) Real Time: The integration over four-momenta remains the same but the free zero-temperature propagators have an additional temperature-dependent term. The free scalar propagator becomes

$$D_\beta(k)=\frac{i}{k^2-m^2+i\epsilon}+\frac{2\pi}{e^{\beta E}-1}\delta(k^2-m^2) \tag{2.24}$$

while the free fermion propagator becomes

$$S_\beta(k)=\frac{i}{\not k-m+i\epsilon}-\frac{2\pi}{e^{\beta E}+1}(\not k+m)\delta(k^2-m^2) \tag{2.25}$$

and the free photon propagator, when the gauge-fixing term $(1/2\xi)(\partial_\mu A^\mu)^2$ is used, becomes

$$D_\beta(k)^{\mu\nu}=\left[\frac{-i}{k^2+i\epsilon}-\frac{2\pi}{e^{\beta E}-1}\delta(k^2)\right][g^{\mu\nu}-(1-\xi)k^\mu k^\nu/(k^2+i\epsilon)]\,, \tag{2.26}$$

where in each case $E=(\vec{k}^2+m^2)^{1/2}$. There are analogous additional temperature-dependent terms for massive vector propagators and ghost propagators.

The vertices are the same as in zero-temperature field theory for both formalisms. The ultraviolet divergences in the finite-temperature field theory are just those of the zero-temperature field theory.[12] Consequently the zero-temperature renormalization procedure eliminates all ultraviolet divergences and finite-temperature effects do not introduce new ultraviolet divergences.

For illustrative purposes, let us calculate for $kT\ll m_e$ the electron one-loop contribution to the photon mass from the diagram in Fig. 1.[13] This turns out to be the "largest" contribution. Since at zero temperature the mass of the photon is exactly zero, the only nonzero contribution must be due to temperature effects. The photon self-energy in the imaginary-time formalism is given by

$$\pi_{\mu\nu}(p^\rho=0)=-\frac{e^2}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}\left[\gamma_\mu\frac{i}{\not k-m_e}\gamma_\nu\frac{i}{\not k-m_e}\right]\,, \tag{2.27}$$

where $k^0=\omega_n=(2n+1)\pi/(-i\beta)$. Since finite-temperature effects do not introduce new ultraviolet divergences and since $\pi_{\mu\nu}(p^\rho=0)$ is zero for $T=0$ the divergences inherent in Eq. (2.27), when treated consistently, should cancel. To handle the divergences consistently, we work in $4-2\epsilon$ dimensions and at the end let $\epsilon\to 0$. Hence $d^3k\to d^{3-2\epsilon}k$ and $\mathrm{Tr}(\gamma_\mu\gamma_\nu)=(4-2\epsilon)g_{\mu\nu}$. Upon evaluating the trace in Eq. (2.27) and noting the parity of the integrands we obtain

$$\pi_{\mu\nu}(p^\rho=0)=0 \text{ for } \mu\neq\nu\,,$$

$$\pi_{00}(p^\rho=0)=\frac{-4e^2\beta(1-\epsilon/2)}{\pi^2}\sum_{n=-\infty}^{\infty}\int\frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}}\left[\frac{-2(2n+1)^2}{[(2n+1)^2+(\beta E/\pi)^2]^2}+\frac{1}{(2n+1)^2+(\beta E/\pi)^2}\right]\,,$$

$$\pi_{ii}(p^\rho=0)=\frac{-4e^2\beta(1-\epsilon/2)}{\pi^2}\sum_{n=-\infty}^{\infty}\int\frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}}\left[\frac{2k_i^2\beta^2/\pi^2}{[(2n+1)^2+(\beta E/\pi)^2]^2}-\frac{1}{(2n+1)^2+(\beta E/\pi)^2}\right]\,,\quad i=1,2,3 \tag{2.28}$$

where $E=(\vec{k}^2+m_e^2)^{1/2}$. Using the identity

$$\sum_{n=-\infty}^{\infty}\frac{1}{(2n+1)^2+x^2}=\frac{\pi}{2x}\tanh\frac{\pi x}{2} \tag{2.29}$$

and its derivatives, we obtain

$$\pi_{00}(p^\rho=0)=e^2\beta\left[1-\frac{\epsilon}{2}\right]\int\frac{d^{3-2\epsilon}k}{(2\pi)^3}\operatorname{sech}^2\left[\frac{\beta E}{2}\right],$$
$$\pi_{ii}(p^\rho=0)=2e^2\beta\left[1-\frac{\epsilon}{2}\right]\int\frac{d^{3-2\epsilon}k}{(2\pi)^3}\left\{\frac{k_i^2}{E^2}\left[\frac{1}{\beta E}\tanh\left[\frac{\beta E}{2}\right]-\tfrac{1}{2}\operatorname{sech}^2\left[\frac{\beta E}{2}\right]\right]-\frac{1}{\beta E}\tanh\left[\frac{\beta E}{2}\right]\right\}. \tag{2.30}$$

Since for x large $\operatorname{sech}x\approx 2e^{-x}$ the integral for π_{00} converges and we obtain the electric mass of the photon in a thermal background at $T\,(\ll m_e/k)$ to be

$$[m_\gamma^{\rm el}]^2=\pi_{00}(p^\rho=0)=4\alpha m_e^2\left[\frac{2kT}{\pi m_e}\right]^{1/2}e^{-m_e/kT} \tag{2.31}$$

which agrees with the result in Ref. 4 up to a factor of 2. For $T\approx 3$°K, the temperature of the observed thermal background

$$m_\gamma^{\rm el}\approx 10^{-4\times 10^4}\ {\rm eV} \tag{2.32}$$

which clearly is not observationally relevant.

The first and third terms in the expression for π_{ii} diverge since $\tanh x\approx 1$ for x large. However if dimensional regularization is employed the divergences exactly cancel. Furthermore, one finds that the finite parts of $\pi_{ii}(p^\rho=0)$ also cancel so that the magnetic mass of the photon vanishes, as required.

Let us now consider those diagrams, which involve a thermal massless neutrino, that escape the exponential suppression. The lowest-order diagrams involving a neutrino that contribute to the photon self-energy are shown in Fig. 2. To evaluate the photon mass in this case it is easiest to use the real-time formalism. To obtain the nonexponentially suppressed contributions to the photon mass we take all the lines in the diagrams in Fig. 2, apart from the neutrino line, to be at zero temperature and for the neutrino line we only take the temperature-dependent part. Note that the contribution to $\pi_{\mu\nu}(p^\rho=0)$ when all lines are zero-temperature propagators vanishes from gauge invariance. The computation reduces to evaluating

$$\pi_{\mu\nu}(p^\rho=0)=\int\frac{d^4q}{(2\pi)^4}\frac{2\pi\delta(q^2){\rm Tr}[qA_{\mu\nu}(p^\rho=0,q)]}{e^{\beta|\vec{q}\,|}+1}, \tag{2.33}$$

where $A_{\mu\nu}(p,q)$ is the physical zero-temperature neutrino-photon forward scattering amplitude, q being the neutrino's four-momentum. This follows since the δ function from the thermal part of the neutrino propagator forces the neutrino to be on its mass shell.

The diagrams for $A_{\mu\nu}(p,q)$ are given in Fig. 3. The contributions to $\pi_{\mu\nu}(p^\rho=0)$, not summed, of each diagram is given in Table I for the Feynman gauge in the standard model where

$$I_a(z)\equiv\int_0^1 dy\frac{y^a}{y+(1-y)z},$$
$$J_a(z)\equiv\int_0^1 dy\frac{y^a}{[y+(1-y)z]^2}. \tag{2.34}$$

In Table I $z\equiv m_l^2/M_W^2$ with m_l being the lepton mass. Note that individual diagrams give contributions to the photon mass of $O(\sqrt{\alpha G_F}(kT)^2)$ [e.g., diagram (i)], while others give terms of $O(\sqrt{\alpha G_F}(m_l/M_W)(kT)^2)$ [e.g., diagram (v)]. However upon summing all the contributions and performing exactly all the parametric integrals we obtain

$$\pi^{\mu\mu}(p^\rho=0)=0,\quad \mu \text{ not summed}. \tag{2.35}$$

Hence the nonexponentially suppressed contributions for both the "magnetic" and "electric" photon masses have canceled. One may wonder whether taking one or more lines in the diagrams for $A_{\mu\nu}(p,q)$ to be the temperature-dependent part of their propagators may lead to terms that are not exponentially suppressed. This is not the case, with each massive temperature-dependent line in the diagrams for $A_{\mu\nu}(p,q)$ is associated an exponentially large suppression factor $\gtrsim O(e^{+m/kT})$ where m is either the lepton mass or the W mass (which is equal to the H^+ mass in the Feynman gauge). More on this later.

As a check on our calculation we repeated it in the nonlinear R_ξ gauge of Ref. 14 with $\xi=1$. In this gauge Feynman rules simplify considerably. There are no $H^\pm W^\mp\gamma$

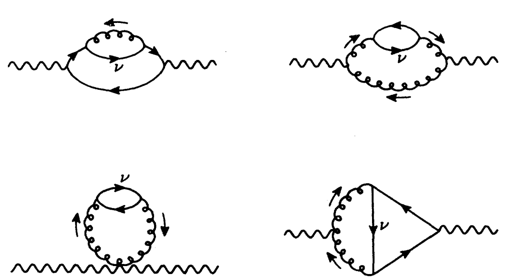

FIG. 2. Two-loop photon self-energy diagrams involving a thermal neutrino. The wavy lines are photons, the curly lines are either a W_μ^+ or its unphysical Higgs partner H^+, while the straight lines are leptons or neutrinos, neutrinos being indicated.

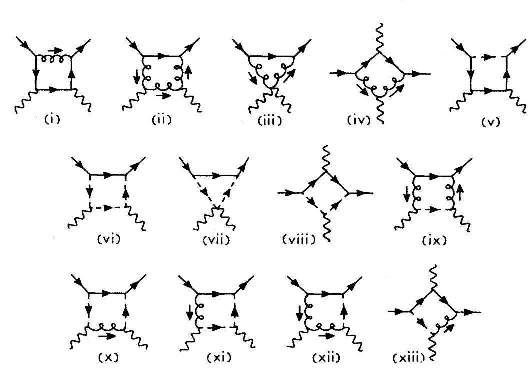

FIG. 3. The one-loop neutrino-photon scattering diagrams. The wavy lines are photons, the curly lines are W_μ^+'s, the dashed lines are H^+'s, and the straight lines are charged leptons when the line is internal and are neutrinos when the line is external.

TABLE I. Diagram-by-diagram contributions to $\pi_{\mu\mu}(p^\rho=0)$. The factor $(7\pi/120\sqrt{2})\alpha G_F\beta^{-4}$ is suppressed for contributions to $\pi_{00}(p^\rho=0)$ while the factor $(7\pi/360\sqrt{2})\alpha G_F\beta^{-4}$ is suppressed for contributions to $\pi_{ii}(p^\rho=0)$, $i=1,2,3$. $z\equiv m_l{}^2/M_W{}^2$.

Diagram number	Feynman gauge	$\xi=1$ in gauge of Ref. 14
i	$4[I_1(z)-2I_2(z)+I_3(z)+z\{J_1(z)-2J_2(z)+J_3(z)\}]$	$=^{a}$
ii	$18I_2(z)-20I_3(z)$	$8I_2(z)-8I_3(z)$
iii	$4I_2(z)$	0
iv	$8[-I_1(z)-I_2(z)+2I_3(z)+z\{-J_1(z)+2J_2(z)-J_3(z)\}]$	$8[-I_1(z)+2I_2(z)-I_3(z)+z\{-J_1(z)+2J_2(z)-J_3(z)\}]$
v	$2z[I_1(z)-2I_2(z)+I_3(z)+z\{J_1(z)-2J_2(z)+J_3(z)\}]$	=
vi	$4z\{I_2(z)-I_3(z)\}$	=
vii	0	0
viii	$4z[-I_1(z)+2I_2(z)-I_3(z)+z\{-J_1(z)+2J_2(z)-J_3(z)\}]$	=
ix	$2J_3(z)$	0
x	0	0
xi	$4z\{J_2(z)-J_3(z)\}$	0
xii	$2z\{J_2(z)-J_3(z)\}$	0
xiii	$8z\{-J_2(z)+J_3(z)\}$	0

[a]The equals sign indicates the contribution in the nonlinear R_ξ gauge for $\xi=1$ is the same as for the Feynman gauge.

couplings so that diagrams (ix)—(xiii) in Fig. 3 do not exist. Diagram (vii) still gives zero contribution. Furthermore the new, more simple form of the $WW\gamma\gamma$ coupling results in diagram (iii) also giving zero. The diagram-by-diagram result in this gauge are also given in Table I. Upon adding the contribution to $\pi_{\mu\nu}(p^\rho=0)$, we find, as before, $\pi_{\mu\nu}(p^\rho=0)=0$.

We should like to point out that checks of electromagnetic gauge invariance greatly simplify in the nonlinear R_ξ gauge.[14] In particular one can easily check the electromagnetic gauge invariance for the neutrino-photon scattering amplitude:

$$p^\mu A_{\mu\nu}(p,q)=0\ . \tag{2.36}$$

The reason for the simplification in the nonlinear R_ξ gauge is that for $\xi=1$ the $W^\pm$ bosons satisfy the tree-level Ward identity[15]

$$p^\mu\left\{\frac{1}{k^2-M_W{}^2}[g_{\nu\rho}(p+2k)_\mu-2g_{\mu\rho}p_\nu+2g_{\mu\nu}p_\rho]\frac{1}{(k+p)^2-M_W{}^2}\right\}=g_{\nu\rho}\left[\frac{1}{k^2-M_W{}^2}-\frac{1}{(k+p)^2-M_W{}^2}\right]\ . \tag{2.37}$$

The term in square brackets is the $W^+W^-\gamma$ vertex and the other factors come from $W^\pm$ propagators. Equation (2.37) is the direct analog of the more familiar Ward identity for fermions

$$p^\mu\left[\frac{1}{\not k-m}\gamma_\mu\frac{1}{\not k+\not p-m}\right]=\frac{1}{\not k-m}-\frac{1}{\not k+\not p+m}\ . \tag{2.38}$$

Using the Ward identity in Eq. (2.37) we checked the gauge invariance of $A_{\mu\nu}(p,q)$ in the nonlinear R_ξ gauge for $\xi=1$ and found Eq. (2.36) to be satisfied.

III. EXPONENTIAL SUPPRESSION OF $m_\gamma^{\rm el}$: GENERAL RESULT

In this section we shall present a general proof that thermal contributions to $m_\gamma^{\rm el}$ from low-temperature backgrounds are suppressed by a factor at least as large as $e^{m_e/2kT}$. The mass of the electron appears in the suppression factor for it is the lightest charged particle. The general treatment of this section will explain why the nonexponentially suppressed contribution to $m_\gamma^{\rm el}$ from massless thermal neutrinos vanished in Sec. II.

To carry out the general proof we consider the spectral decomposition of the photon self-energy, $\pi_{\mu\nu}(P,u)$, at finite temperature in the covariant formulation[9]:

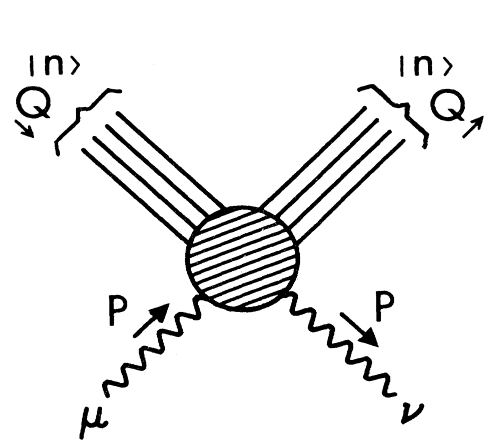

FIG. 4. The forward Compton-type scattering $R(P,Q;P,Q)$ of the state $|n\rangle$, with momentum Q, off the photon, with momentum P.

$$\pi_{\mu\nu}(P,u)=\int d^4x\, e^{iP\cdot x}\sum_n e^{-u\cdot P_n/kT}\langle n\,|\,T(j_\mu(x)j_\nu(0))\,|\,n\rangle \tag{3.1}$$

$$=\int_0^\infty d\sigma^2\left[\int d^4Q\,\theta(Q_0)\delta(Q^2-\sigma^2)e^{-u\cdot Q/kT}T_{\mu\nu}(Q,P)\right], \tag{3.2}$$

where

$$\sum_n\int d^4x\, e^{iP\cdot x}\langle n\,|\,T(j_\mu(x)j_\nu(0))\,|\,n\rangle\delta(P_n-Q)\equiv T_{\mu\nu}(Q,P)\theta(Q^0)\theta(Q^2)\ . \tag{3.3}$$

Each term on the left-hand side of Eq. (3.3) is a Compton-type scattering in the forward direction: $T_{\mu\nu}(Q,P)$ is the sum of all such terms with the same Q

$$T_{\mu\nu}(P,Q)=\sum_{\substack{n\\P_n=Q}} R(P,Q;P,Q)\ , \tag{3.4}$$

where $R(P,Q;P,Q)$ is depicted in Fig. 4. We will proceed to carry out the proof in two steps.

(I) If $T_{\mu\nu}(Q,P)$ is not singular in the limit $P^0=0,\vec{P}\to 0$ then

$$T_{\mu\nu}(P^0=0,\vec{P}\to 0;Q)=0 \tag{3.5}$$

so that $\pi_{\mu\nu}(P_0=0,\vec{P}\to 0)$ vanishes and consequently so does the electric mass of the photon. [Recall that this limit $(P_0=0,\vec{P}\to 0)$ is the limit in which the electric mass of the photon is determined.] The proof is as follows. Electromagnetic gauge invariance implies that

$$P^\mu T_{\mu\nu}(P,Q)=P^\nu T_{\mu\nu}(P,Q)=0\ . \tag{3.6}$$

If $T_{\mu\nu}(P,Q)$ is not singular for $P\to 0$, it can be expanded about $P=0$ so that Eq. (3.6) implies that

$$P^\mu T_{\mu\nu}(0,Q)=0\ . \tag{3.7}$$

The most general form for $T_{\mu\nu}(0,Q)$ is

$$T_{\mu\nu}(0,Q)=Ag_{\mu\nu}+BQ_\mu Q_\nu \tag{3.8}$$

so that Eq. (3.7) implies that $A=B=0$ and consequently Eq. (3.5) is valid. Hence the photon's electric mass is only nonzero if $T_{\mu\nu}(P\to 0,Q)$ is singular.

(II) $T_{\mu\nu}$ is singular only if one of the external photon lines is attached to a charged particle in the state $|\,n\rangle$.

The proof of this statement is analogous to the proof of Low's theorem[16] about bremsstrahlung of very-low-energy quanta. An example of the sort of diagram for $T_{\mu\nu}$ which results in $T_{\mu\nu}$ being singular is given in Fig. 5 where the particle coupling to the photon is a scalar. In this case $T_{\mu\nu}(P,Q)$ has a factor

$$e(2q+P)_\mu\frac{1}{(P+q)^2-m^2} \tag{3.9}$$

which, since the scalar is on-shell ($q^2=m^2$), reduces to

$$e(2q+P)_\mu\frac{1}{2P\cdot q+P^2}\ . \tag{3.10}$$

Clearly in this case as $P\to 0$, $T_{\mu\nu}$ is singular. The same singularity appears for the external photon attached to a charged fermion or vector in the state $|\,n\rangle$.

More generally, a photon can be attached to any external particle i charged or uncharged, via higher-order interactions as depicted in Fig. 6. For this sort of diagram $T_{\mu\nu}(P,Q)$ has a factor

$$j_\mu(P+q,q)\frac{1}{2P\cdot q+P^2}\ , \tag{3.11}$$

where $j_\mu(P+q,q)$ is represented by the cross-hatched blob in Fig. 6 and where the singular factor $1/(2P\cdot q+P^2)$ comes from the propagator. While the propagator factor can give rise to a singularity as $P\to 0$ one must investigate the behavior of $j_\mu(P+q,q)$ as $P\to 0$. Two things can happen.

(A) $j_\mu(q,q)\neq 0$. In this case $T_{\mu\nu}(P\to 0,Q)$ is singular and there is a contribution to $m_\gamma^{\rm el}$. However $j_\mu(q,q)\neq 0$ is just the condition for particle i to be charged, and consequently there is a charged particle in $|\,n\rangle$.

(B) $j_\mu(q,q)=0$. In this case $T_{\mu\nu}(P\to 0,Q)$ is not singular and there is no contribution to $m_\gamma^{\rm el}$. However $j_\mu(q,q)=0$ is precisely the condition for particle i to be uncharged. Instead of being attached to an external charged line, the photon can be attached to an internal charged line as was the case in our example involving the thermal neu-

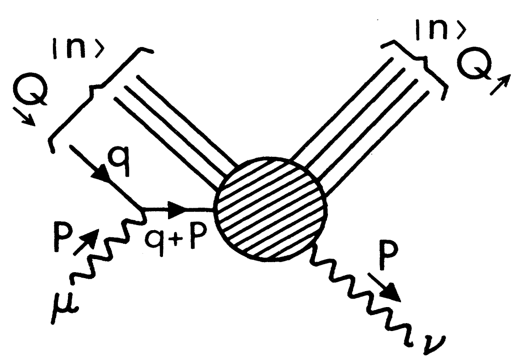

FIG. 5. An example that causes $T_{\mu\nu}(P,Q)$ to be singular as $P\to 0$.

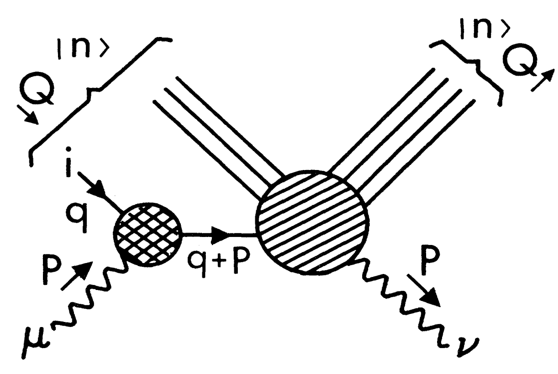

FIG. 6. The class of diagrams that causes $T_{\mu\nu}(P,Q)$ to be singular as $P\to 0$.

trino. However this sort of diagram does not result in the appearance of a singularity[16] as $P \rightarrow 0$ and does not give rise to a contribution to the photon's mass.

To complete the proof recall the form of $\pi_{\mu\nu}(P,u)$ given in Eq. (3.1). If one goes to the rest frame of the fluid, each term has a factor $e^{-P_n^0/kT}$ where P_n^0 is the energy of the state $|n\rangle$. Since from statement (II) only those terms that have at least one charged particle in the state $|n\rangle$ can give rise to a nonzero contribution to $\pi_{\mu\nu}(P^0=0,\vec{P}\rightarrow 0;u)$ and since the electron is the lightest charged particle $P_n^0 \geq m_e$. Therefore the smallest suppression factor is $e^{m_e/kT}$. Consequently the photon's electric mass is suppressed by at least $e^{-m_e/2kT}$ [recall $m_\gamma^{\rm el} \sim (\pi_{00})^{1/2}$].

In summary the chain of argument is as follows. If $T_{\mu\nu}(P\rightarrow 0,Q)$ is not singular, then it is zero from electromagnetic gauge invariance which in turn implies $\pi_{\mu\nu}(P\rightarrow 0,u)$ is zero and $m_\gamma^{\rm el}$ is zero. $T_{\mu\nu}(P\rightarrow 0,Q)$ is only singular if there are charged particles in the state $|n\rangle$ attached to the photon. However since each term in $T_{\mu\nu}$ is suppressed by $e^{-P_n^0/kT}$ and since the lightest charged particle is the electron with mass m_e all such contributions to $m_\gamma^{\rm el}$ are suppressed by at least $e^{-m_e/2kT}$.

IV. SUMMARY AND CONCLUSIONS

We have shown that all contributions to the thermal mass of the photon are suppressed by a large exponential factor $e^{m_e/2kT}$ when the background temperature T is low, $kT \ll m_e$. The crucial point is that contributions to the thermal mass of the photon come only from sets of diagrams in which one or more charged lines is taken to be thermal. Since all charged particles have masses greater than or equal to m_e these lines are accompanied by huge Boltzmann suppression factors at least as large as $e^{m_e/kT}$. While individual diagrams in which only zero-mass or nearly zero-mass (and consequently zero-charge) lines are taken to be thermal give contributions that are not suppressed by large exponential factors, when they are summed the nonexponentially suppressed terms cancel. This latter result follows from (1) electromagnetic gauge invariance and (2) the absence of infrared singularities associated with the vanishing of the photon's momentum for the constituent subdiagrams that make up the photon self-energy in which zero-charge particles are taken to be thermal.

ACKNOWLEDGMENTS

We wish to thank Ranabir Chakrabarti, Darwin Chang, Michael Levine, Ling-Fong Li, Chris Maxwell, and Sergei Petcov for many useful discussions. This work was partially supported by the U.S. Department of Energy (PBP and DGU) and partially supported by the National Science Foundation (JFN).

---

[1]D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981) and references therein.

[2]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[3]L. F. Abbott and M. B. Gavela, Nature 299, 187 (1982).

[4]R. M. Woloshyn, Phys. Rev. D 27, 1393 (1983).

[5]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in *Elementary Particle Theory* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1969), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[6]E. R. Williams, J. E. Faller, and H. A. Hill, Phys. Rev. Lett. 26, 721 (1971). There is a better experimental limit for the "magnetic" mass of the photon but that is expected still to be zero at finite temperatures.

[7]Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. 98B, 265 (1981); G. B. Gelmini and M. Roncadelli, Phys. Lett. 99B, 411 (1981); H. M. Georgi, S. L. Glashow, and S. Nussinov, Nucl. Phys. B193, 297 (1981).

[8]Page 303 of Ref. 2.

[9]H. A. Weldon, Phys. Rev. D 26, 1394 (1982) and references therein.

[10]A. D. Linde, Rep. Prog. Phys. 42, 389 (1979); E. S. Fradkin, Proc. Lebedev Phys. Inst. 29, 7 (1965) (English translation by Consultants Bureau, New York, 1967).

[11]For reviews of finite-temperature field theory see L. Dolan and R. Jackiw, Phys. Rev. D 13, 87 (1974); C. Bernard, *ibid.* 9, 3312 (1974); S. Weinberg, *ibid.* 9, 3357 (1974). See also Ref. 1.

[12]S. Weinberg, Ref. 11.

[13]M. B. Kislinger and P. D. Morley, Phys. Rev. D 13, 2764 (1976); Refs. 1 and 4.

[14]M. B. Gavela, G. Girardi, C. Malleville, and P. Sorba, Nucl. Phys. B193, 257 (1981); K. Fujikawa, Phys. Rev. D 7, 393 (1973); M. Bace and N. D. Hari Dass, Ann. Phys. (N.Y.) 94, 349 (1975).

[15]After completion of this paper we received from N. M. Monyonko and J. H. Reid, Report No. TRI-PP-82-52, 1982 (unpublished) in which they show that the electromagnetic Ward identity for $W^{\pm}$ bosons is satisfied in the nonlinear R_ξ gauge for all ξ.

[16]F. E. Low, Phys. Rev. 110, 974 (1958).