Conservation laws in the monopole-induced baryon-number-violating processes

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(Received 4 March 1983)

Monopole-induced baryon-number-violating processes are analyzed using the conservation laws for the ordinary and the chiral charge densities. It is shown that in the strictly massless limit reactions of the form $u_1+M\to u_2^c+d_3^c+e^++M$ are ruled out by these conservation laws. This, however, does not mean that the baryon-number-violating processes are suppressed, since reactions of the type $u_1+u_2+M\to d_3^c+e^++M$ may take place even if the incoming u_1 and u_2 do not have any appreciable overlap in their wave functions. The role of the Adler-Bell-Jackiw anomaly in the baryon-number-violating processes is investigated. It is shown that the baryon-number violation takes place because of nontrivial boundary conditions at the monopole core, and is independent of the existence of the Adler-Bell-Jackiw anomaly. We may have chirality-conserving as well as chirality-nonconserving baryon-number-violating processes. It is also shown that the inclusion of extra Coulomb energies, e.g., weak or electromagnetic Coulomb energies, cannot qualitatively change the baryon-number-violating effects.

It has been pointed out by Rubakov[1] and Callan[2,3] that grand-unification monopoles of the 't Hooft-Polyakov type[4] may catalyze baryon-number-violating processes at a strong-interaction rate. Two different but equivalent ways have been proposed to understand the process. Both approaches focus on the $J=0$ partial-wave amplitude for a fermion in the presence of a monopole. In the first approach,[1,2] one shows that the theory reduces to a massless Schwinger model, which can then be exactly solved, and one finds a nonzero vacuum expectation value for a baryon-number-violating condensate. In this approach, the helicities carried by various fields in the condensate may be read out in a straightforward manner. However, it is not easy to see the kinematical constraint,[5] which allows only a definite helicity state of a particle to be ingoing, and the opposite helicity state to be outgoing. Also, the mechanism of baryon-number violation is not clear in this picture. In the other approach,[3] the theory is mapped onto an equivalent boson theory, and fermions are represented as solitons in these boson fields. In this picture, it is straightforward to see how the helicity state of a particle is related to whether it is ingoing or outgoing. One can also construct a time history of a process involving initial- and final- state solitons, which looks like a baryon-number-violating scattering process. But in this picture we do not get any constraint on the helicities of the initial- and final-state particles, besides the kinematical constraints. As a result, it is not immediately clear from this picture which scattering processes are allowed, and which are not. In particular, the process one fermion + monopole $\to$three fermions + monopole, which may be ruled out by combining the results of the first approach with the kinematical constraint on the helicities, does not seem to be ruled out by the second approach.

In this paper we work in the soliton approach, and show that the effective boson theory has some exact conservation laws. The conserved quantities are related to the chiral and the ordinary charges in the original fermion theory. We show that of the four ordinary charges and four chiral charges that can be constructed out of the fermionic fields, three ordinary charges and one chiral charge are exactly conserved, one ordinary charge and two chiral charges are locally conserved, but may flow into or out of the monopole core, and hence are globally nonconserved, and one chiral charge is locally and hence also globally nonconserved due to the anomaly. Using the four exact conservation laws, one may rule out many processes, in particular the process $u_{1R}+M\to u_{2R}^c+d_{3L}^c+e_L^++M$, where the initial- and the final-state fermions are free ingoing and outgoing waves, respectively. However, this does not imply suppression of baryon-number-violating processes, since, as we shall show, reactions of the form $u_{1R}+u_{2R}+M\to d_{3L}^c+e_L^++M$ may take place even if the ingoing u_1 and u_2 do not have an appreciable overlap in their wave functions, as opposed to the claim by Grossman *et al.*[6]

Using our formalism we can trace the origin of baryon-number violation. We show that the baryonic charge is a linear combination of charges, some of which are exactly conserved, and some of which are nonconserved at the boundary.[7] Thus the nonconservation of the baryonic charge comes solely from the boundary conditions and has nothing to do with the anomaly. In reactions of the type $u_{1R}+u_{2R}+M\to d_{3L}^c+e_L^++M$, the baryon number, as well as the total helicity, is nonconserved. Hence both the nontrivial boundary condition at the monopole core and the anomaly are responsible for this process. However, there also exist allowed reactions of the form $u_{1R}+d_{3L}+M\to u_{2R}^c+e_L^++M$, where the total helicity carried by the initial- and the final-state particles are the same. The anomaly plays no role in such processes. As far as our knowledge goes, this type of helicity-conserving processes were first noted by Seo,[8] who has given a list of all the possible baryon-number-violating processes. All the processes listed by him obey the conservation laws that we shall discuss below. We, however, do not agree with his claim that the nonconservation of the baryon number takes place in an extended region around the monopole, rather than at the monopole core. This may be just a matter of semantics.

Using the same conservation laws, we find a sufficient condition to be satisfied by a charge in order that the violation of the charge is not catalyzed by a monopole. We also show that the effect of including any extra Coulomb interaction, due to the presence of other gauge fields, cannot qualitatively change the results of Callan and Rubakov, if the generator corresponding to the extra gauge field commutes with the SU(2) subgroup in which the monopole lies. Hence the inclusion of these energies is irrelevant for a qualitative analysis. This again contradicts the investigation by Grossman *et al.*[6] Thus we conclude that the monopole catalysis of proton decay takes place at a typical strong-interaction rate.

Let us consider a system of SU(2) monopole with two Dirac doublets of massless fermions $\binom{\psi_1}{\chi_1}$ and $\binom{\psi_2}{\chi_2}$.[9] For SU(5) monopoles we shall identify these doublets with $\binom{d_3^c}{e^-}$ and $\binom{u_1}{u_2^c}$. As was shown by Callan, in the $J=0$ partial-wave amplitude, the system may be described by an equivalent boson theory of four scalar fields Φ_1, Φ_2, Q_1, and Q_2, with the Hamiltonian

$$H=\int_0^\infty dr\left[\frac{1}{2}\sum_{i=1}^{2}(\Pi_i{}^2+P_i{}^2+\Phi_i'^2+Q_i'^2)+\frac{c}{r^2}(\Phi_1+\Phi_2+Q_1+Q_2)^2\right], \tag{1}$$

where c is a constant. Here Π_i and P_i are the momenta conjugate to Φ_i and Q_i. Various fermion field bilinears may be expressed in terms of the fields Φ_i, Q_i, Π_i, and P_i in the effective boson theory. Table I summarizes the operator correspondences for all the charges and the radial currents.

The fields Φ_i and Q_i satisfy the boundary conditions

TABLE I. Operator correspondences of various charge densities and current densities in the original theory and the effective boson theory. In this table, the upper (lower) component fields ψ_i (χ_i) refer to the eigenstate of the generator $\hat{x}\cdot\vec{T}$ with eigenvalue $+1$ (-1).

Operators in the original theory ($4\pi r^2\times$)	Operators in the boson theory ($(\sqrt{\pi})^{-1}\times$)
$\bar{\psi}_i\gamma^0\psi_i$	$\Phi_i'(r,t)$
$\bar{\chi}_i\gamma^0\chi_i$	$-Q_i'(r,t)$
$\bar{\psi}_i\gamma^0\gamma^5\psi_i$	$\Pi_i(r,t)$
$\bar{\chi}_i\gamma^0\gamma^5\chi_i$	$P_i(r,t)$
$\bar{\psi}_i\hat{x}\cdot\vec{\gamma}\psi_i$	$-\Pi_i(r,t)$
$\bar{\chi}_i\hat{x}\cdot\vec{\gamma}\chi_i$	$P_i(r,t)$
$\bar{\psi}_i\hat{x}\cdot\vec{\gamma}\gamma^5\psi_i$	$-\Phi_i'(r,t)$
$\bar{\chi}_i\hat{x}\cdot\vec{\gamma}\gamma^5\chi_i$	$-Q_i'(r,t)$

$$\Phi_i(r=0)=Q_i(r=0)\,,\qquad \Phi_i'(r=0)=-Q_i'(r=0)\,. \tag{2}$$

There are altogether four ordinary charge densities and four chiral charge densities listed in Table I. We calculate the commutator of each of the eight charges formed out of these charge densities, with the Hamiltonian, to see which of these charges are conserved. When we use the boundary conditions (2), and the extra constraint that $\Phi_1+Q_1+\Phi_2+Q_2$ must vanish at $r=0$ in order to keep the total energy finite, we find that the following charges are conserved:

$$\begin{aligned}
S_1&=\int_0^\infty 4\pi r^2dr(\bar{\psi}_1\gamma^0\psi_1+\bar{\chi}_1\gamma^0\chi_1)=\int_0^\infty(\Phi_1'-Q_1')dr/\sqrt{\pi}\,,\\
S_2&=\int_0^\infty 4\pi r^2dr(\bar{\psi}_2\gamma^0\psi_2+\bar{\chi}_2\gamma^0\chi_2)=\int_0^\infty(\Phi_2'-Q_2')dr/\sqrt{\pi}\,,\\
S_3&=\int_0^\infty 4\pi r^2dr\sum_{i=1}^{2}(\bar{\psi}_i\gamma^0\psi_i-\bar{\chi}_i\gamma^0\chi_i)=\int_0^\infty(\Phi_1'+\Phi_2'+Q_1'+Q_2')dr/\sqrt{\pi}\,,\\
S_4&=\int_0^\infty 4\pi r^2dr(\bar{\psi}_1\gamma^0\gamma^5\psi_1+\bar{\chi}_1\gamma^0\gamma^5\chi_1-\bar{\psi}_2\gamma^0\gamma^5\psi_2-\bar{\chi}_2\gamma^0\gamma^5\chi_2)=\int_0^\infty(\Pi_1+P_1-\Pi_2-P_2)dr/\sqrt{\pi}\,.
\end{aligned} \tag{3}$$

The following charges are nonconserved only through the boundary terms:

$$\begin{aligned}
N_1&=\int_0^\infty 4\pi r^2dr(\bar{\psi}_1\gamma^0\psi_1-\bar{\chi}_1\gamma^0\chi_1-\bar{\psi}_2\gamma^0\psi_2+\bar{\chi}_2\gamma^0\chi_2)=\int_0^\infty(\Phi_1'+Q_1'-\Phi_2'-Q_2')dr/\sqrt{\pi}\,,\\
\dot{N}_1&\propto(\dot{\Phi}_1+\dot{Q}_1-\dot{\Phi}_2-\dot{Q}_2)\,|_{r=0}\,,\\
N_2&=\int_0^\infty(\bar{\psi}_1\gamma^0\gamma^5\psi_1-\bar{\chi}_1\gamma^0\gamma^5\chi_1)4\pi r^2dr=\int_0^\infty(\Pi_1-P_1)dr/\sqrt{\pi}\,,\\
\dot{N}_2&\propto(\Phi_1'-Q_1')\,|_{r=0}\,,\\
N_3&=\int_0^\infty(\bar{\psi}_2\gamma^0\gamma^5\psi_2-\bar{\chi}_2\gamma^0\gamma^5\chi_2)4\pi r^2dr=\int_0^\infty(\Pi_2-P_2)dr/\sqrt{\pi}\,,\\
\dot{N}_3&\propto(\Phi_2'-Q_2')\,|_{r=0}\,.
\end{aligned} \tag{4}$$

Finally, the charge

$$L_1=\int_0^{\infty}(\bar{\psi}_1\gamma^0\gamma^5\psi_1+\bar{\chi}_1\gamma^0\gamma^5\chi_1+\bar{\psi}_2\gamma^0\gamma^5\psi_2+\bar{\chi}_2\gamma^0\gamma^5\chi_2)4\pi r^2dr=\int_0^{\infty}(\Pi_1+P_1+\Pi_2+P_2)dr/\sqrt{\pi} \tag{5}$$

fails to commute with the Hamiltonian because of the presence of the Coulomb term. This is the effect of the anomaly. Conservation of S_1, S_2, and S_3 implies the conservation of total number of fermions of type 1, the total number of fermions of type 2, and the total T_3 charge, respectively, where T_3 is the unbroken generator of the SU(2) subgroup. Conservation of S_4 implies that the total helicity of particle type 1 minus the total helicity of particle type 2 must be conserved. Note that of all the chiral charges, only L_1 fails to be conserved because of the anomaly. The above conservation laws may also be derived by using the equations of motion, or by using the current conservation law ($\partial_0 J^0+\partial_r J^r=0$ or proportional to the anomaly).

As was pointed out by Callan,[3] fermions may be represented by solitons in the fields Φ_i and Q_i. Solitons corresponding to various fermions are shown in Fig. 1. Let us consider the soliton corresponding to the field ψ_i. If it moves with a constant velocity v ($v>0$ if it moves outward), we have

$$v=-\int_0^{\infty}\dot{\Phi}_i dr/\sqrt{\pi}=-\int_0^{\infty}\Pi_i dr/\sqrt{\pi}\ , \tag{6}$$

which shows that the helicity of a particle is determined by whether it is moving outward or inward. We can write down the following general rule:

$$\text{helicity}=-\text{sign of the } T_3 \text{ charge}\times v\ . \tag{7}$$

With all the conservation laws and helicity constraints in mind, we may write down the following allowed processes:

$$\psi_{1R}+\psi_{2R}+M\to\psi_{1L}+\psi_{2L}+M\ , \tag{8}$$

$$\psi_{1R}+\chi^c_{1R}+M\to\psi_{2L}+\chi^c_{2L}+M\ , \tag{9}$$

$$\psi_{1R}+\psi^c_{2L}+M\to\chi_{1R}+\chi^c_{2L}+M\ , \tag{10}$$

etc. A process of the form

$$\psi_{1R}+M\to\psi^c_{2R}+\psi_{1L}+\psi_{2L}+M \tag{11}$$

is not allowed by the conservation of S_4. In fact, for the process $\psi_{1R}+M$, there is no final state of the form $M+$ free fermions, which is allowed by all four conservation laws. It is interesting to see what happens when we have only a $J=0$ right-handed ψ_1 in the initial state. We shall come back to this question later.

Let us now investigate reactions (8) and (9) in some detail. All the conservation laws that we have derived so far are valid in the true quantum-mechanical sense, i.e., the matrix element of the operator between any two states is conserved. We shall now look at the system at the classical level and study its time development. In (8), the charges N_2 and N_3 fail to be conserved, although N_2-N_3 is conserved. As noted in Eq. (4), this violation must be accompanied by a nonzero value of $(\Phi'_1-Q'_1+\Phi'_2-Q'_2)$ at the origin. In reaction (9), on the other hand, N_2 and N_3 are conserved, but N_1 is violated. Hence this must be accompanied by a nonzero time derivative of $\Phi_1+Q_1-\Phi_2-Q_2$ at the origin.

We show the time sequences for reactions (8) and (9) in Figs. 2 and 3, respectively. The reader can verify that the charges N_1, N_2, and N_3 are nonconserved only at the boundary in the time sequences described in Figs. 2 and 3. Conservation of these charges at finite r forces the individual fields to carry fractional helicity. Although the two scattering processes look very different, they basically take place through the same mechanism. In fact, we may define

$$\begin{bmatrix}\psi'_1\\ \chi'_1\end{bmatrix}=\begin{bmatrix}\psi_{1R}\\ \chi_{1R}\end{bmatrix}+\begin{bmatrix}\chi^c_{2L}\\ -\psi^c_{2L}\end{bmatrix},\quad \begin{bmatrix}\psi'_2\\ \chi'_2\end{bmatrix}=\begin{bmatrix}\chi^c_{1R}\\ -\psi^c_{1R}\end{bmatrix}+\begin{bmatrix}\psi_{2L}\\ \chi_{2L}\end{bmatrix} \tag{12}$$

and then reduce the theory to an effective boson theory in terms of the primed variables. The following reactions are then equivalent:

$$\begin{aligned}&\psi'_{1R}+\psi'_{2R}+M\to\psi'_{1L}+\psi'_{2L}+M\\ &\qquad\equiv\psi_{1R}+\chi^c_{1R}+M\to\chi^c_{2L}+\psi_{2L}+M\ ,\\ &\psi'_{1R}+\chi'^c_{1R}+M\to\psi'_{2L}+\chi'^c_{2L}+M\\ &\qquad\equiv\psi_{1R}+\psi_{2R}+M\to\psi_{2L}+\psi_{1L}+M\ ,\end{aligned} \tag{13}$$

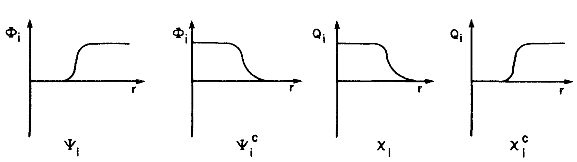

FIG. 1. Solitons corresponding to various fields.

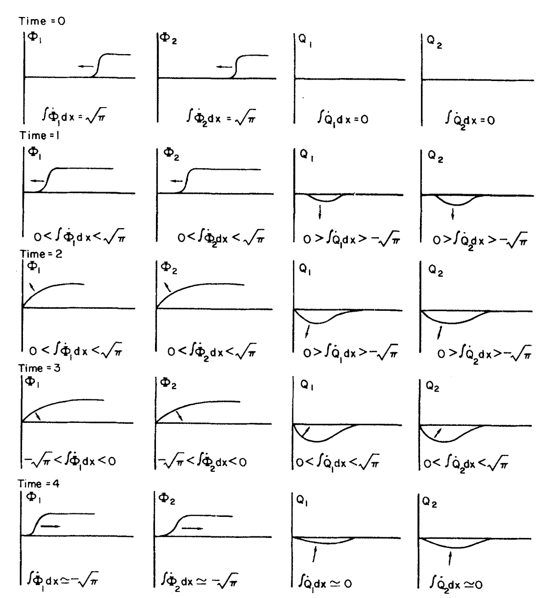

FIG. 2. Time development of the process $\psi_{1R}+\psi_{2R}+M\to\psi_{1L}+\psi_{2L}+M$.

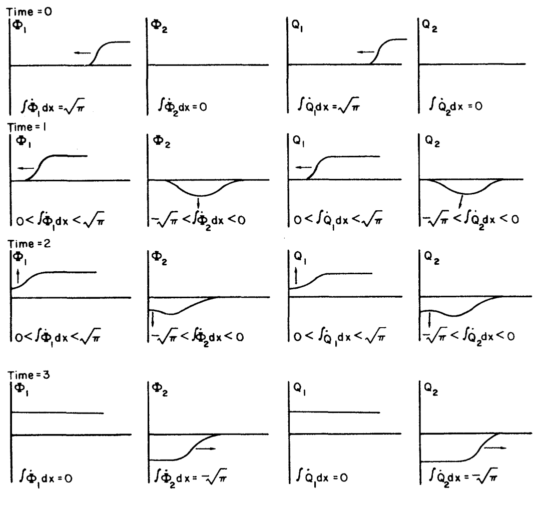

FIG. 3. Time development of the process $\psi_{1R} + \chi^c_{1R} + M \rightarrow \psi_{2L} + \chi^c_{2R} + M$.

and hence in terms of the new boson fields reaction (8) will have the time sequence of Fig. 3, whereas reaction (9) will have the time sequence of Fig. 2.

The point we want to emphasize is that both reactions (8) and (9) take place via the combined effect of the anomaly and the nontrivial boundary condition at the monopole core, even though reaction (8) is just a helicity-flip amplitude. This can be easily seen by noting that both reactions (8) and (9) violate conservation of charges (N_2+N_3 and N_1, respectively) which are anomaly free, and hence must flow into the monopole core.

For an SU(5) monopole, reaction (9) reduces to

$$d^c_{3R} + e^+_R + M \rightarrow u_{2L} + u_{1L} + M \tag{14}$$

which violates baryon number. The origin of this violation may be understood by noting that the baryonic charge

$$\int 4\pi r^2 dr(-\bar{\psi}_1\gamma^0\psi_1 + \bar{\psi}_2\gamma^0\psi_2 - \bar{\chi}_2\gamma^0\chi_2)/3 = -\frac{1}{6}\left[\frac{N_1+S_3}{2} + S_1\right] + \frac{1}{6}(S_3 - N_1) \tag{15}$$

is violated through the boundary terms.

For reaction (10), L_1 is conserved and the process takes place even if we switch off the anomaly term [set $C=0$ in (1)] without changing the boundary conditions (2). This can be seen easily by noting that for reaction (10), the Coulomb term vanishes identically at all times at all points in space, if the two incoming particles travel together. This clearly shows that it is the nontrivial boundary conditions at the core, rather than the anomaly, which is of fundamental importance in the baryon-number-nonconserving processes. Constructing the time sequence for reaction (10) is left as an exercise to the reader.

Let us now go back to the reaction $\psi_{1R} + M$. Any final state involving free outgoing fermions, that carries the same S_4 charge as the initial state, must leave a net S_1, S_2, or S_3 charge at the monopole, which then spreads over an infinite radius around the monopole core. Thus in this case it is not possible to get a final state of the form monopole + free fermions. We can still gain some knowledge about baryon-number nonconservation in this process by doing a classical analysis of the reaction. This time we do not know what the classical final state is, we must actually solve the equations of motion to find the final state. We define

$$\begin{aligned} A &= (\Phi_1+\Phi_2+Q_1+Q_2)/2 , \\ B &= (\Phi_1+\Phi_2-Q_1-Q_2)/2 , \\ C &= (\Phi_1-\Phi_2+Q_1-Q_2)/2 , \\ D &= (\Phi_1-\Phi_2-Q_1+Q_2)/2 . \end{aligned} \tag{16}$$

B, C, and D satisfy the free field equations of motion. A satisfies the equation

$$\ddot{A} - A'' = -(4c/r^2)A . \tag{17}$$

The boundary conditions on the various fields are

$$\begin{aligned} &A(0)=0, \quad A'(0)=0 , \\ &B(0)=C'(0)=D(0)=0 . \end{aligned} \tag{18}$$

With these boundary conditions and the equations of motion, we may study the scattering of solitary waves in A, B, C, and D fields. The result has been summarized in Fig. 4. Of these, the solitary waves in B, C, and D travel with the velocity of light all through (since they are free fields) and come back undistorted. The solitary wave in A, on the other hand, may suffer a time delay and may also be distorted by scattering. For the present purpose, we may neglect both these effects.

We may now construct the initial and final states of any scattering process by superposing the various diagrams in Fig. 4. This is allowed, since the equations of motion are linear in the fields. In particular, we may verify the correctness of reactions (8)–(10) using these diagrams. If we now study the scattering of a Φ_1 soliton from the core, we get the final state shown in Fig. 5. We find that the scattering process conserves all the charges S_1, S_2, S_3, and S_4, but the outgoing solitons carry half fermionic charge. Although this analysis does not tell us what are the possible final states, we can interpret the classical result as a time evolution of the expectation values of different operators in a given state. Note that $\langle \Delta B \rangle$ is nonzero in this scattering, where B is the total baryonic charge outside the monopole core.

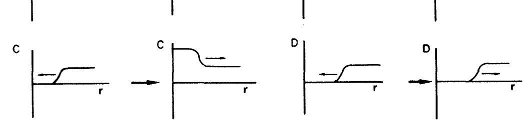

FIG. 4. Classical scattering of A, B, C, and D solitons from the core.

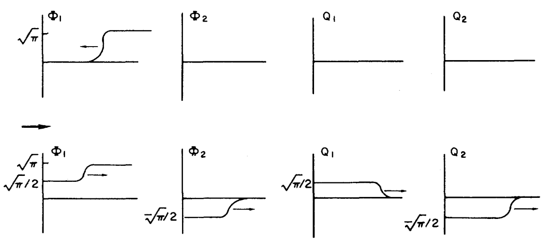

FIG. 5. Classical scattering of ψ_{1R} from the core.

We can probably get a clearer picture in the classical analysis by giving the fermions a small mass. The mass term will eventually drive the Φ_i and Q_i fields at $r=0$ to be integral multiples of $\sqrt{\pi}$. In this case S_4 is not conserved, and we may have final states of the form monopole + free fermions. But we are going to argue now that it is the massless limit which is more relevant for proton decay. It is clear from the way we constructed Fig. 5 from Fig. 4 that if we have incoming ψ_{1R}, ψ_{2R} separated by a distance, we shall still get ψ_{1L}, ψ_{2L} in the final state, only the final-state solitons are now spread out over a distance (two-step solitons). This is true for any reaction of the form two fermions + monopole→two fermions + monopole. Thus, for example, in the reaction $u_{1R}+u_{2R}+M\rightarrow e_L^+ + d_{3L}^c + M$, even if the two incoming u quarks have a small overlap in their wave function, we still get a baryon-number-violating process. If the quarks have mass m, we expect this to happen so long as the two incoming u quarks are separated by a distance less than m^{-1}, since we expect the mass term to become operative only if we wait for a time of order m^{-1}. This shows that the massless limit is probably a good approximation for monopole-induced proton decay, since the radius of the proton is small compared to the Compton wavelength of the quarks.

Let us now turn to study the effect of introducing extra Coulomb interactions. What we mean by extra Coulomb interactions is the following. In SU(5) gauge theory, for example, we have three other diagonal generators, besides the unbroken generator T_3 belonging to the SU(2) subgroup in which the monopole lies. They are

$$\begin{bmatrix} -1 & & & & \\ & 0 & & & \\ & & \frac{1}{2} & & \\ & & & \frac{1}{2} & \\ & & & & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & & & & \\ & -1 & & & \\ & & \frac{1}{2} & & \\ & & & \frac{1}{2} & \\ & & & & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & \frac{1}{2} & & \\ & & & \frac{1}{2} & \\ & & & & -1 \end{bmatrix}. \tag{19}$$

There are massless gauge bosons associated with these generators. [The last generator does not have a massless gauge particle associated with it if the electroweak SU(2) is broken. But we must include its effect at a distance $<m_{\text{weak}}^{-1}$.] The presence of these gauge bosons will introduce new Coulomb energies in the Hamiltonian, and if they produce an energy barrier that destroys the scattering solutions that we constructed before, we may lose the baryon-number-violating effect. We shall show that this can never happen.

Let us consider the Coulomb energy contribution from a particular generator T. If $A(r,t)$ be the total T charge inside a sphere of radius r, the extra Coulomb energy contribution to the Hamiltonian is given by

$$C_1 \int dr[A(r,t)]^2/r^2 \,, \tag{20}$$

C_1 being a constant. $A(r,t)$ may be calculated at any time t by summing the total T charge lying between the origin and a distance r from the origin, with the total T charge that has flown into the origin in the time $-\infty$ to t. Both can be calculated in terms of the fields Φ_i, Q_i, Π_i, and P_i, using the expressions for the charge densities and the radial currents from Table I. The radial current at $r=0$ turns out to vanish for all the three generators. The charge densities, on the other hand, may be expressed in terms of the fields Φ_i', Q_i', $\dot{\Phi}_i$, and $\dot{Q}_i$. Finiteness of the $\int \dot{\Phi}_i^2$, $\int \Phi_i'^2$, $\int \dot{Q}_i^2$, and $\int Q_i'^2$ terms in H requires that in the original solution, without the extra Coulomb energies, any time or space derivatives of the fields are bounded by

$$K r^{-1/2+\epsilon} \tag{21}$$

near the origin. ϵ is a positive constant. Then $A(r,t)$ is bounded by

$$A(r,t) < K' r^{1/2+\epsilon} \tag{22}$$

which guarantees that the extra Coulomb energy (20) will always be finite if we evaluate it using the original solution for the fields. In other words, the extra Coulomb energy cannot produce any infinite energy barrier to the baryon-number-violating processes.

The key point to the above conclusion is the vanishing of the radial T current at the core. This may be understood as follows. From Table I and the boundary conditions (2), we may conclude that

$$\begin{aligned} (\bar{\psi}_{iL}\hat{x}\cdot\vec{\gamma}\psi_{iL} + \bar{\chi}_{iL}\hat{x}\cdot\vec{\gamma}\chi_{iL})\,|_{r=0} &= 0 \,, \\ (\bar{\psi}_{iR}\hat{x}\cdot\vec{\gamma}\psi_{iR} + \bar{\chi}_{iR}\hat{x}\cdot\vec{\gamma}\chi_{iR})\,|_{r=0} &= 0 \,, \end{aligned} \quad i=1,2 \tag{23}$$

which implies that the total flow of fermionic current into the origin for any SU(2) doublet must vanish independently for the left-handed and the right-handed parts. Loosely speaking this implies that an ingoing $\psi_{iL(R)}$ into the core must be accompanied by an outgoing $\chi_{iL(R)}$ and vice versa. Now, all the generators of the extra Abelian subgroups commute with the full SU(2) subgroup in which the monopole lies. Hence, two members of the doublet must always carry the same T charge, and the total T current into the origin must necessarily vanish.

The presence of these extra Coulomb interaction terms, however, may produce anomalies in the currents N_1, N_2, and N_3 (but not in the conserved currents S_i), and hence produce a new source of violation of these currents.[10]

We may use the above result to find out which charges can be violated by monopole catalysis. As we have just seen, any charge, which commutes with the full SU(2) subgroup, must necessarily be conserved. On the other hand if T_3 is the unbroken generator of the SU(2) subgroup, then the total T_3 charge must also be conserved (conservation of S_3). Thus if any charge can be expressed as a linear combination of T_3 and another charge, which commutes with the full SU(2) subgroup in which the monopole lies, then the monopole cannot catalyze the nonconservation of that charge.

In conclusion, we may state the following results:

(1) In the scattering of fermions from the monopole, the conservation of S_1, S_2, S_3, S_4, defined in Eq. (3) must be satisfied. They imply conservation of total number of particles of type 1 (e^- and d_3^c), total number of particles of type 2 (u_1 and u_2^c), total T_3 charge, where T_3 is the unbroken generator of the SU(2) subgroup in which the monopole lies, and the total helicity carried by particles of type 1 minus the total helicity carried by particles of type 2. In counting the number of particles of a given type, we should count -1 for antiparticles, whereas, in counting the total helicity carried by particles of a given type, we should count $-1(+1)$ for left- (right-) handed particles, irrespective of whether they are particles or antiparticles. Some of the allowed reactions are

$$u_{1R}+u_{2R}+M \rightarrow e_L^+ + d_{3L}^c + M \ ,$$

$$e_L^- + u_{2R} + M \rightarrow u_{1R}^c + d_{3L}^c + M \ ,$$

etc.

(2) The nontrivial boundary conditions at the monopole core are important for any scattering, including the helicity-flip amplitude $\psi_{1R}+\psi_{2R}+M \rightarrow \psi_{1L}+\psi_{2L}+M$. This process violates the conservation of chiral charge S_3, which is free from anomaly, as well as the chiral charge L_1, defined in Eq. (5), which is anomalous. Hence this process can take place only if the charge S_3 flows into the monopole core. In fact, there exists helicity-conserving processes like $e_L^- + u_{2R} + M \rightarrow u_{1R}^c + d_{3L}^c + M$, where anomaly does not play any role, since the charge L_1 is conserved in this process.

(3) We have shown that processes like $u_{1R}+M \rightarrow u_{2L}+e_L^+ + d_{3L}^c + M$ are not allowed in the limit where the quarks are massless. This does not imply the suppression of baryon-number-violating processes, since in the massless limit processes like $u_{1R}+u_{2R}+M \rightarrow e_L^+ + d_{3L}^c + M$ do not have suppression, even if the incoming u_1 and u_2 quarks do not have any appreciable overlap in their wave function. Another reaction, which is ruled out by the conservation laws, is $u_{1R}+u_{2R}+M \rightarrow U_{1L} + U_{2L} + M$.

(4) We have shown that if any charge, free from anomaly, can be expressed as a linear combination of the diagonal generator of the SU(2) subgroup, and another charge, which commutes with the generators of the full SU(2) subgroup, then the conservation of that charge cannot be violated by monopole catalysis. [Here the SU(2) subgroup refers to the subgroup in which the monopole lies.]

(5) We have shown that the inclusion of the extra Coulomb energy, due to the interaction of the matter fields with the other diagonal massless vector fields of the full grand unification gauge group, cannot qualitatively change the results of Rubakov and Callan, although it may certainly affect the quantitative result. This result is true for any grand unified theory, so long as the generators corresponding to the extra Abelian gauge fields commute with the full SU(2) subgroup in which the monopole lies.

I wish to thank W. A. Bardeen, S. Das, S. Dawson, A. S. Goldhaber, S. Rao, and A. N. Schellekens for many useful discussions during various stages of this work.

*Operated by Universities Research Association, Inc. under contract with the United States Department of Energy.

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[7]I wish to thank S. Das for first pointing out to me that the baryon number current is, in general, nonzero at the origin as a consequence of the boundary conditions.

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[9]At any space-time point, the fields ψ_i and χ_i refer to the eigenstates of the unbroken U(1) generator with eigenvalues $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. In the standard spherically symmetric gauge, ψ_i and χ_i may be expressed in terms of the two-component spinors $\xi_i(r,t)$ as

$$\psi_i(\vec{r},t)=\tfrac{1}{2}(I+\hat{r}\cdot\vec{\sigma})\xi_i(\vec{r},t) \ ,$$

$$\chi_i(\vec{r},t)=\tfrac{1}{2}(I-\hat{r}\cdot\vec{\sigma})\xi_i(\vec{r},t) \ .$$

[10]I wish to thank A. S. Goldhaber for a discussion on this point.